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Homework 11 :- Fluid Mechanics

41. $\rho_{Gold} = 1.93 \times 10^4 \ kg/m^3$

$m = 31.103 g$

$= (31.103 g)\left(\frac{1 kg}{10^3 g}\right) = 31.103 \times 10^{-3} kg$

Using $\rho = \frac{m}{V}$

Substitute $1.93 \times 10^4 \ kg/m^3$ for ρ, $31.103 \times 10^{-3} kg$ for m in the above expression of density and solve for V as,

$$1.93 \times 10^4 \ kg/m^3 = \frac{31.103 \times 10^{-3} kg}{V}$$

$$V = \frac{31.103 \times 10^{-3} kg}{1.93 \times 10^4 \ kg/m^3} = 1.61 \times 10^{-6} m^3$$

$$= (1.61 \times 10^{-6} m^3)\left(\frac{10^6 cm^3}{1 m^3}\right)$$

$$= 1.61 \ cm^3$$

Hence volume of 1 troy ounce of pure gold is 1.61 cm

45.

Using $p = p_o + \rho g h$

$p = F/A$ and $\vec{F} = m\vec{a}$

$\rho_{coffee} = 1000\ kg/m^3$

$h = 7.5\ cm$.

$= (7.5\ cm)\left(\frac{1\ m}{10^2\ cm}\right) = 7.5\times10^{-2}\ m$

Substitute $1000\ kg/m^3$ for ρ, $9.8\ m/s^2$ for g, $7.5\times10^{-2}\ m$ for h in the exp. of p (take $p_o = 0$) and solve as,

$$p = (1000\ kg/m^3)(9.8\ m/s^2)(7.5\times10^{-2}\ m) = 735\ N/m^2$$

$m = 375\ g$

$= (375\ g)\left(\frac{1\ kg}{10^3\ g}\right) = 375\times10^{-3}\ kg$

Substitute $375\times10^{-3}\ kg$ for m, $9.8\ m/s^2$ for g in the exp. of force and solve as,

$$F = (375\times10^{-3}\ kg)(9.8\ m/s^2) = 3.67\ N$$

$A = \pi r^2$

Substitute $3.67\ N$ for F and value of area from above calculation and solve as,

$$p = \frac{3.67\ N}{(\pi) r^2}$$

Equate the two eq. of pressure and solve for r as,

$$735\ N/m^2 = \frac{3.67\ N}{(\pi) r^2}$$

$$r^2 = \frac{3.67\ N}{(735\ N/m^2)\pi}$$

$$r = \sqrt{\frac{3.67\ N}{(735\ N/m^2)\pi}} = 0.0399\ m$$

$$= (0.0399\ m)\left(\frac{10^2\ cm}{1\ m}\right) = 3.99\ cm$$

Hence inside radius of the coffee mug is 3.99 cm

51. $P = P_0 + \rho g h$

$\vec{F} = m\vec{a}$

$\rho_{mercury} = 1.36 \times 10^4\ kg/m^3$

$1\ atm = 101325\ N/m^2$ and $P_0 = 0$

Substitute $101325\ N/m^2$ for P, $9.8\ m/s^2$ for g, $1.36 \times 10^4\ kg/m^3$ for ρ in the above exp. and solve for h as,

$$101325\ N/m^2 = (1.36 \times 10^4\ kg/m^3)(9.8\ m/s^2)h$$

$$h = \frac{101325\ N/m^2}{(1.36 \times 10^4\ kg/m^3)(9.8\ m/s^2)}$$

$$= 0.760\ m$$

$$= (0.760\ m)\left(\frac{10^3\ mm}{1\ m}\right) = 760\ mm$$

Hence the height of mercury column for 1 atm pressure is 760 mm.

52. $P = h\rho g$ ...... (1)

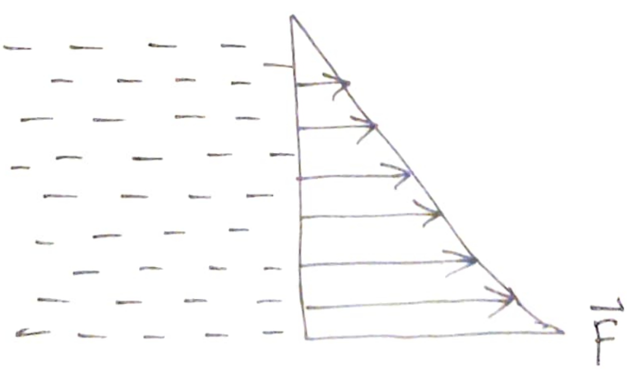
$$h = (11.0\,km)\left(\frac{10^3\,m}{1\,km}\right) = 1.10\times10^4\,m$$

Substitute $(1.10\times10^4\,m)$ for h, $(1030\,kg/m^3)$ for ρ and $(9.80\,m/s^2)$ for g in eq.(1) and calculate the pressure exerted

$$P = h\rho g$$
$$= (1.10\times10^4\,m)(1030\,kg/m^3)(9.80\,m/s^2)$$
$$= 1.11\times10^8\,Pa$$

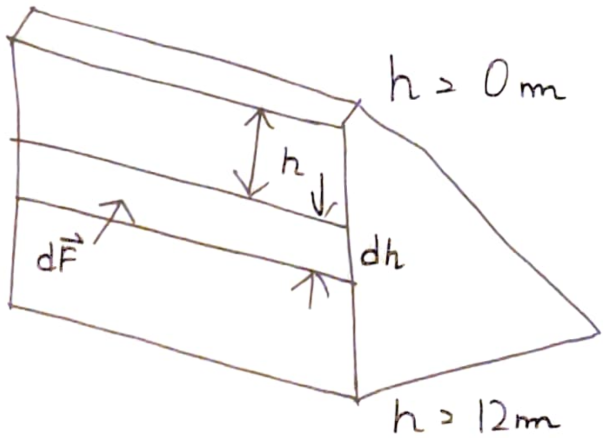
The pressure exerted by the ocean at the bottom of Mariana's Trench is 1.11×10^8 Pa

55. Using $\frac{dp}{dh} = -\rho g$, $p = p_0 + \rho g h$ and $p = F/A$



(a) $dA = (w)(dh)$

$P = \rho(g)(h)$



$$\rho(g)(h) = \frac{dF}{(w)(dh)}$$

$$dF = \rho(g)(w)(h)(dh)$$

$$\int dF = \int \rho(g)(w)(h)(dh)$$

$$F = \rho(g)(w)\int_{0\,m}^{12\,m} h\,dh = \rho(g)(w)\left[\frac{h^2}{2}\right]_{0\,m}^{12\,m}$$

Substitute $1000\ kg/m^3$ for ρ, $9.8\ m/s^2$ for g, $10\ m$ for w in the above ~~cal~~ calculation and solve as,

$$F = (1000\ kg/m^3)(9.8\ m/s^2)(10\ m)\left[\frac{144}{2}\ m^2\right]$$

$$= 7.06 \times 10^6\ N$$

Hence the net force acting on the dam is $7.06 \times 10^6\ N$

(b) Pressure in a fluid increases with depth and so does its resultant force. Hence in order for dam to sustain the pressure force, the thickness of the dam wall increases with depth.

57. $p = p_0 + \rho g h$

$h = 300 \text{ mm}$

$= (300 \text{ mm})\left(\dfrac{1 \text{ m}}{10^3 \text{ mm}}\right) = 0.3 \text{ m}$

$\rho_{mercury} = 1.36 \times 10^4 \text{ kg/m}^3$

Substitute $1.36\times10^4 \text{ kg/m}^3$ for ρ, 0.3 m for h in above exp. for p and solve as,

$p = (1.36 \times 10^4 \text{ kg/m}^3)(g)(0.3 \text{ m})$

$= (4080 \text{ kg/m}^2)(g)$

$\rho_{water} = 1000 \text{ kg/m}^3$

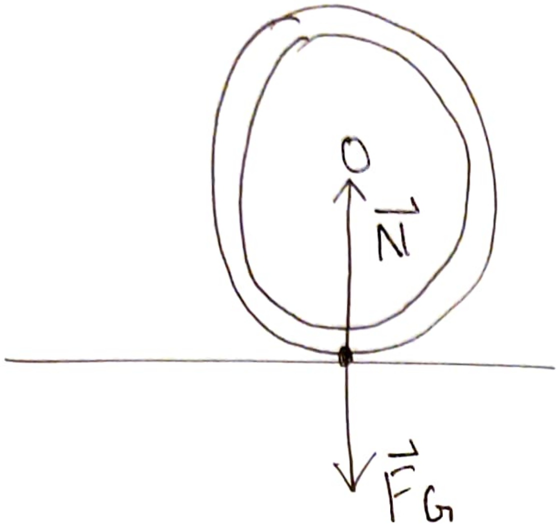
Substitute 1000 kg/m^3 for ρ and equate the obtained pressure in the exp. of pressure and solve for h as,

$(4080 \text{ kg/m}^2)(g) = (1000 \text{ kg/m}^3)(g)h$

$$h = \frac{4080 \text{ kg/m}^2}{1000 \text{ kg/m}^3} = 4.08 \text{ m}$$

Hence in order to measure the blood pressure of 300 mm, the water column ~~to~~ must be 4.08 m tall.

58. Using $p = F/A$ and $\vec{F} = m\vec{a}$



Substitute 80.0 kg for m, 9.81 m/s^2 for g in the exp. of force

$F = (80.0\,kg)(9.81\,m/s^2) = 784.8\,N$

$N = 784.8\,N$

Substitute $3.50 \times 10^5\,N/m^2$ for p, 784 N for F in the exp. of p and solve for A as,

$$3.50 \times 10^5\,N/m^2 = \frac{784\,N}{A}$$

$$A = \frac{784\,N}{3.50 \times 10^5\,N/m^2} = 2.2411 \times 10^{-3}\,m^2$$

Hence the area in contact with the ground is $2.2411 \times 10^{-3}\,m^2$

65. (a) $f = \frac{\rho_{body}}{\rho_{fw}}$

Substitute $995\ kg/m^3$ for ρ_{body} and $1000\ kg/m^3$ for ρ_{fw} and solve for f.

$$f = \frac{995\ kg/m^3}{1000\ kg/m^3} = 0.995$$

Hence, fraction of the person's body submerged in fresh water is 0.995 or 99.5%.

(b) $f = \frac{\rho_{body}}{\rho_{sw}}$

Substitute $995\ kg/m^3$ for ρ_{body} and $1027\ kg/m^3$ for ρ_{sw} and solve for f

$$f = \frac{995\ kg/m^3}{1027\ kg/m^3} = 0.9689$$

Hence, fraction of the person's body submerged in salt water is 0.969 or 96.9%

67. (a) $m_{displaced} = 390.0g - 350.5g = 39.50g$

Hence, the mass of fluid displaced is 39.50g

(b) $\rho = \frac{m}{V}$ ......(1)

From eq. (1) volume of iron is

$$V = \frac{m_i}{\rho_i}$$

Substitute $7.8\ g/cm^3$ for ρ_i and $390g$ for m_i in the above eq. and get,

$$V = \frac{390g}{7.8g/cm^3} = 50\ cm^3$$

Hence, the volume of iron is $50\ cm^3$

(c) From eq (1),

$$\bar{\rho}_o = \frac{m_f}{V_f}$$

Substitute $39.50g$ for m_f and $50\ cm^3$ for V_f in above eq. and get,

$$\rho^o = \frac{39.50g}{50cm^3} = 0.79g/cm^3$$

Hence, the density of fluid is $0.79\ g/cm^3$ and the fluid is ethyl alcohol.

69. (a)

$$\text{fraction submerged} = \frac{\rho_{woman}}{\rho_a}$$

The submerged percent is:

$100\% - 4\% = 96\%$

$\bar{\rho}_{woman} = (\text{fraction submerged})(\rho_a)$

Substitute $1 \times 10^3\ kg/m^3$ for ρ_a and 0.96 for fraction submerged,

$\bar{\rho}_{woman} = (0.96)(1 \times 10^3\ kg/m^3) = 960\ kg/m^3$

Hence, the density of the woman is $960\ kg/m^3$

(b) Substitute $1.025 \times 10^3\ kg/m^3$ for ρ_a and $960\ kg/m^3$ for $\bar{\rho}_{obj}$ in the expression of the fraction submerged,

$$\text{fraction submerged} = \frac{960\ kg/m^3}{1.025 \times 10^3\ kg/m^3}$$

$$= 0.9366$$

Therefore the percent of her volume above sea water is,

$$= (1 - 0.9366) \times 100 = 6.34\%$$

Hence 6.34% of woman body volume is above the sea water. Therefore, she indeed floats more ~~is~~ in sea water.

74. Using $V_{air} = V_{Lf} - V_{Le}$

(a) $\text{fraction submerged} = \dfrac{\bar{\rho}_{obj}}{\rho_{fl}}$

$$\rho = \frac{m}{V}$$

Combine the expressions; the exp. of frac. Submerged is,

$$\text{fraction submerged} = \frac{m/V}{\rho_{fl}}$$

The submerged portion of the man is 97% when his lungs are empty so,

$$0.97 = \frac{m/V_{le}}{\rho_{fL}} \Rightarrow V_{le} = \frac{m}{0.97\,\rho_{fL}}$$

Since 95% is submerged when his lungs are full,

$$0.95 = \frac{\frac{m}{V_{le}}}{\rho_{fL}} \Rightarrow V_{lf} = \frac{m}{0.95\,\rho_{fL}}$$

$$V_{air} = \frac{m}{0.95\rho_{fl}} - \frac{m}{0.97\rho_{fL}} = \frac{m}{\rho_{fl}}\left(\frac{1}{0.95} - \frac{1}{0.97}\right)$$

Substitute 75 kg for m and 1×10^3 kg/m³ for ρ_w in above exp.

$$V_{air} = \frac{m}{\rho_{fl}}\left(\frac{1}{0.95} - \frac{1}{0.97}\right) = \frac{75\,kg}{1\times10^3\,kg/m^3}\left(\frac{0.02}{0.95\times0.97}\right)$$

$$= 1.63\times10^{-3}\,m^3 = 1.63\,L$$

Hence, the volume of air is 1.63 L

(b) The deep breath has volume of 2 L, So the volume of 1.63 L is reasonable.

75. Average flow rate $= v/z$

Substitute 100 km/h for v and 10 km/L for z,

$$\text{Average flow rate} = \frac{100 \text{ km/h}}{10 \text{ km/L}}$$

$$= \left(\frac{10 \text{ L}}{1 \text{h}}\right)\left(\frac{1 \text{h}}{3600 \text{s}}\right)\left(\frac{1000 \text{ cm}^3}{1 \text{L}}\right) = 2.78 \text{ cm}^3/\text{sec}$$

Hence, the average flow rate of the gasoline is 2.78 cm^3/sec.

77. $\bar{v} = Q/A$

(a) $A = (20 \text{ m})(20 \text{ m}) = 400 \text{ m}^2$

Substitute 300,000 L/s for Q and 400 m^2 for A in the above expression

$$\bar{v} = \left(\frac{300{,}000 \text{ L/s}}{400 \text{ m}^2}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{L}}\right) = 0.75 \text{ m/s}$$

Hence, the fluid velocity when the river narrows to 20m wide and averages 20 m deep is 0.75 m/s

(b) $A = (60 \text{ m})(40 \text{ m}) = 2400 \text{ m}^2$

Substitute 300,000 L/s for Q and 2400 m^2 for A in $\bar{v} = Q/A$

$$\bar{v} = \left(\frac{300{,}000 \text{ L/s}}{2400 \text{ m}^2}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{L}}\right) = 0.13 \text{ m/s}$$

Hence, when river widens to 60 m and its depth increases to an average of 40 m, the fluid velocity is 0.13 m/s

85. $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Acc. to ques, $h_1 = h_2$

Modified eq.,

$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$

Or,

$p_2 = p_1 + \frac{1}{2}\rho (v_1^2 - v_2^2)$

Substitute 15.0 mm Hg for p_1, 1.29 kg/m³ for ρ, 200 km/h for v_1 and 700 km/h for v_2

$$p_2 = \left\{ (15.0 \text{ mm Hg}) \left(\frac{133 \text{ N/m}^2}{1.00 \text{ mm Hg}} \right) + \frac{1}{2}(1.29 \text{ kg/m}^3)\left[(200 \text{ km/h})^2 - (700 \text{ km/h})^2\right] \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right)^2 \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 \right\}$$

$$= (15.0 \text{ mm Hg}) \left(\frac{133 \text{ N/m}^2}{1.00 \text{ mm Hg}} \right) + \frac{1}{2}(1.29 \text{ kg/m}^3) \left[(55.55 \text{ m/s})^2 - (194.4 \text{ m/s})^2\right]$$

$$= (-20.4 \times 10^3 \text{ N/m}^2) \left(\frac{1.00 \text{ mm Hg}}{133 \text{ N/m}^2} \right) = -153 \text{ mm Hg}$$

Hence, the pressure will be -153 mm Hg

87. (a) Rearrange the flow rate eq. in terms of volume.

$$V = Q/A$$

$$V_1 = Q/A_1$$

Substitute 40×10^{-3} m^3/s for Q and 0.045 m^2

$$V_1 = \frac{40 \times 10^{-3} \text{ m}^3/\text{s}}{\pi\, 0.045 \text{ m}^2} = 6.29 \text{ m/s}$$

$$V_2 = Q/A_2$$

Substitute 40×10^{-3} m^3/s for Q and 0.015 m^2

$$V_2 = \frac{40 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.015 \text{ m}^2)} = 56.6 \text{ m/s}$$

The pressure difference:

$$P_1 - P_2 = \tfrac{1}{2}\rho (V_2^2 - V_1^2)$$

Substitute 6.29 m/s for V_1, 56.6 m/s for V_2 and 1000 kg/m^3 for ρ.

$$P_1 - P_2 = \tfrac{1}{2}(1000 \text{ kg/m}^3)\left((56.6 \text{ m/s})^2 - (6.29 \text{ m/s})^2\right)$$

$$= 1.58 \times 10^6 \text{ N/m}^2$$

Therefore the pressure difference or pressure drop due to change in area is $1.58 \times 10^6 \; N/m^2$

(b) $V = \sqrt{2gh}$

$\Rightarrow h = V^2/2g$

Substitute 56.6 m/s for V (final velocity) and 9.81 m/s² for g

$$h = \frac{(56.6 \; m/s)^2}{2(9.81 \; m/s^2)} = 163 \; m$$

Therefore the max. height where water can rise is ~~19~~ 163 m

89. Acc. to Bernoulli's eq,

$$P_0 + \frac{mg}{A} + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_0 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

Now, at the top there is no flow hence the velocity is zero.

$$\frac{mg}{\rho A} + g h_1 = g h_2 + \frac{1}{2} V_2^2$$

$$\frac{1}{2} V_2^2 = \frac{mg}{\rho A} + g h_1 - g h_2$$

$$V_2 = \sqrt{2g\left(\frac{m}{\rho A} + (h_1 - h_2)\right)}$$

Substitute 9.8 m/s² for g, 0.5 m for h_1, 0.15 m for h_2, 1000 kg/m³ for ρ, 20 kg for m and 0.1 m² for A in the above velocity eq.

$$V_2 = \sqrt{2(9.8\ \text{m/s}^2)\left(\frac{(20\ \text{kg})}{(1000\ \text{kg/m}^3)(0.1\ \text{m}^2)} + ((0.5\ \text{m}) - (0.15\ \text{m}))\right)}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)\left(\frac{(20\ \text{kg})}{(1000\ \text{kg/m}^3)(0.1\ \text{m}^2)} + (0.35\ \text{m})\right)}$$

$$= \sqrt{10.78\ \text{m}^2/\text{s}^2} = 3.28\ \text{m/s}$$

Hence the velocity of water as it leaves the spout is 3.28 m/s

(b) $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Substitute 1.5 m for h and 9.8 m/s² for g in time eq.

$$t = \sqrt{\frac{2(1.5\ \text{m})}{(9.8\ \text{m/s}^2)}} = \sqrt{0.306\text{s}^2} = 0.55\text{s}$$

Thus, the time taken is 0.55 s

$x = vt$

Substitute 3.28 m/s for v and 0.55 s for t in above eq.

$x = (3.28\ \text{m/s})(0.55\ \text{s}) = 1.81\ \text{m}$

Hence, the distance from the spout the water hits the floor is 1.81 m