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MA-485

MIDTERM TEST #1

NOTE:- Answering questions that I find easy first

PROBLEM 3

Ans:-

$$P(A) = \frac{1}{6}$$

$$P(B) = \frac{4}{36} = \frac{1}{9}$$

$$P(A \cap B) = \frac{1}{36}$$

$$P(A) \times P(B) = \frac{1}{54}$$

Since  $P(A) \times P(B) \neq P(A \cap B)$ ,  $\therefore$  events A & B  
are NOT  
independent!

### PROBLEM - 4

Ans:-

(a) The probability that ~~me~~ I wake up from the alarm, given that all 3 alarm clocks were set up is:-  $1 - (\text{Probability that all 3 alarm clocks fail to wake me up})$ .

$$\Rightarrow 1 - 0.2 \times 0.15 \times 0.1$$

$$\Rightarrow 1 - 0.003$$

$$\Rightarrow 0.997 \text{ is the required probability!}$$

(b) The probability that I will wake up, given that I need at least 2 alarm clocks to wake me up is:-  $P(\text{exactly 2 work}) + P(\text{all 3 work})$

$$\Rightarrow 0.8 \times 0.85 \times 0.1 + 0.85 \times 0.9 \times 0.2 + 0.8 \times 0.9 \times 0.15 + 0.8 \times 0.85 \times 0.9$$

$$\Rightarrow 0.068 + 0.153 + 0.108 + 0.612$$

$$\Rightarrow 0.941 \text{ is the required probability!}$$

(3)

### PROBLEM - I

Ans. Given that there are 15 different books on a book shelf.

(a) The books can be arranged in a line in  $15!$  ways.

$$\Rightarrow \underline{\underline{1,307,674,368,000 \text{ ways.}}}$$

(a) 5 books are of Mathematics, 6 of History & 4 of Physics.

No. of ways by which the same subject books lie together is:-  $3! \times 6! \times 4!$

$$\Rightarrow 6 \times 120 \times 24 \times 24$$

$$\Rightarrow \underline{\underline{12441600 \text{ ways.}}}$$

(a) 5 books can be chosen in  ${}^{15}C_5$  ways.

$$\therefore \frac{15!}{10! \cdot 5!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10!}{10! \times 5 \times 4 \times 3 \times 2 \times 1 \times 10!}$$

$$= \underline{\underline{3003}}$$



(A) Total Math books = 5.  
Total books = 15

So if 5 books are drawn and the probability for all of them to be Math is:-

$$\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \times \frac{2}{12} \times \frac{1}{11} = \frac{1}{3003}$$

(2) Probability that exactly 2 are Math is:-

$$\frac{{}^5C_2 \times {}^{10}C_3}{{}^{15}C_5} = \frac{10 \times 120}{3003}$$

$$= \frac{1200}{3003}$$

$$= 0.3996$$

## PROBLEM 2

Ans:  $P(A) = \frac{1}{3}$

$$P(A^c \cup B^c) = \frac{11}{12} \quad \& \quad P(A \cup B) = \frac{1}{2}$$

(5)

To find :  $P(B) = ?$

$$P(A^c \cup B^c) = P(\bar{A} \cap \bar{B})$$

$$= 1 - P(A \cap B)$$

$$\therefore P(A^c \cup B^c) = 1 - P(A \cap B)$$

$$\frac{11}{12} = 1 - P(A \cap B)$$

$$P(A \cap B) = 1 - \frac{11}{12}$$

$$= \frac{1}{12}$$

$$\therefore P(A \cap B) = \frac{1}{12}$$

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$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{1}{2} = \frac{1}{3} + P(B) - \frac{1}{12}$$

$$\therefore P(B) = \frac{1}{2} - \frac{1}{3} + \frac{1}{12}$$

(6)

$$= \frac{6 - 4 + 1}{12}$$

$$= \frac{3}{12} = \frac{1}{4}$$

$$\therefore P(B) = \underline{\underline{\frac{1}{4}}}$$

### PROBLEM - 5

Ans. Box A (first) = 6 dimes and 2 Pennies.  
Box B = 4 dimes & 2 Pennies.

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

Event  $E$  = selected coin is a dime.

Probability that Box A is selected and coin is a dime,  $P(A \cap E) = \frac{6}{12} = \frac{1}{2}$

Probability that Box B is selected and coin is a dime,  $P(B \cap E) = \frac{4}{12} = \frac{1}{3}$

(7)

(a) Using Baye's Theorem, we get:-

Probability that the selected coin is a prime,  
 $P(E) = P(A) \times P(A|E) + P(B) \times P(B|E)$

$$= \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{3}{8} + \frac{1}{6}$$

$$= \frac{13}{24}$$

(b) Using Baye's Theorem again, we get:-

Probability that Box A is taken, given that the selected coin is a dime,  $P(A|E)$

$$\Rightarrow \frac{P(A) \times P(A|E)}{P(A) \times P(A|E) + P(B) \times P(B|E)}$$

$$\text{Probability, } P(A|E) = \frac{\frac{1}{2} \times \frac{3}{4}}{\frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3}}$$

$$= \frac{\frac{3}{4}}{\frac{13}{12}} = \frac{9}{13}$$