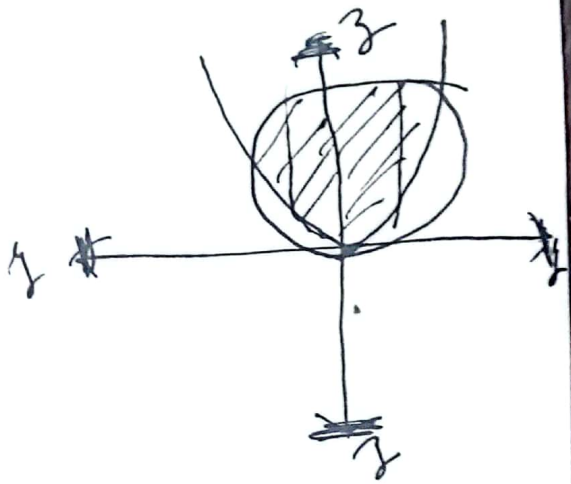
$$\left(\frac{d}{dx}+\frac{d}{dy}+\frac{d}{dy}\right)\nabla\cdot\vec{F} - \nabla^2(P\hat{i}+Q\hat{j}+R\hat{k})$$

$$\nabla(\nabla\cdot\vec{F}) - \nabla^2\vec{F}$$

$$\text{grad}(\text{div}\,\vec{F}) - \nabla^2\vec{F}$$

Ex 13.6

55. Find the area of the part of the sphere $x^2+y^2+z^2 = 4z$ that lies inside the paraboloid $z = x^2+y^2$



$$x^2+y^2 = r^2$$
$$z = r^2$$

$$x^2+y^2+z^2 = 4z$$
$$r^2+z^2 = 4z$$
$$z+z^2 = 4z$$
$$z^2 = 4z-z$$
$$z^2 = 3z$$
$$z^2-3z = 0$$
$$z(z-3) = 0$$
$$z=0 \;;\; z=3$$
$$z = r^2$$
$$0 = r \qquad \pm\sqrt{3} = r$$
$$3 = r^2 \;; \qquad 0 \le r \le \sqrt{3}$$

surface area =

$$\iint_D \sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}\,dA$$

for this, we need to write the sphere in terms of z..

$x^2 + y^2 + z^2 = 4z$

$x^2 + y^2 + z^2 - 4z + 4 - 4 = 0$

$x^2 + y^2 + (z-2)^2 - 4 = 0$

$x^2 + y^2 + (z-2)^2 = 4$

$(z-2)^2 = 4 - x^2 - y^2$

$z - 2 = \sqrt{4 - x^2 - y^2}$

$z = 2 + \sqrt{4 - x^2 - y^2}$

$$\frac{dz}{dx} = 0 + \frac{1}{2\sqrt{4 - x^2 - y^2}} \cdot (0 - 2x - 0)$$

$$= \frac{-x}{\sqrt{4 - x^2 - y^2}}$$

$$\frac{dz}{dy} = 0 + \frac{1}{2\sqrt{4 - x^2 - y^2}} \cdot (0 - 0 - 2y)$$

$$= \frac{-y}{\sqrt{4 - x^2 - y^2}}$$

$$S = \iint_D \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x^2 - y^2}}\right)^2 + \left(\frac{-y}{\sqrt{4 - x^2 - y^2}}\right)^2}\, dA$$

$$= \iint_D \sqrt{1 + \frac{x^2}{4 - x^2 - y^2} + \frac{y^2}{4 - x^2 - y^2}}\, dA$$

$$= \iint_D \sqrt{\frac{4 - x^2 - y^2 + x^2 + y^2}{4 - x^2 - y^2}}\, dA$$

$$= \iint_D \frac{2}{\sqrt{4-(x^2+y^2)}}\, dA$$

$$dA = r\, dr\, d\theta$$

$$0 \le r \le \sqrt{3} \; ; \; 0 \le \theta \le 2\pi$$

$$= \int_0^{2\pi} \int_0^{\sqrt{3}} \frac{2}{\sqrt{4-r^2}}\, r\, dr\, d\theta$$

$$u = 4 - r^2 \quad ; \; u = 4 - 0 = 4$$

$$du = 0 - 2r\, dr \; ; \; u = 4 - 3 = 1$$

$$-du = 2r\, dr$$

$$= \int_0^{2\pi} \int_4^1 \frac{-du}{\sqrt{u}}\, d\theta$$

$$= \int_0^{2\pi} \int_1^4 u^{-\frac{1}{2}}\, du\, d\theta$$

$$= \int_0^{2\pi} \left(2\sqrt{u}\right)_1^4 d\theta$$

$$= \int_0^{2\pi} (2 \times 2 - 2)\, d\theta$$

$$= 2(2\pi - 0) = \underline{4\pi}$$

41. Ans Find the area of the surface. The part of the surface $y = 4x + z^2$ that lies between the planes $x = 0$, $x = 1$, $z = 0$ & $z = 1$.