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PH 222-2A

RECITATION #3

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1. Ans :- Force experienced by the charged particle situated in the uniform electric field is, $\vec{F} = q\vec{E}$.

Here is $\vec{F}$ force experienced by charged particle, q is charge of the particle and $\vec{E}$ is the uniform external electric field.

Relation between external force, mass of the body and acceleration of the body is, $\vec{F} = m\vec{a}$

(a) Force on proton can be calculated by :-

$$\vec{F} = q\vec{E}.$$

Substituting 1.602×10^{-19} C for q and $200\ \hat{i}$ N/C for $\vec{E}$.

$$\vec{F} = 1.602 \times 10^{-19} C \times 200\ \hat{i}\ N/C$$
$$= 3.204 \times 10^{-17}\ \hat{i}\ N$$

Rearranging the equation of force for acceleration,

$$\vec{a} = \frac{\vec{F}}{m}.$$

Substituting $3.204 \times 10^{-17}\ \hat{i}$ N for $\vec{F}$ and 1.67×10^{-27} kg for m.

$$\vec{a} = \frac{3.204 \times 10^{-17}\ \hat{i}\ N}{1.67 \times 10^{-27}\ kg}$$
$$= 1.916 \times 10^{10}\ m/s^2$$

$\therefore$ Force on proton is $\vec{F} = 3.204 \times 10^{-17}\ \hat{i}$ N and acceleration of proton is $\vec{a} = 1.916 \times 10^{10}\ \hat{i}\ m/s^2$.

(b) Force on electron can be calculated by the equation,

$$\vec{F} = q\vec{E}.$$

Substituting -1.602×10^{-19} C for q and $200\ \hat{i}$ N/C for $\vec{E}$

$$\vec{F} = (-1.602 \times 10^{-19} C)(200\ \hat{i}\ N/C) = -3.204 \times 10^{-17}\ \hat{i}\ N$$

Here the negative sign shows that force is acting in opposite direction of the electric field.

Rearrange the equation of force for acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$

Substituting $3.204 \times 10^{-17}\,\hat{i}$ N for $\vec{F}$ and 9.12×10^{-31} kg for m.

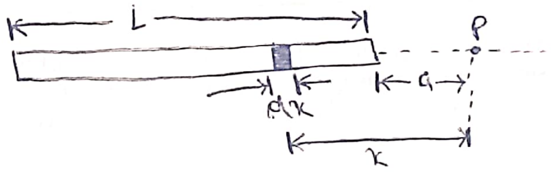
$$\vec{a} = \frac{(-3.204 \times 10^{-17}\,\hat{i}\ \text{N})}{(9.12 \times 10^{-31}\ \text{kg})}$$

$$= -3.51 \times 10^{13}\,\hat{i}\ \text{m/s}^2$$

Here negative sign shows that direction of acceleration of electron is in opposite direction of the electric field.

$\therefore$ Force on electron is $\vec{F} = -3.204 \times 10^{-17}\,\hat{i}$ N and acceleration of electron is $\vec{a} = -3.51 \times 10^{13}\,\hat{i}\ \text{m/s}^2$.

2. Ans:



The electric field for the elementary length dx at P is $dE = \frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda\,dx}{x^2}\right)$ $[\because dq = \lambda\,dx]$

Thus, the electric field for the whole rod is

$$E = \int dE$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_a^{L+a} \frac{\cancel{\lambda}\,dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(-\left(\frac{1}{x}\right)_a^{L+a}\right)$$

$$= \frac{-\lambda}{4\pi\varepsilon_0}\left[\frac{1}{L+a} - \frac{1}{a}\right]$$

$$\Rightarrow \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{L+a}\right]$$

$$\Rightarrow \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{L}{a(L+a)}\right]$$

$$\therefore E = \frac{\lambda L}{4\pi\varepsilon_0\, a(L+a)}$$

3. Ans.

$$\vec{dE} = \frac{d\theta\,(\vec{r} - \vec{r}_1)}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_1|^3}$$

We know that $\theta \propto L$.

$$\therefore \theta = \lambda L$$

$d\theta = \lambda\, dL$ (for an elementary charge $d\theta$).

$$d\theta = \lambda R\, d\theta$$

$$\vec{r}_1 = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$\vec{r} - \vec{r}_1 = -R\cos\theta\,\hat{i} - R\sin\theta\,\hat{j}$$

$$|\vec{r} - \vec{r}_1| = \{R^2\cos^2\theta + R^2\sin^2\theta\}^{1/2}$$

$$= R$$

$$\vec{E}(\theta) = \int_{-\theta_0}^{\theta_0} \frac{KLR\,d\theta\,(-R\cos\theta\,\hat{i} - R\sin\theta\,\hat{j})}{R^3}$$

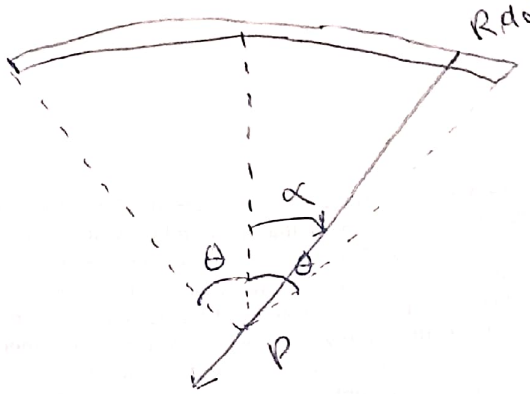
$$E_x(\theta) = \frac{-KL}{R}\int_{-\theta_0}^{\theta_0} \cos\theta \, d\theta = \frac{-2KL}{R}\sin\theta_0$$

$$E_y(\theta) = \frac{-KL}{R}\int_{-\theta_0}^{\theta_0} \sin\theta \, d\theta = 0$$

$$\vec{E}(\theta) = \frac{-2KL}{R}\sin\theta$$

3. Ans :-

Q = total charge



To find :- Electric field at P.

Charge density $\lambda = \frac{Q}{R(2\theta)} = \frac{Q}{2R\theta}$

$$d\vec{E} = \frac{K(dq)}{R^2} = \frac{K(\lambda)(R\,d\alpha)}{R^2} = \frac{K\lambda(d\alpha)}{R}$$

Due to symmetry, the horizontal components get cancelled.

$$d\vec{E}_y = \frac{K\lambda}{R}(d\alpha) \cdot \cos\alpha$$

$$\therefore E_{net} = \int_{-\theta}^{+\theta} \frac{K\lambda}{R}\cos\alpha \, d\alpha$$

$$= \frac{K\lambda}{R}(2\sin\theta) = \frac{K\lambda}{R} \cdot 2\sin\theta$$

$$= \frac{K}{R} \cdot \frac{Q}{2R\theta} \cdot 2\sin\theta$$

$\Rightarrow E_{net} = \frac{KQ}{R^2} \cdot \frac{\sin\theta}{\theta}$ along vertically downward direction.

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