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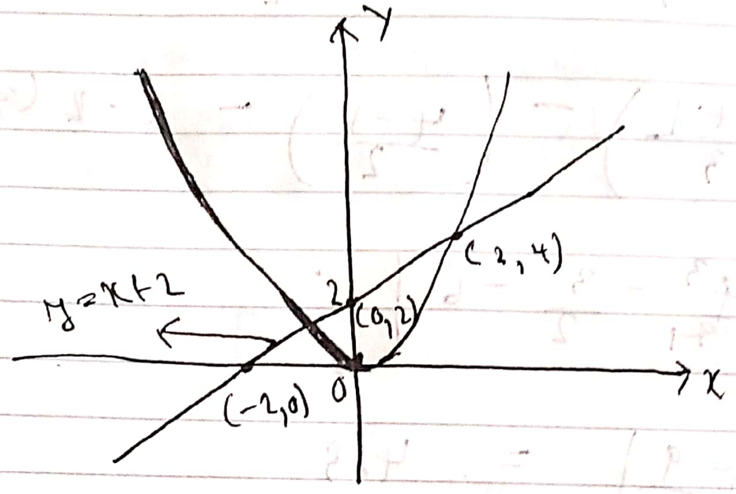
MATH - 141

EXAM - 2

SPRING 2021

1. Ans :- Given curves are :- $y = x^2$ & $x = y - 2$ or $y = x + 2$



Intersecting both curves :-

$$x^2 = x + 2$$
$$\Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow x^2 - 2x + x - 2 = 0$$
$$\Rightarrow x(x-2) + 1(x-2) = 0$$
$$\Rightarrow (x-2)(x+1) = 0$$

$$\therefore x = 2 \quad \& \quad x = -1$$

Now, The area between the two curves :-

$$A = \int_a^b |f(x) - g(x)| \, dx$$

$$= \int_{x=-1}^{2} |x^2 - x - 2| \, dx$$

P.T.O

$$\Rightarrow \left| \int_{x=-1}^{2} x^2\,dx - \int_{x=-1}^{2} x\,dx - 2\int_{x=-1}^{2} dx \right|$$

$$\Rightarrow \left| \left(\frac{x^3}{3}\right)_{-1}^{2} - \left(\frac{x^2}{2}\right)_{-1}^{2} - 2(x)_{-1}^{2} \right|$$

$$= \left| \left(\frac{8-(-1)^3}{3}\right) - \left(\frac{2^2-(-1)^2}{2}\right) - 2(2-(-1)) \right|$$

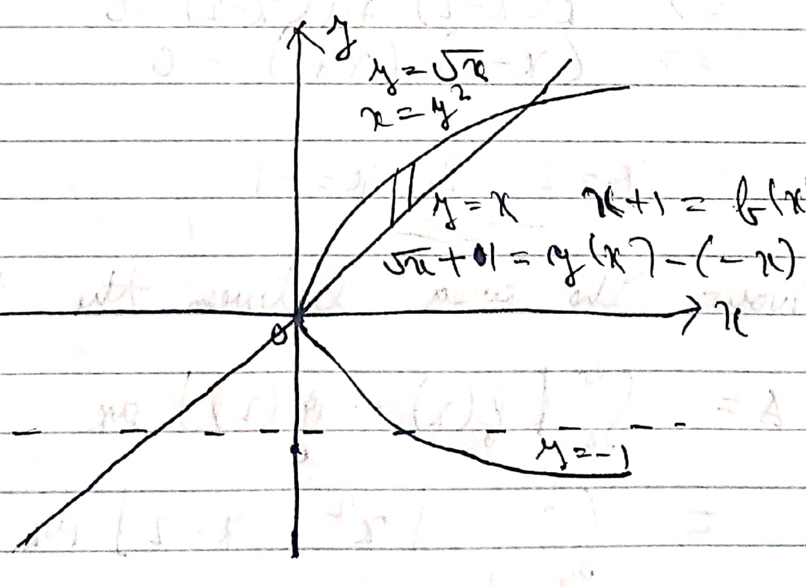
$$= \left| \left(\frac{8+1}{3}\right) - \left(\frac{4-1}{2}\right) - (-2(3)) \right|$$

$$= \left| \frac{\cancel{9}^{3}}{\cancel{3}_{1}} - \frac{3}{2} - 6 \right|$$

$$= \left| \frac{-9}{2} \right| = 4.5$$

$\therefore$ Area $(A) = 4.5$

2. Ans. Given Curves :- $x = y^2$ & $x = y$



P.T.O

To find the intercept points :-

$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \; ; \; x - 1 = 0$

$x = 1$

$\therefore$ The volume of the solid from the revolution bellow curved $g(x)$ & $f(x)$ rotated about $y = d$ on $[a, b]$ is $V = \int_a^b \pi \left[(g(x) - d)^2 - (f(x) - d)^2 \right] dx$

$\therefore V = \int_0^1 \pi \left[(\sqrt{x} + 1)^2 - (x+1)^2 \right] dx$

$= \pi \int_0^1 \left[(x + 2\sqrt{x} + 1) - (x^2 + 2x + 1) \right] dx$

$= \pi \int_0^1 (x + 2\sqrt{x} + \cancel{1} - x^2 - 2x - \cancel{1})\, dx$

$= \pi \int_0^1 (2\sqrt{x} - x^2 - x)\, dx$

$= \pi \left[\frac{2 x^{3/2}}{3/2} - \frac{x^3}{3} - \frac{x^2}{2} \right]_0^1$

$= \pi \left[\frac{4}{3}(1) - \frac{1}{3} - \frac{1}{2} - 0 \right]$

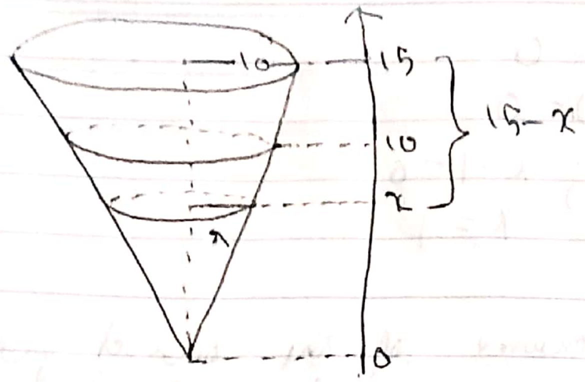
$= \pi \left[1 \cdot \frac{\cancel{3}}{\cancel{3}} - \frac{1}{2} \right]$

$= \pi \left[1 - \frac{1}{2} \right]$

$= \pi \cdot \frac{1}{2} = \frac{\pi}{2}$

3. Ans: NOTE - Will finish the ones I am confident in & come back to this later!

4. Ans:



$$\frac{r}{10} = \frac{x}{15}$$

$$r = \frac{2x}{3}$$

Work, $W = \int_0^{10} (15-x)\, 62.4 \left(4\pi x^2/9\right) dx$

$$= \frac{83.2}{3}\pi \int_0^{10} (15x^2 - x^3)\, dx$$

$$= \frac{208{,}000\pi}{3} \text{ ft.lb}$$

5. Ans: Average value $= \frac{1}{b-a} \int_a^b f(x)\, dx$

$$\Rightarrow \frac{1}{1-(-1)} \int_{-1}^{1} \frac{x^2}{(x^2+3)^2}\, dx$$

$$\Rightarrow \frac{1}{2} \int_{-1}^{1} \frac{x^2}{(x^2+3)^2}\, dx$$

P.T.O

(5)

let $u = x^3 + 3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\Rightarrow \frac{1}{2}\int \frac{du/3}{u^2}$

$\Rightarrow -\frac{1}{6}\left(\frac{1}{u}\right)$

$\Rightarrow \left[\frac{-1}{6(x^3+3}\right]_{-1}^{1}$

$\Rightarrow -\frac{1}{6}\left(\frac{1}{4} - \frac{1}{2}\right)$

$\Rightarrow -\frac{1}{6} \times -\frac{1}{4}$

$\Rightarrow \frac{1}{24}$

6. Ans: Given curve :- $y = \frac{x^4}{4} + \frac{1}{2x^2}$, $1 \le x \le 3$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left[\frac{x^4}{4} + \frac{1}{2x^2}\right]$

$= \frac{4x^3}{4} + \frac{(-2)}{\cancel{8}_4 x^3}$

$= x^3 - \frac{1}{4x^3}$

P.T.O

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \left(x^3 - \frac{1}{4x^3}\right)^2$$

$$= \left(\frac{4x^6 - 1}{4x^3}\right)^2$$

$$= \frac{16x^{12} + 1 - 8x^6}{16x^6}$$

$\therefore$ The length of the curve, $L = \int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$$\Rightarrow \int_1^3 \sqrt{1 + \frac{16x^{12} + 1 - 8x^6}{16x^6}}\, dx$$

$$\Rightarrow \int_1^3 \frac{1}{4x^3} \sqrt{16x^6 + 16x^{12} + 1 - 8x^6}\, dx$$

$$\Rightarrow \int_1^3 \frac{1}{4x^3} \sqrt{(4x^6 + 1)^2}\, dx$$

$$\Rightarrow \int_1^3 \frac{1}{4x^3} (4x^6 + 1)\, dx$$

$$\Rightarrow \int_1^3 x^3 + \frac{1}{4} x^{-3}\, dx$$

$$\Rightarrow \left(\frac{x^4}{4} - \frac{1}{8x^2}\right)_1^3$$

$$\Rightarrow \left[\frac{3^4}{4} - \frac{1}{8(3)^2} - \left(\frac{1}{4} - \frac{1}{8}\right)\right]$$

P.T.O

$\Rightarrow \frac{81}{4} - \frac{1}{72} - \frac{1}{8}$

$\Rightarrow \frac{1458 - 1 - 9}{72} = \frac{1448}{72}$

$\therefore$ Exact length of the curve, $L = \underline{\underline{20.11}}$

7. Ans. $y = \sqrt{4-x^2}$, $-1 \le x \le 1$, about the x-axis,

Surface area is $= 2\pi \int_a^b y\sqrt{1+(y')^2}\,dx$

$y' = \frac{d}{dx}(4-x^2)^{\frac{1}{2}}$

$= \frac{1}{2}(4-x^2)^{\frac{1}{2}-1} \cdot (-2x)$

$= \frac{-x}{\sqrt{4-x^2}}$

Surface area:- $2\pi \int_{-1}^{1} \sqrt{4-x^2} \sqrt{1+\frac{x^2}{4-x^2}}\,dx$

$\Rightarrow 2\pi \int_{-1}^{1} \cancel{\sqrt{4-x^2}} \cdot \frac{\sqrt{4-\cancel{x^2}+\cancel{x^2}}}{\cancel{\sqrt{4-x^2}}}\,dx$

$\Rightarrow 2\pi \int_{-1}^{1} \sqrt{4}\,dx$

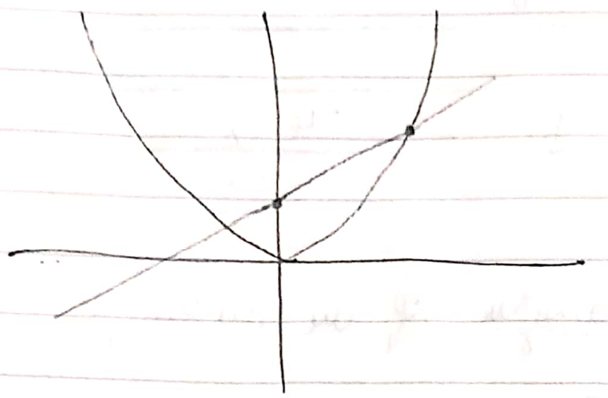
$\Rightarrow 4\pi \int_{-1}^{1} dx$

$\Rightarrow 4\pi(1+1)$

$\Rightarrow \underline{\underline{8\pi}}$

P.T.O

8. Ans. Given curve :- $y = x^2$, $y = x+6$



Finding the intersection :-

$$x^2 = x+6 \Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$
$$\Rightarrow (x-3)(x+2) = 0$$
$$\therefore x = -2, 3$$

Area bounded, $A = \int_{-2}^{3} (x+6-x^2)\, dx$

$$= \left(\frac{x^2}{2} + 6x - \frac{x^3}{3}\right)_{-2}^{3}$$

$$= \frac{9}{2} + 18 - 9 - 2 + 12 - \frac{8}{3}$$

$$= \frac{125}{6}$$

x - coordinate of centroid $= \frac{1}{\pi} \int_{-2}^{3} x(6+x-x^2)\, dx$

$$= \frac{1}{\pi}\left[3x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_{-2}^{3}$$

$$= \frac{\cancel{6}^{1}}{\cancel{125}} \times \frac{\cancel{125}}{12\,2} = \frac{1}{2}$$

y-coordinate of centroid $= \frac{1}{A}\int_{-2}^{3} \frac{1}{2}\{(6+x)^2 - (x^2)^2\}dx$

$$\Rightarrow \frac{1}{2A}\int_{-2}^{3}(36+12x+x^2-x^4)\,dx$$

$$\Rightarrow \frac{1}{2A}\left[36x+6x^2+\frac{x^3}{3}-\frac{x^5}{5}\right]_{-2}^{3}$$

$$\Rightarrow \frac{1}{\cancel{2}\times\cancel{125}} \times \cancel{6}^{8} \times \frac{\cancel{500}^{4}}{\cancel{3}} = \underline{\underline{4}}$$

9. Ans:

$$\frac{dy}{dx} = 2e^y + xe^y$$

$$\frac{dy}{dx} = e^y[2+x]$$

$$\frac{dy}{e^y} = (2+x)\,dx$$

$$\Rightarrow \int e^{-y}\,dy = \int (2+x)\,dx$$

$$\Rightarrow \frac{e^{-y}}{(-1)} = 2x + \frac{x^2}{2} + C$$

As $y(0) = 0$

$$-1 = 0 + 0 + C$$

$$\therefore \underline{\underline{C = -1}}$$

$$\therefore -e^{-y} = 2x + \frac{x^2}{2} - 1$$

P.T.O

OR

$$e^{-y} = 1 - 2x - \frac{x^2}{2}$$

$$\frac{1}{e^{y}} = 1 - 2x - \frac{x^2}{2}$$

$$\frac{1}{1-2x-\frac{x^2}{2}} = e^{y}$$

$$\therefore y = \log\left(\frac{1}{1-2x-\frac{x^2}{2}}\right)$$