MA-485

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Exercise 1.1

Ans:- There are 26 letters and 10 digits.
Hence, number of possible license plates

$$= C_{26,2} \times C_{10,4}$$

$$= \frac{26!}{24!\,2!} \times \frac{10!}{6!\,4!}$$

$$= \frac{26 \times 25 \times \cancel{24!}}{\cancel{24!}\ 2!} \times \frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}\ 4!}$$

$$= \frac{26 \times 25}{2 \times 1} \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$$

$$= 13 \times 25 \times 30 \times 7$$

$$= 13 \times 25 \times 210$$

$$= 68250$$

Exercise 1.2

Ans:- No of letters in PROBABILITY = 11
Since B & I are repeated 2 times each and all other letters are single,
Hence, the number of distinguishable permutations of letters in the word PROBABILITY

$$= \frac{11!}{2!\,2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2!}}{2! \times \cancel{2!}}$$

$$= 9979200$$

## ~~Exercise~~ Exercise 1.3

Ans:- There are 2 types of fruit and 5 days.

No of ways Judi puts one piece of fruit $= C_{5,2}$

$$= \frac{5!}{3!\,2!}$$

$$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!}$$

$$= \frac{\cancel{20}^{10}}{\cancel{2}_1 \times 1} = 10$$

## Exercise 1.4

Ans:- Ann needs to decide which 3 days are with oranges, $C_{8,3} = \frac{8!}{5!\,3!}$

$$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times 3!}$$

$$= \frac{8 \times 7 \times \cancel{6}^{2}}{\cancel{3}_1 \times \cancel{2}_1 \times 1} = 56 \text{ ways}$$

5 days are left.

She needs to decide which 2 days are with apples, $C_{5,2} = \frac{5!}{3!\,2!}$

$$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}\,2!}$$

$$= \frac{\cancel{20}^{10}}{\cancel{2}_1 \times 1} = 10 \text{ ways}$$

3 days are left and she needs to ~~decide~~ pick any other fruit other than apples or oranges, $C_{3,3} = \frac{\cancel{3!}^{1}}{\cancel{3!}_1} = 1$ way

$\therefore$ Total no of ways $= 56 \times 10 \times 1$
$= \underline{\underline{560}}$ ways

Exercise 1.5

Ans:- If Lili invites Kevin & Gerry, then she has to select the remaining 4 from 18 ways $= C_{18,4}$

$$= \frac{18!}{14!\,4!}$$

$$= \frac{18 \times 17 \times 16 \times 15 \times \cancel{14!}}{\cancel{14!}\,4!}$$

$$= \frac{\cancel{18}^{6\,3} \times 17 \times \cancel{16}^{4} \times 15}{4 \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 12 \times 17 \times 15$$
$$= \underline{\underline{3060}}$$

If Lili excludes Kevin & Gerry, all 6 are selected from 18 ways $= C_{18,6}$

$$= \frac{18!}{12!\,6!}$$

$$= \frac{\cancel{18}^{6} \times 17 \times \cancel{16} \times \cancel{15} \times 14 \times 13 \times \cancel{12!}}{\cancel{12!} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 6 \times 17 \times 1 \times 1 \times 14 \times 13$$

$$= \underline{\underline{18564}}$$

Hence, possible ways are :- $3060 + 18564 = \underline{\underline{21624}}$

Exercise 1.6

Ans:- Here, set $A = \{1, 2, \ldots\ldots 20\}$.
If we select 5 numbers as $x_1, x_2, x_3, x_4$ & $x_5$.

We need probability $Q(x_{smallest} > 6)$
$= Q(\text{All } x_i > 6)$
where $i = 1$ to $5$.

(a) When 5 numbers are ordered, so in the favourable space we need to select numbers from 7 to 20. i.e. 14 possibilities.

$\Rightarrow Q(x_{smallest} > 6)$ considering order $= \frac{14}{20} \times \frac{13}{19} \times \frac{12}{18} \times \frac{11}{17} \times$

$\frac{10}{16}$

$\Rightarrow 0.1291$

(b) When numbers are not in order $\Rightarrow$ total sample space $\Rightarrow C_{20,5}$

We need to select 5 out of 14 as we do not need to select any numbers from $\{1, 2, 3, 4, 5, 6\}$

$\Rightarrow \frac{C_{14,5}}{C_{20,5}}$

$$\Rightarrow \frac{\frac{14!}{9!\,5!}}{\frac{20!}{15!\,5!}}$$

$$\Rightarrow \frac{\frac{14\times13\times12\times11\times10\times\cancel{9!}}{\cancel{9!}\,\cancel{5!}}}{\frac{20\times19\times18\times17\times16\times\cancel{15!}}{\cancel{15!}\,\cancel{5!}}}$$

$$\Rightarrow 0.1291$$

Exercise 1.7

Ans, sample = $\{1, 2, \ldots, 20\}$

A = All five numbers are even.

Since even numbers are

$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$$P(\text{All five numbers are even}) = \frac{C_{10,5}}{C_{20,5}}$$

$$= \frac{\frac{10!}{5!\,\cancel{5!}}}{\frac{20!}{15!\,\cancel{5!}}}$$

$$\Rightarrow \frac{\frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}}}{\frac{20 \times 19 \times 18 \times 17 \times 16 \times \cancel{15!}}{\cancel{15!}}}$$

$$\Rightarrow \frac{\cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times \cancel{6}}{\cancel{20} \times 19 \times \cancel{18} \times 17 \times \cancel{16}}$$

$$\Rightarrow \frac{21}{19 \times 17 \times 4} \Rightarrow \frac{21}{19 \times 68}$$

$$\Rightarrow \frac{21}{1292}$$

Exercise 1.8

Ans, 6 couples = 6 men & 6 women

So, total people = 12

P( 4 people are chosen where two men and two women)

$$= \frac{C_{6,2} \times C_{6,2}}{C_{12,4}}$$

$$\Rightarrow \frac{\frac{6!}{4!\,2!} \times \frac{6!}{4!\,2!}}{\frac{12!}{8!\,4!}}$$

$$\Rightarrow \frac{\frac{6\times5\times\cancel{4!}}{\cancel{4!}\,2!} \times \frac{6\times5\times\cancel{4!}}{\cancel{4!}\times2!}}{\frac{12\times11\times10\times9\times\cancel{8!}}{\cancel{8!}\,4!}}$$

$$\Rightarrow \frac{\frac{\cancel{30}^{15}}{2} \times \frac{\cancel{30}^{15}}{2}}{\frac{\cancel{12}^{3}\times11\times\cancel{10}^{5}\times9}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}}$$

$$\Rightarrow \frac{\cancel{225}}{11\times\cancel{5}\times\cancel{9}\times1} = \frac{5}{11}$$

Exercise 1.9

Ans:- There are 200 trout. 50 are tagged.

$\therefore$ The probability that a trout is tagged is $\frac{\cancel{50}}{\cancel{200}_4} = 0.25$

We need to find $P(X=5)$ given $n=50$ and $p=0.25$.

The formula is $P(X=x) = C_{n,x} \times p^x \times (n-p)^{n-x}$

$$P(X=5) = C_{50,5} \times (0.25)^5 \times (1-0.25)^{50-5}$$

$$\Rightarrow \frac{(50 \times 49 \times 48 \times 47 \times 46)}{(5 \times 4 \times 3 \times 2 \times 1)} \times 0.25^5 \times 0.75^{45}$$

$$\Rightarrow 2{,}118{,}760 \times 0.25^5 \times 0.75^{45}$$

$$\Rightarrow 0.0049$$