12.8 19 Ans. $\iint_R xy\,dA$, where R is the region in the first quadrant bounded by the lines $y=x$ & $y=3x$ & the hyperbolas $xy=1$, $xy=3$; $x=\frac{u}{v}$, $y=v$.

$y = x$

$v = \frac{u}{v}$

$v^2 = u$

$v = \pm\sqrt{u}$

$y = 3x$

$v = \frac{3u}{v}$

$v^2 = 3u$

$v = \pm\sqrt{3u}$

$\sqrt{u} \le v \le \sqrt{3u}$

$xy = 1$

$\frac{u}{\cancel{v}} \cdot \cancel{v} = 1$

$u = 1$

$xy = 3$

$\frac{u}{\cancel{v}} \cdot \cancel{v} = 3$

$u = 3$

$1 \le u \le 3$

$$dA = \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 0 & 1 \end{vmatrix}$$

$$= \frac{1}{v} - 0 = \frac{1}{v}\,dv\,du = dA$$

$$\therefore \int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} u \frac{1}{v}\,dv\,du$$

$$= \int_1^3 u \left(\ln|v|\right)_{\sqrt{u}}^{\sqrt{3u}} du$$

$$= \int_1^3 u \left(\ln|\sqrt{3u}| - \ln|\sqrt{u}|\right) du$$

$$= \int_1^3 u \ln\left|\sqrt{\frac{3u}{u}}\right| du$$

$$= \ln\sqrt{3}\left(\frac{u^2}{2}\right)_1^3 = \frac{\ln\sqrt{3}}{2}(9-1)$$

(60)

$= 4 \ln|\sqrt{3}|$

$= 4 \ln|3^{\frac{1}{2}}|$

$= \cancel{4}^{2} \cdot \frac{1}{\cancel{2}_{1}} \ln|3|$

$= \underline{\underline{2 \ln|3|}}$