(1)

NAME :- SHREYAS SRINIVASA

MA-485

BLAZER ID :- SSRINIVA

# HW2

## Exercise 0.1

Ans :- (a) No. of ways that we draw four cards from an ordinary deck of 52 cards with replacement :-

There are 52 ways to select a card in each case here :-

$52 \times 52 \times 52 \times 52$

$= 52^4$

$\therefore$ There are 7,311,616 ways to select the cards here.

(b) As we are selecting without replacement, the total no. of ways to select 4 cards here is :- $52 \times 51 \times 50 \times 49$

$= 6,497,400$

# Exercise 0.2

**Ans:** The probability that we win the jackpot is computed here as:-

$$\frac{(\text{No of ways that 5 correct balls are selected from the 5 correct ones})}{(\text{Total ways to select 5 balls from 69 balls})} \times \frac{1}{\text{Total number of red balls}}$$

$$= \frac{1}{C_{69,5}} \times \frac{1}{26}$$

$$= \frac{1}{\frac{69!}{64!\,5!}} \times \frac{1}{26}$$

$$= \frac{1}{\frac{69 \times 68 \times 67 \times 66 \times 65 \times 64!}{64!\; 5 \times 4 \times 3 \times 2 \times 1}} \times \frac{1}{26}$$

$$= \frac{1}{69 \times 17 \times 67 \times 11 \times 13} \times \frac{1}{26}$$

$$= \frac{1}{292,201,338}$$

$$= 3.422978 \times 10^{-9}$$

Exercise 0.4

Ans:- (a) Total no of arrangements

$= 6!$

No of favorable ways $= 2 \times 3! \, 3!$

$$\text{Probability} = \frac{2 \times 3! \, 3!}{6!}$$

$$= \frac{\cancel{2} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{3!}}{\cancel{6} \times 5 \times \cancel{4} \times \cancel{3!}}$$

$$= \frac{1}{2 \times 5 \times 1 \times 1}$$

$$= \frac{1}{10}$$

$$= 0.1$$

(d) Total arrangements = 7!

Favorable arrangements = $2 \times 4! \times 3!$

Probability $= \dfrac{2 \times 4! \times 3!}{7!}$

$= \dfrac{\cancel{2} \times \cancel{4!} \times \cancel{3} \times 2 \times 1}{7 \times \cancel{6} \times 5 \times \cancel{4!}}$

$= \dfrac{2}{7 \times 5 \times 1}$

$= \dfrac{2}{35}$

$= 0.057142857$

Exercise 0.5

Ans:- (a) P( Equal Tails / Equal Heads)

$= C_{10,5} \times (0.5)^5 \times (0.5)^5$

$= \dfrac{10!}{5!\,5!} \times \dfrac{0.03125}{0.03125}$

$= \dfrac{\cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times 0.0009765$

(5)

$= 6 \times 7 \times 6 \times 0.0009765625$

$= 36 \times 7 \times 0.0009765625$

$= 0.24609375$

(ii) $P(x \leq 4) = \sum_{x=0}^{4} C_{10,x} \times 0.5^{x} \times 0.5^{10-x}$

$= 0.3770$

Exercise 0.6

Ans:- Sample space :- $\{ (1,1), (1,2), \ldots, (1,6), (2,1), (2,2), \ldots, (2,6), (3,1), (3,2), \ldots, (3,6), \ldots, (6,1), (6,2), \ldots, (6,6) \}$

Total no of sample space $= 6^2 = 36$.

Sum is at least 10 = Sum is 10 + Sum is 11 + Sum is 12

$= \{(4,6), (6,4), (5,5), (5,6), (6,5), (6,6)\}$

So, Probability of the sum being at least 10 $= \frac{\cancel{6}\, 1}{\cancel{36}\, 6}$

$= \frac{1}{6}$

Exercise 0.3

Ans. (19) The 4 members from the 20 members for the first committee can be selected in $C_{20,4}$ ways as the order in which the members can be selected is not important

The 7 members for the second committee can be selected from the remaining 16 members in $C_{16,3}$ ways, as the order is not important.

Using the multiplication rule:

$$C_{20,4} \times C_{16,3}$$

$$= \frac{20!}{\cancel{16!}\,4!} \times \frac{\cancel{16!}}{13!\,3!}$$

$$= \frac{\cancel{20}\times 19\times \cancel{18}\times 17\times \cancel{16}\times 15\times 14\times \cancel{13!}}{\cancel{13!}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1\times \cancel{3}\times \cancel{2}\times 1}$$

$$= 5\times 19\times 17\times 8\times 15\times 14$$

$$= 2,713,200 \text{ ways}$$

(b) The 4 members for the first committee can be selected from the 20 members in $C_{20,4}$ ways, as the order in which the members can be selected is not important.

Since the committees can overlap, the 3 members for the 2nd committee can be selected in $C_{20,3}$ ways, as the order in which the members are selected is not important.

∴ Using multiplication rule:-

$$C_{20,4} \times C_{20,3}$$

$$= \frac{20!}{16!\,4!} \times \frac{20!}{17!\,3!}$$

$$= \frac{\overset{5}{\cancel{20}} \times 19 \times \overset{3}{\cancel{18}} \times 17 \times \cancel{16!}}{\cancel{16!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{\overset{10}{\cancel{20}} \times 19 \times 18 \times \cancel{17!}}{\cancel{17!} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 5 \times 19 \times 17 \times 10 \times 19 \times 18$$

$$= 5{,}523{,}300 \text{ ways}$$