

23. An $\iint_R \frac{x-2y}{3x-y} dA$, where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, and $3x-y=8$.

$$\text{Let } x-2y=u \text{ \& } 3x-y=v$$

$$\therefore u=4 \text{ \& } u=0.$$

$$\therefore 0 \leq u \leq 4$$

$$\& \quad v=1 \text{ \& } v=8$$

$$\therefore 1 \leq v \leq 8$$

$$\text{To find } dA :- \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix}$$

$$x-2y=u$$

$$x = u + 2y$$

$$x = u + 2(3x-v)$$

$$x = u + 6x - 2v$$

$$-u + 2v = 5x$$

$$-\frac{u+2v}{5} = x$$

$$y = 3x - v$$

$$y = 3\left(-\frac{u+2v}{5}\right) - v$$

$$= -\frac{3u+6v}{5} - \frac{v}{1}$$

$$y = -\frac{3u+v}{5} \quad (61)$$

$$\therefore dA = \begin{vmatrix} -\frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & \frac{1}{5} \end{vmatrix} = -\frac{1}{25} - \left(\frac{2}{5} \times -\frac{3}{5} \right)$$

$$= -\frac{1}{25} + \frac{6}{25}$$

$$= \frac{5}{25} = \frac{1}{5} dv du$$

$$\therefore \iint_R \frac{x-2y}{3x-y} dA = \int_0^4 \int_1^8 \frac{u}{v} \cdot \frac{1}{5} dv du$$

$$= \frac{1}{5} \int_0^4 u (\ln |v|)^8 du$$

$$= \frac{1}{5} \int_0^4 u (\ln |8| - \ln |1|) du$$

$$= \frac{\ln \left| \frac{8}{1} \right|}{5} \left(\frac{u^2}{2} \right)_0^4$$

$$= \frac{\ln |8|}{5} \cdot \frac{1}{2} \cdot \left(\frac{8}{1} \right)$$

$$= \frac{8}{5} \ln |8|$$