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(1)

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CS 355
SPRING 2023
HW 1

1. Ans: The power set is:-

$P = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$

Power set has $2^4 = 16$ elements as shown above.

2. Ans: Let X be an element of 2^A.
$\therefore$ X is a subset of A.

We are given that A is a subset of B.
$\therefore$ X is also a subset of B.

$\therefore$ X also belongs to 2^B.

Since every element $X \in 2^A$ is also an element of 2^B, we can say that $2^A \subset 2^B$.

3. Ans: (i) $\Omega \setminus (A \cup B) = \{1,2,3,4,5,6,7,8,9,10,11,12\} \setminus \{\{2,4,6,8,10,12\} \cup \{1,3,6,9,12\}\}$

$\Rightarrow \Omega \setminus (A \cup B) = \{1,2,3,4,5,6,7,8,9,10,11,12\} \setminus \{2,4,6,8,10,12,1,3,9\}$

$\Rightarrow \Omega \setminus A \cup B = \{5,7,11\}$

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(ii) $A \cap B = \{2,4,6,8,10,12\} \cap \{1,3,6,9,12\}$

$A \cap B = \{6, 12\}$

(iii) $A \cup B = \{2,4,6,8,10,12\} \cup \{1,3,6,9,12\}$
$A \cup B = \{1,2,3,4,6,8,9,10,12\}$

4. Ans:- Undergrad :- $A^c \cap B^c = (A \cup B)^c$

Let $P = (A \cup B)^c$ & $Q = A^c \cap B^c$

Let x be an arbitrary element of P, then we have $x \in P \Rightarrow x \in (A \cup B)^c$
$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in A^c$ and $x \in B^c$
$\Rightarrow x \in A^c \cap B^c$
$\Rightarrow x \in Q$

$\therefore P \subset Q$ —— (i)

Again, let y be an arbitrary element of Q, then we have :-

$y \in Q \Rightarrow y \in A^c \cap B^c$

$\Rightarrow y \in A^c$ and $y \in B^c$

$\Rightarrow y \notin A$ and $y \notin B$
$\Rightarrow y \notin (A \cup B)$
$\Rightarrow y \in (A \cup B)^c$
$\Rightarrow y \in P$
$\therefore Q \subset P$ — (ii)

Combining (i) and (ii), we get :-

$P = Q$, i.e. $(A \cup B)^c = A^c \cap B^c$

Hence :- $A^c = (A^c \cap B) \cup (A^c \cap B^c)$

$(A^c \cap B) \cup (A^c \cap B^c) = A^c \cap (B \cup B^c)$ (since from associative law)

$= A^c \cap U$ (since from complement law)

$= A^c$ (from identity law)

$\therefore A^c = (A^c \cap B) \cup (A^c \cap B^c)$