STUDY GUIDE
March 23, 2023

There are eight questions, for a total of 100 points.

There are formula hints for different equations at the end of the test papers.

For decimal fractions, show at least three digits, rounding correctly.

For full credit you should set up the calculation on the test as part of the answer.

Question 1 10 points

Jerry goes to work each day by two different routes. As he leaves the apartment there is a traffic light. If he has a green light, he takes route A and travels for 10-20 minutes, uniformly distributed. If he has a red light, he takes route B and travels 10-15 minutes, uniformly distributed.

He has timed the traffic light and the light is green for 2 minutes, and a red light for 3 minutes. He arrives at the traffic light at uniformly random times each day. (The traffic light is green with probability 2/5 and the traffic light is red with probability 3/5.)

What is the PDF of his travel time and what is his expected travel time from the traffic light to work?

Question 2 10 points

Let X, Y be uniform random variables, both in [0,2].

What is $P(X + Y \leq 1)$? and $P(Y - X < 1)$?

Question 3 20 points

Let X and Y be two random variables, with joint distribution as in the table:

		Y	
		1	2
X	1	.1	.3
	2	.1	.5

(a) What is the PMF for X and the PMF for Y?

(b) What is $E[X]$, $E[Y]$ and $E[XY]$

(c) What is $Var(X)$ and $Var(Y)$?

(d) Are X and Y independent?

Question 4 10 points

Let X and Y be random variables with the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} cx^2y^5, & \text{if } 0 \leq x \leq 3, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

What is the value of c? [5 points]

Answer: ....

Are X and Y independent? Give a reason.[5 points]

Answer: ....

Question 5 20 points

Two continuous uniform random variables X and Y have a joint PDF given by

$$f_{X,Y}(x,y) = \begin{cases} c(2x+3y), & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of c?

(b) Find the marginal PDFs of X and Y, i.e., $f_X(x)$ and $f_Y(y)$

(c) What is $E[2X+3Y]$?

Question 6 10 points

In the town Imaginaryville yearly precipitation is modeled as a normal random variable with mean 200 inches and standard deviation 6 inches.

Answers involving Φ may be left as function calls without using any function lookup. So, $\Phi(.5)$ could be a correct answer. Note that the argument in $\Phi()$ should be non-negative.

(a) (3 points) What is the probability in a randomly chosen year that the precipitation R is below 212 inches?

Answer: ....

(b) (3 points) What is the probability in a randomly chosen year that the precipitation is above 208 inches?

Answer: ....

(c) (4 points) What is the probability in a randomly chosen year that the precipitation is between 195 inches and 202 inches?

Answer: ....

Question 7 10 points

All answers should be left in terms of Φ.

Let X and Y be independent Standard Normal Distribution random variables

(a) [2 points] What is the probability that $|X| < .55$?

(b) [2 points] What is the probability that $-.1 < Y < 0.6$?

(c) [3 points] What is μ_{3X-2Y}?

(d) [3 points] What is σ_{6X} and $Var(8Y)$?

Question 8 10 points

The time that a PC power supply burns out can be modeled by an exponential random variable with a mean of 4 years. What is the probability that your PC power supply will burn out between two years and three years after purchase?

END OF SAMPLE TEST

ADDITIONAL QUESTIONS

Question 9 Let X, Y be uniform random variables, both in [0,1], let $Z = X - Y$.

What is $P(\frac{1}{2} \leq Z \leq \frac{3}{2})$?

Question 10 Alvin throws darts at a circular target of radius r and is equally likely to hit any point on the circular target.

Let X be the distance of Alvin's hit from the center of the target.

(a) 6 points: Find the PDF of X.

Hint: First calculate the CDF

Answer: ....

(b) 3 points: Find the mean of X.

Answer: ....

(c) 1 point: Find the variance of X.

Answer: ....

Question 11 Ocean water temperature near Antarctica during its summer is modeled as a normal random variable with mean equal to 38 degrees Fahrenheit and standard deviation equal to 8 degrees Fahrenheit. What is the probability that the temperature at a randomly chosen time will be above 7 degrees Celsius?

Hint(1): The temperatures are not in the same units. Use $F(t) = \frac{5}{9}C + 32$ to convert.

Hint(2): Leave the answer as a call to the routine Φ

Question 12 I drive home on the interstate, which means every day I have to drive up the entrance ramp and merge in to traffic. Let X = amount of time (in seconds) before the next gap in traffic comes so I can merge onto the interstate. X is known to have an exponential distribution with the average amount of time equal to 30 seconds, i.e., the PDF of X is

$$f_X(x) = \begin{cases} \frac{1}{30}e^{-\frac{1}{30}x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

a. Find the probability that I will have to wait more than 20 and less than 40 seconds for the next gap in traffic.

   Hint: The answer should be expressed in terms of the exponential e.

b. What is the expected waiting time?

Question 13 Let X be uniformly distributed in the unit interval [0,1]. Let $Y = g(X)$ where

$$g(x) = \begin{cases} 2 & \text{if } x \leq \frac{1}{3}\} \\ 1 & \text{if } x > \frac{1}{3}\} \end{cases}$$

(a) 5 points: Derive the PMF of Y and find its expected value.

(b) 5 points: Find the expected value using the expected value rule,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dy$$

Question 14 Mickey is taking two classes. They both require several papers, but never in the same week. An English paper takes 30-50 minutes to write, uniformly distributed, and a Philosophy paper takes 40-70 minutes, also uniformly distributed.

The professors get together each week to decide which class requires a paper, throwing dice, and the probability of having an English paper is .6, and the probability of having a philosophy paper is .4.

What is the expected value of the time Mickey will spend on a paper each week?

Question 15 Let X be a random variable with PDF

$$f_X(x) = \begin{cases} 0 & x < 0 \\ cx^3 & 0 \leq x \leq 4 \\ 0 & x > 3 \end{cases}$$

(a) [5 points] What is c?

(b) [5 points] What is $E[X]$?

Hints

- Exponential distribution: $f(x) = \lambda e^{-\lambda x}, E[X] = \frac{1}{\lambda}$.
  CDF of exponential distribution: $F(x) = P(X \leq x) = 1 - e^{-\lambda x}$
- For X, Y normal random variables

$$\mu_{X+Y} = \mu_X + \mu_Y$$
$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$$
$$P(X < x) = \Phi\left(\frac{x - \mu_X}{\sigma_X}\right)$$

- $E(X) = \sum_{x \in X} xp(x)$ or $E(X) = \int_{x \in X} xp(x)\, dx$
- $\Phi(x) = 1 - \Phi(-x)$
- $var(X) = E[X^2] - (E[X])^2$