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# MATH - 141

## SECTION - 5.5
### Practice Homework

7. $\int x\sqrt{1-x^2}\,dx$

Ans. Let $u = 1-x^2$

$du = -2x\,dx$

$-\frac{du}{2} = x\,dx$

Substituting, we get :-

$\int \sqrt{u}\cdot\left(-\frac{1}{2}du\right)$

$\Rightarrow -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$\Rightarrow -\frac{1}{3}(1-x^2)^{3/2} + C$

11. $\int \cos(\pi t/2)\,dt$

Ans. Let $u = \frac{\pi t}{2}$

$du = \frac{\pi}{2}dt$

$\therefore dt = \frac{2\,du}{\pi}$

$\therefore \int \cos\left(\frac{\pi t}{2}\right)dt = \int \cos u\cdot\left(\frac{2}{\pi}du\right)$

$(1) \quad = \frac{2}{\pi}\sin u + C = \frac{2}{\pi}\sin\left(\frac{\pi t}{2}\right) + C$

17. $\int \cos^3\theta \sin\theta \, d\theta$

Ans: Let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$\sin\theta \, d\theta = -du$

$$\int \cos^3\theta \sin\theta \, d\theta = \int u^3(-du)$$

$$= -\frac{1}{4} \cdot u^4 + C$$

$$= -\frac{1}{4}\cos^4\theta + C$$

18. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx$

Ans: Let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \, dx$

$\Rightarrow \frac{1}{\sqrt{x}} \, dx = 2\,du$

$$\therefore \int \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx = \int \sin u \cdot (2\,du)$$

$$= 2\int \sin u \, du$$

$$= -2\cos u + C$$

$$\Rightarrow -2\cos\sqrt{x} + C$$

21. $\int \frac{(\ln x)^2}{x} dx$

Ans:- Set $u = \ln x$

$du = \frac{dx}{x}$

$\therefore \int \frac{(\ln x)^2}{x} dx = \int u^2 \, du$

$= \frac{1}{3} u^3 + C$

$= \frac{1}{3} (\ln x)^3 + C$

23. $\int \sec^2\theta \tan^3\theta \, d\theta$

Ans:- Set $u = \tan\theta$.

$du = \sec^2\theta \, d\theta$

$\int \sec^2\theta \tan^3\theta \, d\theta = \int u^3 \, du$

$= \frac{1}{4} u^4 + C$

$= \frac{1}{4} \tan^4\theta + C$

25. $\int e^x \sqrt{1+e^x} \, dx$

Ans:- Set $u = 1+e^x$

$du = e^x dx$

$\therefore \int e^x \sqrt{1+e^x} \, dx = \int \sqrt{u} \, du = \frac{2}{3} u^{3/2} + C$

$\Rightarrow \frac{2}{3}(1+e^x)^{3/2} + C$

29. $\int 5^t \sin(5^t)\, dt$

Ans= Set $u = 5^t$

$du = 5^t \ln 5\, dt$

$\frac{du}{\ln 5} = 5^t\, dt$

$$\int 5^t \sin(5^t)\, dt = \int \sin u \left(\frac{1}{\ln 5} du\right)$$

$$= -\frac{1}{\ln 5} \cos u + C$$

$$= -\frac{1}{\ln 5} \cos(5^t) + C$$

31. $\int \frac{(\arctan x)^2}{x^2+1}\, dx$

Ans= Set $u = \arctan x$

$du = \frac{1}{x^2+1}\, dx$

$$\int \frac{(\arctan x)^2}{x^2+1}\, dx = \int u^2\, du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{1}{3}(\arctan x)^3 + C$$



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37. $\int \sinh^2 x \cosh x \, dx$

Ans:- Let $u = \sinh x$

$du = \cosh x \, dx$

$\therefore \int \sinh^2 x \cosh x \, dx = \int u^2 \, du$

$= \frac{u^3}{3} + C$

$= \frac{1}{3} \sinh^3 x + C$

43. $\int \frac{dx}{\sqrt{1-x^2} \sin^{-1} x}$

Ans:- Let $u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}} dx$

$\therefore \int \frac{dx}{\sqrt{1-x^2} \sin^{-1} x} = \int \frac{1}{u} du$

$= \ln |u| + C$

$= \ln |\sin^{-1} x| + C$

45. $\int \frac{1+x}{1+x^2} dx$

Ans:- Let $u = 1+x^2$

$du = 2x \, dx$

$\therefore \int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$

$= \tan^{-1}x + \int \frac{\frac{1}{2}\,du}{u}$

$= \tan^{-1}x + \frac{1}{2}\ln|u| + C$

$= \tan^{-1}x + \frac{1}{2}\ln|1+x^2| + C$

$= \tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$

$[\because 1+x^2 > 0]$

53. $\int_0^1 \cos(\pi t/2)\,dt$

Ans: Let $u = \frac{\pi t}{2}$

$du = \frac{\pi}{2}\,dt$

when $t = 0$, $u = 0$;
when $t = 1$, $u = \frac{\pi}{2}$

$\therefore \int_0^1 \cos(\pi t/2)\,dt$

$\Rightarrow \int_0^{\pi/2} \cos u \left(\frac{2}{\pi}\,du\right)$

$\Rightarrow \frac{2}{\pi}\left[\sin u\right]_0^{\pi/2}$

$\Rightarrow \frac{2}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right) = \frac{2}{\pi}(1-0)$

$= \frac{2}{\pi}$

61. $\int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x)\, dx$

Ans:- According to the property of definite integrals:-

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is even} \\ 0, & \text{if } f(x) \text{ is odd} \end{cases}$$

Here, $f(x) = x^3 + x^4 \tan x$

$f(-x) = (-x)^3 + (-x)^4 \tan(-x)$

$= -x^3 - x^4 \tan x$

$f(-x) = -[x^3 + x^4 \tan x]$

$f(-x) = -f(x)$

$\therefore$ The function is odd

$\therefore \int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x)\, dx = 0$

62. $\int_0^{\pi/2} \cos x \sin(\sin x)\, dx$

Ans:- Let $\sin x = u$

$\cos x\, dx = du$

$\therefore \int_0^{\pi/2} \sin u\, du = [-\cos u]_0^{\pi/2} = -\cos\frac{\pi}{2} - (-\cos 0)$

(7)

$\Rightarrow \quad -0+1$

$\Rightarrow \quad \underline{\underline{1}}$

71. $\int_0^1 \frac{e^z+1}{e^z+z}\,dz$

Ans:- Let $u = e^z + z$

$du = (e^z+1)\,dz$

When $z=0$, $u=1$;

when $z=1$, $u=e+1$

$\therefore \int_0^1 \frac{e^z+1}{e^z+z}\,dz = \int_1^{e+1} \frac{1}{u}\,du$

$= [\ln|u|]_1^{e+1}$

$= \ln|e+1| - \ln|1|$

$= \underline{\underline{\ln(e+1)}}$

77. Evaluate $\int_{-2}^{2} (x+3)\sqrt{4-x^2}\,dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

Ans:- $\int_{-2}^{2} (x+3)\sqrt{4-x^2} = \int_{-2}^{2} (x\sqrt{4-x^2} + 3\sqrt{4-x^2})\,dx$

$= \underbrace{\int_{-2}^{2} x\sqrt{4-x^2}\,dx}_{1} + \underbrace{\int_{-2}^{2} 3\sqrt{4-x^2}\,dx}_{2}$

Solving for ① :-

$$\int_{-2}^{2} x\sqrt{4-x^2}\,dx$$

Set $u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

When $x = -2$, $u = 4-(-2)^2$

$= 4-4$

$= \underline{\underline{0}}$

When $x = 2$, $u = 4-(2)^2$

$= 4-4$

$= \underline{\underline{0}}$

$\therefore \int_{-2}^{2} \sqrt{4-x^2}\,x\,dx = \int_0^0 -\sqrt{u}\,\frac{du}{2}$

$= \underline{\underline{0}}$ [$\because$ Since both integral bounds are 0, the value of this integral would be 0.]

Solving for ② by interpreting it in terms of area :-

$$\int_{-2}^{2} 3\sqrt{4-x^2}\,dx = 3\int_{-2}^{2}\sqrt{4-x^2}\,dx$$

$y = \sqrt{4-x^2}$ is upper half of the circle from $-2$ to $2$ with radius 2 area of semicircle $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi(2)^2 = 2\pi$$

$$\therefore 3\int_{-2}^{2}\sqrt{4-x^2}\,dx = 3(2\pi) = \underline{\underline{6\pi}}$$

$\therefore$ Final value of the complete integral :-

$$\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx = 0 + 6\pi = \underline{\underline{6\pi}}$$