$$S = \int_0^1\int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dz}\right)^2}\, dx\, dz$$

$$\frac{dy}{dx} = 4 \quad ; \quad \frac{dy}{dz} = 2z$$

$$= \int_0^1\int_0^1 \sqrt{1+16+4z^2}\, dx\, dz$$

$$= \int_0^1\int_0^1 \sqrt{17+4z^2}\, dx\, dz$$

$$= \int_0^1 \sqrt{17+4z^2}\, (x)_0^1\, dz$$

$$= \int_0^1 \sqrt{17+4z^2}\, dz$$

$\sqrt{bx^2+a}$; we can substitute $x = \frac{\sqrt{a}}{\sqrt{b}}\tan(u)$

This is a trig substitution.

$$x = \frac{\sqrt{17}}{2}\tan(u)$$

$$z = \frac{\sqrt{17}}{2}\tan(u)$$

$$dz = \frac{\sqrt{17}}{2}\sec^2(u)\, du$$

$$= \int_0 \sqrt{17+4\times\frac{17}{4}\tan^2(u)}\cdot\frac{\sqrt{17}}{2}\sec^2(u)\, du$$

$$= \int \sqrt{17(1+\tan^2 u)} \cdot \frac{\sqrt{17}}{2}\sec^2(u)\,du$$

$$= \int \sqrt{17}\cdot \sec u \cdot \frac{\sqrt{17}}{2}\sec^2(u)\,du$$

$$= \frac{17}{2}\int \sec^3(u)\,du$$

$$uv - \int v\,du$$

$$u = \sec u; \quad du = \sec u \tan u$$

$$dv = \sec^2 u\,du$$

$$v = \tan u$$

$$\int \sec^3(u)\,du = \sec u \tan u - \int \tan u \cdot \sec u \tan u$$

$$= \sec u \tan u - \int \sec u \tan^2 u$$

$$= \sec u \tan u - \int \sec u\,(\sec^2 u - 1)$$

$$= \sec u \tan u - \int \sec^3 u + \int \sec u$$

$$2\int \sec^3 u\,du = \sec u \tan u + \int \sec u$$

$$= \sec u \tan u + \int \frac{\sec u \cdot (\sec u + \tan u)}{(\sec u + \tan u)}\,du$$

$$= \sec u \tan u + \int \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u}\,du$$

$$t = \sec u + \tan u$$

$$dt = \sec u \tan u + \sec^2 u\,du$$

$$= \sec u \tan u + \int \frac{dt}{t}$$

$$2\int \sec^3 u \, du = \sec u \tan u + \ln|t|$$

$$\int \sec^3 u \, du = \frac{\sec u \tan u}{2} + \frac{\ln|\sec u + \tan u|}{2}$$

$$= \frac{17}{4} \left[\sec u \tan u + \ln|\sec u + \tan u| \right]$$

$$z = \frac{\sqrt{17}}{2} \tan(u)$$

$$\tan^{-1}\left(\frac{2}{\sqrt{17}} z\right) = u$$

$$= \frac{17}{4} \left(\sec\left(\tan^{-1}\left(\frac{2}{\sqrt{17}} z\right)\right) \cdot \cancel{\tan}\left(\cancel{\tan^{-1}} \frac{2}{\sqrt{17}} z\right) \right.$$

$$+ \ln\left(\sec\left(\tan^{-1}\left(\frac{2}{\sqrt{17}} z\right)\right) + \right.$$

$$\left.\left. \cancel{\tan}\left(\cancel{\tan^{-1}}\left(\frac{2}{\sqrt{17}} z\right)\right)\right)\right)$$

Let $y = \tan^{-1}\left(\frac{2}{\sqrt{17}} z\right)$

$$\frac{2}{\sqrt{17}} z = \tan y$$

$$\frac{4}{17} z^2 = \frac{\sin^2 y}{\cos^2 y}$$

$$\frac{4}{17} z^2 + 1 = \frac{\sin^2 y}{\cos^2 y} + 1$$

$$\int \sec x \, dx = \int \frac{\sec x \cdot (\sec x + \tan x)}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x \, dx$

$$= \int \frac{du}{u} = \ln|u| = \ln|\sec x + \tan x|$$

Continued

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$$\frac{4}{17} z^2 + 1 = \frac{\sin^2 y + \cos^2 y}{\cos^2 y}$$

$$\frac{4}{17} z^2 + 1 = \frac{1}{\cos^2 y}$$

$$\frac{4}{17} z^2 + 1 = \sec^2 y$$

$$\sqrt{\frac{4}{17} z^2 + 1} = \sec\left(\tan^{-1}\left(\frac{2}{\sqrt{17}} z\right)\right)$$

$$= \frac{17}{4} \left[\sqrt{\frac{4}{17} z^2 + 1} \cdot \frac{2}{\sqrt{17}} z + \ln\left| \sqrt{\frac{4}{17} z^2 + 1} + \frac{2}{\sqrt{17}} z \right| \right]_0$$

$$= \frac{17}{4}\left[\sqrt{\frac{4}{17}+\frac{1}{1}}\cdot\frac{2}{\sqrt{17}} + \ln\left|\sqrt{\frac{4}{17}+\frac{1}{1}}+\frac{2}{\sqrt{17}}\right|\right.$$

$$- \ln|1+0|$$

$$= \frac{17}{4}\left[\sqrt{\frac{4+17}{17}}\cdot\frac{2}{\sqrt{17}} + \ln\left|\sqrt{\frac{4+17}{17}}+\frac{2}{\sqrt{17}}\right|\right)$$

$$= \frac{17}{4}\left[\sqrt{\frac{21}{17}}\cdot\frac{2}{\sqrt{17}} + \ln\left|\frac{\sqrt{21}}{\sqrt{17}}+\frac{2}{\sqrt{17}}\right|\right)$$

$$= \frac{\sqrt{21}}{4} + \frac{17}{4}\ln\left|\frac{\sqrt{21}+2}{\sqrt{17}}\right|$$

29. Ans- $x = u+v$, $y = 3u^2$, $z = u-v$ $(2,3,0)$

$$r(u,v) = (u+v)\hat{i} + (3u^2)\hat{j} + (u-v)\hat{k}$$

$$r_u = \hat{i} + 6u\hat{j} + \hat{k}$$

$$r_v = \hat{i} - \hat{k}$$