CS 355-2C

HW-7

NAME :- SHREYAS SRINIVASA
BLAZER ID :- SSRINIVA

1. Ans :-

```
import random
import matplotlib.pyplot as plt

def simulate():
    result = []
    for i in range(1000):
        X = random.uniform(7,30)
        result.append(X)
    plt.hist(result, bins=12)
    plt.show()

simulate()
```

Solution

given :-
Un
Let
i

PDF

At 6 AM
and co
hour

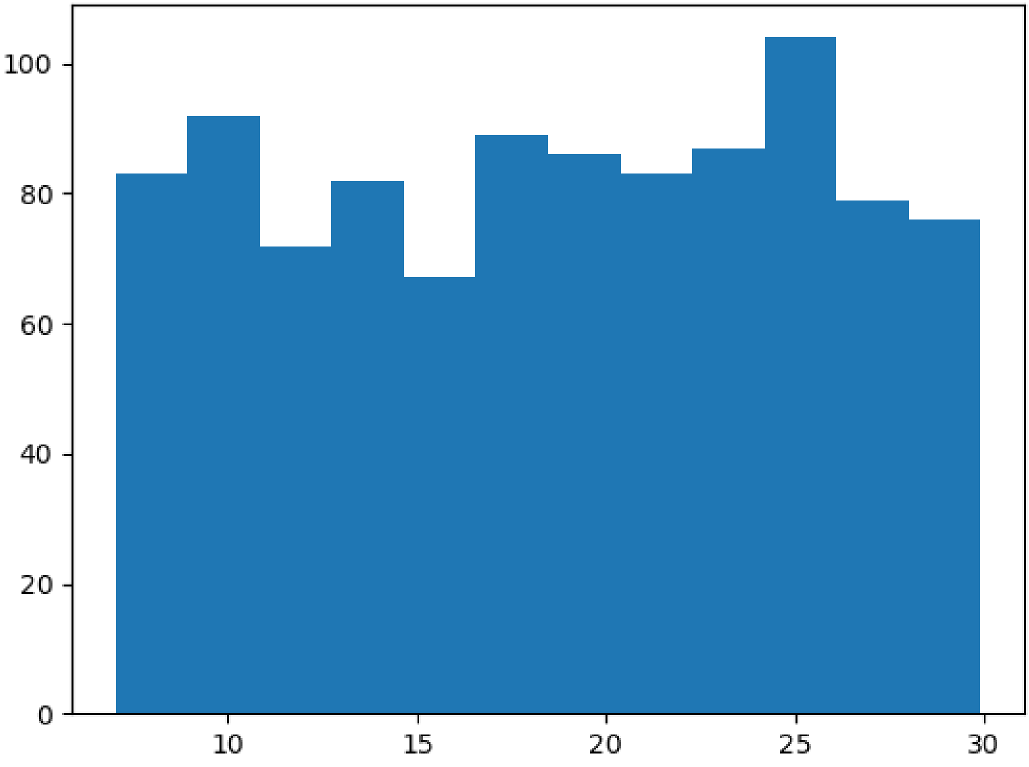
HW7Q1.py

C: > Users > zoope > OneDrive > Desktop > STUDENT > UAB > SPRING 2023 > CS 355 > HW7Q1.py > ...

```python
import random
import matplotlib.pyplot as plt

def simulate():
    result = []
    for i in range(1000):
        X = random.uniform(7, 30)
        result.append(X)
    plt.hist(result, bins=12)
    plt.show()

simulate()
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE Python Debug Console

```
(base) C:\Users\zoope\OneDrive\Desktop\STUDENT\UAB\SPRING 2023\CS 355> c: && cd "c:\Users\zoope\OneDrive\Desktop\STUDENT\UAB\SPRING 2023\CS 355" && cmd /C "C:\Users\zoope\anaconda3\python.exe c:\Users\zoope\.vscode\extensions\ms-python.python-2023.4.1\pythonFiles\lib\python\debugpy\adapter/../..\debugpy\launcher 55006 -- "C:\Users\zoope\OneDrive\Desktop\STUDENT\UAB\SPRING 2023\CS 355\HW7Q1.py" "

(base) C:\Users\zoope\OneDrive\Desktop\STUDENT\UAB\SPRING 2023\CS 355>
```



100
80
60
40
20
0
10
15
20
25
30

Solution to problem :-

Given :- X = time of arrival
Uniform distribution is
between 7:10 & 7:30.
i.e. 20 minutes.

PDF of X is :-

pdf

$$f_X(x) = \begin{cases} \frac{1}{20}, & 7:10 \le x \le 7:30 \\ 0, & \text{otherwise} \end{cases}$$

(7,30)

Y = waiting time

At 6 AM, the train starts and comes every quarter hour.

②

(3)

when a person the station at 7:10 AM, they can take the train at 7:15 AM or 7:30 AM which means that they would have to wait for 5 or 15 minutes.

Let A and B be the events:

$A = \{7{:}10\text{ AM} \le X \le 7{:}15\text{ AM}\}$

$B = \{7{:}15\text{ AM} \le X \le 7{:}30\text{ AM}\}$

Y given A is :-

$$f_{Y|A}(y) = \begin{cases} \frac{1}{5} \; ; & 0 < y < 5 \\ 0 \; ; & \text{otherwise} \end{cases}$$

Y given B is:

$$f_{Y|B}(y) = \begin{cases} \frac{1}{15} ; 0 < y < 15 \\ 0 ; \text{otherwise} \end{cases}$$

The train comes at every quarter hour.

$\therefore P(A) = \frac{1}{4}$ & $P(B) = \frac{1}{4}$

$$\therefore f_Y(y) = P(A) f_{Y/A}(y) + P(B) f_{Y|B}(y)$$

$$\therefore f_Y(y) = \frac{1}{4} \cdot \frac{1}{5} + \frac{3}{4} \cdot \frac{1}{15} ;$$

$$= \frac{1}{10} ; \quad 0 \le y \le 5$$

If a person takes the train at 7:30, then $f_{Y|A}(y) = 0$,

which means that if he boards the train at 7:15, then the probability of waiting time is zero.

$$\therefore f_Y(y) = \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot \frac{1}{15}$$

$$f_Y(y) = \frac{1}{20} \; ; \; 5 \leq y \leq 15$$

2. Ans:- (a) The expected value of X is given by:-

$E[X] = \int x f_X(x)\,dx$ from $\boxed{-\infty \to \infty}$

$$E[X] = \int_1^3 \left(\frac{x}{4}\right) dx$$

$$= \left[\frac{x^2}{8}\right] \text{ from } 1 \to 3$$

$$= 1.5$$

The probability of event A $\{x > 2\}$, can be calcul-
-ated as:-

$$P(A) = \int_2^3 \left(\frac{x}{4}\right) dx$$

$$= \left[\frac{x^2}{8}\right] \text{ from } 2 \to 3$$

$$= \frac{1}{8}$$

The conditional PDF of X given A is given by:-

$$\boxed{0 \to \infty}$$

$$f_{X|A}(x) = \frac{P(A \text{ and } X = x)}{P(A)}$$

$$= \frac{f_X(x)}{P(A)}$$

$$= \frac{\frac{x}{4}}{\frac{1}{8}}$$

$$= \frac{x}{2}, \text{ for } [redacted] \rightarrow 2 < x < 3$$

The expected value of X given A is given by :-

$$E[X|A] = \int x \, f_{X|A}(x) \, dx$$

$\boxed{\text{from } -\infty \rightarrow \infty}$

$$= \int_2^3 \left(\frac{x^2}{2}\right) dx$$

$$= \left[\frac{x^3}{6}\right] \text{ from } 2 \to 3$$

$$= \frac{19}{6}$$

(b) We can find the distribution of Y as follows:-

$$F_Y(y) = P(Y \leq y)$$

$$= P(X^2 \leq y)$$

$$= P(-\sqrt{y} \leq X \leq \sqrt{y})$$

Since X is only defined for $1 < X < 3$, the range of Y is $1 < Y < 9$.

$$\therefore F_Y(y) = \begin{cases} 0, & \text{for } y < 1 \\ \int \frac{(\sqrt{y})^3}{4} dx, & \text{for } 1 \le y \le 4 \\ \int_1^3 \frac{x}{4} dx + \int_3^{\sqrt{y}} \frac{x}{4} dx, & \text{for } 4 \le y < 9 \\ 1, & \text{for } y \ge 9 \end{cases}$$

We can differentiate $F_Y(y)$ to get the PDF of Y:-

$$f_Y(Y) = \frac{1}{8\sqrt{y}}, \text{ for } 1 \le y < \boxed{4}$$

(9)

$$\left(\frac{1}{4}\right) \int (\sqrt{y})^{\frac{3}{4}} \frac{x}{4}\, dx, \text{ for } 4 \leq y < 9,$$

$$0, \text{ otherwise}$$

We can find the expected value of Y :-

$$E[Y] = \int y\, f_Y(y)\, dy$$

from $-\infty \to \infty$

$$= \int_1^4 y \left(\frac{1}{8\sqrt{y}}\right) dy + \int_4^9 y \left[\frac{1}{4} \int (\sqrt{y})^{\frac{3}{4}} \frac{x}{4}\, dx\right] dy$$

10

$= \frac{1}{16}\left[y^{\frac{3}{2}}\right]$ from $1 \to 4 +$

$\boxed{\left(\frac{1}{16}\right)\int_4^9 y^{\frac{3}{2}}\, dy}$

$= \frac{1}{16}\left[y^{\frac{3}{2}}\right]$ from $1 \to 4 + \frac{1}{16}$ $\boxed{\left[\left(\frac{2}{5}\right)y^{\frac{5}{2}}\right] \text{from } 4 \to 9}$

$= \frac{23}{4}$

To find the variance of Y:

$var(Y) = E[Y^2] - E[Y]^2$

$\therefore E[Y^2] = \int y^2 f_Y(y)\, dy$

from $-\infty \to \infty$

$= E[Y^2] = \int_1^4 y^2 ($

$+ \int_4^9 y^2 [($

$= \left(\frac{1}{16}\right)\left[y^{\frac{5}{2}}\right.$

$$= E[Y^2] = \int_1^4 y^2\left(\frac{1}{8\sqrt{y}}\right) dy$$

$$+ \int_4^9 y^2\left[\left(\frac{1}{4}\right)\int_{(\sqrt{y})}^{\frac{3}{4}} \frac{x}{4}\, dx\right] dy$$

$$= \left(\frac{1}{16}\right)\left[y^{\frac{5}{2}}\right] \text{ from } 1 \to 4 +$$

$$\left(\frac{1}{16}\right)\int_4^9 y^{\frac{5}{2}}\, dy$$

(12)

$= \frac{1}{16}\left[y^{\frac{5}{2}}\right]$ from $1 \to 4$ +

$\left(\frac{1}{16}\right)\left[\left(\frac{2}{7}\right)y^{\frac{7}{2}}\right]$ from $4 \to 9$

$= \frac{221}{16}$

3. Ans. (a) $\int_1^2$

$= \int_1^2 \frac{c}{x^2} dx$

$= c \times$

$= c\,Y$

$\therefore$ Variance of Y is :-

$var(Y) = E[Y^2] - E[Y]^2$

$= \frac{221}{16} - \left(\frac{23}{4}\right)^2$

$= \frac{121}{16}$

13

3. Ans.: (a) $\int_1^2 f(x)\,dx = 1$

$= \int_1^2 \frac{k}{x^2}\,dx = 1$

$= k \times \left| -\frac{1}{x} \right|_1^2 = 1$

$= k \times \left(1 - \frac{1}{2}\right) = 1$

$\therefore k = 2$

(b) The probability is computed as :-

$$P(A) = P(X > 1.5)$$

$$P(A) = \int_{1.5}^{2} \frac{2}{x^2}\,dx$$

$$P(A) = 2 \times \left| -\frac{1}{x} \right|_{1.5}^{2}$$

$$P(A) = 2 \times \left(\frac{1}{1.5} - \frac{1}{2} \right)$$

$$P(A) = 2 \times \left(\frac{2}{3} - \frac{1}{2} \right)$$

$$P(A) = 2 \times \frac{1}{6}$$

$$= \frac{1}{3}$$

15

The conditional PDF for X given A is obtained using Bayes theorem as:-

$$f(x|A) = \frac{f(x)}{P(A)}$$

$$f(x|A) = \frac{3 \times 2}{x^2}$$

$$f(x|A) = \frac{6}{x^2}, \; 1.5 < x < 2$$

Scanned with CamScanner