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MA-485

HW-6

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Exercise 1.1

Ans:- In the experiment of rolling two dice, the sample space is as follows:

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

Let X be the minimum of the two numbers obtained. The possible values X can take are 1, 2, 3, 4, 5, and 6.

The probability mass function of X.

$$P(X=1) = P\left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), \\ (1,6), (2,1), (3,1), (4,1), (5,1), (6,1) \end{array} \right\}$$

$$= \frac{11}{36}$$

$P(X=2) = P\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),$
$(4,2),(5,2),(6,2)\}$

$= \frac{9}{36}$

$P(X=3) = P\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),$
$(6,3)\}$

$= \frac{7}{36}$

$P(X=4) = P\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$
$= \frac{5}{36}$

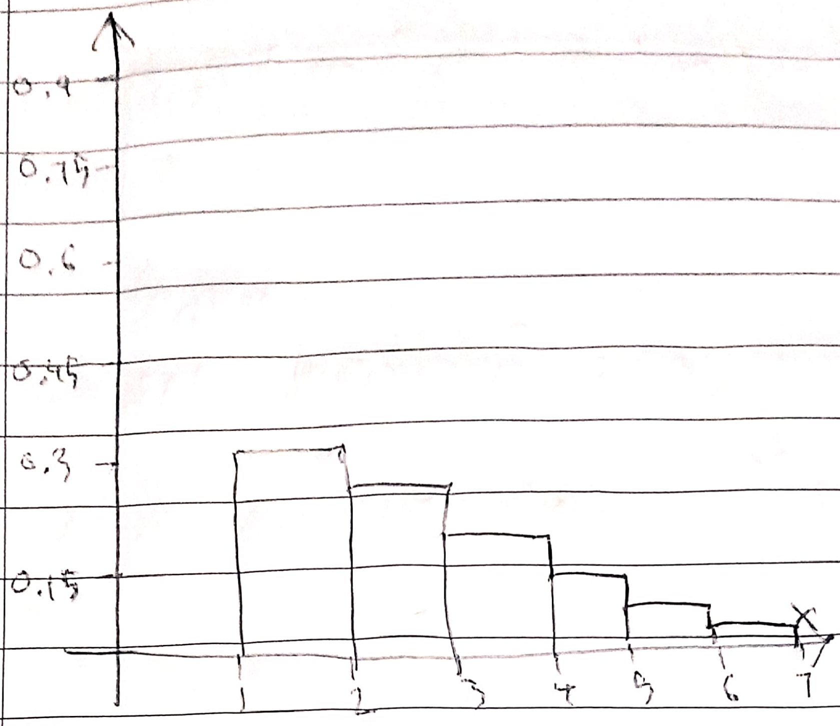
$P(X=5) = P\{(5,5),(5,6),(6,5)\}$

$= \frac{3}{36}$

$P(X=6) = P\{(6,6)\} = \frac{1}{36}$

$\therefore$ The probability distribution function of X is as follows:-

x	1	2	3	4	5	6	Total
$P(x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$	1



Exercise 1.2

Ans

$P(\text{first roll}) = \frac{1}{6}$

$P(\text{second roll})$

$= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

$P(\text{third roll}) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

$P(\text{fourth roll}) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{125}{1296}$ & so on...

$P(X \text{ rolls until } \cancel{\text{get}} \text{ a } 6) = \left(\frac{5}{6}\right)^{x-1} \times \frac{1}{6}$

Exercise 1.3

Ans: From the interval $(0, 1)$:-

P (selecting a point less than $\frac{2}{5}$) $= \frac{2}{5}$

No of points, $n = 5$

$x \sim$ Binomial $\left(n = 5, p = \frac{2}{5}\right)$

$$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$$

$$= 1 - C^{5,0} \times \left(\frac{2}{5}\right)^0 \times \left(\frac{3}{5}\right)^5 - C^{5,1} \times \left(\frac{2}{5}\right)^1 \times \left(\frac{3}{5}\right)^4$$

$$= 1 - \left(\frac{3}{5}\right)^5 - 5 \times \left(\frac{2}{5}\right)^1 \times \left(\frac{3}{5}\right)^4$$

$$= 1 - \frac{243}{3125} - \frac{810}{3125}$$

$$= 1 - \frac{243 - 810}{3125}$$

$$= \frac{2072}{3125}$$

$$P(x \geq 2) = 0.66304$$

Exercise 1.4

Ans The certain rare blood type can be found only in 0.05% of the people.

Hence probability of a person having this rare blood type is, $p = \frac{0.05}{100} = 0.0005$

& $n = 4000$.

The average number of people having this rare blood group is :- $\lambda = np = 4000 \times 0.0005$
$= \underline{\underline{2}}$

Let X be the number of people who have this rare blood group.
$X \sim P(2)$.

The probability of atmost two people in a group of randomly selected 4000 people that will have this rare blood type is :-

$$P(X \leq 2) = \sum_{x=0}^{2} \frac{e^{-2} 2^x}{x!} = P(x=0) + P(x=1) + P(x=2)$$

$$P(X \leq 2) = \frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!}$$

$$= e^{-2} + 2 \times e^{-2} + 2 \times e^{-2}$$

$P(X \leq 2) = 0.1353 + 0.2707 + 0.2707$
$= 0.6767$

$P(X \leq 2) = 0.6767$

Exercise 1.5

Ans. Here, $n = 150$, $P(\text{show-up}) = 0.99$

$\therefore \lambda = np = 150 \times 0.99$
$= 148.5$

$P(\text{All have seats}) = P(X \leq 148)$
$= 0.505$ [$\because$ From poisson calculator]