SHREYAS SRINIVASAN
012551187

02/23/2021

HWK 1

Section 7.1

1. $\int x e^{2x}\, dx$; $u = x$, $dv = e^{2x}\, dx$

Ans: $du = dx$
$v = \frac{1}{2} e^{2x}$

$$\int x e^{2x}\, dx = \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x}\, dx$$
$$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$$

3. $\int x \cos 5x\, dx$

Ans. Let $u = x$
$dv = \cos 5x\, dx$
$du = dx$

$v = \frac{1}{5} \sin 5x$

$$\therefore \int x \cos 5x\, dx = \frac{1}{5} x \sin 5x - \int \frac{1}{5} \sin 5x\, dx$$
$$= \frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + C$$

5. $\int t e^{-3t}\, dt$

Ans: Let $u = t$
$dv = e^{-3t}\, dt$

P.T.O

$du = dt$
$v = -\frac{1}{3}e^{-3t}$

$\therefore \int t e^{-3t}\, dt = -\frac{1}{3} t e^{-3t} - \int -\frac{1}{3} e^{-3t}\, dt$
$= -\frac{1}{3} t e^{-3t} + \frac{1}{3}\int e^{-3t}\, dt$
$= -\frac{1}{3} t e^{-3t} - \frac{1}{9} e^{-3t} + C$

7. $\int (x^2 + 2x) \cos x \, dx$

Ans: Let $u = x^2 + 2x$
$dv = \cos x \, dx$

$du = (2x+2)\, dx$
$v = \sin x + C$

$I = \int (x^2 + 2x) \cos x \, dx$

$= (x^2 + 2x) \sin x - \int (2x+2) \sin x \, dx$

Let $U = 2x+2$
$dV = \sin x \, dx$

$dU = 2\, dx$

$V = -\cos x + C$

$\therefore \int (2x+2) \sin x \, dx = -(2x+2)\cos x - \int -2\cos x \, dx$
$= -(2x+2) \cos x + 2 \sin x$

P.T.O

$\therefore\ I = (x^2 + 2x)\sin x + (2x+2)\cos x - 2\sin x + C$

11. $\int t^4 \ln t \, dt$

Ans: let $u = \ln t$,
$dv = t^4 \, dt$

$du = \frac{1}{t} dt$

$v = \frac{1}{5} t^5$

$\therefore \int t^4 \ln t \, dt = \frac{1}{5} t^5 \ln t - \int \frac{1}{5} t^5 \cdot \frac{1}{t} dt$

$= \frac{1}{5} t^5 \ln t - \int \frac{1}{5} t^4 dt$

$= \frac{1}{5} t^5 \ln t - \frac{1}{25} t^5 + C$

12. $\int \tan^{-1} 2y \, dy$

Ans: $\int \tan^{-1}(2y) \cdot 1 \, dy$

$\int f \cdot g' = f \cdot g - \int f' \cdot g$

$f = \tan^{-1}(2y)$

$g' = 1$

$f' = \frac{1}{1+(2y)^2} \cdot 2$

$= \frac{2}{1+4y^2}$

P.T.O

$$g = \int g' \cdot dy$$

$$g = \int 1 \cdot dy$$

$$g = y$$

$$\therefore \int \tan^{-1}(2y) \cdot dy = \tan^{-1}(2y) \cdot y - \int \frac{2}{1+4y^2} \cdot y \, dy$$

$$= \underbrace{\tan^{-1}(2y) \cdot y}_{1} - \underbrace{\int \frac{2y}{1+4y^2} \cdot dy}_{2}$$

Solving (2) :-

$$\int \frac{2y}{1+4y^2} dy = \frac{1}{4} \int \frac{8y}{1+4y^2} \cdot dy$$

Let $h(y) = 1 + 4y^2$

$$h'(y) = 8y \cdot dy$$

$$\therefore \int \frac{2y}{1+4y^2} dy = \frac{1}{4} \ln(1+4y^2)$$

$$\therefore \int \tan^{-1}(2y) \, dy = y \cdot \tan^{-1}(2y) - \frac{1}{4} \ln(1+4y^2) + C$$

P.T.O

17. $\int e^{2\theta} \sin 3\theta \, d\theta$

Ans. let $u = \sin 3\theta$

$dv = e^{2\theta} \, d\theta$

$du = 3 \cos 3\theta \, d\theta$

$v = \frac{1}{2} e^{2\theta}$

$I = \int e^{2\theta} \sin 3\theta \, d\theta$

$= \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{2} \int e^{2\theta} \cos 3\theta \, d\theta$

let $U = \cos 3\theta$

$dV = e^{2\theta} \, d\theta$

$dU = -3 \sin 3\theta \, d\theta$

$V = \frac{1}{2} e^{3\theta} + C$

$\therefore \int e^{2\theta} \cos 3\theta \, d\theta = \frac{1}{2} e^{2\theta} \cos 3\theta + \frac{3}{2} \int e^{2\theta} \sin 3\theta \, d\theta$

$I = \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta - \frac{9}{4} \int e^{2\theta} \sin 3\theta \, d\theta$

$= \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta - \frac{9}{4} I$

$\therefore \frac{13}{4} I = \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta + C_1$

$I = \frac{e^{2\theta}}{13} (2 \sin 3\theta - 3 \cos 3\theta) + C$

$\left[\because C = \frac{4}{13} C_1 \right]$

23. $\int_0^{1/2} x \cos \pi x \, dx$

Ans. Let $u = x$
$dv = \cos \pi x \, dx$

$du = dx$
$v = \frac{1}{\pi} \sin \pi x$

$\therefore \int_0^{1/2} x \cos \pi x \, dx = \left[\frac{1}{\pi} x \sin \pi x \right]_0^{1/2}$

$- \int_0^{1/2} \frac{1}{\pi} \sin \pi x \, dx$

$= \frac{1}{2\pi} - 0 - \frac{1}{\pi} \left[-\frac{1}{\pi} \cos \pi x \right]_0^{1/2}$

$= \frac{1}{2\pi} + \frac{1}{\pi^2} (0 - 1)$

$= \frac{1}{2\pi} - \frac{1}{\pi^2}$

$= \frac{\pi - 2}{2\pi^2}$

27. $\int_1^5 \frac{\ln R}{R^2} \, dR$

Ans. Let $u = \ln R$
$dv = \frac{1}{R^2} dR$

$du = \frac{1}{R} dR$

P.T.O

$v = -\frac{1}{R}$

$$\int_1^5 \frac{\ln R}{R^2}\,dR = \left[-\frac{1}{R}\ln R\right]_1^5 - \int_1^5 -\frac{1}{R^2}\,dR$$

$$= -\frac{1}{5}\ln 5 - 0 - \left[\frac{1}{R}\right]_1^5$$

$$= -\frac{1}{5}\ln 5 - \left(\frac{1}{5} - 1\right)$$

$$= \frac{4}{5} - \frac{1}{5}\ln 5.$$

29 $\int_0^{\pi} x \sin x \cos x \, dx$

Ans $\sin 2x = 2\sin x \cos x$

$\therefore \int_0^{\pi} x \sin x \cos x \, dx = \frac{1}{2}\int_0^{\pi} x \sin 2x \, dx$

Let $u = x,$
$dv = \sin 2x \, dx$

$du = dx,$
$v = -\frac{1}{2}\cos 2x$

$\therefore \frac{1}{2}\int_0^{\pi} x \sin 2x \, dx = \frac{1}{2}\left[-\frac{1}{2}x\cos 2x\right]_0^{\pi} -$

$\frac{1}{2}\int_0^{\pi} -\frac{1}{2}\cos 2x \, dx$

$= -\frac{1}{4}\pi - 0 + \frac{1}{4}\left[\frac{1}{2}\sin 2x\right]_0^{\pi}$

$= -\frac{\pi}{4}$

33. $\int_0^{\pi/3} \sin x \ln(\cos x)\, dx$

Ans:- Let $u = \ln(\cos x)$

$dv = \sin x\, dx$

$du = \frac{1}{\cos x}(-\sin x)\, dx$

$v = -\cos x$

$\therefore \int_0^{\pi/3} \sin x \ln(\cos x)\, dx = \Big[-\cos x \ln(\cos x)\Big]_0^{\pi/3} - \int_0^{\pi/3} \sin x\, dx$

$= -\frac{1}{2}\ln\frac{1}{2} - 0 - \Big[-\cos x\Big]_0^{\pi/3}$

$= -\frac{1}{2}\ln\frac{1}{2} + \left(\frac{1}{2} - 1\right)$

$= \frac{1}{2}\ln 2 - \frac{1}{2}$

39. $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\, d\theta$

Ans: Let $x = \theta^2$

$dx = 2\theta\, d\theta$

$\therefore \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\, d\theta = \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^2 \cos(\theta^2) \cdot \frac{1}{2}(2\theta\, d\theta)$

$= \frac{1}{2}\int_{\pi/2}^{\pi} x \cos x\, dx$

P.T.O

(a)

$u = x$
$dv = \cos x \, dx$

$du = dx$
$v = \sin x$

$\therefore \frac{1}{2}\int_{\pi/2}^{\pi} x \cos x \, dx = \frac{1}{2}\left([x \sin x]_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} \sin x \, dx \right)$

$= \frac{1}{2}[x \sin x + \cos x]_{\pi/2}^{\pi}$

$= \frac{1}{2}(\pi \sin \pi + \cos \pi) - \frac{1}{2}\left(\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2}\right)$

$= \frac{1}{2}(\pi . 0 - 1) - \frac{1}{2}\left(\frac{\pi}{2} 1 + 0\right)$

$= -\frac{1}{2} - \frac{\pi}{4}$

$= \frac{-2 - \pi}{4}$

Exercise 7.2

1. $\int \sin^2 x \cos^3 x \, dx$

Ans. $\int \sin^2 x \cos^2 x \cos x \, dx$

$\Rightarrow \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$

Let $u = \sin x$
$du = \cos x \, dx$

P.T.O

$\Rightarrow \int u^2(1-u^2)\,du$

$\Rightarrow \int (u^2 - u^4)\,du$

$\Rightarrow \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$\Rightarrow \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

3. $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$

Ans: $\int_0^{\pi/2} \sin^7\theta \cos^4\theta \cos\theta \, d\theta$

$\Rightarrow \int_0^{\pi/2} \sin^7\theta \, (1-\sin^2\theta)^2 \cos\theta \, d\theta$

Let $u = \sin\theta$
$du = \cos\theta\, d\theta$

$\Rightarrow \int_0^1 u^7(1-u^2)^2\,du$

$\Rightarrow \int_0^1 u^7(1-2u^2+u^4)\,du$

$\Rightarrow \int_0^1 (u^7 - 2u^9 + u^{11})\,du$

$\Rightarrow \left[\frac{1}{8}u^8 - \frac{1}{5}u^{10} + \frac{1}{12}u^{12}\right]_0^1$

$\Rightarrow \left(\frac{1}{8} - \frac{1}{5} + \frac{1}{12}\right) - 0$

$\Rightarrow \frac{15-24+10}{120}$

$\Rightarrow \frac{1}{120}$

P.T.O

$$\Rightarrow \frac{1}{\sqrt{2}} + \frac{1}{2}\left(\log|\sqrt{2}+1| + 0\right) \quad [\because \log 1 = 0]$$

$$\therefore I = \frac{1}{\sqrt{2}} + \frac{1}{2}\log|\sqrt{2}+1|$$

10. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sin B \cot B}{\sec B}\, dB$

Ans: $$\int_{\pi/6}^{\pi/3} \frac{\cancel{\sin B} \cdot \frac{\cos B}{\cancel{\sin B}}}{\frac{1}{\cos B}}\, dB$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \cos^2 B\, dB$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{1}{2}\left[1 + \cos 2B\right] dB$$

$$\Rightarrow \frac{1}{2}\left[\left(\frac{\pi}{3} - \frac{\pi}{6}\right) + \left(\frac{\sin 2B}{2}\right)_{\pi/6}^{\pi/3}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{\pi}{6} + \frac{1}{2} \times 0\right]$$

$$= \frac{1}{2} \times \frac{4\pi}{6} = \frac{\pi}{3}$$

7. $\int_0^{\pi/2} \cos^2\theta \, d\theta$

Ans. $\int_0^{\pi/2} \frac{1}{2}(1+\cos 2\theta)\, d\theta$

$\Rightarrow \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$\Rightarrow \frac{1}{2}\left[\left(\frac{\pi}{2}+0\right) - (0+0)\right] = \frac{\pi}{4}$

9. $\int_0^{\pi} \cos^4(2t)\, dt$

Ans. $\int_0^{\pi} [\cos^2(2t)]^2\, dt$

$\Rightarrow \int_0^{\pi} \left[\frac{1}{2}(1+\cos(2\cdot 2t))\right]^2 dt$

$\Rightarrow \frac{1}{4}\int_0^{\pi} [1+2\cos 4t + \cos^2(4t)]\, dt$

$\Rightarrow \frac{1}{4}\int_0^{\pi} [1+2\cos 4t + \frac{1}{2}(1+\cos 8t)]\, dt$

$\Rightarrow \frac{1}{4}\int_0^{\pi} \left(\frac{3}{2}+2\cos 4t + \frac{1}{2}\cos 8t\right) dt$

$\Rightarrow \frac{1}{4}\left[\frac{3}{2}t + \frac{1}{2}\sin 4t + \frac{1}{16}\sin 8t\right]_0^{\pi}$

$\Rightarrow \frac{1}{4}\left[\left(\frac{3}{2}\pi + 0 + 0\right) - 0\right]$

$\Rightarrow \frac{3}{8}\pi$