

02/13/2022

CS-355

HW #4

NAME :- SHREYAS

SRINIVASA

BLAZER ID :- LSSRINIVA

1. Ans:-

Those rolls with a sum of 11 are:-

1, 5, 5 [with 3 permutations]

1, 4, 6 [with 6 permutations]

2, 3, 6 [with 6 permutations]

2, 4, 5 [with 6 permutations]

3, 3, 5 [with 3 permutations]

3, 4, 4 [with 3 permutations]

Thus, there are $\frac{(3+6+6+6+3+3)}{27}$ ways to produce a sum of 11.

Those rolls with a sum of 12 are:-

1, 5, 6 [with 6 permutations]

2, 5, 5 [with 3 permutations]

2, 4, 6 [with 6 permutations]
 3, 4, 5 [with 6 permutations]
 3, 3, 6 [with 3 permutations]
 4, 4, 4 [with 1 permutation]

\therefore There are $(6+3+6+6+3+1) = 25$ ways to produce a sum of 12.

since $27 > 25$, a sum of 11 is more probable.

2. Ans: import math

def f(x):

return math.sin(x) * math.cos(x)

def num-integral(f, a, b, num):

h = (b-a)/num

x = [a + i * h for i in range(num+1)]

y = [f(x[i]) for i in range(num+1)]

return h * (sum(y) - 0.5 * (y[0] + y[num]))

a = 0

b = 4 * math.pi

num = 1000

area = num-integral(f, a, b, num)

if area < 0:

area = -area

print("Approximate area:", area)

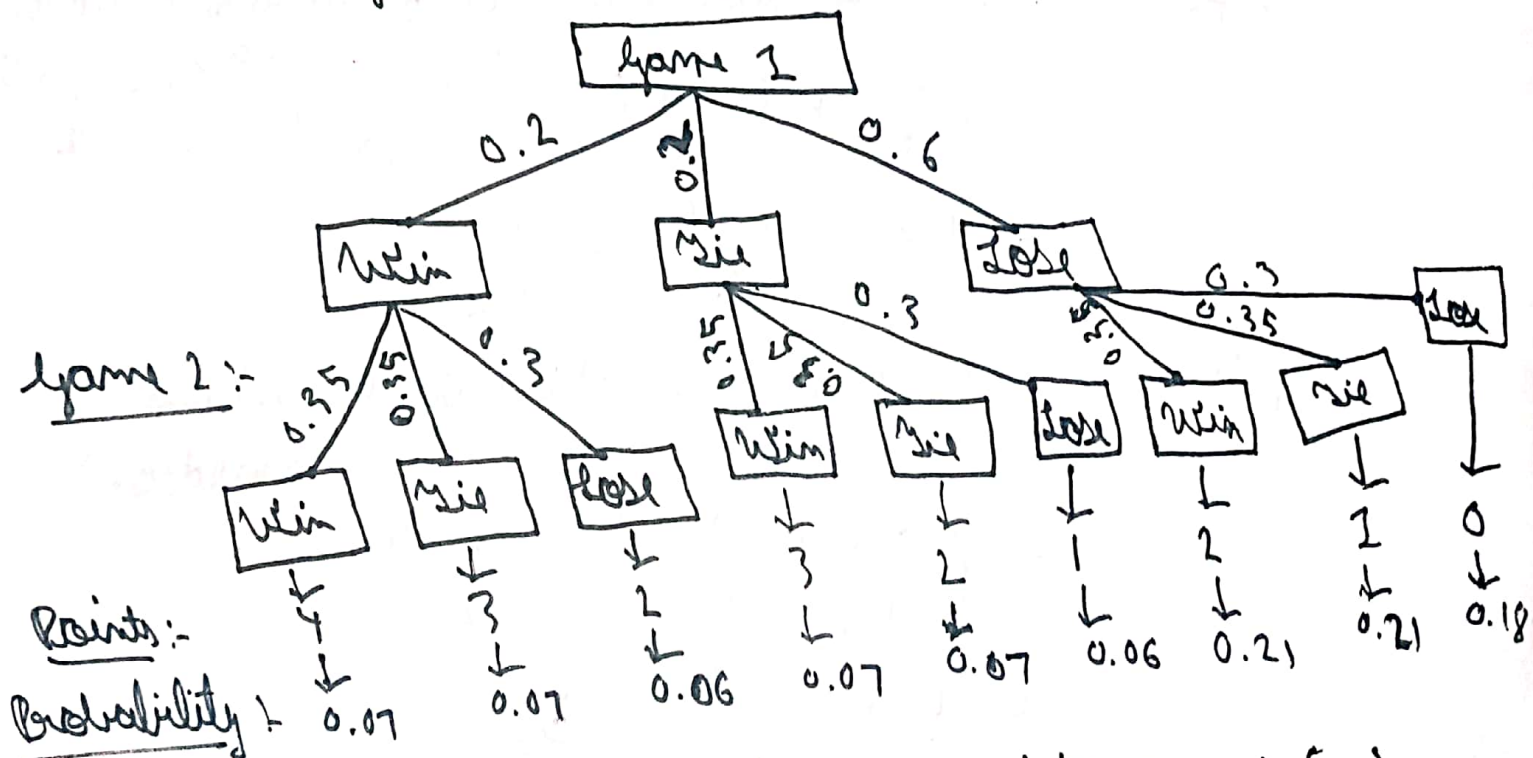
3. Ans:- Probability of not ~~losing~~ losing the first game = 0.4
 Probability of not losing the second game = 0.7
 So, probability of losing the first game and second game is $(1-0.4) = 0.6$ and $(1-0.7) = 0.3$ respectively.

As the team is equally likely to win or tie in any game, so:-

$$P(\text{win}) = P(\text{tie}) = 0.2 \text{ for first game}$$

$$P(\text{win}) = P(\text{tie}) = 0.35 \text{ for second game}$$

\therefore We can create the following probability tree diagram:-



So, PMF of the number of points earned 'x' is:-

$$P(X=x) = \begin{cases} 0.18 & \text{for } x=0 \\ 0.21 & \text{for } x=1 \\ 0.34 & \text{for } x=2 \\ 0.14 & \text{for } x=3 \\ 0.07 & \text{for } x=4 \end{cases}$$

- Supporting, sharing and database Activities
- Worked on Incident Management and Change Management through service now.
- Testing the environment and identifying bugs during software upgrades

4. Ans: Total number of children in the family = 7
 Let X be a random variable denoting the number of girl children in the family.

Then the sample space of the random variable X is given by $R_X = \{2, 3, 4, 5, 6, 7\}$

Let p be the probability that each natural child is a girl. i.e. $p = \frac{1}{2}$

$$\begin{aligned} P(X=2) &= P(\text{no girl child in 5 natural children}) \\ &= \binom{5}{0} p^0 (1-p)^5 = \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(1 - \frac{1}{2}\right)^5 \\ &= 1 \times 1 \times \left(\frac{1}{2}\right)^5 = \frac{1}{32} \end{aligned}$$

$$\begin{aligned} P(X=3) &= P(\text{one girl child in 5 natural children}) \\ &= \binom{5}{1} \left(\frac{1}{2}\right)^1 \left(1 - \frac{1}{2}\right)^4 \\ &= 5 \times \left(\frac{1}{2}\right)^5 = \frac{5}{32} \end{aligned}$$

$$\begin{aligned} P(X=4) &= P(\text{two girl children in 5 natural children}) \\ &= \binom{5}{2} \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^3 \\ &= 10 \times \left(\frac{1}{2}\right)^5 = \frac{10}{32} \end{aligned}$$

$$\begin{aligned}
 P(X=5) &= P(\text{three girl children in 5 natural children}) \\
 &= \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(1 - \frac{1}{2}\right)^2 \\
 &= 10 \times \left(\frac{1}{2}\right)^5 = \frac{10}{32}
 \end{aligned}$$

$$\begin{aligned}
 P(X=6) &= P(\text{four girl children in 5 natural children}) \\
 &= \binom{5}{4} \left(\frac{1}{2}\right)^4 \left(1 - \frac{1}{2}\right)^1 \\
 &= 5 \times \left(\frac{1}{2}\right)^5 = \frac{5}{32}
 \end{aligned}$$

$$\begin{aligned}
 P(X=7) &= P(5 \text{ girl children in 5 natural children}) \\
 &= \binom{5}{5} \left(\frac{1}{2}\right)^5 \left(1 - \frac{1}{2}\right)^0 \\
 &= 1 \times \frac{1}{32} = \frac{1}{32}
 \end{aligned}$$

Thus, the pmf of the random variable X is given by

X	2	3	4	5	6	7
$P(X)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

5. Ans: Let Probability of X , $P(X=k) = \frac{1}{10}$

$$= 0.1, k=0, 1, 2, \dots, 9$$

X	0	1	2	3	4	5	6	7	8	9
$P(X)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

(a) Let $Y = X \bmod(3)$

Hence the values of Y for all values of X is:-

X	0	1	2	3	4	5	6	7	8	9
Y	0	1	2	0	1	2	0	1	2	0

\therefore Possible values of Y are 0, 1 & 2.

Finding the probability of Y for all its values:-

Let $Y = 0$,

$$\begin{aligned} P(Y=0) &= P(X=0) + P(X=3) + P(X=6) + P(X=9) \\ &= 0.1 + 0.1 + 0.1 + 0.1 \\ &= 0.4 \end{aligned}$$

Let $Y = 1$,

$$\begin{aligned} P(Y=1) &= P(X=1) + P(X=4) + P(X=7) \\ &= 0.1 + 0.1 + 0.1 \\ &= 0.3 \end{aligned}$$

Let $Y = 2$,

$$\begin{aligned} P(Y=2) &= P(X=2) + P(X=5) + P(X=8) \\ &= 0.1 + 0.1 + 0.1 \\ &= 0.3 \end{aligned}$$

Hence, Probability of mass function of Y is:-

Y	0	1	2
$P(Y)$	0.4	0.3	0.3

(b) Let $Y = 5 \bmod (X+1)$

Hence the values of Y for all values of X :-

X	0	1	2	3	4	5	6	7	8	9
$X+1$	1	2	3	4	5	6	7	8	9	10
Y	0	1	2	1	0	5	5	5	5	5

\therefore Possible values of Y are 0, 1, 2 and 5.

~~Guiding the~~ probability of Y for all its values :-

Let $Y=0$,

$$\begin{aligned}P(Y=0) &= P(X=0) + P(X=4) \\&= 0.1 + 0.1 \\&= 0.2\end{aligned}$$

Let $Y=1$,

$$\begin{aligned}P(Y=1) &= P(X=2) + P(X=3) \\&= 0.1 + 0.1 \\&= 0.2\end{aligned}$$

Let $Y=2$,

$$\begin{aligned}P(Y=2) &= P(X=2) \\&= 0.1\end{aligned}$$

Let $Y=5$,

$$\begin{aligned}P(Y=5) &= P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) \\&= 0.1 + 0.1 + 0.1 + 0.1 + 0.1 \\&= 0.5\end{aligned}$$

Hence, Probability mass function of Y :-

Y	0	1	2	5
$P(Y)$	0.2	0.2	0.1	0.5