MP 40

A GENERAL PURPOSE METHOD FOR ELECTRIC AND MAGNETIC COMBINED PROBLEMS FOR 2D, AXISYMMETRIC AND TRANSIENT SYSTEMS

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ABSTRACT

The non-linear analysis of electrical devices is often limited by the complexity of the power supply. In effect, in the case of solid conductors, the voltage is required, while in the case of stranded conductors, the current must be known. These restrictions are due to the nature of the field equations. A new formulation is proposed which offers the possibility to choose the power supply as well as any kind of connection between components. Two types of "electromagnetic conductors" are considered: solid conductors, in which eddy current can be induced, and stranded conductors without eddy current. An equation combining the magnetic potential, the current, and the voltage is established for each conductor. This equation is used in a conventional circuit analysis method. The magnetic field and circuit equations can be solved simultaneously in the resulting system. An implicit method is used to dicretize the equations in time. A Newton-Raphson linearization algorithm is used to handle problems that include materials with non linear properties.
This formulation has been implemented in the commercial program FLUX2D. To illustrate one of the many applications, an A.C. electromagnet is studied.

INTRODUCTION

To obtain, for instance, a given current density in a solid conductor, an iterative solution is performed. In effect, only voltage source is available on such conductors. To avoid these iterations, which can take a long time, the idea is to solve simultaneously the magnetic and electric equations. In the literature, we find two types of approach: the integro-differential method[1] and the direct method[2,3,4,5]. The direct method seems to be more efficient(speed, lower memory requirements)[3].
The proposed method, based on the direct method, is intended to be more general than the others ones. Electric circuits can include resistances, inductances, current or voltage sources and "electromagnetic conductors". The interest of this work is that we can use in the same problem solid or stranded conductors with any type of connection between the components. Furthermore 2D and axisymmetric cases have been taken into account.
Computational results are compared to reference values for an A.C. electromagnet with shaded rings.

I. EQUATIONS

We are going to set the equations for the components we want to use. We distinguish two types of conductors: the "electromagnetic conductors" (which belong to the finite element part) and the others (sources, passive conductors like R, L). Furthermore, two types of "electromagnetic conductors" have been considered: solid and stranded conductors.

A. Solid conductors

These conductors can develop eddy-currents. Maxwell's equations applied to two-dimensional domains lead to the equation:

$$\mathrm{curl}\,(\,\nu\ \mathrm{curl}\,A\,) = J \qquad (1)$$

where ν is the reluctivity of the conductor, A the magnetic vector potential and J the current density. A relation links the current density to the vector potential and to the electric scalar potential V:

$$J = -\,\sigma\, dA/dt - \sigma\, \mathrm{Grad}V \qquad (2)$$

By substituting (2) in (1), the field equation becomes:

$$\mathrm{curl}\,(\,\nu\ \mathrm{curl}\,A\,) + \sigma\, \partial A/\partial t + \sigma\, \mathrm{Grad}V = 0 \qquad (3)$$

where σ is the conductivity of the conductor.

A.1) The 2D cartesian case

In the two dimensional model, the magnetic vector potential and the current density are reduced to the z components. It follows that the value of the electric scalar potential is constant on the cross-section of a two-dimensional conductor. Its value is a linear function of the z-coordinate. If L is the length of the solid conductor (along the z coordinate), L GradV represents the potential difference ΔV induced between the ends of the conductor. Equation (2) can be integrated on each solid conductor region k, where GradV is constant. We obtain:

$$\int_{S_k} j\,dS_k = I_k = \int_{S_k} \sigma\, \partial A/\partial t\, dS_k - \left(\mathrm{Grad}V_k\right)\int_{S_k} \sigma\, dS_k \qquad (4)$$

where I_k is the total current through the solid conductor k. If we consider that:

$$\Delta V_k = L_k\, \mathrm{Grad}V_k$$

$$R_k = L_k \Big/ \int_{S_k} \sigma_k\, dS_k \qquad (5)$$

where L_k is the length of the k^{th} solid conductor and R_k its d.c. resistance. We can deduce the relation between the total current I_k and the potential difference ΔV_k of the k^{th} solid conductor

$$\Delta V_k = R_k\, I_k + R_k \int_S \sigma\ \partial A/\partial t\, dS_k \qquad (6)$$

Using the Galerkin method, we can discretize (3) and (6):

$$[S]\,[A] - [G]\, d[A]/dt - [C]\,[\Delta V] = [0]$$
$$[\Delta V] = [R]\,[I] + [R]\,[C]^T\, d[A]/dt \qquad (7)$$

where A, ΔV, I represent the potential vector at each node (N nodes), the potential difference for each solid conductor (M solid conductors) and the global current respectively.The matrices are defined by:

$S_{ij}\,(N\mathrm{x}N) = \int_S \{\,\nu \mathrm{Grad}\beta_i\ \mathrm{Grad}\beta_j\,\}\ L\, dS$ with $A = \sum \beta_j A_j$

$G_{ik}(N\mathrm{x}N) = \int_S \sigma\, \beta_i\, \beta_j\, L\, dS$

$C_{ik}(N\mathrm{x}M) = \int_S \sigma\, \beta_i\ dS$

$R_{kk}(M\mathrm{x}M) = R_k$ resistance of the k^{th} conductor where ν, σ, L are the reluctivity, the conductivity and the thickness of the studied problem.

A.2) The axisymmetric case

In this case, the unknown is $A^*=rA$. Equation (3) gives:

$$\partial(\nu/r\partial A^*/\partial z)/\partial z + \partial(\nu/r\partial A^*/\partial r)/\partial r - \sigma/r\partial A^*/\partial t - \sigma \text{ Grad}V = 0 \quad (8)$$

If we notice that ($r\text{Grad}V_k$) is constant on each region k, we have:

$$\Delta V^*_k = 2\pi r \text{ Grad}V_k$$
$$R^*_k = 2\pi / \int \sigma/r \, dS_k$$

Integrating equation (2) leads to :

$$\Delta V^*_k = R^*_k I_k + R^*_k \int s/r \, \partial A^*/\partial t \, dSk \quad (9)$$

Using the Galerkin method, we can discretize (8) and (9):

$$[S^*][A^*] - [G^*] d[A^*]/dt - [C^*][\Delta V^*] = [0]$$
$$[\Delta V^*] = [R^*][I] + [R^*][C^*]^T d[A^*]/dt \quad (10)$$

The matrices are defined by :

$$S^*_{ij} (NxN) = \int 2\pi/r (\nu \text{ Grad}\beta_i \text{ Grad}\beta_j) dS$$
$$G^*_{ij} (NxM) = \int 2\pi/r \, \beta_i \, \beta_j \, dS$$
$$C^*_{ij} (NxM) = \int \sigma/r \, \beta_i \, dS$$

B. Stranded conductors

These conductors are caracterised by the lack of eddy-currents. They are considered to be too thin to allow eddy-currents to appear.

If we have a region k with NS_k filaments crossed by a current I_k and of area S_k, the current density will be considered as constant:

$$J_k = NS_k.I_k / S_k \quad (11)$$

In this case, Maxwell's equations lead to:

$$\text{curl}(\nu \text{ curl } A) = NS_k.I_k / S_k \quad (12)$$

B.1) The 2D cartesian case

A relation between the total current I_k and the potential difference ΔV_k is obtained from (6), applied to each filament:

$$\Delta V_k = \Sigma \left(R_i I_k + \int_{S_i} R_i \sigma \, \partial A/\partial t \, dS_i\right) \; i=1, NS_k \quad (13)$$

where S_i is the cross sectional area of a filament.

$S_i = \lambda S_k / NS_k$ where λ is a coefficient of filling

$$R_i = L_k / \int_{S_i} \sigma \, dS_i$$

The value of ΔV_k represents the sum of the potential difference in each filament of the k^{th} region. If a coil is modeled with two regions, the potential difference at the end of the coil will be given by the sum of the potential difference on these two regions. If the value of σ is constant on a filament, we can write:

$$\Delta V_k = \Sigma \left(L/(\sigma_i S_i) I_k + \int_{S_i} L_k / S_i \, \partial A/\partial t \, dS_i \right) \; i = 1, NS_k \quad (14)$$

We notice that:

$$\Sigma L/\sigma_i S_i = (L \, NS/\lambda S_k) \, \Sigma \, 1/\sigma_i = (L \, NS/\lambda S_k) \, NS/S_k \int_{Sk} 1/\sigma \, dS_k$$

$$\Sigma \int_{Si} L/S_i \, \partial A/\partial t \, dS_i \approx L \, NS/S_k \int_{Sk} \partial A/\partial t \, dS_k$$

Finally, we have:

$$\Delta V_k = R'_k I_k + L_k \, NS_k / S_k \int_{S_i} \partial A/\partial t \, dS_k \quad (15)$$

$$R'_k = (1/\lambda_k) \, L_k \, (NS_k^2 / S_k^2) \int_{S_i} 1/\sigma \, dS_k$$

Using the Galerkin method, we can discretize (12) and (15):

$$[S][A] = [C'][I] \quad (16)$$
$$[\Delta V] = [R'][I] + [C']^T d[A]/dt$$

We use the same notation as previously. The matrices are defined by :

$$S_{ij}(NxN) = \int_S \nu \text{ Grad}\beta_i \text{ Grad}\beta_j \, L \, dS$$
$$C'_{ik}(NxF) = -(NS \, L / S_k) \int_S \beta_i \, dS$$

$R'_{kk}(FxF)=R_k$ resistance of the k^{th} conductor

where NS, F are the number of wires and the number of fine wires respectively.

B.2) The axisymmetric case

In the axisymmetric case, Maxwell's equation is written:

$$\partial(\nu/\partial A/\partial z)/\partial z + \partial(\nu/r\partial rA/\partial r)/\partial r = -j \quad (17)$$

A similar development to the previous one in solid conductors leads to:

$$\Delta V^*_k = R'^*_k I_k + 2\pi \, NS/S_k \int \partial A/\partial t \, dS_k \quad (18)$$

with

$$R'^*k = 2\pi/\lambda \, (NS^2/S^2_k) \int r/\sigma \, dS$$

Using the Galerkin method, we can discretize (17) and (18):

$$[S^*][A^*] = [C'^*][I]$$
$$[\Delta V^*] = [R'^*][I] + [C'^*]^T d[A^*]/dt \quad (19)$$

We use the same notation as previously. The matrices are defined by :

$$S^*_{ij}(NxN) = \int_S 2\pi/r (\nu \text{ Grad}\beta_i \text{ Grad}\beta_j) dS$$
$$C'^*_{ik}(NxF) = -2\pi (NS / S_k) \int_S \beta_i \, dS$$

$R'^*_{kk}(FxF)=R'^*_k$ resistance of the k^{th} stranded conductor

C. Electrical network equations

A representation with a matrix of impedances is chosen to simulate all kind of circuits with impedances, inductances and sources. The equations are :

$$[E_m] = [Z_m]\,[I_m] + [L_m]\, d[I_m]/dt \qquad (20)$$

where E_m, Z_m, L_m, I_m represent the vector of voltage sources in each mesh m, the matrix of resistances (symmetric), the matrix of inductances(symmetric) and the vector of the current in each mesh. The matrices $[Z_m]$ and $[L_m]$ are calculated by:

$Z_{ij}(KxK)$:
- Z_{ii} addition of resistances in the mesh i
- Z_{ij} addition of resistances wich belongs to the meshes i and j with the sign "+" if the currents of meshes i and j are in the same way, with the sign "-" otherwise

$L_{ij}(KxK)$ is the same matrix of Z for inductances. K is the number of electric meshes.

Now that the equations are set, we can put them together to obtain the coupling system.

II. COUPLING THE FIELD AND CIRCUIT EQUATIONS

We put the electrical network equations in the previously studied equations for the two types of magnetic conductors.

A. Coupling with solid conductors

We have to put (7) in (20) so as to take into account the solid conductor in the electric circuit. Due to memory space and rapidity, we have chosen to keep the potential difference ΔV as unknown. The system to solve is:

$$\begin{aligned}
&[S]\,[A] - [G]\, d[A]/dt - [C]\,[\Delta V] &&= [0] \\
&[R]\,[C]^T d[A]/dt - [\Delta V] + [R]\,[I] &&= [0] \qquad (21)\\
&[D]\,[\Delta V] + [Z'_m]\,[I_m] + [Lm]\, d[Im]/dt &&= [E_m]
\end{aligned}$$

where the matrix [D] is defined by the relation:
$[I] = [D]^T [I_m]$

$D_{ij}(KxM)$:
- +1 if conductor i belongs to mesh j with right way
- -1 if conductor i belongs to mesh jwith wrong way
- 0 if conductor i does not belong to mesh j

$[Z'_m]$ is the matrix of resistances but Z'_{ii} is the addition of resistances in the mesh i without the resistances of solid conductors.

The system (21) of equations can be symmetrical:

$$\begin{aligned}
&[S]\,[A] - [G]\, d[A]/dt - [C]\,[\Delta V] &&= [0] \\
&-[C]^T d[A]/dt + [R^{-1}]\,[\Delta V] - [D]^T [I_m] &&= [0] \\
&- [D]\,[\Delta V] - [Z'_m]\,[I_m] - [L_m]\, d[I_m]/dt &&= -[E_m] \qquad (22)
\end{aligned}$$

(the matrix [R] is diagonal and strictly positive by definition)

B. Coupling with stranded conductors

The same method is used. We obtain:

$$\begin{aligned}
&[S]\,[A] - [C']\,[I] = [0] \qquad (23)\\
&[E_m] = [Z_m]\,[I_m] + [L_m]\, d[I_m]/dt + [D'][C']^T d[A]/dt
\end{aligned}$$

As previously, we have:
$[I] = [D']^T [I_m]$

This system of equations can be symmetrical:

$$\begin{aligned}
&[S]\,[A] - [C']\,[D']^T [I_m] &&= [0] \\
&-[D']\,[C']^T d[A]/dt - [Z_m]\,[I_m] - [L_m]\, d[I_m]/dt &&= -[E_m]
\end{aligned} \qquad (24)$$

C. Coupling with "both"

The system of equations (23) can be solved simultaneously with the system of equations (24). We obtain :

$$\begin{aligned}
&[S]\,[A] - [G]\, d[A]/dt - [C]\,[\Delta V] - [C']\,[D']^T [Im] = [0] \\
&-[C]^T d[A]/dt + [R^{-1}]\,[\Delta V] - [D]^T [I_m] = [0] \\
&-[D'][C']^T d[A]/dt - [D][\Delta V] - [Z'_m]\,[I_m] - [L_m]\, d[I_m]/dt = -[E_m]
\end{aligned} \qquad (25)$$

The equation actually solved depends on the application and the results needed. If the supplies are sinusoïdal, the permanent state can be obtained in one resolution. The complex notation is used to write the system. We can transform the **d/dt** operator by $j\omega$ (where ω is the angular frequency), and divide lines 2 and 3 of (25) by $i\omega$ to get the complex equations to solve:

$$\begin{bmatrix} [S] - i\omega\,[G] & -[C] & -[C']\,[D']^T \\ -[C]^T & \frac{[R^{-1}]}{i\omega} & -\frac{[D]^T}{i\omega} \\ -[D']\,[C']^T & \frac{[D]}{i\omega} & -\frac{[\dot{Z}_m]}{i\omega} \end{bmatrix} \begin{bmatrix} [A] \\ [\Delta V] \\ [I_m] \end{bmatrix} = \begin{bmatrix} [0] \\ [0] \\ -\frac{[E_m]}{i\omega} \end{bmatrix} \qquad (26)$$

If we are interested in the beginning of phenomena, a step by step algorithm is required. We have to discretize in time equation (25). We use the implicit method:

$$\left.\frac{dA}{dt}\right|_{t+\Delta t} = \frac{A(t+\Delta t) - A(t)}{\Delta t}$$

where Δt is the time step

In replacing $d[A]/dt$ and $d[I_m]/dt$ by their equivalent, we obtain:

$$\begin{bmatrix} [S] - \frac{[G]^T}{\Delta t} & -[C] & -[C']\,[D']^T \\ -[C]^T & +[R]^{-1}\Delta t & -[D]^T \Delta t \\ -[D']\,[C']^T & -[D]\,\Delta t & -[Z'_m]\,\Delta t - [L_m] \end{bmatrix} \begin{bmatrix} A \\ \Delta V \\ I_m \end{bmatrix}_{t+\Delta t} =$$

$$\begin{bmatrix} \frac{[G]^T}{\Delta t}[A]_t \\ -[C]^T [A]_t \\ -[E]_{t+\Delta t}\Delta t - [L_m]\,[I_m]_t - [D']\,[C']^T [A]_t \end{bmatrix} \qquad (27)$$

We can notice that this system is symmetric.

III. EXAMPLE

To show the interest of this new formulation, we modeled an A.C. electromagnet with shaded rings.

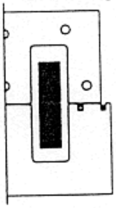


fig 1: cross-section

III.1 Description

The cross-section is represented on figure 1 and a view of the ring is given on figure 2:

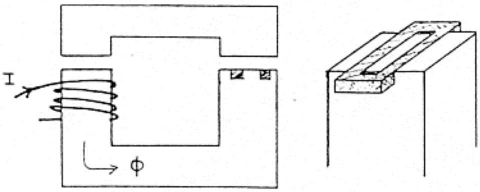


fig 2 : specific views

Due to the symmetry, half of the cross-section is modeled in 2D.The main coil is fed with a voltage supply. The two shaded rings are solid conductors and are short-circuited. For the electric part of the problem, two electric meshes have been considered. The first circuit is composed of a voltage supply(half of the real voltage, ALI1), one half of a coil(the stranded conductor, COIL) and one resistance(RES1) for the end of the coil. The second circuit is composed of two solid conductors(SOL1 and SOL2) and a resistance(RES2) that represents the end of the shaded rings as shown on figure 3.

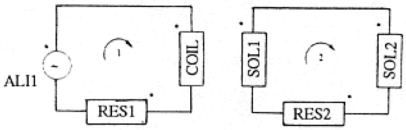


fig 3:electric circuits

III.2 Results

The computational results are compared to values gained from an iterative method already validated. The compared values are :

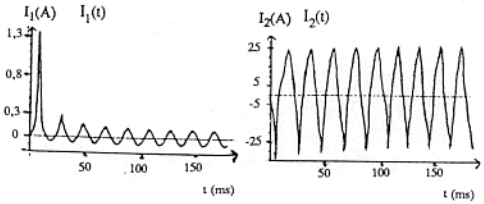
- I_1 current in the coil
- I_2 current in the shaded rings

	FLUX2D circuit	iterative method	difference (in %)
I_1 (A)	0.0778	0.0795	2.1
I_2 (A)	19.1	18.2	4.9

The two computational results are in good agreement.

We have noticed the evolution in time of the two currents.

fig 4: currents



CONCLUSION

Thus, we have presented a new formulation which allows the inclusion of the circuit equations in the finite element analysis in two dimensions(2D). The formulation wants to be general enough to represent all kind of conductors with eddy-current or not. The electric equations allow all connections between these conductors and sample conductors as resistance, inductance and voltage or current sources. The supplies can be of any type: constant, trapezoïdal, sinusoïdal or defined by the user. Furthermore, the axisymmetric problem has also been implemented with the case of non-linear characteristic in material(Newton-Raphson algorithm).

To illustrate this formulation, a real example has been treated. It is an A.C. electromagnet with shaded rings in short-circuit. This example is interesting because it uses at the same time solid and stranded conductors. The computational results show good agreements with values gained by an iterative method already validated.

To further develop this general formulation, we intend to add capacitances and diodes as possible components of the external circuit. So far, with this formulation, we have already modelled situations such as induction heating processes and asynchronous machines.

ACKNOWLEDGMENTS

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