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CS - 355

HW - 6

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Problem 1 Ans:- Given that X and Y be normal random variables with means 0 and 1 respectively and variances 1 and 4 respectively.

i.e. $X \sim N(\mu = 0, \sigma^2 = 1)$ and $Y \sim N(\mu = 1, \sigma^2 = 4)$

now,

(a) $P(X \leq 1.5)$ can be found by $Z = \frac{X - \mu}{\sigma}$, where $\mu = 0$ & $\sigma = 1$.

$\therefore Z = \frac{X}{1}$

$\therefore$ Probability can be found by using the standard normal distribution table :-

$P(X \leq 1.5) = P(Z \leq \frac{1.5}{1}) = P(Z \leq 1.5) \approx \underline{0.9332}$

We use the same approach to find $P(X \leq -1)$:-

$P(X \leq -1) = P(Z \leq \frac{-1-0}{1}) = P(Z \leq -1)$

$\approx \underline{0.1587}$

(b) PDF of $\frac{(Y-1)}{2}$:-

Let $u = \frac{y-1}{2}$

$y = 2u+1$

$dy = 2\,du$

$\therefore J = \frac{dy}{du} = 2$

By method of transformation, the probability density function of $V = \frac{Y-1}{2}$ is obtained as follows:-

$$f_V(u) = f_Y(2u+1)\,|J|$$

$$= \frac{2}{\sigma_Y\sqrt{2\pi}}\, e^{\frac{-1}{2\sigma_Y^2}(2u+1-\mu_Y)^2}$$

$$= \frac{2}{2\sqrt{2\pi}}\, e^{\frac{-1}{2\times 4}(2u+1-1)^2}$$

$$= \frac{1}{\sqrt{2\pi}}\, e^{\frac{-4}{8}(u)^2}$$

$$f_V(u) = \frac{1}{\sqrt{2\pi}}\, e^{\frac{-1}{2}(u)^2}, \quad -\infty < u < \infty$$

$\therefore$ This implies that $U = \frac{Y-1}{2}$ follows standard normal distribution.

(c) $P(-1 \leq Y \leq 1) = P\left(\frac{-1-\mu_Y}{\sigma_Y} \leq \frac{Y-\mu_Y}{\sigma_Y} \leq \frac{1-\mu_Y}{\sigma_Y}\right)$

$= P\left(\frac{-1-1}{2} \leq Z \leq \frac{1-1}{2}\right)$

$= P(-1 < Z < 0)$

$= P(Z < 0) - P(Z < -1)$

$= 0.5 - 0.1587$

$= \underline{\underline{0.3413}}$

Problem 2 Ans: Formula to convert celsius to fahrenheit:-

$(F-32) \times \frac{5}{9} = C$

Here, $F = 59°F$

$\therefore C = (59-32) \times \frac{5}{9}$

$= \cancel{27}^{\,3} \times \frac{5}{\cancel{9}_{\,1}}$

$= \underline{\underline{15°C}}$

$\mu = 10°C$

$\sigma = 10°C$

$P(x \leq 15°)$

$\therefore$ Z score $= \frac{x-\mu}{\sigma} = \frac{15-10}{10} = \underline{\underline{0.5}}$

$\therefore P(X \leq 15) = P(Z \leq 0.5)$

$= 0.6915$

$\therefore$ Required Probability $= 0.6915$

Problem 3 Ans - The joint PDF of two continuous random variables X and Y is :-

$$f_{X,Y}(x,y) = \begin{cases} c(x+y) & , \ 0 \leq x \leq 2 \\ 0 & , \ \text{otherwise} \end{cases}$$

(a) To find c.

We know that,

$$\int_x \int_y f_{x,y}(x,y)\, dy\, dx = 1$$

$$\Rightarrow \int_0^2 \int_0^2 c(x+y)\, dy\, dx = 1$$

$$\Rightarrow c \int_0^2 \int_0^2 (x+y)\, dy\, dx = 1$$

$$\Rightarrow c \int_0^2 \left[xy + \frac{y^2}{2} \right]_0^2 dx = 1$$

$$\Rightarrow R\int_0^2\left[\left(2x+\frac{2^2}{2}\right)-0\right]dx=1$$

$$\Rightarrow R\int_0^2 (2x+2)\,dx=1$$

$$\Rightarrow 2c\int_0^2 (x+1)\,dx=1$$

$$\Rightarrow 2c\left[\frac{x^2}{2}+x\right]_0^2=1$$

$$\Rightarrow 2c\left[\left(\frac{2^2}{2}+2\right)-0\right]=1$$

$$\Rightarrow 2c\,[2+2]=1$$

$$\Rightarrow 2c\times 4=1$$

$$\Rightarrow 8c=1$$

$$c=\frac{1}{8}$$

(b) The marginal distribution of variable x is :-

$$f_X(x)=\int_y f_{x,y}(x,y)\,dy$$

$$f_X(x)=\int_0^2 \frac{1}{8}(x+y)\,dy$$

$$f_X(x) = \frac{1}{8}\left[xy + \frac{y^2}{2}\right]_0^2$$

$$f_X(x) = \frac{1}{8}\left[\left(x \times 2 + \frac{2^2}{2}\right) - 0\right]$$

$$f_X(x) = \frac{1}{8}(2x+2)$$

$$f_X(x) = \frac{1}{8} \times 2(x+1)$$

$$f_X(x) = \frac{1}{4}(x+1)$$

The marginal distribution of variable y is :-

$$f_Y(y) = \int_x f_{x,y}(x,y)\,dx$$

$$f_Y(y) = \int_0^2 \frac{1}{8}(x+y)\,dx$$

$$f_Y(y) = \frac{1}{8}\left[\frac{x^2}{2} + yx\right]_0^2$$

$$f_Y(y) = \frac{1}{8}\left[\left(\frac{2^2}{2} + 2y\right) - 0\right]$$

$$f_Y(y) = \frac{1}{8}(2 + 2y)$$

$$f_Y(y) = \frac{1}{4}(1 + y)$$

(c) The expected value of variable X is:-

$$E(X) = \int_x x f_x(x)\,dx$$

$$E(X) = \int_0^2 x \times \frac{1}{4}(x+1)\,dx$$

$$E(X) = \frac{1}{4}\int_0^2 (x^2+x)\,dx$$

$$E(x) = \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2$$

$$E(x) = \frac{1}{4}\left[\left(\frac{2^3}{3} + \frac{2^2}{2}\right) - 0\right]$$

$$E(x) = \frac{1}{4}\left(\frac{8}{3} + 2\right)$$

$$E(x) = \frac{1}{4}\left(\frac{8+6}{3}\right)$$

$$E(x) = \frac{1}{4} \times \frac{14}{3}$$

$$E(x) = \frac{7}{6}$$

(10)

The expected value of variable Y is :-

$$E(Y) = \int_y y \, f_y(y) \, dy$$

$$E(Y) = \int_0^2 y \times \frac{1}{4}(1+y) \, dx$$

$$E(Y) = \frac{1}{4}\left[\frac{y^2}{2} + \frac{y^3}{3}\right]_0^2$$

$$E(Y) = \frac{1}{4}\left[\frac{2^2}{2} + \frac{2^3}{3}\right]$$

$$E(Y) = \frac{1}{4}\left[2 + \frac{8}{3}\right]$$

$$E(Y) = \frac{1}{4} \times \frac{14}{3}$$

$$E(Y) = \frac{7}{6}$$

$$E(2X+3Y) = E(2X) + E(3Y)$$

$$E(2X+3Y) = 2E(X) + 3E(Y)$$

$$E(2X+3Y) = 2 \times \frac{7}{6} + 3 \times \frac{7}{6}$$

$$E(2X+3Y) = \frac{14+21}{6}$$

$$E(2X+3Y) = \frac{35}{6}$$