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MA-485

HW-5

02/22/2022

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Exercise 4.1

Ans:- Let the outcome on the red and blue dice be $x$ and $y$.

$$\therefore P(A) = P(x=4) = \frac{1}{6}$$

$$P(B) = P(x+y=7) = P(x=1, y=6) + P(x=2, y=5) + P(x=3, y=4) + P(x=4, y=3) + P(x=5, y=2) + P(x=6, y=1)$$

$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{56}$$

$$= \frac{\cancel{6}\,1}{\cancel{36}\,6} = \frac{1}{6}$$

$P(A \cap B) = P(x=4, y=3) = \frac{1}{36}$

since $P(A) \times P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$,

which is equal to $P(A \cap B)$.

$\therefore$ Events A & B are independent.

Exercise 4.2

Ans:- A & B are independent.

$P(C) = P(A).P(\bar{B}) + P(\bar{A}).P(B)$

$= \frac{3}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$ → By symmetry.

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Computing the conditional probability :-

$$P(C \mid A) = \frac{P(C \cap A)}{P(A)} = \frac{\cancel{(3/6)} \cdot (3/6)}{\cancel{(3/6)}}$$

$$= \frac{1}{2}$$

$$= P(C)$$

$$P(C \mid B) = \frac{P(B \cap A)}{P(A)} = \frac{(3/6) \cdot (3/6)}{(3/6)}$$

$$= \frac{1}{2} = P(C)$$

$\therefore$ All the events are pairwise indep-
-endent.

For jointly independent, we must
have :-

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C).$$

$$P(A)\,P(B)\,P(C) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

For the probability of the intersection, we get :-

$P(A \cap B \cap C) = P(\phi) = 0$, because if both rolls are odd, the sum must be even.

$\therefore$ We see that they are not jointly independent.

Exercise 4.3

Ans. $P(A) = 0.7 \rightarrow$ probability of missile 1 hitting.

$P(B) = 0.85 \rightarrow$ probability of missile 2 hitting.

$P(C) = 0.9 \rightarrow$ probability of missile 3 hitting.

$\therefore P(A)^c = 1 - P(A)$

$P(A)^c = 0.3 \rightarrow$ probability of missile 1 not hitting.

$P(B)^c = 0.15 \rightarrow$ probability of missile 2 not hitting.

$P(C)^c = 0.1 \rightarrow$ probability of missile 3 not hitting.

$\therefore$ The probability that all missiles hit the target $= P(A) \times P(B) \times P(C)$

$\Rightarrow (0.7) \times (0.85) \times (0.9)$

$\Rightarrow 0.5355$

$\therefore$ The probability that at least one of them hits the target

$= 1 - P(A) \times P(B) \times P(C)$

$= 1 - 0.5355$

$= 0.4645$

The probability that exactly one hits the target:-

$$\Rightarrow P(A) \times P(B)^c \times P(C)^c + P(A)^c \times P(B) \times P(C)^c + P(A)^c \times P(B)^c \times P(C)$$

$$= 0.7 \times 0.15 \times 0.1 + 0.3 \times 0.85 \times 0.1 + 0.3 \times 0.15 \times 0.9$$

$$= \underline{\underline{0.0765}}$$

Exercise 4.4

Ans Let n be the number of trials required to get at least one 6.

Let X denote the number of 6s, which is a binomial distribution with $p = \frac{1}{6}$ & $q = \frac{5}{6}$.

$$P(X \geq 1) = 1 - P(X = 0)$$

[∵ Since minimum number = 07.

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$$0.95 = 1 - {}^{n}C_{0} \times \left(\frac{1}{6}\right)^{0} \times \left(\frac{5}{6}\right)^{n}$$

$$0.95 = 1 - \left(\frac{5}{6}\right)^{n}$$

$$0.05 = \left(\frac{5}{6}\right)^{n}$$

=> calculating value of n

$$0.05 = (0.833)^{n}$$

$$\left[\because \frac{5}{6} = 0.833\right]$$

$$\therefore (0.833)^{16} \simeq 0.05$$

$$\therefore \underline{n = 16}$$

$\therefore$ We need at least 16 ~~~~ trials of the dice, to get 6 with 95% possibility.

Exercise 4.5

Ans:- $P(B|A) = 1$

$P(B|A^c) = \frac{1}{3}$

P(she knows the answer) = 0.80
P(she doesn't know the answer)
= 0.20

To find :- P(she knows the answer | marks question correctly)

Let A = She knows the answer
$A^c$ = She doesn't know the answer
B = She marks the correct answer.
$B^c$ = She doesn't mark the correct answer.

To find: P(A|B).

$$P(A|B) = \frac{P(B|A).P(A)}{P(B|A).P(A) + P(B|A^c).P(A^c)}$$

[∵ Using Bayes' Theorem]

$$= \frac{1 \times 0.80}{0.80 + \frac{1}{3} \times 0.20} = \frac{0.80}{0.866}$$

$$= \underline{\underline{0.9230}}$$

∴ The probability of her knowing the answer, given that she marks the correct answer is :- $\underline{\underline{0.9230}}$

Exercise 4.6

Ans Probability of people that are Conservatives, $P(C) = 0.4$

Probability of people that are liberals, $P(L) = 0.45$

Probability of people that are Independents = 0.15

Probability of Conservatives who voted, $P(V|C) = 0.7$

Probability of Liberals who voted, $P(V|L) = 0.8$

Probability of Independents who voted, $P(V|I) = 0.5$

$$P(L|V) = \frac{P(L) \times P(V|L)}{P(V)}$$

$$P(V) = P(V|C) \times P(C) + P(V|L) \times P(L) + P(V|I) \times P(I)$$

$$P(V) = 0.7 \times 0.4 + 0.8 \times 0.45 + 0.5 \times 0.15$$

$$= 0.715$$

Thus by Bayes' theorem, the required probability is :-

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$$P(L|V) = \frac{P(L) \times P(V|L)}{P(V|C) \times P(C) + P(V|L) \times P(L) + P(V|I) \times P(I)}$$

Substituting the values, we get:-

$$P(L|V) = \frac{0.45 \times 0.8}{0.7 \times 0.4 + 0.8 \times 0.45 + 0.5 \times 0.15}$$

$$\therefore P(L|V) = \frac{0.45 \times 0.8}{0.715}$$

$$= 0.5035$$

Hence the probability that a voter selected at random is a liberal is 0.5035

Exercise 4.7

Ans. P( Actual cancer | positive result)

$$= \frac{P(\text{Actual} \cap \text{positive})}{P(\text{positive result})}$$

$$= \frac{\frac{1}{5000} \times 0.92}{\frac{1}{5000} \times 0.92 + \frac{4999}{5000} \times \frac{1}{5000}}$$

$$= \frac{0.000184}{0.000184 + \frac{(4999 \times 5000) \times 5000}{25000000}}$$

$$= \frac{0.000184}{0.000184 + 0.00019996}$$

$$= \frac{0.000184}{0.00038396}$$

$$= 0.4792$$

No it is not a good diagnos-
-tic procedure because there is
less than a 90% chance for a
person with a positive test
result to actually have cancer.

Exercise 4.8

Ans.:- The probability for a meteor
to strike UAB would probab-
-ly be around the same.