PH-222-2A
RECITATION #5

$\boxed{CA}$ $\oint_S E \hat{r} . \hat{r}\, dA = \frac{q_{en}}{\varepsilon_0}$

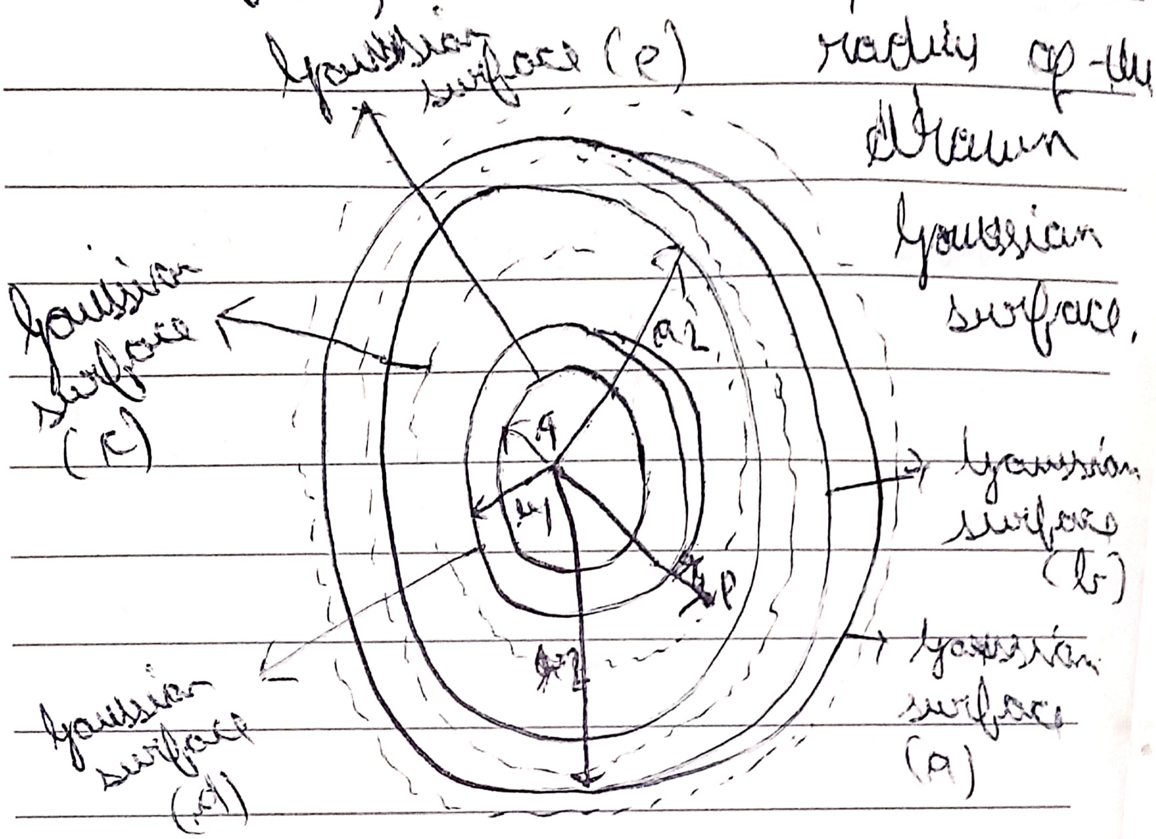
Charge in terms of volume charge density, $q = \rho V$

Volume of the spherical shell hav-
-ing charge distribution ρ_1,
$V_1 = \frac{4}{3}\pi (b_1^3 - a_1^3)$; a_1 is the inner radius & b_1 is the outer radius of the inner shell.

Volume of the spherical shell having charge distribution ρ_2, $V_2 = \frac{4}{3}\pi (b_2^3 - a_2^3)$;

a_2 is the inner radius & b_2 is the outer radius of the outer shell.

Surface area A of the Gaussian surface, $A = 4\pi r^2$; r is the radius of the drawn Gaussian surface.



(a) Electric field at space point P at r distance from the common center ($r > b_2$) as,

$$q_{en} = \rho_1 V_1 + \rho_2 V_2$$

substituting $\frac{4}{3}\pi(b_1^3 - a_1^3)$ for V_1

& $\frac{4}{3}\pi(b_2^3 - a_2^3)$ for V_2, we get:

$$q_{en} = \rho_1\left(\frac{4}{3}\pi\left(b_1^3 - a_1^3\right)\right) + \rho_2\left(\frac{4}{3}\pi\left(b_2^3 - a_2^3\right)\right)$$

Electric field is in radial direction and constant throughout the surface, so $\vec{E}$ is constant.

Flux expression can further be written as,

$$EA = \frac{q_{en}}{\epsilon_0}$$

Substituting $4\pi r^2$ for A,

$$\rho_1\left(\frac{4}{3}\pi\left(b_1^3 - a_1^3\right)\right) + \rho_2\left(\frac{4}{3}\pi\left(b_2^3 - a_2^3\right)\right)$$

for q_{en} in the above expression :-

④

$$\Rightarrow E(4\pi r^2) = \frac{\left(\rho_1\left(\frac{4}{3}\pi(b_1^3 - a_1^3)\right) + \rho_2\left(\frac{4}{3}\pi(b_2^3 - a_2^3)\right)\right)}{\epsilon_0}$$

Rearrange the above expression for electric field,

$$E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(b_2^3 - a_2^3)}{3\epsilon_0 r^2}$$

Thus electric field when $r > b_2$ is

$$E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(b_2^3 - a_2^3)}{3\epsilon_0 r^2}$$

(ii) Electric field at space point P at r distance from the common center ($a_2 < r < b_2$) as :-

$q_{en} = \rho_1 V_1 + \rho_2 V_2$

Here V_2 is the volume of the outer shell which has a_2 as inner radius and r as the outer radius.

Substituting $\frac{4}{3}\pi(b_1^3 - a_1^3)$ for V_1 and $\frac{4}{3}\pi(r^3 - a_2^3)$ for V_2'

in the above expression,

$$q_{en} = \rho_1\left(\frac{4}{3}\pi(b_1^3 - a_1^3)\right) + \rho_2\left(\frac{4}{3}\pi(r^3 - a_2^3)\right)$$

$\vec{E}$ is constant because electric field is in a radial direction.

Flux, $EA = \frac{q_{en}}{\epsilon_0}$

(6)

substituting $4\pi r^2$ for A,

$$\rho_1\left(\frac{4}{3}\pi(b_1^3 - a_1^3)\right) + \rho_2\left(\frac{4}{3}\pi(r^3 - a_2^3)\right) \text{ for}$$

q_{en} in the above expression,

$$E(4\pi r^2) = \frac{\left(\rho_1\left(\frac{4}{3}\pi(b_1^3-a_1^3)\right) + \rho_2\left(\frac{4}{3}\pi(r^3-a_2^3)\right)\right)}{\varepsilon_0}$$

Solving for E :-

$$E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(r^3 - a_2^3)}{3\varepsilon_0 r^2}$$

Thus electric field when $a_2 < r < b_2$ is :-

$$E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(r^3 - a_2^3)}{3\varepsilon_0 r^2}$$

(c) Electric field at space point p at r distance from the common center ($b_1 < r < a_2$) as,

$$q_{en} = \rho_1 V_1$$

Substituting $\frac{4}{3}\pi(b_1^3 - a_1^3)$ for V_1 in the above expression,

$$q_{en} = \rho_1 \left(\frac{4}{3}\pi(b_1^3 - a_1^3)\right)$$

$\vec{E}$ is constant because electric field is in radial direction and constant throughout the surface.

Thus, $EA = \dfrac{q_{en}}{\varepsilon_0}$

Substituting $4\pi r^2$ for A, $\rho_1\left(\frac{4}{3}\pi(b_1^3 - a_1^3)\right)$ for q_{en}

in the above expression,

⑧

$$E(4\pi r^2) = \frac{\rho_1\left(\frac{4}{3}\pi(b_1^3 - a_1^3)\right)}{\varepsilon_0}$$

Solving for E,

$$E = \frac{\rho_1(b_1^3 - a_1^3)}{3\varepsilon_0 r^2}$$

Thus, electric field $b_1 < r < a_2$

is $E = \dfrac{\rho_1(b_1^3 - a_1^3)}{3\varepsilon_0 r^2}$

(d) Electric field at space point p at r distance from the common center ($a_1 < r < b_1$)

as, $q_{enc} = \rho_1 V_1'$

Here V_1' is the volume of the inner shell which has a_1 as the inner radius and r as the outer radius.

Substituting $\frac{4}{3}\pi(r^3 - a_1^3)$ for V_1' in the above expression,

$$q_{en} = \rho_1 \left(\frac{4}{3} \pi (r^3 - a_1^3) \right)$$

$\vec{E}$ is constant because electric field is in the radial direction.

Thus, $EA = \dfrac{q_{en}}{\varepsilon_0}$

Substituting $4\pi r^2$ for A,

$\rho_1 \left(\frac{4}{3} \pi (r^3 - a_1^3) \right)$ for q_{en}:-

$$E(4\pi r^2) = \frac{\rho_1 \left(\frac{4}{3} \pi (r^3 - a_1^3) \right)}{\varepsilon_0}$$

Solving for E :-

$$E = \frac{\rho_1 (r^3 - a_1^3)}{3 \varepsilon_0 r^2}$$

$\therefore$ Electric field when $a_1 < r < b_1$,

is $$E = \frac{\rho_1 (r^3 - a_1^3)}{3 \varepsilon_0 r^2}$$

(2) Electric field at space point p at r distance from the common center ($r < a_1$) as,

$$q_{en} = 0.$$

Hence, the electric field will also be zero by gauss' law.

Thus the electric field when $r < a_1$ is $\underline{E = 0}$