02/19/2022 PH - 222 - 2A

①

RECITATION #6

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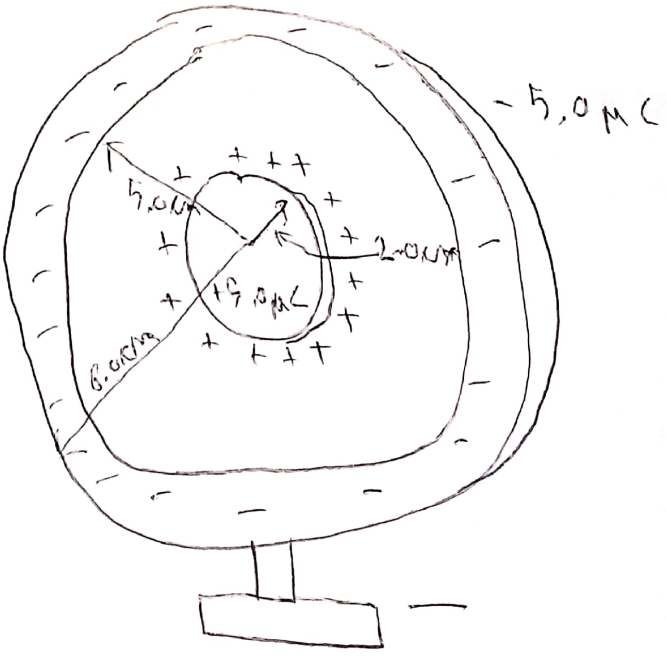
Ans:- Electric field function of the given system is :-

$$\vec{E} = \begin{cases} \frac{kQ}{r^2}\hat{r}, & 2.0\,cm < r < 5.0\,cm \\ 0, & elsewhere \end{cases}$$

The potential difference between two points a and b is :-

$$V_B - V_A = -\int_a^b \vec{E} \cdot d\vec{r} \quad — ①$$



(a) The electric field outside the system is zero. Substituting $E = 0$ in equation ①, the potential difference is zero.

$$V_B - V_A = 0V$$

If A is chosen to be at infinity,

$$V_A = 0V.$$

∴ The electric potential at the surface of the spherical shell is 0V.

(b) For the electric potential at the surface of the inner sphere:-

$$V_B - V_A = -\int_{a=5,0cm}^{b=2,0cm} \vec{E}\cdot d\vec{s}$$

$$V_B - V_A = \int_{5,0cm}^{2,0cm} \left(\frac{kQ}{r^2}\hat{r}\right)\cdot(\hat{r}\,dr)$$

$$V_B - V_A = kQ\left[\frac{1}{r}\right]_{5,0cm}^{2,0cm}$$

$$V_B - V_A = kQ\left[\frac{1}{(2,0cm)\left(\frac{1m}{100cm}\right)} - \frac{1}{(5,0cm)\left(\frac{1m}{100cm}\right)}\right]$$

$$V_B - V_A = 30kQ \rightarrow (2)$$

Here, V_B and V_A are the potential at the sphere and spherical shell respectively.

But, $V_A = 0$.

Substituting $Q = 5\mu C$ and $k = 9\times10^9\ Nm^2C^{-2}$ in equation (2).

$$V_a = 30\times9\times10^9\ Nm^2C^{-2}\times5\mu C\times\frac{1C}{10^6\mu C}$$

$$V_a = 13.5\times10^5\ V$$

$\therefore$ The potential at the inner sphere is

$$V_B = 13.5\times10^5\ V.$$

(c) The electric field at a point at a distance r from the inner surface of the electric field is:-

$$\frac{kQ}{r^2}\hat{r}.$$

The electric potential at that point is:-

$$V = -\int_{\infty}^{r}\vec{E}\cdot d\vec{r}$$

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2} \hat{r} \cdot \hat{r}\, dr$$

$$V = -\int_{\infty}^{5.0\,cm} \vec{E} \cdot d\vec{r} - \int_{5.0\,cm}^{r} \frac{kQ}{r^2} dr$$

Since, the electric field in the limit $\infty \to 5.0\,cm$ is zero;

$$\therefore V = -\int_{5.0\,cm}^{r} \frac{kQ}{r^2} dr$$

$$V = kQ \left[\frac{1}{r} \right]_{5.0\,cm}^{r}$$

$$V = kQ \left[\frac{1}{r} - \frac{1}{5.0\,cm} \right]$$

Substituting, $Q = 5\,\mu C$ & $k = 9 \times 10^9\, Nm^2C^{-2}$, the electric potential inside the space between the spherical shell and the sphere is :-

$$V = (9 \times 10^9\, Nm^2 C^{-2})(5\,\mu C)\left(\frac{1}{r} - \frac{1}{5.0\,cm}\right)$$

(d) The inner sphere is metallic and all the charge resides on the outer surface of the sphere.

$\therefore$ There is no electric field inside the sphere.

The electric potential inside the sphere is :-

$$V = -\int_{\infty}^{x} \vec{E} \cdot d\vec{r}$$

$$\therefore V = -\int_{\infty}^{5.0\,cm} \vec{E} \cdot d\vec{r} - \int_{5.0\,cm}^{2.0\,cm} \vec{E} \cdot d\vec{r} - \int_{2.0\,cm}^{x} \vec{E} \cdot d\vec{r}$$

The only non-zero term in the above equation is :-

$$V = -\int_{5.0\,cm}^{2.0\,cm} \vec{E} \cdot d\vec{r}$$

Solving this as we solved in part (b),

$$V = 13.5 \times 10^{5}\,V.$$

The potential inside the sphere is constant and equal to the potential at the surface of the sphere.

(c) Let P be a point at a distance x from the surface of a spherical shell.

Then the potential at P is :-

$$V = -\int_{\infty}^{x} \vec{E} \cdot d\vec{r}$$

Since the electric field outside the spherical shell is zero.

$\therefore$ The potential at a point outside the spherical shell is $V = 0\,V$.