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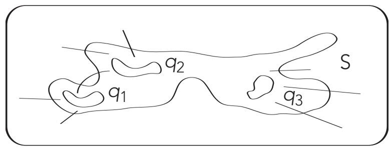
Lab 1:
Electric Fields, Flux and Gauss' Law



. . . before Maxwell people considered physical reality
. . . as material points . . . After Maxwell they considered physical reality as continuous fields. . .

—A. Einstein

OBJECTIVES

- To understand how electric field lines can be used to describe the magnitude and direction of an electric field in a small region of space.
- To discover how the electric flux passing through a small area is related to the magnitude of the electric field, as well as the area and its orientation relative to the direction of the electric field.
- To understand why $\Sigma E \cos\theta \, \Delta A$ over a closed surface is proportional to the number of lines of flux passing through the surface.
- To understand the relationship between the flux passing through a closed surface and the charge enclosed by that surface (Gauss' law) for a two-dimensional situation.
- To explore how Gauss' law predicts where excess charge can be found on a conductor.

OVERVIEW

You have learned that electric field is defined as a vector that describes the direction and magnitude of the force exerted on a positive test charge *per unit charge*. Although the electric field can be calculated using Coulomb's law, it is very difficult when many charges are present at different locations. However, it is possible to calculate the electric field using a completely different formulation of Coulomb's law. This formulation is known as Gauss' law, and it involves relating the electric field surrounding a collection of charges to the net amount of charge inside a closed surface. The Gauss' law formulation is a very powerful tool for calculating electric fields in situations where the distribution of charge is very *symmetric*. Gauss' law can be proven to be mathematically equivalent to

Coulomb's law. In this lab you are going to take an approach to Gauss' law that is not so mathematical.

Before exploring Gauss' law, however, you will see how to represent the electric field in a region of space by using *electric field lines*. Next you will define electric flux, a quantity that is related to the electric field lines passing through a surface. You will try to discover Gauss' law by drawing closed "surfaces" around various charges or groups of charges and seeing how many electric field lines pass in and out of the surface.

Reality Check: Always remember that the electric field, and the various schemes for representing it in space, are just aids to representing electrostatic forces mathematically. The force is the only real physical thing that can be measured! Gauss' law can seem confusing at times, but it has given us a much better understanding of interactions between charged particles.

INVESTIGATION 1: ELECTRIC FIELD LINES, ELECTRIC FLUX AND GAUSS' LAW

Electric Field Lines

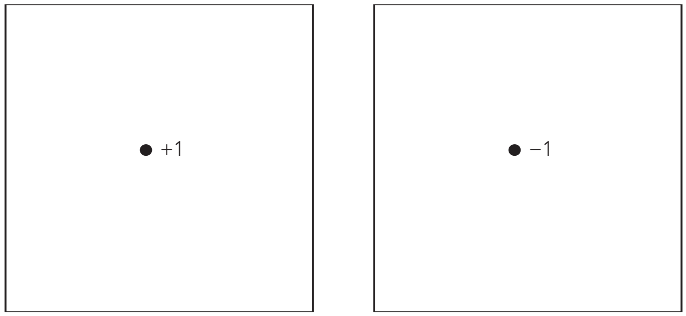
In Lab 1, you represented the electric field produced by a configuration of electric charges by arrows with magnitude and direction at each point in space. This is the *conventional representation* of a "vector field." An alternative representation of the vector field involves the use of *electric field lines*. Unlike an electric field vector that is an arrow located at a point in space with magnitude and direction, electric field lines are continuous. Instead of defining electric field lines formally, you can discover what they represent by using a computer simulation.

You will need:

- field line simulation program (or printouts of electric field lines generated using the simulation)
- computer

Activity 1-1: Simulation of Electric Field Lines from Point Charges

1. Open the field line simulation program. (Or use a printout of the electric field lines from such a program.)
2. Set a single $+1$ (arbitrary unit) charge somewhere on the computer screen and run the program. After a few minutes sketch the field lines in the space below, on the left. Then place a -1 charge somewhere on the screen, and sketch the field lines associated with it on the right. Show the direction of the electric field on each line by placing an arrow on it.



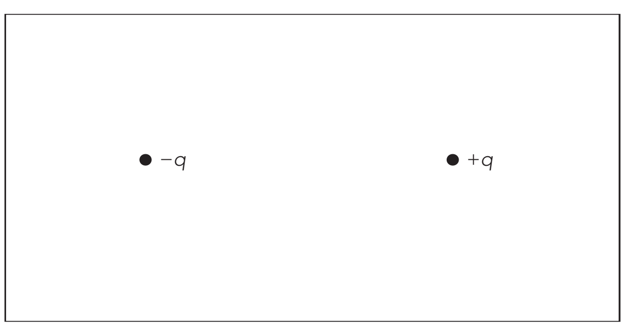
Question 1-1: How many lines are there in the drawings? Are the lines more dense (closely spaced) or less dense near the charge? Explain. How do the directions of the lines depend on the sign of the charge?

3. Try another magnitude of charge. You don't need to sketch the result.

Question 1-2: What is the magnitude of your new charge? How many lines are shown in the simulation? How do the number of lines compare to the number in Question 1-1?

Question 1-3: If you know the magnitude of the charges, describe the rule for telling how many lines will come out of or into a charge in this simulation. Explain your answer based on your observations.

4. Repeat the exercise using two charges of the *same magnitude* but opposite sign. After the simulated lines are drawn, sketch them in the space below. Indicate how much charge you used on the diagram.



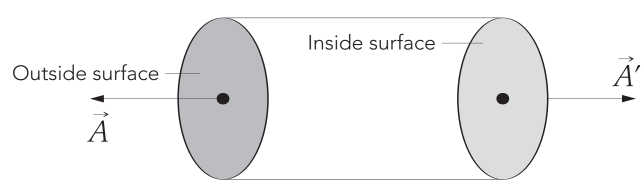
Question 1-4: Are the lines beginning or ending on a charge more dense near it or far away from it? How does the direction of the lines depend on the sign of the charge?

Question 1-5: Summarize the properties of electric field lines. What does the number of lines signify? What does the direction of a line at each point in space represent? What does the density of the lines represent?

Defining Electric Flux

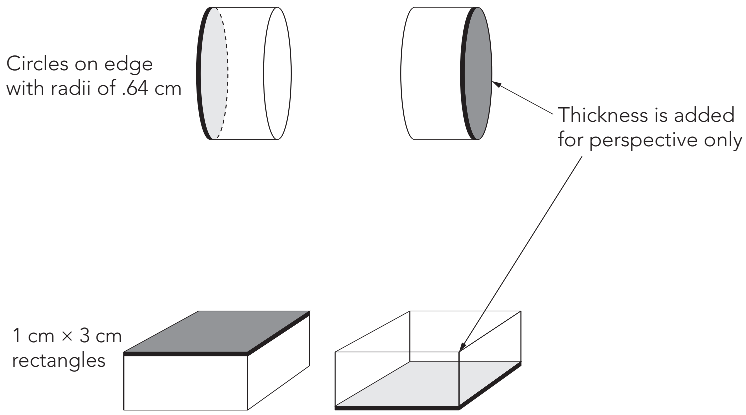
We can think of an electric charge as having a number of electric field lines diverging from it or converging on it in such a way that the number of lines is proportional to the magnitude of the charge. Now we can explore the mathematics of enclosing charges within surfaces and seeing how many electric field lines pass through the surface. *Electric flux* is defined as a measure of the number of electric field lines passing through a surface. In defining "flux" we are constructing a mental model of lines streaming out from the surface area surrounding each unit of charge like streams of water or rays of light. Physicists do not really think of charges as having anything real streaming out from them, but the mathematics that best describes the forces between charges is the same as the mathematics that describes streams of water or rays of light. Let's explore the behavior of this mathematical model.

It should be obvious that the number of field lines passing through a surface depends on how that surface is oriented relative to the electric field lines. The orientation of a small surface of area A can be defined by a normal vector that is perpendicular to the surface and has a magnitude equal to the surface area, as shown in the figure below. By convention, the normal vector points away from the *outside* of the surface. The normal vectors are pictured for small surfaces of area A that make up the ends of a cylindrical box. In the picture the inside of each surface is light gray and the outside is dark gray.



Activity 1-2: Drawing Normal Vectors

Use the definition of normal to an area given above to draw normal "area" vectors to the surfaces shown below. Label each vector as $\vec{A}$. Let the length of the normal vector in cm be equal in magnitude to each area in cm^2.



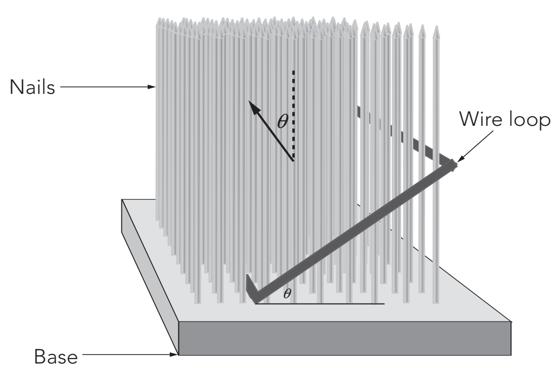
By convention, if an electric field line passes from the inside to the outside of a closed surface we say the flux is positive. If the field line passes from the outside to the inside of a surface, the flux is—by definition—negative.

Prediction 1-1: How does the flux through a surface depend on the angle between the normal vector to the surface and the electric field lines? For example, what is the electric flux when the angle is 90 degrees? What happens to the electric flux as you rotate the surface at various angles between 0 degrees (or 0 radians) and 180 degrees (or π radians) with respect to the electric field vectors? What do you think?

In order to test your predictions in a quantitative way, you can use a mechanical model of some electric field lines and of a surface. The model is made with a 10 by 10 array of nails poking up at 1/4″ intervals through a piece of foil covered foam insulation. The "surface" is a wire loop painted white on the "outside." You will need

- 100 nails (approximately 4″ long) to represent field lines
- 5″ × 5″ square of Styrofoam
- 5″ × 5″ square of ¼″ graph paper affixed to the Styrofoam to make an evenly spaced grid for the nails
- square wire loop (4″ × 4″) to represent a "surface"
- protractor
- data logging software
- *RealTime Physics Electricity and Magnetism* experiment configuration files

The finished mechanical model for flux is shown below.



Activity 1-3: Flux as a Function of Angle

1. Use your mechanical model, a protractor and some calculations to fill in the data table below. **Note:** The angle, θ, is between the normal to the area (surface of loop) and the direction of the field lines (nails). If the normal is pointing generally in the same direction as the field, the angle is between 0 and 90°. If the normal is pointing generally opposite to the field, the angle is more than 90°, and the flux is negative.
2. Open the experiment file **Flux vs. Angle (L02A1-3),** and enter your data for flux vs. angle.
3. Find the mathematical relationship between flux and angle. Guess the mathematical relationship (there are a few hints in this lab), and try to confirm your guess by plotting a new graph of flux as a function of angle and comparing it to the graph of your data. (Create a new **calculated column.** Use the function of angle that you want to test, and adjust the parameters until you get the best agreement with the graph of the actual data. Try different functions until you have found the one that describes the functional relationship between your data and the angle. (Use the angle in radians for trigonometric functions.)
4. Alternatively, you can do a manual fit to the trigonometric function A sin (B θ + C) + D, and find the parameters A, B, C, and D.

Φ (flux = number of lines)	θ (deg) (normal pointing up)	Φ (flux = number of lines)	θ (deg) (normal pointing down)
64		−8	
56		−16	
48		−24	
40		−32	
32		−40	
24		−48	
16		−56	
8		−64	
0			

Print all of your graphs and affix to these sheets.

Question 1-6: What is the mathematical relationship between flux and angle? Explain based on your observations.

A Mathematical Representation of Flux through a Surface

By definition, the relationship between flux, Φ, and angle, θ, for a uniform electric field $\vec{E}$ is $\Phi = E\,A\cos\theta$, where θ is the angle between the normal to the surface and the electric field vector, $\vec{E}$. Flux is a scalar quantity.

If the electric field is not uniform or if the angle between the surface and electric field varies from point to point on the surface, then flux needs to be calculated by breaking the surface into infinitely many infinitesimally small areas (so that $\Delta\Phi = E\cos\theta\,\Delta A$) and then adding the fluxes through all the areas. This gives

$$\Phi = \sum\Delta\Phi = \sum E\,\Delta A\cos\theta \qquad \text{[flux through a surface]}$$

Some surfaces like that of a sphere or that representing a rectangular box are closed surfaces as they have no holes or breaks in them.

In order to study the amount of flux passing through closed surfaces, you will need

- field line simulation program (or field line printouts from one)
- computer

Activity 1-4: Discovering Gauss' Law in Flatland

Since it is charge that produces the electric field, the electric flux passing through a closed surface must depend on the enclosed charge. But what is the dependence? Suppose you lived in a two-dimensional world in which all charges and electric field lines were constrained to lie in a flat, two-dimensional space. Of course, mathematicians call such a space a plane. You can open the field line simulation program again and set the program to sketch lines for some nutty creative mix of charges (or use already printed ones). Don't be too creative or the lines will take forever to sketch out! You should do the following:

1. Open up the simulation, place some positive *and* negative charges at different places on the screen and start the program to calculate and display the electric field lines in two dimensions.

2. In the space below, sketch your computer screen showing the configuration of the charges and associated "e-field" lines, or include a screen shot of the computer screen.

3. Draw arrows on each of the lines indicating in which direction a *small* positive test charge would move along each line. **Note:** "small" means that the test charge does not exert large enough forces on the charges that create the e-field to cause the field to change noticeably when it is brought near them.

Having done all this preparation, you should be ready to discover how the net number of lines enclosed by a surface is related to the net charge enclosed by the surface.

4. Draw three two-dimensional closed "surfaces" in pencil on your diagram, above. Some of them should enclose charge, and some should avoid enclosing charge.
5. Draw at least one more closed "surface" that encloses charges, but zero *net* charge.

You are ready to discover how the net number of lines enclosed by a surface is related to the net charge enclosed by the surface.

6. Count the *net* lines of flux coming out of (leaving) each "surface." (**Note:** The *net* number of lines is defined as the number of lines coming out of the surface minus the number of lines going in.)
7. Fill in the table below for three different Flatland "surfaces."

	Charge enclosed by the arbitrary surface			Lines of flux in and out of the surface		
	$+q$	$-q$	q_{net}	Lines_{out}	Lines_{in}	Lines_{net}
1						
2						
3						
4						

Question 1-7: What is the apparent relationship between the net flux (net number of lines) passing through an imaginary surface and the net charge enclosed by the two-dimensional "surface"? Explain, based on your simulated observations.

Gauss' Law in Three Dimensions

If you were to repeat the simulated exploration you just performed in a three-dimensional space, what do you think would be the appropriate expression for Gauss' law? You will explore this in the next activity.

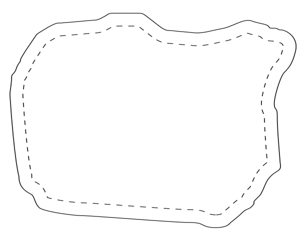
Activity 1-5: Statements of Gauss' Law

Question 1-8: Express the three-dimensional form of Gauss' law in words. **Hints:** What would be the three-dimensional equivalent of the closed curves you drew in steps (4) and (5)? Does it still make sense in three-dimensions to talk about the net number of lines coming out of a closed surface?

Question 1-9: Using the results from Activities 1-3 and 1-4, and your answer to Question 1-8, express Gauss' law using an equation. (**Hint:** The net flux should be on one side of the equation, and the other side should have an expression for the net charge enclosed by the surface. Don't worry about the actual value of the proportionality constant.

INVESTIGATION 2: USING GAUSS' LAW

An electrical conductor has some of its electrical charges free to move about (i.e., not bound to atoms). If a free charge in a conductor experiences an electric field, it will move under the influence of that field. *Thus*, we can conclude that if there are no charges moving within a conductor, the electric field within the conductor is zero.



Let's consider the conductor shown above that has been touched by a charged black plastic, Teflon, or rubber rod so that it has an excess of negative charge on it. Where does this charge go if it is free to move about? Is it distributed uniformly throughout the conductor? If we know that $\vec{E} = 0$ everywhere inside a conductor, we can use Gauss' law to figure out where the excess charge on a conductor is located. This is the subject of the next two activities.

Activity 1-6: Where is the Excess Charge in a Conductor?

Question 1-10: Consider the conductor in the diagram above, with an excess charge Q. As just argued above, there is zero electric field within the conductor. What is the amount of excess charge enclosed by the dashed Gaussian surface drawn just below the surface of the conductor? **Hint:** Use Gauss' law, and the fact that $\vec{E} = 0$ everywhere inside the conductor.

Question 1-11: If the conductor has excess charge and it can't be within the Gaussian surface according to Gauss' law, then what's the only place this excess charge can be?