

SPRING 2006 — MA 227 — FINAL

Name: _____

1. PART I

There are 10 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

- (1) Find the cross product of the vectors $\langle 1, 2, -3 \rangle$ and $\langle -1, 0, 1 \rangle$.

- (2) Find the derivative of the vector function $\langle 1, 2 \cos t, t \sin t \rangle$.

- (3) A particle starts at the origin at time $t = 0$. Its velocity is given by $v(t) = \langle t, t^2, e^{-2t} \rangle$. What is the position vector of the particle at time T ?

- (4) Find the partial derivatives of the function $f(x, y) = xy \log(x + y)$.

- (5) Find the gradient of $f(x, y, z) = \frac{x-y}{1+z^2}$.

- (6) Find the linearization $L(x, y)$ of $f(x, y) = x/y$ at the point $\langle 1, 1 \rangle$.

- (7) Write down the iterated integral for $\iint_D 3y dA$ where D is the triangular region with vertices $(0, 0)$, $(1, 1)$ and $(2, 0)$. You do not have to compute the integral.

- (8) Evaluate $\iint_D dA$ where $D = \{(x, y) : 0 \leq x^2 + y^2 \leq 4, 0 \leq y, 0 \leq x\}$.

- (9) Find the Jacobian of the transformation $x = 6u^2 - 2v^2$, $y = 2u^2 + 3v^2$.

- (10) Compute $\operatorname{div} \mathbf{F}$ when $\mathbf{F}(x, y, z) = \langle \log(xy), 2 \cos(xyz), -2e^{xy} \rangle$.

2. PART II

There are 5 problems in Part II, each worth 12 points. On Part II problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

- (1) At what point does the curve $r(t) = \langle 2t, t, t^2 \rangle$ have maximum curvature? What is the curvature at this point?

- (2) Find the local maximum and minimum values and saddle points of

$$f(x, y) = x^3 + xy^2 + 3x^2 + y^2.$$

- (3) Find the mass m of the solid E lying below the paraboloid $x^2 + y^2 + z = 4$ but above the plane $z = 0$. The density function for E is $f(x, y, z) = \sqrt{x^2 + y^2}$. Argue why the center of mass is on the z -axis. Evaluate the z -component of the center of mass $\bar{z} = M_{xy}/m$.

- (4) Evaluate the integral $\iiint_E y^2 dV$ where E is the region bounded by the ellipsoid $x^2/4 + y^2/9 + z^2/16 = 1$ by transforming the domain of integration to a ball.

- (5) Let C be the curve consisting of the sides of the triangle with vertices $(0, 0)$, $(1, 1)$, and $(0, 3)$. Evaluate $\int_C (xydx + x^2dy)$ by using Green's Theorem as well as by direct integration.