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NAME :- SHREYAS SRINIVASA

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HW4

Exercise 1.1

Ans:- Let X be the number on the top face of die 1 and Y be the number of the top face of die 2.

The probability of $X = x$, where $x = 1, 2, 3, 4, 5, 6$ is:-

$$P(X=1) = P(X=2) = P(X=3) = P(X=4) = P(X=5) = P(X=6) = \frac{1}{6}$$

The probability of $Y = y$ where $y = 1, 2, 3, 4, 5, 6$ is :-

$P(Y=1) = P(Y=2) =$

$P(Y=3) = P(Y=4) = P(Y=5) = P(Y=6)$
$= \frac{1}{6}$

When 2 dice are tossed, there are 2 totals which are divisible by 5 and they are 5, 10.

There are 2 combinations which give a total of 5 (2+3 and 3+2) and one combination which gives 10 (5+5)

The combinations that give a total that is divisible by 5 are
$\{(2,3), (3,2), (5,5)\}$

$P(\text{divisible by } 5) = P(X=2, Y=3)$
$+ P(X=3, Y=2)$
$+ P(X=5, Y=5)$

$= P(X=2)P(Y=3) +$
$P(X=3)P(Y=2) +$
$P(X=5)P(Y=5)$

because X and Y are independent.

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$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{3}{36}$$

The combination for which the total is divisible by 5 and both dice land on 5 is (5, 5).

The probability of getting a total that is divisible by 5 and the dice land on 5 is $\frac{1}{}$

$P(\text{both } 5 \cap \text{divisible by } 5)$

$= P(X = 5, Y = 5)$

$= P(X = 5)\, P(Y = 5)$

X and Y are independent

$= \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

The conditional probability that both the dice have landed on 5 given that the total of their top faces is found to be divisible by 5 is:-

P(both 5 | divisible by 5)

$$= \frac{P(\text{both } 5 \cap \text{divisible by } 5)}{P(\text{divisible by } 5)}$$

$$= \frac{1/36}{3/36}$$

$$= \frac{1}{3}$$

The probability that both of them have landed on 5 is $\frac{1}{3}$

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Exercise 1.2

Ans:- (a) P(First Head | 2 Heads)

$$= \frac{n(\text{First Head} \cap 2 \text{ Heads})}{n(2 \text{ Heads})}$$

$$= \frac{n\{HHT, HTH\}}{n(\{HHT, HTH, THH\})}$$

$$= \frac{2}{3}$$

(b) P(First Head | ≥ 2 Heads)

$$= \frac{P(\text{First Head} \cap \geq 2 \text{ Heads})}{P(\geq 2 \text{ Heads})}$$

$$= \frac{n\{HHT, HTH, HHH\}}{n(\{HHT, HTH, THH, HHH\})}$$

$$= \frac{3}{4}$$

Exercise 1.3

Ans:- Given event that :-
A = Married man votes
B = Married woman votes

$P(B) = 0.5 \quad P(A) = 0.4$
$P(B|A) = 0.7$

(a) Probability that a married husband and ~~wife~~ wife both vote, i.e. $P(A \cap B)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B|A) \cdot P(A)$$

$$\Rightarrow P(A \cap B) = 0.7 \times 0.4$$
$$\Rightarrow P(A \cap B) = 0.28$$

Required probability is 0.28

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(b) Probability that a man votes given that his wife votes, i.e. $P(A|B)$

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.28}{0.5}$$

$$\Rightarrow P(A|B) = 0.56$$

Required Probability = 0.56

Exercise 1.4

Ans. Given :-

A : Patient in hospital with myocardial infarction.

B : Patient in hospital have had strokes.

$\therefore P(B|A) = 0.20, \; P(B|A') = 0.35$

$P(A) = 0.40$

$P(A') = 0.60$

Probability of patients that have had strokes :-

$P(B) = P(B \cap A) + P(B \cap A')$

$P(B) = P(A) . P(B|A) + P(A') . P(B|A')$

$P(B) = 0.40 \times 0.20 + 0.60 \times 0.35$

$P(B) = 0.08 + 0.21$

$P(B) = 0.28$

$\therefore$ Percentage of patients have had strokes is 28 %.

Exercise 11.5

Ans: Given :-

E_1 :- Output from machine I

E_2 :- Output from machine II

E_3 :- Output from machine III (9)

E_1, E_2 & E_3 are mutually disjoint events.

& A :- output is defective.

$\therefore P(E_1) = 0.50$; $P(E_2) = 0.30$;
$P(E_3) = 0.20$

& $P(A|E_1) = 0.04$, $P(A|E_2) = 0.02$
$P(A|E_3) = 0.03$.

Probability of total defective, i.e. $P(A)$.

By using Baye's theorem.

$$P(A) = \sum_{i=1}^{3} P(E_i) . P(A|E_i)$$

$$P(A) = P(E_1) . P(A|E_1) + P(E_2) . P(A|E_2) + P(E_3) . P(A|E_3)$$

$$\Rightarrow P(A) = 0.50 \times 0.04 + 0.30 \times 0.02 + 0.20 \times 0.03$$

$P(A) = 0.02 + 0.006 + 0.006$

$P(A) = 0.032$

∴ The proportion of total output that is defective is 0.032

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