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(1)

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MATH-141

HWK-3

EXERCISE 4.4

1. Ans: (a) $\lim_{x \to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type $\frac{0}{0}$.

(b) $\lim_{x \to a} \frac{f(x)}{p(x)} = 0$ because the numerator approaches 0 while the denominator becomes large.

(c) $\lim_{x \to a} \frac{h(x)}{p(x)} = 0$ because the numerator approaches a finite number while the denominator becomes large.

(d) (i) If $\lim_{x \to a} p(x) = \infty$ and $f(x) \to 0$ through positive values, then $\lim_{x \to a} \frac{p(x)}{f(x)} = \infty$. [For eg:-, take $a=0$, $p(x) = \frac{1}{x^2}$, and $f(x) = x^2$.] (ii) If $f(x) \to 0$ through negative values, then $\lim_{x \to a} \frac{p(x)}{f(x)} = -\infty$. [For eg, take $a=0$, $p(x) = \frac{1}{x^2}$, and $f(x) = -x^2$.] (iii) If $f(x) \to 0$ through both positive & negative values, then the limit might not exist. [For eg: take $a = 0$, $p(x) = \frac{1}{x^2}$ & $f(x) = x$.]

V.T.O

(e) $\lim_{x \to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type $\frac{\infty}{\infty}$.

9. Ans This limit has the form $\frac{0}{0}$.

$\therefore \lim_{x \to 4} \frac{x^2 - 2x - 8}{x - 4}$

$\Rightarrow \lim_{x \to 4} \frac{(x-4)(x+2)}{x-4}$

$\Rightarrow \lim_{x \to 4} (x+2) = 4 + 2 = \underline{\underline{6}}$

13. Ans This limit has the form $\frac{0}{0}$.

$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{1 - \sin x}$

Using L'Hospital's rule, we get :-

$\lim_{x \to \frac{\pi}{2}} \frac{-\sin x}{-\cos x}$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \tan x$

$\Rightarrow \underline{\underline{-\infty}}$

P.T.O

15. Ans This limit has the form $\frac{0}{0}$.

$$\therefore \lim_{t \to 0} \frac{e^{2t} - 1}{\sin t}$$

Using L'Hospital's rule:-

$$\lim_{t \to 0} \frac{2e^{2t}}{\cos t} = \frac{2(1)}{1} = \underline{\underline{2}}$$

19. Ans. This limit has the form $\frac{\infty}{\infty}$.

$$\therefore \lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$$

Using L'Hospital's rule, we get:-

$$\lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}}$$

$$\Rightarrow \lim_{x \to \infty} \frac{2}{\sqrt{x}}$$

$$\Rightarrow \underline{\underline{0}}$$

24 Ans $\lim_{t \to 0} \frac{8^t - 5^t}{t}$

$$\Rightarrow \frac{8^0 - 5^0}{0} = \frac{0}{0} \text{ form.}$$

Using L'Hospital's rule:-

$$\lim_{t \to 0} \left[\frac{\frac{d}{dt}(8^t - 5^t)}{\frac{d}{dt}(t)} \right]$$

$$\therefore \lim_{t \to 0} \left[\frac{\ln 8 . 8^t - \ln 5 . 5^t}{1} \right] \quad \left[\because \frac{d}{dt}(a^t) = \log a \cdot a^t\right]$$

$$\Rightarrow \lim_{t \to 0} \left[\frac{\ln . 2^3 . 8^t - \ln(5) . 5^t}{1} \right]$$

$$\Rightarrow \lim_{t \to 0} \left[3 \ln 2 . 8^t - \ln(5) . 5^t \right]$$

$$\Rightarrow 3 \ln 2 . 8^0 - \ln(5) . 5^0$$

$$[\because a^0 = 1]$$

$$\Rightarrow 3 \ln 2 - \ln(5)$$

$$\Rightarrow 0.4700$$

$$\therefore \lim_{t \to 0} \frac{8^t - 5^t}{t} = 0.4700$$

27. Ans. This limit has the form $\frac{0}{0}$.

Using L' Hospital's rule, $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$ becomes :-

$$\lim_{x \to 0} \frac{e^x - 1}{2x}.$$

PTO

Applying the rule again, we get :-

$$\lim_{x \to 0} \frac{e^x}{2}$$

$$\Rightarrow \underline{\underline{\frac{1}{2}}}$$

32. Ans This limit is in the form of $\frac{\infty}{\infty}$.

Using L'Hospital's rule, we get :-

$$\lim_{x \to \infty} \frac{\frac{d}{dx}(\ln x)^2}{\frac{d}{dx} x}$$

$$\therefore \lim_{x \to \infty} \frac{2 \ln x \cdot \frac{1}{x}}{1}$$

This is again in the $\frac{\infty}{\infty}$ form.

$\therefore$ Using L'Hospital's rule again, we get -

$$\lim_{x \to \infty} \frac{\frac{d}{dx}(2 \ln x)}{\frac{d}{dx}(x)}$$

$$\Rightarrow \lim_{x \to \infty} \frac{\left(\frac{2}{x}\right)}{1} = \frac{2}{\infty} = \underline{\underline{0}}$$

P.T.O

35. Ans: This limit can be evaluated by substituting 0 for x.

$$\therefore \lim_{x\to 0} \frac{\ln(1+x)}{\cos x + e^x - 1} = \frac{\ln 1}{1+1-1}$$

$$\Rightarrow \frac{0}{1} = 0$$

37. Ans: This limit has the form $\frac{0}{\infty}$, so L'Hospital's rule does not apply.

As $x \to 0^+$, $\arctan(2x) \to 0$ & $\ln x \to \infty$,

$$\therefore \lim_{x\to 0^+} \frac{\arctan(2x)}{\ln x} = 0$$

43 Ans This limit has the form $0 \cdot \infty$.

Let us change it to the form $\frac{0}{0}$.

$$\therefore \lim_{x\to\infty} x \sin(\pi/x) = \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x}$$

Using L'Hospital's rule, we get:-

$$\lim_{x\to\infty} \frac{\cos(\pi/x)(-\pi/x^2)}{-1/x^2}$$

$$\Rightarrow \pi \lim_{x\to\infty} \cos(\pi/x)$$

$$\Rightarrow \pi(1) = \pi$$

P.T.O

57. Ans- $y = x^{\sqrt{x}}$

$$\ln y = \sqrt{x}\ln x$$

$$\therefore \lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \sqrt{x}\ln x$$

$$= \lim_{x \to 0^+} \frac{\ln x}{x^{-1/2}}$$

Using L'Hospital's rule, we get:-

$$\lim_{x \to 0^+} \frac{1/x}{-\frac{1}{2}x^{-3/2}}$$

$$\Rightarrow -2 \lim_{x \to 0^+} \sqrt{x}$$

$$\Rightarrow 0$$

$$\therefore \lim_{x \to 0^+} x^{\sqrt{x}} = \lim_{x \to 0^+} e^{\ln y}$$

$$= e^0$$

$$= \underline{\underline{1}}$$

Exercise 7.8

1. (a) Ans- Since $y = \frac{x}{x-1}$ has an infinite discontinuity at $x = 1$,

$\therefore \int_1^2 \frac{x}{x-1}$ is a Type 2 improper integral.

(b) Ans- Since $\int_0^\infty \frac{1}{1+x^3}\,dx$ has an infinite interval of integration, it is an improper integral of Type 1.

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(c) Ans Since $\int_0^{\infty} x^2 e^{-x^2}\,dx$ has an infinite interval of integration, it is an improper integral of Type I.

(d) Ans Since $y = \cot x$ has an infinite discontinuity at $x = 0$, $\int_0^{\pi/4} \cot x\,dx$ is a Type 2 improper integral.

5. Ans $$\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t \to \infty} \int_3^t (x-2)^{-3/2}\,dx$$

$$= \lim_{t \to \infty} \left[-2(x-2)^{-1/2} \right]_3^t$$

Let $u = x - 2$
$du = dx$

$$\therefore \lim_{t \to \infty} \left(\frac{-2}{\sqrt{t-2}} + \frac{2}{\sqrt{1}} \right)$$

$$\Rightarrow 0 + 2 = \underline{\underline{2}}$$

$\therefore$ This integral is convergent.

7. Ans $$\int_{-\infty}^{0} \frac{1}{3-4x}\,dx = \lim_{t \to -\infty} \int_t^0 \frac{1}{3-4x}\,dx$$

$$= \lim_{t \to -\infty} \left[-\frac{1}{4} \ln|3-4x| \right]_t^0$$

$$= \lim_{t \to -\infty} \left[-\frac{1}{4}\ln 3 + \frac{1}{4}\ln|3-4t| \right]$$

$= \infty$ $\therefore$ This integral is divergent.

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<u>9 b_{ii}</u> $\int_2^\infty e^{-5p}\,dp = \lim_{t\to\infty}\int_2^t e^{-5p}\,dp$

$$= \lim_{t\to\infty}\left[-\frac{1}{5}e^{-5p}\right]_2^t$$

$$= \lim_{t\to\infty}\left(-\frac{1}{5}e^{-5t}+\frac{1}{5}e^{-10}\right)$$

$$= 0+\frac{1}{5}e^{-10}$$

$$= \frac{1}{5}e^{-10}$$

$\therefore$ This integral is convergent.

<u>15 b_{ii}</u> $\int_0^\infty \sin^2\alpha\,d\alpha = \lim_{t\to\infty}\int_0^t \frac{1}{2}(1-\cos 2\alpha)\,d\alpha$

$$= \lim_{t\to\infty}\left[\frac{1}{2}\left(\alpha-\frac{1}{2}\sin 2\alpha\right)\right]_0^t$$

$$= \lim_{t\to\infty}\left[\frac{1}{2}\left(t-\frac{1}{2}\sin 2t\right)-0\right]$$

$$= \infty$$

$\therefore$ This integral is divergent.