

## COMPUTER SCIENCE TEST

Time—170 minutes







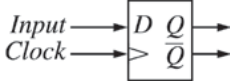
70 Questions

The pages at the beginning of the Computer Science Test containing the “Notation, Conventions, and Definitions” were revised in 2010. Test editions with a copyright date of 2010 or later will contain the following revised pages. Note that revisions were made to # 4, 5, 7, 8, 9, 10, 11, and 13.

### Notation, Conventions, and Definitions:

In this test a reading knowledge of modern programming languages is assumed. The following notational conventions and definitions are used.

1. All numbers are assumed to be written in decimal notation unless otherwise indicated.
2.  $\lfloor x \rfloor$  denotes the greatest integer that is less than or equal to  $x$ .
3.  $\lceil x \rceil$  denotes the least integer that is greater than or equal to  $x$ .
4.  $g(n) = O(f(n))$  means there exist  $C > 0$  and  $N > 0$  such that  $|g(n)| \leq Cf(n)$  for all  $n > N$ .  
 $g(n) = \Omega(f(n))$  means there exist  $C > 0$  and  $N > 0$  such that  $g(n) \geq Cf(n)$  for all  $n > N$ .  
 $g(n) = \Theta(f(n))$  means  $g(n) = O(f(n))$  and  $g(n) = \Omega(f(n))$ .  
 $g(n) = o(f(n))$  means  $g(n)/f(n) \rightarrow 0$  as  $n \rightarrow \infty$ .  
 $g(n) = \omega(f(n))$  means  $g(n)/f(n) \rightarrow \infty$  as  $n \rightarrow \infty$ .
5.  $\exists$  denotes “there exists”.  
 $\forall$  denotes “for all”.  
 $\rightarrow$  denotes “implies”;  $\supset$  is also used to denote “implies”; and  $\Rightarrow$  is also used to denote “implies”.  
 $\neg$  denotes “not”;  $\bar{A}$  is also used to denote “not  $A$ ”.  
 $\vee$  denotes “inclusive or”;  $+$  also denotes “inclusive or”, e.g.,  $P + Q$  can denote “ $P$  or  $Q$ ”.  
 $\oplus$  denotes “exclusive or”.  
 $\wedge$  denotes “and”; also, juxtaposition of statements can denote “and”, e.g.,  $PQ$  can denote “ $P$  and  $Q$ ”.  
A Boolean formula is satisfiable if it is true under some assignment of Boolean values for its variables.  
A Boolean formula is a tautology (or is valid) if it is true under all assignments of Boolean values for its variables.
6.  $\emptyset$  denotes the empty set.  
If  $A$  and  $B$  denote sets, then:  
 $A \cup B$  is the set of all elements that are in  $A$  or in  $B$  or in both;  
 $A \cap B$  is the set of all elements that are in both  $A$  and  $B$ ;  
 $A - B$  is the set of all elements in  $A$  that are not in  $B$ ;  
 $\bar{A}$  is the set of all elements not in  $A$  that are in some specified universal set;  
 $|A|$  is the cardinality of  $A$ .
7. In a string expression,  $\epsilon$  denotes the empty string.  
A string also denotes the singleton set containing just that string.  
If  $S$  and  $T$  denote sets of strings, then:  
 $ST$  denotes the concatenation of  $S$  and  $T$ ;  
 $S + T$  denotes  $S \cup T$ ;  
 $S^n$  denotes  $\underbrace{SS \dots S}_n$ ;  
 $S^*$  denotes  $\epsilon + S + S^2 + S^3 + \dots$ ;  
 $S^+$  denotes  $S + S^2 + S^3 + \dots$ .

8. The complement of a language  $L$  over an alphabet  $\Sigma$  is defined to be  $\Sigma^* - L$ .
9. Unless otherwise indicated in a question, an undirected graph  $G = (V, E)$  has no self-loops or multiedges (multiple edges between the same pair of vertices). In addition, assume that  $|V| > 0$ .
10. In a logic diagram:
-  represents an AND gate.
  -  represents an inclusive OR gate.
  -  represents an exclusive OR gate.
  -  represents a NOT gate.
  -  represents a NAND gate.
  -  represents a NOR gate.
11.  represents a  $D$ -type flip-flop, which stores the value of its  $D$  input when clocked.
12. In a finite automaton diagram, states are represented by circles, with final (or accepting) states indicated by two concentric circles. The start state is labeled "Start". An arc from state  $s$  to state  $t$  labeled  $a$  indicates a transition from  $s$  to  $t$  on input  $a$ . A label  $a/b$  indicates that this transition produces an output  $b$ . A label  $a_1, a_2, \dots, a_k$  indicates that the transition is made on any of the inputs  $a_1, a_2, \dots, a_k$ .
13. Unless otherwise stated, concurrent programs execute on a platform in which shared memory is not cached, assignment statements execute atomically, and the scheduling algorithm is fair, but no other assumptions are made. The following mechanisms are used to control concurrency:
- `fork()` creates a new child process with memory and register contents initially identical to the parent process. The operation returns 0 in the child process and 1 in the parent process.
  - `join()` blocks until all child processes have terminated.
- For a semaphore  $s$ :
- the operation  $P(s)$  blocks until  $s > 0$  and then decrements  $s$  before continuing.
  - the operation  $V(s)$  increments  $s$  and wakes up one process blocked on  $s$  (if any).
- A binary semaphore is a semaphore restricted to have a value of either 0 or 1, with an initial value of 1. A mutual exclusion lock  $m$ , with accompanying `lock(m)` and `unlock(m)` operations, is synonymous with a binary semaphore.
14. A loop invariant for a `while` statement
- `while B do S`
- is an assertion that is true each time the guard  $B$  is evaluated during execution of the `while` statement.

22. Consider a relational database schema with the following instance of a relation  $R(A, B, C)$ .

$A$	$B$	$C$
1	1	1
1	2	3
2	2	4

Assuming the instance above is valid, which of the following functional dependencies must be FALSE?

- I.  $A \rightarrow B$
  - II.  $A \rightarrow C$
  - III.  $AB \rightarrow C$
- (A) I only      (B) II only      (C) I and II      (D) I and III      (E) II and III
- 

23. A computer memory system has a 64KB byte-addressable main memory with 16-bit addresses. This same system has a one-level cache memory that can hold 16 blocks of data, where each block contains 16 bytes. Assuming this is a direct-mapped cache, to which cache block will main memory address  $9A81_{16}$  map?

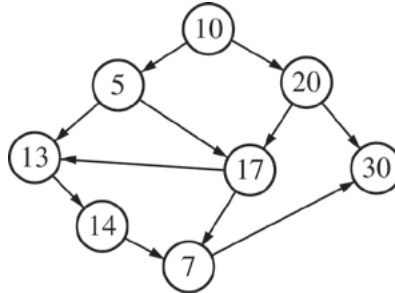
- (A)  $0_{16}$       (B)  $1_{16}$       (C)  $8_{16}$       (D)  $9_{16}$       (E)  $A_{16}$
- 

24. Let  $M_0, M_1, M_2, \dots$ , be an effective enumeration of all Turing machines. Which of the following problems is (are) decidable?

- I. Given a natural number  $n$ , does  $M_n$  starting with an empty tape halt in fewer than  $n$  steps?
  - II. Given a natural number  $n$ , does  $M_n$  starting with an empty tape halt in exactly  $n$  steps?
  - III. Given a natural number  $n$ , does  $M_n$  starting with an empty tape halt after at least  $n$  steps?
- (A) I only      (B) II only      (C) III only      (D) I and II      (E) I and III
-

29. Five wireless nodes— $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$ —are arranged in a line. Any pair of adjacent nodes are within range of each other, and nonadjacent nodes do not interfere. All nodes execute the Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol with Ready to Send/Clear to Send (RTS/CTS) exchange prior to transmission. If the link bandwidth is  $L$  bits per second for successful transmissions and reliable transmission is desired, what is the maximum possible total bandwidth attainable for this network, in bits per second?
- (A)  $L$       (B)  $2L$       (C)  $3L$       (D)  $5L$       (E)  $6L$
- 

30. Consider the following directed graph.



Which of the following is a topological sort of the nodes of the graph?

- (A) 5, 7, 10, 13, 14, 17, 20, 30  
(B) 10, 5, 13, 14, 7, 30, 17, 20  
(C) 10, 5, 13, 17, 20, 14, 7, 30  
(D) 10, 5, 20, 13, 17, 30, 14, 7  
(E) 10, 20, 5, 17, 13, 14, 7, 30
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31. Consider a brute-force password-guessing attack that can submit authentication requests at a rate of one every millisecond. Assume that a password consists of 1–6 characters from a 10-symbol alphabet. In the average case, approximately how many seconds would it take to determine the password using this type of attack?

- (A) 100      (B) 500      (C) 555      (D) 1,000      (E) 1,111
- 

32. Consider the following two problems.

**Nearest Neighbors:** Given an unsorted array of  $n$  floating-point numbers as input, return two of the numbers that are closest in value to each other.

**Farthest Neighbors:** Given an unsorted array of  $n$  floating-point numbers as input, return two of the numbers that are farthest in value from each other.

Assume that the only operations allowed on the data are

- comparing the values of two entries in the array and identifying the larger value;
- comparing the distance between two array entries (the absolute value of the difference between the two array entries) with the distance between two other array entries;
- swapping two entries in the array.

Further assume that each allowed operation has unit cost. What are the worst-case optimal asymptotic running times for algorithms that solve the two problems?

	<u>Nearest Neighbors</u>	<u>Farthest Neighbors</u>
(A)	$\Theta(n \log n)$	$\Theta(n)$
(B)	$\Theta(n \log n)$	$\Theta(n \log n)$
(C)	$\Theta(n^2)$	$\Theta(n)$
(D)	$\Theta(n^2)$	$\Theta(n \log n)$
(E)	$\Theta(n^2)$	$\Theta(n^2)$

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33. Which of the following typically occurs when a procedure call is executed on a processor?

- I. Program counter is updated.
  - II. Stack pointer is updated.
  - III. Data cache is flushed to avoid aliasing problems.
- (A) I only      (B) II only      (C) I and II only      (D) I and III only      (E) I, II, and III
- 

34. The routing table below uses the longest prefix match in its routing decisions.

Network Address/Length	Next Hop
196.94.8.0/21	<i>S</i>
196.94.16.0/24	<i>T</i>
196.94.0.0/16	<i>U</i>

What are the next hops for each of the following two destination addresses?

- |     | Next Hop to<br><u>196.94.10.7</u> | Next Hop to<br><u>196.94.34.9</u> |
|-----|-----------------------------------|-----------------------------------|
| (A) | <i>S</i>                          | <i>T</i>                          |
| (B) | <i>S</i>                          | <i>U</i>                          |
| (C) | <i>T</i>                          | <i>T</i>                          |
| (D) | <i>T</i>                          | <i>U</i>                          |
| (E) | <i>U</i>                          | <i>U</i>                          |
- 

35. Consider a recursive algorithm for sorting an array of  $n \geq 2$  integers that works as follows.

- (a) If there are only 2 elements to be sorted, compare them and swap them if they are out of order.
- (b) Otherwise, do the following steps in order.
  - (1) Recursively sort the first  $n - 1$  elements of the array.
  - (2) In the resulting array, recursively sort the last  $n - 1$  elements.
  - (3) In the resulting array, recursively sort the first 2 elements of the array.

What is the asymptotic running time complexity of this algorithm measured in terms of the number of comparisons made?

- (A)  $\Theta(n \log n)$       (B)  $\Theta(n^2)$       (C)  $\Theta(n^3)$       (D)  $\Theta(2^n)$       (E)  $\Theta(3^n)$
-

37. A hash function  $h$  maps 16-bit inputs to 8-bit hash values. What is the largest  $k$  such that in any set of 1,000 inputs, there are at least  $k$  inputs that  $h$  maps to the same hash value?

- (A) 3      (B) 4      (C) 10      (D) 64      (E) 256
- 

38. Consider three processes P1, P2, and P3 with respective arrival times of 0 ms, 10 ms, and 20 ms and respective processing times of 30 ms, 15 ms, and 30 ms. The three processes are preemptively scheduled on a single-CPU system using the shortest-remaining-processing-time-first scheduling policy. Which of the following shows the order in which the processes complete, from first to last?

- (A) P1      P2      P3  
(B) P1      P3      P2  
(C) P2      P1      P3  
(D) P2      P3      P1  
(E) P3      P1      P2
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39. Which of the following statements about fixed-length and variable-length instruction set architectures (ISAs) is (are) true?

I. Variable-length ISAs allow for a smaller code size over fixed-length ISAs.

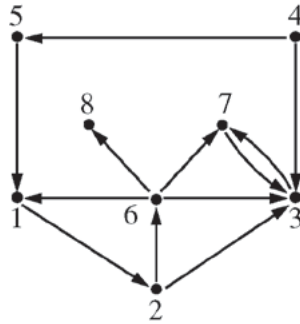
II. Fixed-length ISAs simplify instruction fetch and decode over variable-length ISAs.

III. Variable-length ISAs require more registers than fixed-length ISAs.

- (A) I only      (B) II only      (C) I and II only      (D) II and III only      (E) I, II, and III
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41. Given a directed graph  $G = (V, E)$ , it is convenient to represent the connectivity properties of  $G$  using an associated directed acyclic graph  $G' = (V', E')$ , where the vertices in  $V'$  are the strongly connected components of  $G$  and for  $S, T \in V'$ ,  $(S, T)$  is in  $E'$  if and only if there exist  $u \in S$  and  $v \in T$  such that  $(u, v) \in E$ .

Let  $G$  be the graph shown below.



Which of the following is its associated directed acyclic graph  $G'$ ?

- (A)
- (B)
- (C)
- (D)
- (E)



50. Which of the following statements about caches is (are) true?

- I. A direct-mapped cache can have a lower miss rate than an associative cache of the same size (number of blocks).
  - II. Programs with high spatial locality have a low cache miss rate primarily because the exact same addresses are re-referenced.
  - III. Programs with high temporal locality have a low cache miss rate primarily because the exact same addresses are re-referenced.
- (A) I only      (B) II only      (C) III only      (D) I and II      (E) I and III
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51. A  $k$ -sorted array is a nearly sorted array in which no element is more than  $k$  locations away from its final position in the sorted array. Thus, a 0-sorted array is completely sorted and every array of size  $n$  is  $n$ -sorted.

Suppose that  $A$  is a  $k$ -sorted array of size  $n$ . If insertion sort is used to sort  $A$ , what is the order of growth of the number of comparisons performed by the sorting algorithm in the worst case?

- (A)  $\Theta(k)$       (B)  $\Theta(kn)$       (C)  $\Theta(k^2n)$       (D)  $\Theta(n \log_k n)$       (E)  $\Theta(n^2)$
- 

52. The subtype principle describes when one type may be substituted for another. Which of the following is true?

- (A) An instance of a subtype may be used in any expression in which an instance of the supertype may be used because the subtype must support a superset of the operations supported by the supertype.
  - (B) An instance of a subtype may be used in any expression in which an instance of the supertype may be used because the subtype must support a subset of the operations supported by the supertype.
  - (C) An instance of a supertype may be used in any expression in which an instance of the subtype may be used because the subtype must support a superset of the operations supported by the supertype.
  - (D) An instance of a supertype may be used in any expression in which an instance of the subtype may be used because the subtype must support a subset of the operations supported by the supertype.
  - (E) Instances of subtypes and supertypes may be used interchangeably.
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53. Consider a regular language  $L$  over  $\{0, 1\}$ . Which of the following languages over  $\{0, 1\}$  must also be regular?

I.  $\{w \in L \mid \text{the length of } w \text{ is even}\}$

II.  $\{w \in L \mid \text{the length of } w \text{ is prime}\}$

III.  $\{w \in L \mid \text{the length of } w \text{ is an integer power of } 2\}$

(A) None      (B) I only      (C) III only      (D) I and III only      (E) I, II, and III

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54. In order to create a good solution for the mutual exclusion problem for concurrent processes, which of the following conditions must hold?

I. No process should have to wait forever to enter its critical region.

II. No process running outside of its critical region may block other processes from entering their critical region.

III. There should be no assumptions about the speed or number of CPUs.

(A) None      (B) I and II only      (C) I and III only      (D) II and III only      (E) I, II, and III

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55. What is the negation of the predicate  $\forall x \exists y (p(y) \rightarrow q(x))$ ?

(A)  $\forall x \exists y (p(y) \rightarrow \neg q(x))$

(B)  $\forall x \exists y (p(y) \wedge \neg q(x))$

(C)  $\exists x \forall y (p(y) \rightarrow \neg q(x))$

(D)  $\exists x \forall y (q(x) \rightarrow \neg p(y))$

(E)  $\exists x \forall y (p(y) \wedge \neg q(x))$

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