03/10/2021

SP'21 PHYS-230

TEST 1

NAME :- SHREYAS SRINIVASA

PALOMAR ID :- 012551187

Q1. Ans: $v(t) = a t e^{wt^2}$

(a) Velocity (v) = m/s.

$(a \times sec) . e^{w \times s^2}$ [∵ t = sec]

$\left(\frac{m}{sec^2} \times sec\right) e^{\frac{1}{s^2} \times s^2}$

$= \frac{m}{sec} . e^1$ [where e is constant]

∴ The unit of $a = m/s^2$

& the unit of $w = 1/s^2$

(b) $v(t) = a t e^{wt^2}$

$\frac{dx}{dt} = a t e^{wt^2}$ [∵ $v = dx/dt$]

$\Rightarrow dx = at e^{wt^2} dt$

$\Rightarrow \int dx = a \int_0^t t e^{wt^2} dt$

(1)

P.T.O

$$x(t) = a\int_0^{t^2} e^{\mu w y}\,\frac{dy}{2}$$

$$x(t) = \frac{a}{2}\left(\frac{e^{\mu w y}}{\mu w}\right)_0^{t^2}$$

$$x(t) = \frac{a}{2\mu w}\left(e^{\mu w t^2} - e^0\right)$$

$$x(t) = \frac{a}{2\mu w}\left(e^{\mu w t^2} - 1\right)$$

$t^2 = y$

$2t\,dt = dy$

$t\,dt = \frac{dy}{2}$

$t = 0,\ y = 0$

$t = t,\ y = t^2$

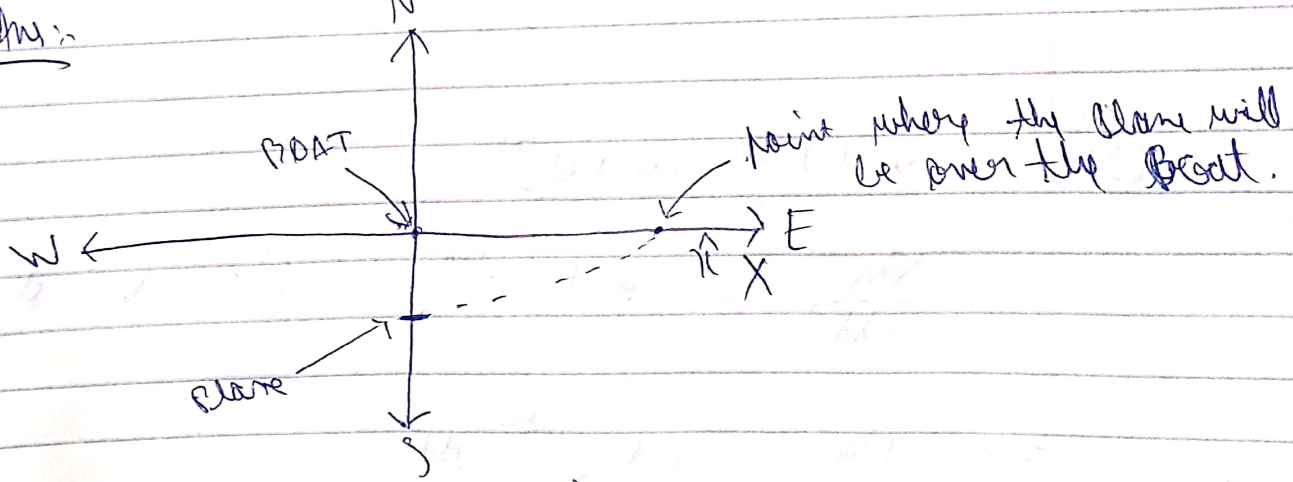
(c) $v(t) = a\,t\,e^{\mu w t^2}$

$$\vec{a}(t) = \frac{dv}{dt} = a\left[t\,e^{\mu w t^2}(\mu w \times 2t) + e^{\mu w t^2}(1)\right]$$

$$\vec{a}(t) = a e^{\mu w t^2}\left(t(2\mu w t) + 1\right)$$

$$\vec{a}(t) = a e^{\mu w t^2}\left(2\mu w t^2 + 1\right)$$

Q2. Ans:-



Velocity of Plane, $\vec{V}_p = 39\,\hat{x}$ m/s

Velocity of Wind, $\vec{V}_w = -25\,\hat{x} + 15\,\hat{y}$ m/s

P.T.O

(3)

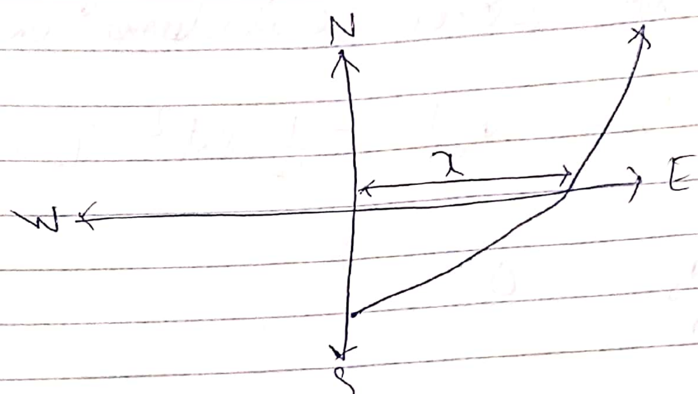
Resultant velocity of the plane due to wind,

$$\vec{V}_p = \vec{V}_p + \vec{V}_w$$

$$= (39-25)\hat{x} + (0+15)\hat{y}$$

$$\vec{V}_p = V_{px}\hat{x} + V_{py}\hat{y}$$

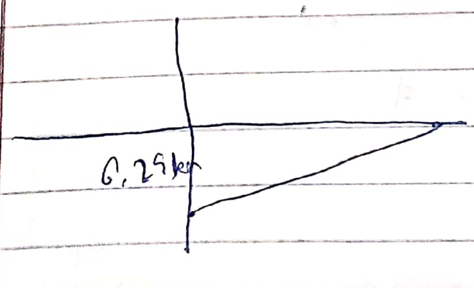
$$= (14\hat{x} + 15\hat{y}) \text{ m/s}$$



Trajectory of Plane

Velocity of Boat, $\vec{V}_B = (V_x \hat{x})$ m/s

(a) Only x could be the shortest distance, since Time taken (when plane will be over the boat) = Time taken by plane to cover 6.25 km distance in Y direction.



In Y direction, velocity of Plane, $V_{py} = 15$ m/s

$$t = \frac{d}{v} = \frac{6250 \text{ m}}{15 \text{ m/s}} = 416.67 \text{ s}$$

In the time t, distance travelled by plane in X direction,

$$X = V_{px} \times t = (14 \times 416.67)\ m$$
$$= 5833.33\ m$$
$$= 5.83\ km$$

(b) To intersect the path of the plane, the boat has to cover x distance in time t.

$$\therefore\ S = v_0 t + \frac{1}{2} a t^2 \quad [\because \text{3rd equation of motion}]$$

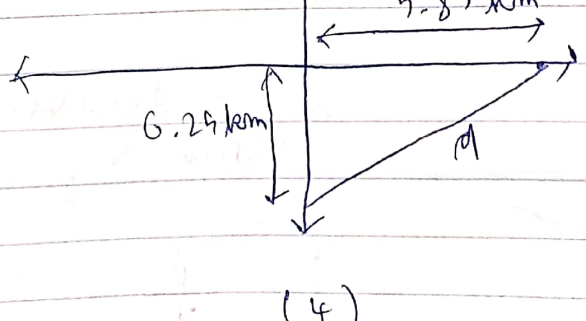
Initial velocity, $V_0 = 0$

$$\therefore\ x = 0 + \frac{1}{2} a t^2$$

$$\therefore\ a_B = \frac{2x}{t^2}$$

$$a_B = 0.672\ m/s^2 \text{ in } x \text{ direction}$$

(c)



(7)

P.T.O

Total distance covered by plane,

$$d = \sqrt{(5.83)^2 + (6.25)^2}$$

$$= \underline{\underline{8.547 \text{ km}}}$$

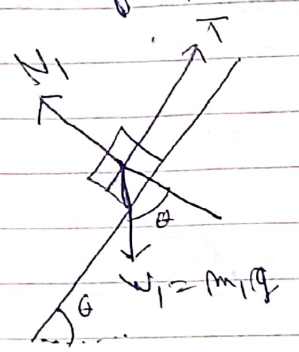
Q3. Ans:- (a) Free body diagram (F.B.D) for block 1 :-

Forces acting on M_1 :-

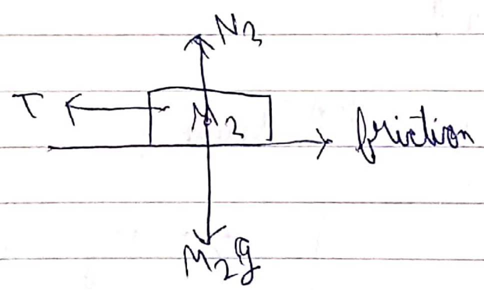
(i) Gravitational force, $W_1 = m_1 g$

(ii) Contact force i.e. normal force $= N_1$

(iii) Tension force $= T$



(b) F.B.D. for block 2 :-



Forces acting on M_2 :-

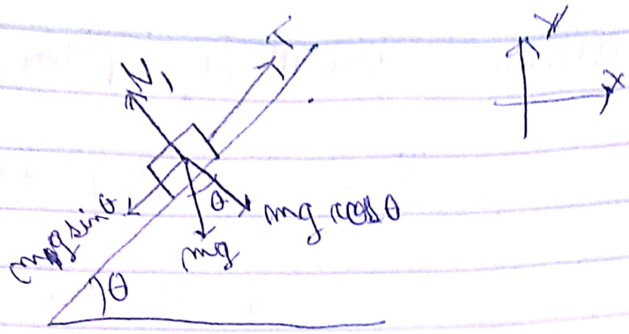
(i) $W_2 = m_2 g$

(ii) N_2 = normal force on M_2 due to support.

(iii) Tension force (T)

(iv) friction = f

(c)



For Equilibrium,
From Newton's 2nd law,

$$\Sigma F_x = 0$$

$$T - m_1 g \sin\theta = 0$$

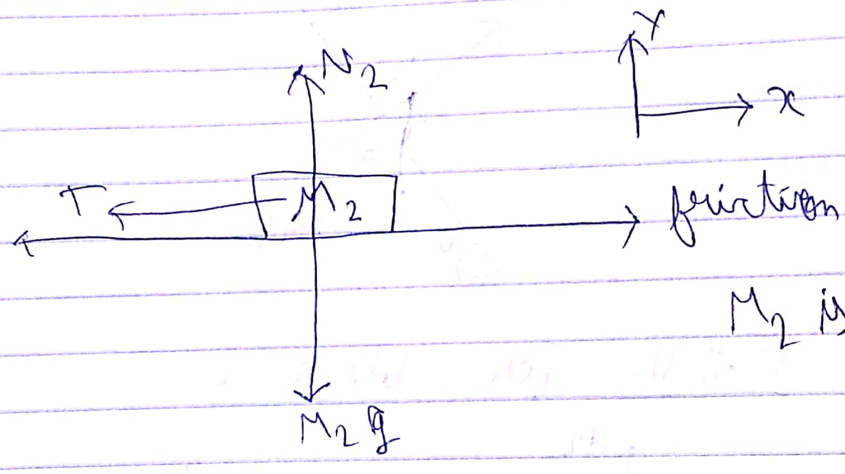
$$\boxed{T = m_1 g \sin\theta} \rightarrow (1)$$

$$\Sigma F_y = 0$$

$$N_1 - mg\cos\theta$$

$$\boxed{N_1 = mg\cos\theta} \quad - (2)$$

(d)



M₂ is in Equilibrium

From Newton's 2nd law :-

$$\Sigma F_x = 0$$

$$T - \text{friction} = 0$$

$$T = \text{friction}$$

$$\Sigma F_y = 0$$

$$N_2 - M_2 g = 0$$

$$N_2 = M_2 g$$

P.T.O

Here T = static friction

Static friction $= M_1 g \sin\theta$

$f_{static} = M_1 g \sin\theta$

Q 4. Ans: (a) The particle is speeding up, during the intervals from 0 s to 3 s & from 5 s to 6 s respectively, since acceleration is positive in these intervals.

(b) The particle is slowing down during the interval from 8 s to 12 s, since acceleration is negative.

(c) The particle is in constant motion during the time interval from 3 s to 5 s & from 6 s to 8 s respectively, since there is no change in speed.

(d) At $t = 1.5$ s & $t = 10$ s, the speed is zero, since the particle is on the x-axis at these points, which means that y is 0.

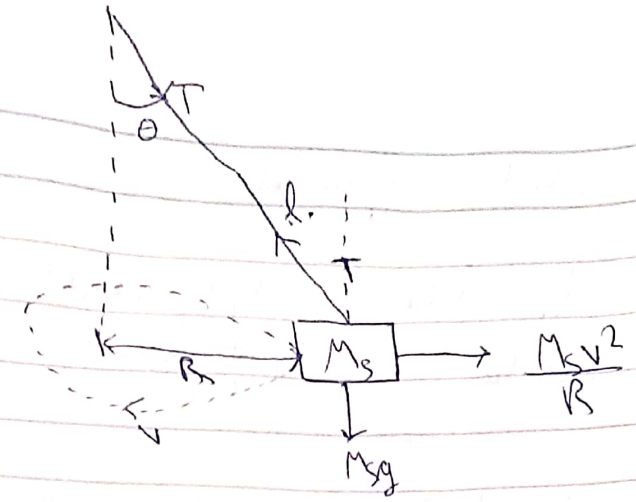
(e) Area of v-t graph $= 10 \times 1 \times 1 + 0.5$

$= 10.5$ m

∴ The Displacement from 4 s to 12 s is 10.5 m

Q 5. Ans:- (a) Free body diagram (F.B.D) of rubber stopper :-

P.T.O

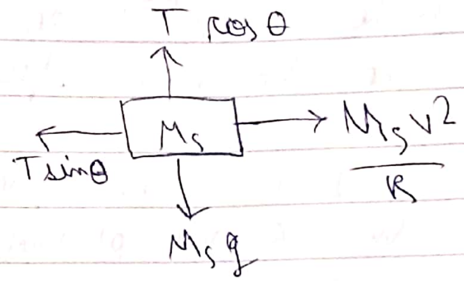


(c) Tension in the string = Weight of the hanging block

$$= \frac{234}{1000} \text{ kg} \times 9.8 \text{ m/s}^2$$

$$= \underline{\underline{2.29 \text{ N}}}$$

(d) $R = l \sin\theta$



Balancing vertical forces :-

$$T \cos\theta = M_s g = 0.145 \times 9.8$$

$$2.29 \cos\theta = 1.421$$

$$\cos\theta = 0.620$$

$$\theta = \cos^{-1}(0.620)$$

$$\theta = \underline{\underline{51.65^\circ}}$$

Balancing Horizontal forces :-

P.T.O

$$T \sin\theta = \frac{M_s v^2}{R} = \frac{M_s v^2}{l \sin\theta}$$

$\therefore$ Speed of the rubber stopper, $v = \sqrt{\frac{T \sin^2\theta \times l}{M_s}}$

$$= \sqrt{\frac{2.29 \times \sin^2(51.65) \times 0.75}{0.145}}$$

$$v = 2.699 \text{ m/s}$$