PH 222 2A

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IN-CLASS EXAM 1

1 Ans: Volume charge density = ρ

$a = 20$ cm

$b = 50$ cm [$\because 2.5 \times 20$]

Using Gauss's law:-

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

(a) $r = \frac{a}{4} < a$

$q_{in} = 0$

$E = 0$

(d) $r = a$ $\quad q_{in} = 0$

$E = 0$

(e) $r = 2a$

$$q_{in} = \rho \times \frac{4\pi}{3\pi}\left((2a)^3 - a^3\right)$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = \frac{\rho\, 4\pi \times 7a^3}{3\varepsilon_0}$$

$$E \times \cancel{4\pi}(2a)^2 = \frac{\cancel{4\pi}\,\rho\, 7a^3}{3\varepsilon_0}$$

$$E = \frac{7\rho\, a^{\cancel{3}}}{3\varepsilon_0 \times 4\cancel{a^2}} = \frac{7\rho\, a}{12\varepsilon_0}$$

$$= \frac{7 \times 4\pi \times 9 \times 10^9 \times 2 \times 10^{-6} \times 0.20}{12}$$

$$E = 2.64 \times 10^4\ N/C$$

(d) $r = b$

$$q_{in} = \rho \times \frac{4\pi}{3}\left((2.5a)^3 - a^3\right)$$

$$= \frac{4\pi}{3}\rho a^3 \times 14.625$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E \times 4\pi b^2 = \frac{4\pi \rho a^3}{3\varepsilon_0} \times 14.625$$

$$E = \frac{\rho a^3 \times 14.625}{3\varepsilon_0 \times (2.5a)^2}$$

$$= \frac{9\times10^9 \times 4\pi \times 0.20 \times 2\times10^{-6} \times 14.625}{3\times 6.25}$$

$$= 3.53\times10^4 \text{ N/C}$$

(e) $x = 2b$

$$E \times 4\pi (2b)^2 = \frac{4\pi \rho A^3 \times 14.625}{3\varepsilon_0}$$

$$E = 8.82 \times 10^3 \ N/C$$

2. Ans From Gauss's law :-

Through any closed surface, $\frac{1}{\varepsilon_0}$ times net charge is enclosed in the closed surface.

$$S_1 = \phi = \frac{-2q_1 - 2q_1 + 3q_1 + q_1}{\varepsilon_0}$$

$$= 0$$

$$S_2 = \phi = \frac{-2q_1}{\varepsilon_0}$$

S_3 : $\phi = \frac{q}{\varepsilon_0}$

S_4 : $\phi = -\frac{4q}{\varepsilon_0}$

S_5 : $\phi = -\frac{2q}{\varepsilon_0}$

S_6 : $\phi = \frac{3q}{\varepsilon_0}$

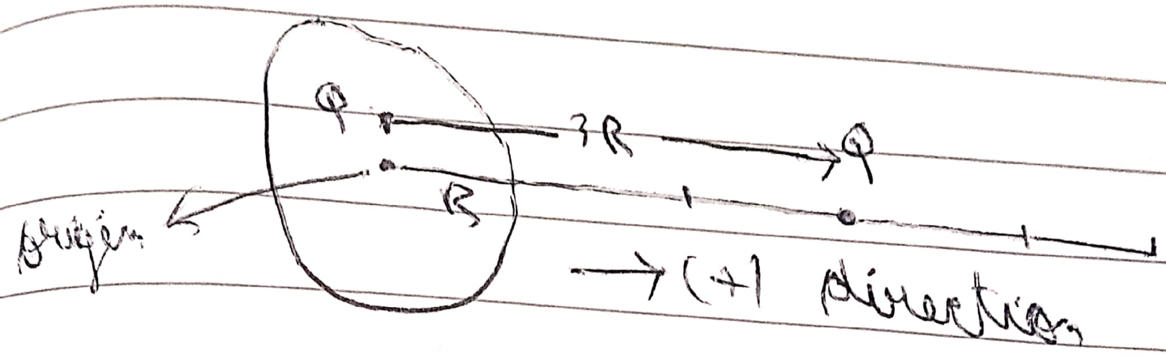
3.Ans: Electric field due to insulating sphere :-

(i) inside the sphere = 0

(ii) outside the sphere = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$

∆ Electric field due to a point charge :- $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$

So,



∴ electric field at $x = \frac{R}{3}$,

(a) So, $\vec{E}_{total} = \vec{E}_{\text{due to sphere at } x = \frac{R}{3}}$

$+ \vec{E}_{\text{due to outside charge at } x}$

$= \left(R - \frac{R}{3}\right) + 2R$

$= \frac{8R}{3}$

[∵ Electric field inside the insulating sphere $= 0$]

$$\therefore \vec{E}_{total} = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\left(\frac{8R}{3}\right)^2} \cdot (-\hat{i})$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{9Q}{64R^2}$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{9Q}{6R^2}$$

directed towards sphere.

(b) Now, at $x = 5R$:-

$\vec{E}'_{total} = \vec{E}_{\text{due to sphere at } x = 5R}$

$+ \vec{E}_{\text{due to charge } Q \text{ at } x = 2R}$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{(5R)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{(2R)^2}$$

$$\vec{E}_{total} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2}\left[\frac{1}{25} + \frac{1}{4}\right]$$

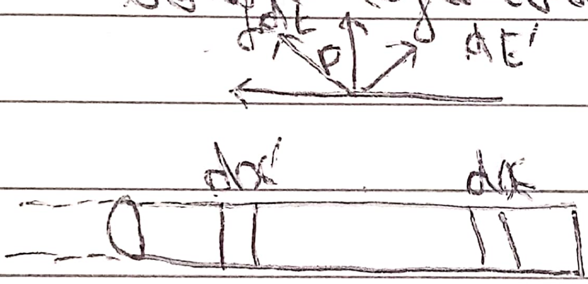
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{29Q}{100R^2}$$

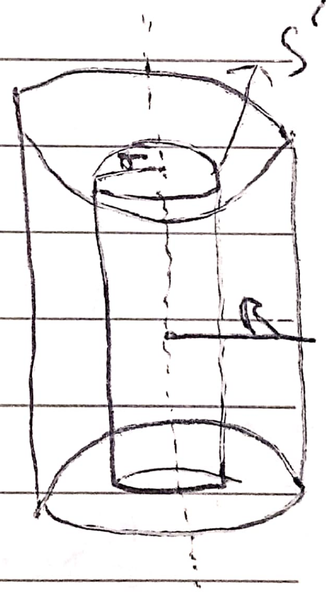
towards (+)ve direction

4. Ans (Will come back to it.)

5. Ans

Considering an infinitely long cylindrical volume.





Considering a point P, where we have to find the direction of electric field.

Considering an element dx. Electric field due to dx is shown as dE.

Considering a symmetrically opposite element dx' and its electric field is shown as dE'.

Electric field by the cylinder is radial because the components along the cylinder cancel out and the radial components add up.

(a) Finding electric field inside the cylinder for $r < R$:-

As $\rho = \alpha r$

Making a gaussian surface S' (check previous page).

Then charge inside S' is:-

$$Q = \iiint \rho \, dV = \boxed{\int_0^{L} \int_0^{2\pi} \int_0^{r} \rho \, dr (r \, d\phi) \, dz}$$

$$Q = \alpha \int_0^{r} r^2 \, dr (2\pi L)$$

$Q = 2\pi$

$= 2$

Using

$\oint E$

$E \int$

$\Rightarrow E$

$$\Phi = 2\pi\alpha L \left| \frac{r^3}{3} \right|_0^r$$

$$= \frac{2\pi\alpha r^3 L}{3}$$

Using Gauss's law :-

$$\oint \vec{E} \cdot d\vec{a} = \frac{q}{\epsilon_0}$$

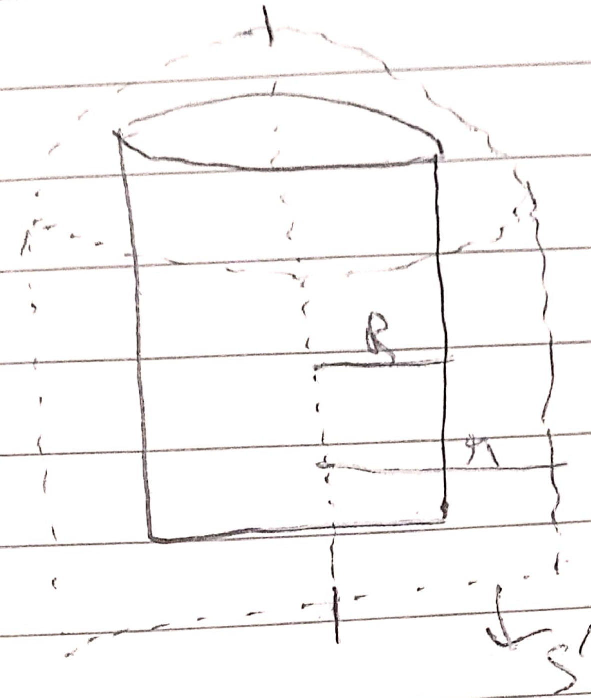
$$E \int da = \frac{q}{\epsilon_0}$$

$$\Rightarrow E(2\pi r L) = \frac{2\pi\alpha r^3 L}{3\epsilon_0}$$

$$E = \frac{\alpha r^2}{3\epsilon_0} \quad , \text{ for } r \leq R$$

d(ϕ) $d\overset{r}{g}$

(b) For $r \geq R$:-



Finding charge enclosed (Q) :-

$$Q = \int_{v} \rho \, dv$$

$$= \int_{0}^{L} \int_{0}^{2\pi} \int_{0}^{r} \alpha r (r \, dr \, d\phi \, dz)$$

$$Q = 2\pi\alpha L \int_{0}^{R} r^2 \, dr$$

$$= \frac{2\pi\alpha L R^3}{3}$$

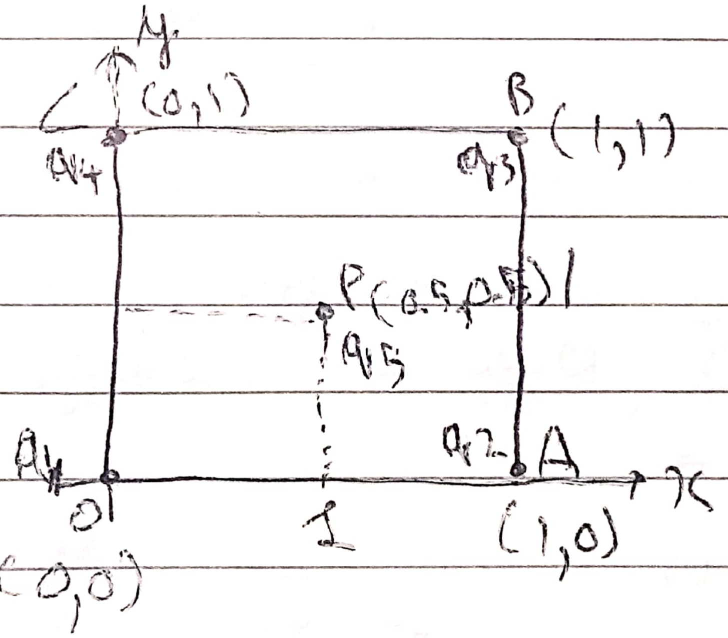
Using Gauss's law on our constructed Gaussian surface, we get:-

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E(2\pi r L) = \frac{2\pi \alpha L R^3}{3\epsilon_0}$$

$$E = \frac{\alpha R^3}{3\epsilon_0 r}, \text{ for } r > R$$

4. Ans.



Given :-

$q_1 = -4\,\mu C$

$q_2 = 5\,\mu C$

$q_3 = 4\,\mu C$

$q_4 = 5\,\mu C$

$q_5 = 2\,\mu C$

Side of the square = 1 m

The co-ordinates of point P are :- (0.5, 0.5)

When there are two charges, then electric force on charge (1) due to charge (2) is :-

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \quad -(1)$$

where, $\vec{r}_1$ = Position vector of charge q_1

$\vec{r}_2$ = Position vector of charge q_2

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/C^2$$

So :-

$$\left.\begin{aligned} \vec{r}_1 &= 0 \\ \vec{r}_2 &= \hat{i} \\ \vec{r}_3 &= \hat{i} + \hat{j} \\ \vec{r}_4 &= \hat{j} \\ \vec{r}_5 &= 0.5\hat{i} + 0.5\hat{j} \end{aligned}\right\} \text{from our figure}$$

(i) Force on q_5 due to q_1 :-

) - (1)

$$\vec{F}_{51} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_5 q_1 (\vec{r}_5 - \vec{r}_1)}{|r_5 - r_1|^3}$$

$$\vec{r}_5 - \vec{r}_1 = 0.5\hat{i} + 0.5\hat{j}$$

$$\left[|\vec{r}_5 - \vec{r}_1| = \sqrt{0.5^2 + 0.5^2} \right.$$

$$\left. = 0.71\text{ m} \right]$$

$$\Rightarrow \vec{F}_{51} = \frac{9\times10^9 \times 2\times10^{-6} \times (-4\times10^{-6}) \cdot (0.5\hat{i} + 0.5\hat{j})}{(0.71)^3}$$

$$\vec{F}_{51} = -0.203646\,(0.5\hat{i} + 0.5\hat{j})$$

$$\vec{F}_{51} = -0.1\hat{i} - 0.1\hat{j} \quad \text{— (2)}$$

(ii) Force on q_5 due to q_2 :-

$$\vec{F}_{52} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_5 q_2 (\vec{r}_5 - \vec{r}_2)}{|\vec{r}_5 - \vec{r}_2|^3}$$

$$\left[\vec{r}_5 - \vec{r}_2 = (0.5\hat{i} + 0.5\hat{j}) - \hat{i} \right.$$

$$\left. = -0.5\hat{i} + 0.5\hat{j} \right]$$

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$$\left[\,|\vec{r_5} - \vec{r_2}| = \sqrt{(-0.5)^2 + (0.5)^2}\right.$$

$$\left. = 0.71\,m \right]$$

$$\Rightarrow \vec{F}_{52} = \frac{9\times10^{9} \times 2\times10^{-6} \times 5\times10^{-6} \times (-0.5\hat{i} + 0.5\hat{j})}{(0.71)^2}$$

$$= 0.25455\,(-0.5\hat{i} + 0.5\hat{j})$$

$$\vec{F}_{52} = -0.13\hat{i} + 0.13\hat{j} \quad (5)$$

(iii) Force on q_5 due to q_3 :-

$$\vec{F}_{53} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_5 q_3 (\vec{r_5} - \vec{r_3})}{|\vec{r_5} - \vec{r_3}|^3}$$

$$\left[\,\vec{r_5} - \vec{r_3} = (0.5\hat{i} + 0.5\hat{j}) - (\hat{i} + \hat{j})\right.$$

$$\left. = -0.5\hat{i} - 0.5\hat{j}\,\right]$$

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$$\left[|\vec{r}_5 - \vec{r}_3| = \sqrt{(-0.5)^2 + (-0.5)^2} \right.$$

$$= 0.71 \}$$

$$\Rightarrow \vec{F}_{53} = \frac{9\times10^9 \times 2\times10^{-6} \times 4\times10^{-6} \times (-0.5\hat{i} - 0.5\hat{j})}{(0.71)^3}$$

$$= 0.203646(-0.5\hat{i} - 0.5\hat{j})$$

$$\vec{F}_{53} = -0.1\hat{i} - 0.1\hat{j} \quad - 4$$

(iii) Force on q_5 due to q_4 :-

$$\vec{F}_{54} = \frac{1}{4\pi\epsilon_0} \frac{q_5 q_4 (\vec{r}_5 - \vec{r}_4)}{|\vec{r}_5 - \vec{r}_4|^3}$$

$$\vec{r}_5 - \vec{r}_4 = (0.5\hat{i} + 0.5\hat{j}) - \hat{i}$$

$$\left[\ \vec{r}_5 - \vec{r}_4 = 0.5\hat{i} - 0.5\hat{j}\right.$$

$$|\vec{r}_5 - \vec{r}_4| = \sqrt{0.5^2 + (-0.5)^2}$$

$$\left. = 0.71 \ \right\}$$

$$\Rightarrow \vec{F}_{54} = \frac{9\times10^{9} \times 2\times10^{-6} \times 5\times10^{-6} \times (0.5\hat{i} - 0.5\hat{j})}{(0.71)^3}$$

$$= 0.25455\,(0.5\hat{i} - 0.5\hat{j})$$

$$\vec{F}_{54} = +0.13\hat{i} - 0.13\hat{j} \quad -(5)$$