NAME :- SHREYAS SRINIVASA
PALOMAR ID :- 012551187

MATH-141

Exercise 6.1

1. Ans $A = \int_{x=1}^{x=8} (y_T - y_B)\, dx$

$$= \int_1^8 \left(\sqrt[3]{x} - \frac{1}{x} \right) dx$$

$$= \left[\frac{3}{4} x^{4/3} - \ln|x| \right]_1^8$$

$$= (12 - \ln 8) - \left(\frac{3}{4} - \ln 1 \right)$$

$$= \frac{45}{4} - \ln 8$$

3. Ans $A = \int_{y=-1}^{y=1} (x_R - x_L)\, dy$

$$= \int_{-1}^{1} [e^y - (y^2 - 2)]\, dy$$

$$= \int_{-1}^{1} (e^y - y^2 + 2)\, dy$$

$$= \left[e^y - \frac{1}{3} y^3 + 2y \right]_{-1}^{1}$$

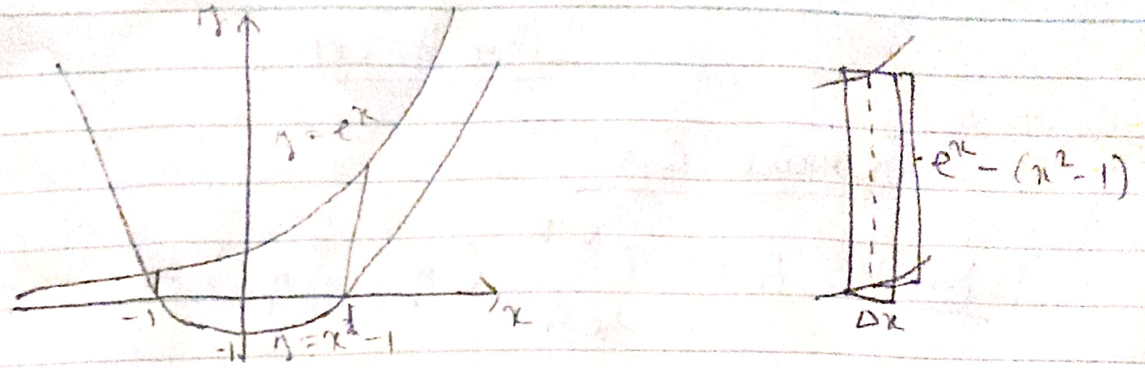
$$= \left(e^1 - \frac{1}{3} + 2\right) - \left(e^{-1} + \frac{1}{3} - 2\right)$$

$$= e - \frac{1}{e} + \frac{10}{3}$$

P.T.O

(2)

5. Ans.



$$A = \int_{-1}^{1} [e^x - (x^2-1)]\, dx$$

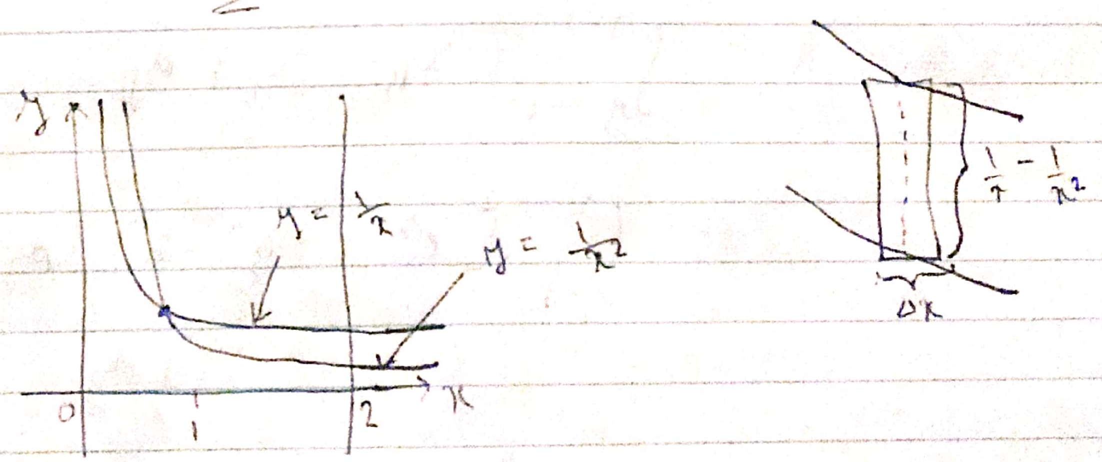
$$= \left[e^x - \frac{1}{3}x^3 + x\right]_{-1}^{1}$$

$$= \left(e - \frac{1}{3} + 1\right) - \left(e^{-1} + \frac{1}{3} - 1\right)$$

$$= e - \frac{1}{e} + \frac{4}{3}$$

$$=$$

6. Ans.



$$A = \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

$$= \left[\ln x + \frac{1}{x}\right]_1^2$$

$$= \left(\ln 2 + \frac{1}{2}\right) - (\ln 1 + 1)$$

$$= \ln 2 - \frac{1}{2} = 0.19 \text{ approx.}$$

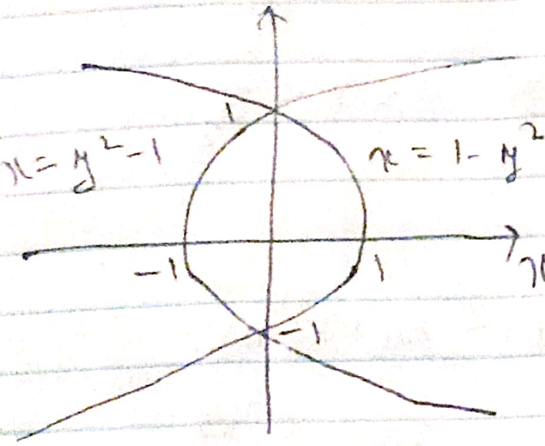
P.T.O

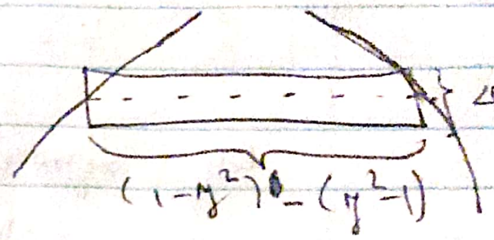
11. Ans. The curves intersect when $1 - y^2 = y^2 - 1$

$\Rightarrow 2 = 2y^2$

$\Rightarrow y^2 = 1$

$y = \pm 1$





$$A = \int_{-1}^{1} \left[(1-y^2) - (y^2-1) \right] dy$$

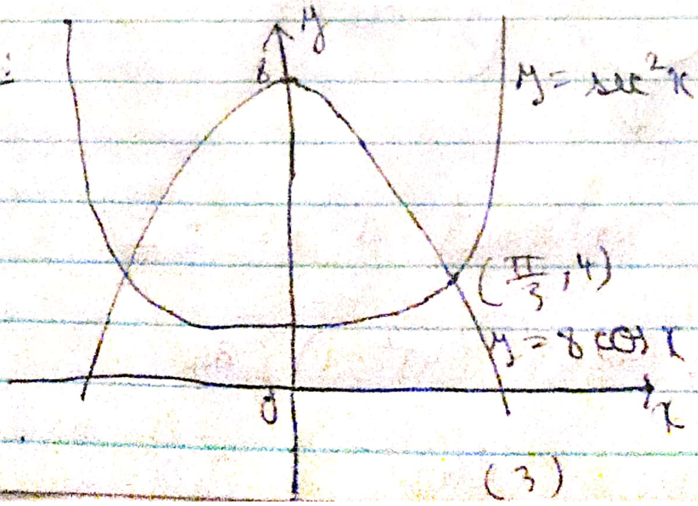
$$= \int_{-1}^{1} 2(1-y^2)\, dy$$

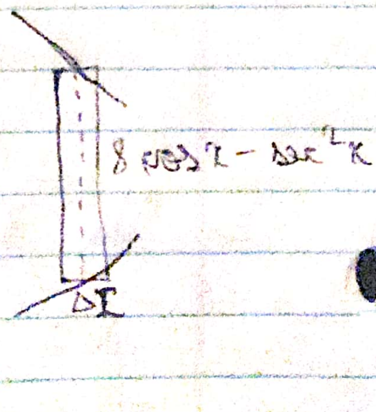
$$= 2 \cdot 2 \int_{0}^{1} (1-y^2)\, dy$$

$$= 4 \left[y - \frac{1}{3} y^3 \right]_0^1$$

$$= 4 \left(1 - \frac{1}{3} \right) = \underline{\underline{\frac{8}{3}}}$$

15. Ans:





P.T.O

The curves intersect when $8\cos x = \sec^2 x$

$\Rightarrow 8\cos^3 x = 1$

$\Rightarrow \cos^3 x = \frac{1}{8}$

$\Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3}$ for $0 < x < \frac{\pi}{2}$

By symmetry,

$$A = 2\int_0^{\pi/3} (8\cos x - \sec^2 x)\, dx$$

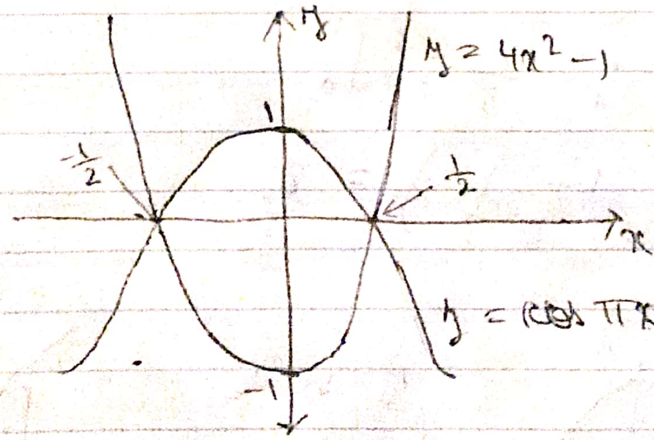
$$= 2\left[8\sin x - \tan x\right]_0^{\pi/3}$$

$$= 2\left(\cancel{8}^{4} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}} - \sqrt{3}\right)$$

$$= 2(3\sqrt{3})$$

$$= 6\sqrt{3}$$

14. diag.



The curves intersect at $x = \pm\frac{1}{2}$.

$$A = \int_{-1/2}^{1/2} \left[\cos \pi x - (4x^2 - 1)\right] dx$$

P.T.O

$$\Rightarrow 2\int_0^{1/2}(\cos \pi x - 4x^2 + 1)\,dx \quad [\because \text{by symmetry}]$$

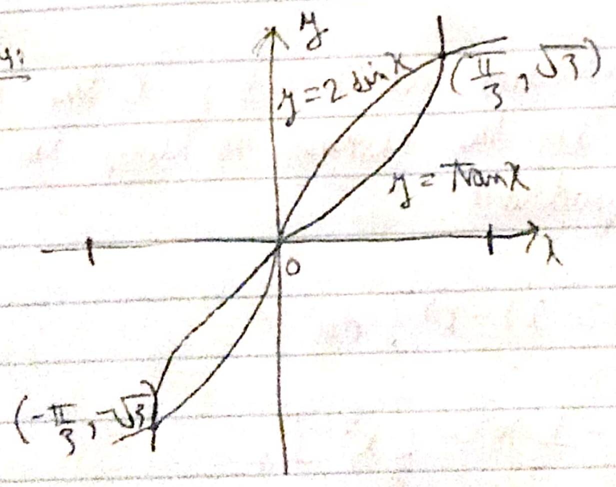
$$\Rightarrow 2\left[\frac{1}{\pi}\sin \pi x - \frac{4}{3}x^3 + x\right]_0^{1/2}$$

$$\Rightarrow 2\left[\left(\frac{1}{\pi} - \frac{1}{6} + \frac{1}{2}\right) - 0\right]$$

$$\Rightarrow 2\left(\frac{1}{\pi} + \frac{1}{3}\right)$$

$$\Rightarrow \frac{2}{\pi} + \frac{2}{3}$$

21. Ans:



The curves intersect when $\tan x = 2\sin x$ (on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$)

$$\Rightarrow \sin x = 2\sin x \cos x$$
$$\Rightarrow 2\sin x \cos x - \sin x = 0$$
$$\Rightarrow \sin x(2\cos x - 1) = 0$$
$$\Rightarrow \sin x = 0 \text{ or } \cos x = \frac{1}{2}$$
$$\Rightarrow x = 0 \text{ or } x = \pm\frac{\pi}{3}$$

$$A = 2\int_0^{\pi/3}(2\sin x - \tan x)\,dx \quad [\because \text{by symmetry}]$$

P.T.O

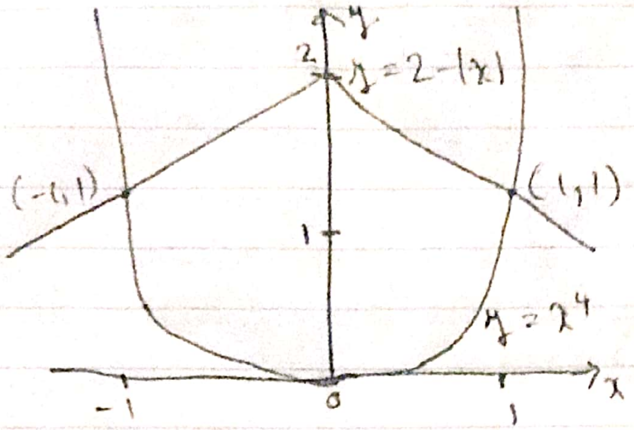
$$\Rightarrow 2\left[-2\cos x - \ln|\sec x|\right]_0^{\pi/3}$$

$$\Rightarrow 2\left[(-1-\ln 2)-(-2-0)\right]$$

$$\Rightarrow 2(1-\ln 2)$$

$$\Rightarrow 2-2\ln 2$$

25. Ans:



The curves intersect at $x = \pm 1$ & the area of the region enclosed by the curves is twice the area enclosed in the first quadrant.

$$A = 2\int_0^1 \left[(2-x)-x^4\right] dx$$

$$= 2\left[2x - \frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1$$

$$= 2\left[\left(2-\frac{1}{2}-\frac{1}{5}\right)-0\right]$$

$$= 2\left(\frac{13}{10}\right)$$

$$= \frac{13}{5}$$

P.T.O

29. Ans. (a) Required area $= 12 + 27 = 39.$

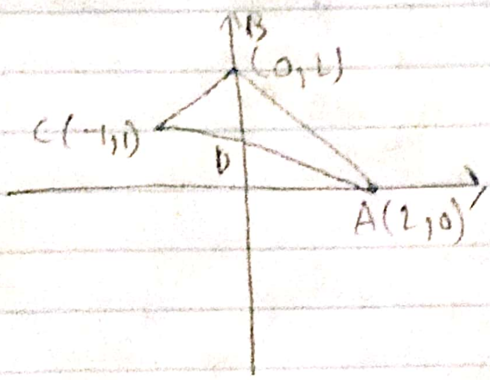
(b) $f(x) \leq g(x)$ for $0 \leq x \leq 2$ & $f(x) \geq g(x)$ for $2 \leq x \leq 5$

$$\therefore \int_0^5 [f(x) - g(x)]\,dx = \int_0^2 [f(x) - g(x)]\,dx + \int_2^5 [f(x) - g(x)]\,dx$$

$$\Rightarrow -\int_0^2 [g(x) - f(x)]\,dx + \int_2^5 [f(x) - g(x)]\,dx$$

$$\Rightarrow -12 + 27 = 15$$

34. Ans.



Equations of lines :- $AB = x + y = 2$
$\Rightarrow y = 2 - x$

$BC = y = 2 + x$

$DC = y = \dfrac{2-x}{3}$

Breaking Δ into 2 parts & integrating, we get :-

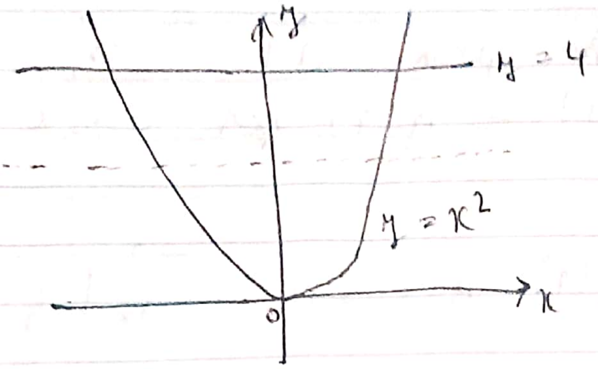
(i) area $(\Delta BCD) = \displaystyle\int_{-1}^{0} (2+x) - \left(\frac{2-x}{3}\right) dx$

$$= \int_{-1}^{0} \frac{4 + 4x}{3}\,dx = \frac{2}{3}$$

(continued on pg. 9)

P.T.O

57. Ans:



Due to symmetry, we consider only the first quadrant, where $y = x^2$

$\Rightarrow x = \sqrt{y}$

We are looking for a number b such that:

$$\int_0^b \sqrt{y}\, dy = \int_b^4 \sqrt{y}\, dy$$

$$\Rightarrow \frac{2}{3}\left[y^{3/2} \right]_0^b$$

$$\Rightarrow \frac{2}{3}\left[y^{3/2} \right]_b^4$$

$$\Rightarrow b^{3/2} = 4^{3/2} - b^{3/2}$$

$$\Rightarrow 2b^{3/2} = 8$$

$$b^{3/2} = 4$$

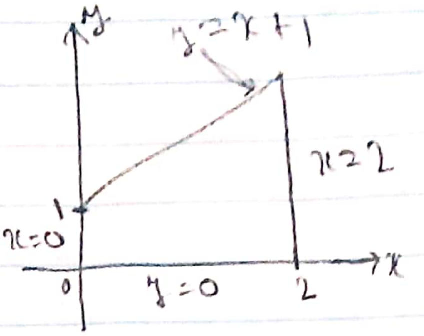
$$b = 4^{2/3}$$

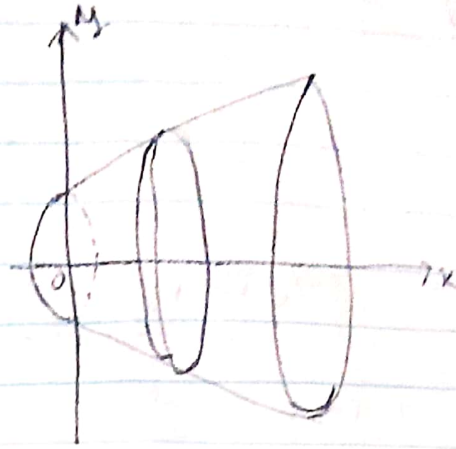
$$b = 2.52 \text{ approx.}$$

Exercise 6.2

P.T.O

1. Ans.





Cross - section is a disk with radius $x+1$, so its area is

$$A(x) = \pi (x+1)^2$$
$$= \pi (x^2+2x+1)$$

$$V = \int_0^2 A(x)\,dx$$
$$= \int_0^2 \pi (x^2+2x+1)\,dx$$
$$= \pi \left[\frac{1}{3}x^3 + x^2 + x \right]_0^2$$
$$= \pi \left(\frac{8}{3} + 4 + 2 \right) = \frac{26\pi}{3}$$

Exercise 6.1 (continued)

34. Ans Area $(\triangle ABD) = \int_0^2 (2-x) - \frac{(2-x)}{3}\,dx$

$$= \int_0^2 \frac{2}{3}(2-x)\,dx$$
$$= \frac{2}{3}\left(2x - \frac{x^2}{2}\right)_0^2$$
$$= \frac{4}{3}$$

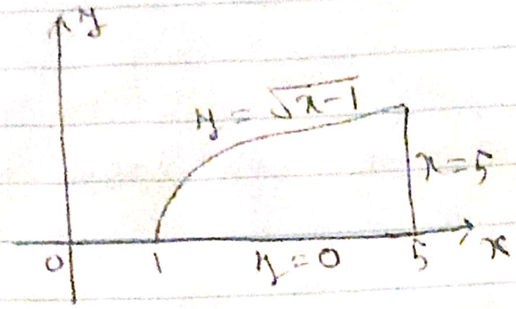
P.T.O

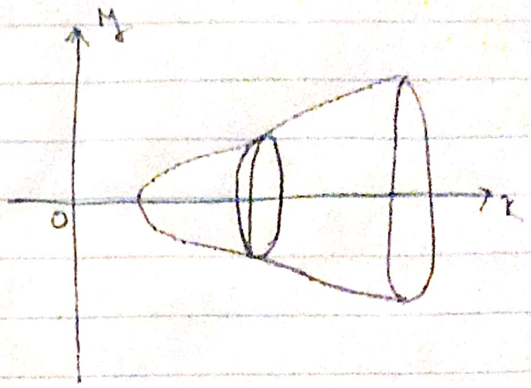
Total area $= \frac{2}{3} + \frac{4}{3}$

$= 2$ units

Exercise 6.2

3. Ans:





Cross-section is a disk with radius $\sqrt{x-1}$, so its area is $A(x) = \pi(\sqrt{x-1})^2 = \pi(x-1)$.

$$V = \int_1^5 A(x)\,dx = \int_1^5 \pi(x-1)\,dx$$

$$= \pi\left[\frac{1}{2}x^2 - x\right]_1^5$$

$$= \pi\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= 8\pi$$