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MA - 489

HW - 7

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Exercise 1.7

Ans:- $F(x) = e^{-\frac{1}{x}}$ for $x > 0$

~~[illegible]~~ If F is a distribution function of the random variable x, then :-

(i) $0 \leq F(x) \leq 1$

(ii) $F(x) < F(y)$ when $x < y$

(iii) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$ and $F(\infty) = \lim_{x \to \infty} F(x) = 1$

Given that :- $F(x) = \begin{cases} e^{-1/x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

Clearly, F(x) satisfies all the above properties.

Hence, it is a distribution function.

In this case, there is a density function obtained by differentiating :-

$$\frac{d}{dx} F(x) = \frac{d}{dx}\left(e^{-\frac{1}{x}}\right) = e^{-\frac{1}{x}} \frac{d}{dx}\left(-\frac{1}{x}\right)$$

$$= e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)$$

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Hence, the density function is $\begin{cases} \frac{1}{x^2} \cdot e^{-\frac{1}{x}} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

Exercise 1.2

Ans:- Suppose X has density function :-

$$f(x) = \begin{cases} c(1-x^2) & , -1 < x < 1 \\ 0 & , \text{elsewhere.} \end{cases}$$

One of the properties of density function is :-

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\Rightarrow c\int_{-1}^{1} (1-x^2)\, dx = 1$$

$$\Rightarrow c\left[\int_{-1}^{1} 1\, dx - \int_{-1}^{1} x^2\, dx\right] = 1$$

$$\Rightarrow c\left[2 - \left(\frac{x^3}{3}\right)_{-1}^{1}\right] = 1$$

$$\Rightarrow c\left[2 - \left(\frac{1}{3} + \frac{1}{3}\right)\right] = 1$$

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$$\Rightarrow \quad k\left(\frac{4}{3}\right) = 1$$

$$\Rightarrow \quad k = \frac{3}{4}$$

$\therefore$ The density of X is :-

$$f(x) = \begin{cases} \frac{3}{4}(1-x^2) & , -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

The distribution function of X is

$$F_X(x) = P(X \le x)$$

$$= \frac{3}{4}\int_{-1}^{x} (1-x^2)\, dx$$

$$= \frac{3}{4}\left[\int_{-1}^{x} 1\, dx - \int_{-1}^{x} x^2\, dx\right]$$

$$= \frac{3}{4}\left[(x+1) - \left(\frac{x^3}{3}\right)_{-1}^{x}\right]$$

$$= \frac{3}{4}\left[(x+1) - \left(\frac{x^3}{3} + \frac{1}{3}\right)\right]$$

$$= \frac{3}{4}\left[\left(x - \frac{x^3}{3} + \frac{2}{3}\right)\right]$$

$$= \frac{3}{4}\left(x - \frac{x^3}{3}\right) + \frac{1}{2}$$

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$$P(X > 0.5) = 1 - P(X \leq 0.5)$$

$$= 1 - \frac{3}{4}\int_{-1}^{0.5} (1 - x^2)\, dx$$

$$= 1 - \frac{3}{4}\left[\int_{-1}^{0.5} 1\, dx + \int_{-1}^{0.5} -x^2\, dx\right]$$

$$= 1 - \frac{3}{4}\left[1.5 - \int_{-1}^{0.5} x^2\, dx\right]$$

$$= 1 - \frac{3}{4}(1.125)$$

$$= \underline{0.15625}$$

$$P(0 < X < 0.5) = \frac{3}{4}\int_{0}^{0.5} (1 - x^2)\, dx$$

$$= \frac{3}{4}\left[\int_{0}^{0.5} 1\, dx + \int_{0}^{0.5} -x^2\, dx\right]$$

$$= \frac{3}{4}\left[0.5 - \int_{0}^{0.5} x^2\, dx\right]$$

$$= \frac{3}{4}(0.45833)$$

$$= \underline{0.34375}$$

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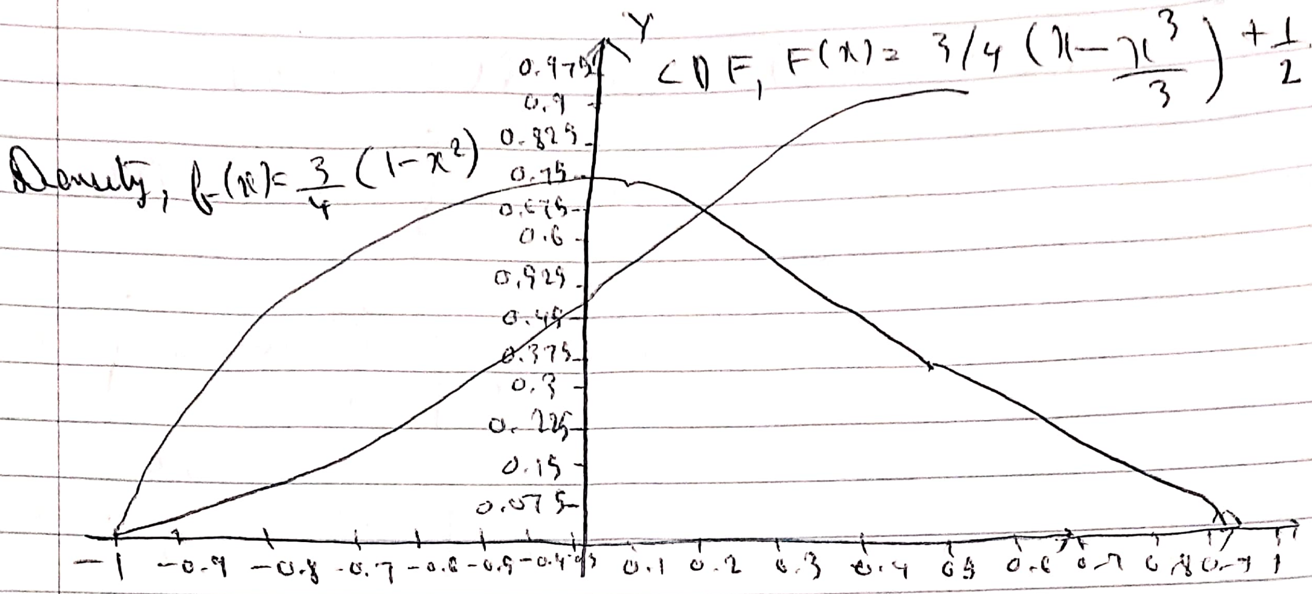

Exercise 1.3

Ans: In the interval from 0 to 1, the longer segment will be at least twice as long as the shorter segment, when the point on this segment is either in the range 0 to 0.33 or in the range 0.67 to 1.

If the point is in the range 0 to 0.33, the second segment from 0.33 to 1 is twice as big as the first segment from 0 to 0.33.

If the point is in the range 0.67 to 1, the first segment from 0 to 0.67 which is twice as big as the second segment which is from 0.67 to 1.

Is this correct reasoning?

Exercise 2.1

Ans Here, parameter $\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{0.693}{3}$

$= 0.231$

(a) $P(X > 9) = e^{-\lambda x}$

$= e^{-0.231 \times 9}$

$= 0.125$

(b) $P(X > 14 \mid X > 8) = \frac{e^{-0.231 \times 14}}{e^{-0.231 \times 8}}$

$= 0.25$

Exercise 2.2

Ans:- $X \sim \exp.(\lambda)$; $\lambda > 0$.

Then, probability density function of X is :-

$f_X(x) = \lambda e^{-\lambda x}$; $x > 0$; $\lambda > 0$.

$P(X > \frac{2}{\lambda}) = \int_{2/\lambda}^{\infty} \lambda e^{-\lambda x} = \lambda \left[\frac{e^{-\lambda x}}{-\lambda} \right]_{\frac{2}{\lambda}}^{\infty}$

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$$= e^{-\lambda \times \frac{2}{\cancel{x}}}$$

$$= \underline{e^{-2}}$$

$$P\left(\left|X - \frac{1}{\lambda}\right| < \frac{2}{\lambda}\right) = P\left(-\frac{1}{\lambda} < X < \frac{3}{\lambda}\right)$$

$$= P\left(0 < X < \frac{3}{\lambda}\right);$$

$$X \geq 0$$

$$= \int_0^{3/\lambda} \lambda e^{-\lambda/x}$$

$$= \left[-e^{-\lambda \cancel{x}}\right]_0^{3/\lambda}$$

$$= \underline{1 - e^{-3}}$$