$$= \iint_D \sqrt{(\sqrt{x})^2+(\sqrt{y})^2+1}\ dA$$

$$= \int_0^1\int_0^1 \sqrt{x+y+1}\ dx\,dy$$

$$f(x,y) = \frac{2}{3}\left(x^{\frac{3}{2}} + y^{\frac{3}{2}}\right)$$

$$f_x = x^{\frac{1}{2}} = \sqrt{x}$$

$$f_y = y^{\frac{1}{2}} = \sqrt{y}$$

<u>13.7 41.a</u> Density $\rho = 870\ kg/m^3$

$$\vec{V} = z\,\hat{i} + y^2\,\hat{j} + x^2\,\hat{k}$$

$$\nabla = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$$

$$\nabla.\vec{V} = \frac{dz}{dx} + \frac{dy^2}{dy} + \frac{dx^2}{dz}$$

$$= 0 + 2y + 0$$

$$= \underline{\underline{2y}}$$

Let $x^2+y^2 = r^2$

$\therefore r^2 = 4$

$r = \pm 2$

$\therefore 0 \le r \le 2$; $0 \le z \le 1$; $0 \le \theta \le 2\pi$

Let $x = r\cos\theta$, $y = r\sin\theta$ & $dV = r\,dr\,d\theta\,dz$

Rate of outward flow through the cylinder is given by:-

$$R = \int_0^1 \int_0^{2\pi} \int_0^2 \rho . (\nabla . \vec{v})\, dV$$

$$= \int_0^1 \int_0^{2\pi} \int_0^2 870 . 2y . \; r\,dr\,d\theta\,dz$$

$$= 1740 \int_0^1 \int_0^{2\pi} \int_0^2 r \sin\theta \; r\,dr\,d\theta\,dz$$

(6)
580
$\times 8$
4640

$$= 1740 \int_0^1 \int_0^{2\pi} \left(\frac{r^3}{3}\right)_0^2 \sin\theta\, d\theta\, dz$$

$$= 1740 \int_0^1 \int_0^{2\pi} \frac{8}{3} \sin\theta\, d\theta\, dz$$

$$= \cancel{1740}^{580} \times \frac{8}{\cancel{3}_1} \int_0^1 (-\cos\theta)_0^{2\pi}\, dz$$

$$= 4640 . (-1 - (-1)) . (1-0)$$

$$= 4640 . (-1+1) . (1)$$

$$= 4640 . (0).$$

$$= \underline{\underline{0}} \text{ kg/s}$$

27.b) $\vec{F}(x,y,z) = y\hat{j} - z\hat{k}$

curl $\vec{F} = \nabla \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & y & -z \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0) = \underline{\underline{0}}$$

$$= \langle 0,0,0 \rangle$$

90

$$\iint_S \vec{F}\cdot d\vec{S} = \iint \operatorname{curl}\vec{F}\cdot d\vec{S}$$

$$= \iint \langle 0,0,0\rangle \cdot d\vec{S}$$

$$= \underline{\underline{0}}$$

9c