NAME :- SHREYAS SRINIVASA
BLAZER ID :- SSRINIVA

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MA-485
MIDTERM TEST #2

Problem 1

Ans:- Possible values of the two dices rolled :-

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

Let X = Sum of 2 dices

$$X = \{2, 3, 4, 5, 6, 7, 3, 4, 5, 6, 7, 8, 4, 5, 6, 7, 8, 9, 5, 6, 7, 8, 9, 10, 11, 7, 8, 9, 10, 11, 12\}$$

Probability function for X :-

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Problem 2

Note :- Will come back to it later.

Problem 3

Ans:- given pdf is :-

$$f(x) = \begin{cases} c(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) We know that for valid pdf,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_0^2 c(2-x)\, dx = 1$$

$$c\left[2x - \frac{x^2}{2}\right]_0^2 = 1$$

$$c[4-2] = 1$$

$$c = \frac{1}{2}$$

(b) CDF of X is $F(x) = P(X \le x)$
$$= \int_0^x f(t)\, dt$$

$$= \int_0^x \frac{1}{2}(2-t)\,dt$$

$$= \frac{1}{2}\left[2t - \frac{t^2}{2}\right]_0^x$$

$$= \frac{1}{2}\left(2x - \frac{x^2}{2}\right)$$

$$= \begin{cases} 0 & ; \quad x < 0 \\ \frac{1}{2}\left(2x - \frac{x^2}{2}\right) & ; \quad 0 \le x < 2 \\ 1 & ; \quad x \ge 2 \end{cases}$$

(1c) $P(X > 1) = 1 - P(X \le 1)$

$$= 1 - F(1)$$

$$= 1 - \frac{1}{2}\left(2 - \frac{1}{2}\right)$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

Problem 4

Ans: Let X be a random variable.

Let $X \sim U(0,1)$, pdf of X

$$f_X(x) = \begin{cases} 1 & ; \ 0 \le x \le 1 \\ 0 & ; \ \text{otherwise} \end{cases}$$

cdf of X is :-

$F_X(x) = x$

Let $Y = \frac{1}{\sqrt{x}}$

(a) range of $x = 0 \to 1$

$y = \frac{1}{\sqrt{x}} : 1 \to \infty$

(b) cdf of y is :-

$$F_Y(y) = P(Y \le y)$$

$$= P\left(\frac{1}{\sqrt{x}} \le y\right)$$

$$= P\left(\sqrt{x} > \frac{1}{y}\right)$$

$$= 1 - P\left(\sqrt{x} \le \frac{1}{y}\right)$$

$$= 1 - P\left(X \le \frac{1}{y}\right)$$

$$= 1 - F_X\left(\frac{1}{\sqrt{y}}\right)$$

$$= 1 - \frac{1}{\sqrt{y}}$$

$$\therefore F_y(y) = 1 - y^{-\frac{1}{2}}$$

(c) pdf of y is :-

$$f_y(y) = \frac{d}{dy}\left(1 - y^{-\frac{1}{2}}\right)$$

$$= 0 - \left(-\frac{1}{2}\right) y^{-\frac{3}{2}}$$

$$f_y(y) = \frac{1}{2} y^{-\frac{3}{2}} \quad ; \quad 1 < y < \infty$$

Problem 5

Note :- Will come back to it later.

Problem 6

Ans. $Z \sim N(0,1)$, $X \sim N(-2,4)$

(a) $P(Z > -1.52) = P(Z < 1.52)$

$$= 0.5 + P(0 < Z < 1.52)$$
$$= 0.5 + 0.4357$$
$$= 0.9357$$

(b) $P(|X| < 2.8) = P(-2.8 < X < 2.8)$

$$= P\left(\frac{-2.8+2}{2} < \frac{X-4}{6} < \frac{2.8+2}{2}\right)$$

$$= P(-0.4 < Z < 2.4)$$

$$= P(0 < Z < 0.4) + P(0 < Z < 2.4)$$

$$= 0.1554 + 0.4918$$

$$= 0.6472$$

(c) $P(Z < 1.5 \mid Z > 0.2) = \dfrac{P(Z < 1.5 \cap Z > 0.2)}{P(Z > 0.2)}$

$$= \frac{P(0.2 < Z < 1.5)}{P(Z > 0.2)}$$

$$= \frac{P(0 < Z < 1.5) - P(0 < Z < 0.2)}{0.5 - P(0 < Z < 0.2)}$$

$$= \frac{0.4332 - 0.0793}{0.5 - 0.0793}$$

$$= \frac{0.3539}{0.4207} = 0.8412$$

(iii) $y = 4x + 3$

$$E(y) = E(4x+3)$$
$$= 4E(x) + 3$$
$$= 4 \times (-2) + 3$$
$$= -8 + 3$$
$$= -5$$

$$V(y) = 4^2 V(x)$$
$$= 4^2 \times 4$$
$$= 64$$

$$\Rightarrow Y \sim N(-5, 64)$$

Problem 5

Ans: Given $X \sim$ Exponential (λ)

such that $t_{1/2} = 6$ years.

$$\therefore P(X > t_{1/2}) = \frac{1}{2}$$

$$\Rightarrow \int_6^\infty \lambda e^{-\lambda x}\, dx = \frac{1}{2} \quad [\because t_{1/2} = 6]$$

$$\Rightarrow \left[\frac{\lambda e^{-\lambda x}}{-\lambda} \right]_6^\infty = \frac{1}{2} \quad \left[\because \int e^{mx} = \frac{e^{mx}}{m}\right]$$

$$\Rightarrow -e^{-\infty} + e^{-6\lambda} = \frac{1}{2}$$

$$\Rightarrow e^{-6\lambda} = \ln\left(\frac{1}{2}\right)$$

$$\Rightarrow \lambda = \frac{\ln 1 - \ln 2}{-6} \quad \left[\because \ln(a/b) = \ln a - \ln b\right]$$

$$\Rightarrow \lambda = \frac{\ln 2}{6}$$

now, $P(X > x) = \int_x^{\infty} \lambda e^{-\lambda x}\, dx$

$$= \left[\frac{\lambda e^{-\lambda x}}{-\lambda}\right]_x^{\infty}$$

$$= -e^{-\infty} + e^{-\lambda x}$$

$$= e^{-\lambda x} \quad \left[\because e^{-\infty} = 0\right]$$

now, $P(X > 18) = e^{-\lambda \times 18}$

$$= e^{-\frac{\ln 2}{\cancel{6}_1} \times \cancel{18}^{3}}$$

$$= e^{-3\ln 2}$$

$$= \underline{\underline{0.125}}$$

$$P(X \geq 14 \mid X > 8) = \frac{P(X \geq 14 \cap X > 8)}{P(X > 8)}$$

$$= P(X \geq 14) \mid P(X > 8)$$

$$= e^{-\lambda \times 14} \mid e^{-\lambda \times 8}$$

$$= e^{-\lambda(14-8)}$$

$$= e^{-\frac{\ln 2}{6} \times 6}$$

$$= e^{-\ln 2}$$

$$= \underline{\underline{0.5}}$$

<u>Problem 2</u>

<u>Ans:</u> (a) The probability of having exactly 2 winning tickets here is computed as:-

$P(X = 2)$ for X = binomial ($n = 200, p = 0.01$)

$$P(x=2) = \binom{200}{2} 0.01^2 \times 0.99^{198}$$

For poisson approximation, mean = np

$$= 200 \times 0.01$$
$$= \underline{\underline{2}}$$

$\therefore$ $P(X=2)$ for Poisson $(2) = \left(\frac{2^2}{2}\right) e^{-2}$

$\Rightarrow$ $\underline{\underline{0.2707}}$ is the required probability

(ii) The chance of winning anything here is computed as:- $P(X \geq 1)$

$= 1 - P(X = 0)$

$= 1 - 0.99^{200}$

Using poisson approximation :-

$1 - e^{-2} = 0.8647$ is the required probability here