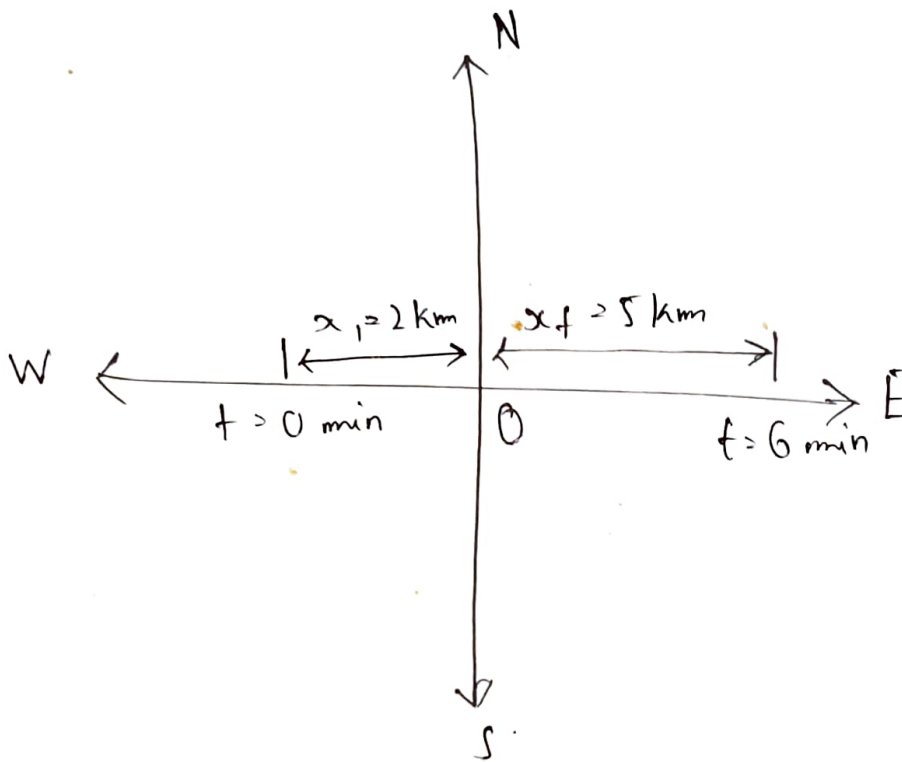


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Homework 2:- 1D Motion

25. The displacement diagram of the car can be shown as.



(a) Initially at time $t = 0$, the position is given as 2.0 km in the west of the traffic light
 $x_i = (-2.0 \text{ km})\hat{i}$

Finally at the time $t = 6.0 \text{ min}$, the position as 5.0 km in the east of the traffic light
 $x_f = (5.0 \text{ km})\hat{i}$

Hence, the car's position with respect to the traffic light initially at time $t = 0$ is $(-2.0 \text{ km})\hat{i}$ and finally at time $t = 6.0 \text{ min}$ is $(5.0 \text{ km})\hat{i}$

(b) Calculate the displacement of cars in between 0 min and 6 min.

$$\text{Displacement} = (5.0 \text{ km}) - (-2.0 \text{ km}) = 7.0 \text{ km}$$

Hence, the car's displacement between 0 min and 6 min is 7.0 km

$$31. \quad \bar{v} (0 \text{ s} - 0.4 \text{ s}) = \frac{4 \text{ m} - 0 \text{ m}}{0.4 \text{ s} - 0 \text{ s}} = 10 \text{ m/s} \quad \left(\bar{v} = \frac{x_f - x_i}{t_f - t_i} \right)$$

Therefore, for time interval from 0 to 0.4 s, the average velocity is 10 m/s

Substitute -2 m for x_f , 4 m for x_i , 0.6 s for t_f and 0.4 s for t_i to find \bar{v}

$$\bar{v} (0.4 - 0.6 \text{ s}) = \frac{-2 \text{ m} - 4 \text{ m}}{0.6 \text{ s} - 0.4 \text{ s}} = -30 \text{ m/s}$$

Therefore, for time interval from 0.4 s to 0.6 s, the average velocity is -30 m/s

Substitute -6 m for x_f , -2 m for x_i , 1 s for t_f and 0.6 s for t_i to find \bar{v}

$$\bar{v} (0.6 \text{ s} - 1 \text{ s}) = \frac{-6 \text{ m} - (-2 \text{ m})}{1 \text{ s} - 0.6 \text{ s}} = -10 \text{ m/s}$$

Therefore, for time interval from 0.6 s to 1 s , the average velocity is -10 m/s

Substitute -4 m for x_f , -6 m for x_i , 1.6 s for t_f and 1 s for t_i to find \bar{v} .

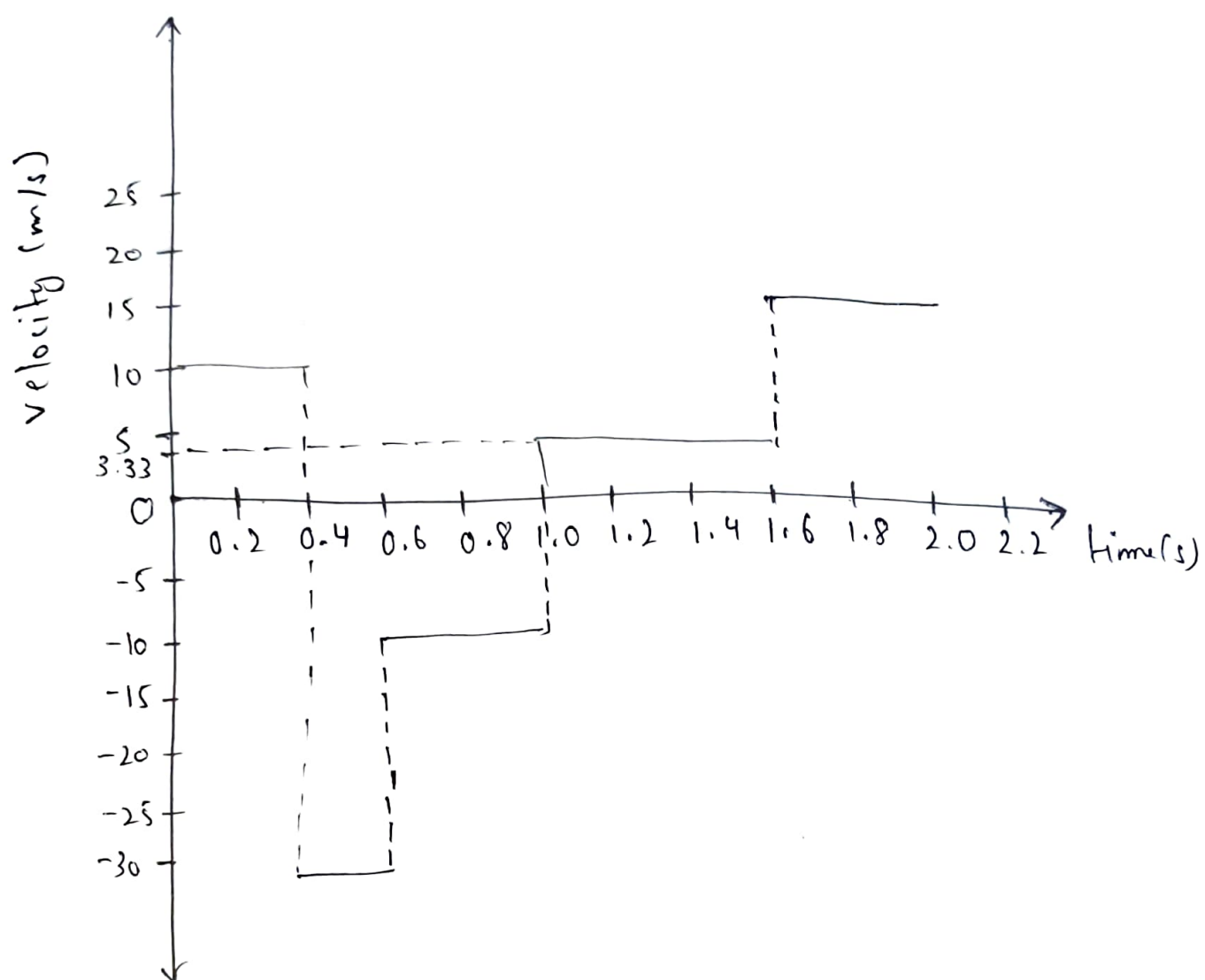
$$\bar{v} (1 \text{ s} - 1.6 \text{ s}) = \frac{-4 \text{ m} - (-6 \text{ m})}{1.6 \text{ s} - 1 \text{ s}} = 3.33 \text{ m/s}$$

Therefore, for time interval from 1 s to 1.6 s the average velocity is 3.33 m/s

Substitute 2 m for x_f , -4 m for x_i , 2 s for t_f and 1.6 s for t_i to find \bar{v} .

$$\bar{v} (1.6 \text{ s} - 2 \text{ s}) = \frac{2 \text{ m} - (-4 \text{ m})}{2 \text{ s} - 1.6 \text{ s}} = 15 \text{ m/s}$$

Therefore, for time interval from 1.6 s to 2 s , the average velocity is 15 m/s



35. The instantaneous velocity of the particle is given by,

$$v(t) = \frac{dx(t)}{dt}$$

The instantaneous speed of the particle is given by,

$$\text{Speed} = |v(t)|$$

The expression of average velocity is given by

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

(a) The particle moves along the x -axis according to expression given by,

$$x(t) = 10t - 2t^2 \text{ m}$$

Then its instantaneous velocity with change in time is given by,

$$v(t) = \frac{d(10t - 2t^2 \text{ m})}{dt} = 10 - 4t \text{ m/s}$$

Substitute 2s for t to find the instantaneous velocity at time $t = 2\text{s}$ and solve,

$$v(2\text{s}) = (10 - (4 \times 2)) \text{ m/s} = 2 \text{ m/s}$$

Similarly, substitute 3s for t to find the instantaneous velocity at time $t = 3\text{s}$ and solve,

$$\begin{aligned} v(3\text{s}) &= (10 - (4 \times 3)) \text{ m/s} \\ &= -2 \text{ m/s} \end{aligned}$$

Hence, the instantaneous velocity at time $t = 2\text{s}$ is 2 m/s and that for time $t = 3\text{s}$ is -2 m/s

(b) The instantaneous velocity at $t = 2\text{s}$ is 2 m/s

$$(\text{Instantaneous speed})_{(2\text{s})} = |v(2\text{s})| = 2 \text{ m/s}$$

The instantaneous velocity at time $t = 3s$ is $-2m/s$

$$\begin{aligned}(\text{Instantaneous speed})(3s) &= |v(3s)| \\&= |-2m/s| \\&= 2m/s\end{aligned}$$

Hence, the instantaneous speed at time $t = 2s$ is $2m/s$ and that for time $t = 3s$ is $2m/s$

(c) Substitute $2s$ for t and solve for the value of the position of the particle along the x-axis.

$$x(2s) = (10 \times 2) - (2(2^2))m = 12m$$

Similarly, substitute $3s$ for t to find the position at time $t = 3s$ and solve,

$$x(3s) = (10 \times 3) - (2(3^2))m = 12m$$

The expression for the average velocity:-

$$\frac{x_f - x_i}{t_f - t_i}$$

Substitute $3s$ for t_f , $2s$ for t_i , $12m$ for final position and $12m$ for initial position, x_i in the expression and solve

$$= \frac{12 \text{ m} - 12 \text{ m}}{3 \text{ s} - 2 \text{ s}} = 0 \text{ m/s}$$

Hence, the average velocity of the particle between time interval $t = 2 \text{ s}$ and $t = 3 \text{ s}$ is 0 m/s .

39.
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

For time interval $0 \text{ s} - 20 \text{ s}$ from the velocity versus time graph is calculated as follows,

Substitute 6 m/s for v_f and 0 m/s for v_i , 20 s for t_f and 0 s for t_i in the above expression.

$$\begin{aligned} \frac{\Delta v}{\Delta t} &= \left(\frac{6 \text{ m/s} - 0 \text{ m/s}}{20 \text{ s} - 0 \text{ s}} \right) = 6/20 \text{ m/s}^2 \\ &= 0.3 \text{ m/s}^2 \end{aligned}$$

For time interval 20 s to 50 s

Substitute 2 m/s for v_f and 6.0 m/s for v_i , 50 s for t_f and 20 s for t_i

$$\begin{aligned} \frac{\Delta v}{\Delta t} &= \left(\frac{2 \text{ m/s} - 6 \text{ m/s}}{50 \text{ s} - 20 \text{ s}} \right) = -4/30 \text{ m/s}^2 \\ &= -0.13 \text{ m/s}^2 \end{aligned}$$

For time interval 50 s to 70 s slope is zero which means there is no change in velocity.

Therefore $a = 0 \text{ m/s}^2$

For interval 70 s - 90 s

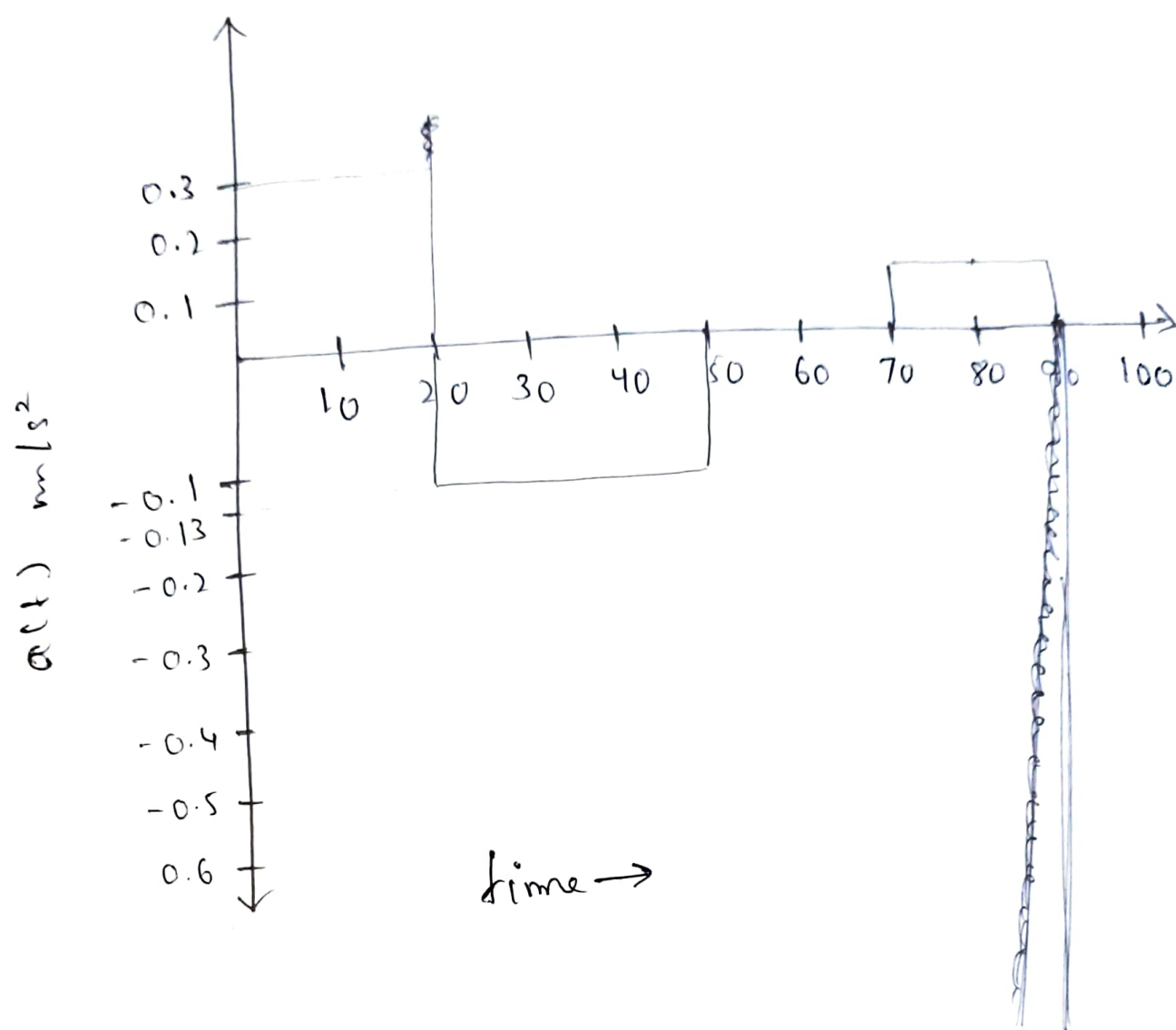
Substitute 4 m/s for v_f and 2.0 m/s for v_i , 90 s for t_f and 70 s for t_i

$$\frac{\Delta v}{\Delta t} = \left(\frac{4 \text{ m/s} - 2 \text{ m/s}}{90 \text{ s} - 70 \text{ s}} \right) = 2/20 \text{ m/s}^2 = 0.1 \text{ m/s}^2$$

For interval 90 s - 100 s

Substitute -2 m/s for v_f and 4.0 m/s for v_i , 100 s for t_f and 90 s for t_i

$$\frac{\Delta v}{\Delta t} = \left(\frac{-2 \text{ m/s} - 4 \text{ m/s}}{100 \text{ s} - 90 \text{ s}} \right) = -6/10 \text{ m/s}^2 = -0.6 \text{ m/s}^2$$



45. Displacement of the particle moving in a straight line is the change in position of the particle and is given by

$$\Delta x = x - x_0$$

The displacement of the particle in terms of position with constant acceleration is given by the expression

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

The relation between velocity and acceleration when the acceleration is constant is given by.

$$v = v_0 + at$$

(a) Substitute 30 m/s for v_0 , 30 m/s^2 for a and 5 s for t in the expression of displacement of the particle

$$\begin{aligned}\Delta x &= (30 \text{ m/s})(5 \text{ s}) + \left(\frac{1}{2}(30 \text{ m/s}^2)(5 \text{ s})^2\right) \\ &= 150 \text{ m} + 375 \text{ m} = 525 \text{ m}\end{aligned}$$

Hence, the displacement of the particle moving in a straight line at $t = 5 \text{ s}$ is 525 m

(b) Substitute 30 m/s for v_0 , 30 m/s^2 for a and 5 s for t in the kinematic equation of motion

$$\begin{aligned}v &= 30 \text{ m/s} + (30 \text{ m/s}^2)(5 \text{ s}) \\ &= 30 \text{ m/s} + 150 \text{ m/s} \\ &= 180 \text{ m/s}\end{aligned}$$

Hence, the velocity of the particle at time $t = 5 \text{ s}$ is 180 m/s

44. The expression of average acceleration is given by,

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1}$$

The relation between velocity and acceleration when the acceleration is constant is given by,

$$v = v_0 + at$$

(a) Substitute -8.0 m/s for v_2 as it is in 'leftward' direction, 5.0 m/s for v_1 , 20 s for t_2 and 10 s for t_1 in the above expression

$$\bar{a} = \frac{(-8.0 \text{ m/s}) - (5 \text{ m/s})}{(20 \text{ s}) - (10 \text{ s})} = -1.3 \text{ m/s}^2$$

Hence, the acceleration of the particle is -1.3 m/s^2

(b) Substitute 5.0 m/s for v because velocity is the final velocity of the particle, -1.3 m/s^2 for a and 10 s for t in the kinematic equation of motion and solve for final velocity,

$$(5.0 \text{ m/s}) = v_0 + (-1.3 \text{ m/s}^2)(10 \text{ s})$$

Rearrange for v_0 and solve,

$$\begin{aligned} v_0 &= (5.0 \text{ m/s}) + (13 \text{ m/s}) \\ &= 18 \text{ m/s} \end{aligned}$$

Hence, the particle initial velocity is 18 m/s

- (c) Substitute 0 m/s for the final velocity, 18 m/s for the initial velocity calculated above and -1.3 m/s^2 for constant acceleration, a and solve for t

$$0 \text{ m/s} = 18 \text{ m/s} + (-1.3 \text{ m/s}^2)t$$
$$= 13.8 \text{ s}$$

Hence, the time for which the velocity of the particle is zero is 13.8 s .

57. From the kinematic equation of motion, the relation between velocity and acceleration when the acceleration is constant is given by,
- $$v = v_0 + at$$

The final position of the motorcycle with constant acceleration is given by the expression.

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

- (a) Substitute 26.8 m/s for v , 0 m/s for v_0 and 3.90 s for t in the kinematic equation of motion mentioned above and solve for a .

$$(26.8 \text{ m/s}) = (0 \text{ m/s}) + (3.90 \text{ a})$$

Rearrange for a and solve

$$(3.90 \text{ s})a = (26.8 \text{ m/s})$$

$$a = \frac{26.8 \text{ m/s}}{3.90 \text{ s}} = 6.87 \text{ m/s}^2$$

Hence, the average acceleration of the powerful motorcycle is 6.87 m/s^2

(b) Substitute 0 m for x_0 , 0 m/s for v_0 , 3.90 s for t and 6.87 m/s^2 for a in the kinematic equation of motion for position and velocity with constant acceleration and solve for final position x .

$$x = (0 \text{ m}) + (0 \text{ m/s})(3.9 \text{ s}) + \frac{1}{2}(6.87 \text{ m/s}^2)(3.9 \text{ s})^2$$

Further solve,

$$\begin{aligned} x &= \frac{1}{2}(6.87 \text{ m/s}^2)(3.90 \text{ s})^2 \\ &= \frac{1}{2}(6.87 \text{ m/s}^2)(15.21 \text{ s}^2) \\ &= 52.25 \text{ m} \end{aligned}$$

Hence, the powerful motorcycle can travel upto 52.25 m in that time.

67. The final position of the rock with constant acceleration when motion is in y direction is given by the expression,

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$y - y_0 = v_0 t + \frac{1}{2} a t^2$$

When the point of release is $y_0 = 0$, then the above kinematic equation for velocity and position becomes and is given by, $y = v_0 t + \frac{1}{2} a t^2$

From the kinematic equation of motion, the relation between velocity and acceleration when the acceleration is constant is given by,

$$v = v_0 + a t$$

(a) Substitute -14.0 m/s for v_0 , 0.5 s for t , and -9.81 m/s^2 for g in the above mentioned kinematic equation for velocity and position and solve,

$$y_1 = (-14.0 \text{ m/s})(0.5 \text{ s}) + \frac{1}{2} (-9.81 \text{ m/s}^2)(0.5 \text{ s})^2$$

Here acceleration is the gravitational acceleration on the rock in the downward direction.

Further solve the expression.

$$y_1 = -7.0 \text{ m} - 1.22625 \text{ m} = -8.23 \text{ m}$$

Substitute -14.0 m/s for v_0 , -9.81 m/s^2 for g and 0.5 s for t_1 in the equation of kinematic motion for velocity and acceleration and solve for v_1 .

$$\begin{aligned} v_1 &= (-14.0 \text{ m/s}) + (-9.81 \text{ m/s}^2)(0.5 \text{ s}) \\ &= -14.0 \text{ m/s} - 4.905 \text{ m/s} \\ &= -18.9 \text{ m/s} \end{aligned}$$

Hence, for $t = 0.5 \text{ s}$, the displacement is -8.23 m and its final velocity is -18.9 m/s

(b) Substitute -14.0 m/s for v_0 , 1.00 s for t_2 and -9.81 m/s^2 for g in the above mentioned kinematic equation for velocity and position and solve,

$$y_2 = (-14.0 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(1.00 \text{ s})^2$$

Here, acceleration is the gravitational acceleration of the rock in the downward direction. Further solve the expression,

$$y_2 = -14.0 \text{ m} - 4.905 \text{ m} = -18.9 \text{ m}$$

Substitute -14.0 m/s for v_0 , -9.81 m/s^2 for g and 1 s for t_2 in the equation and solve

$$\begin{aligned}
 v_2 &= (-14.0 \text{ m/s}) + (-9.81 \text{ m/s}^2)(1.00 \text{ s}) \\
 &= -14.0 \text{ m/s} - 9.81 \text{ m/s} \\
 &= -23.8 \text{ m/s}
 \end{aligned}$$

Hence, for $t = 1.00 \text{ s}$, the displacement is -18.9 m and its final velocity is ~~18.9~~ -23.8 m/s

(c) Substitute -14.0 m/s for v_0 , 1.50 s for t_3 and -9.81 m/s^2 for g in the above mentioned eq. and solve,

$$\begin{aligned}
 y_3 &= (-14.0 \text{ m/s})(1.50 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(1.50 \text{ s})^2 \\
 y_3 &= -21.0 \text{ m} - 11.04 \text{ m} = -32.0 \text{ m}
 \end{aligned}$$

Substitute -14.0 m/s for v_0 , -9.81 m/s^2 for g and 1.50 s for t_3 and solve,

$$\begin{aligned}
 v_3 &= (-14.0 \text{ m/s}) + (-9.81 \text{ m/s}^2)(1.50 \text{ s}) \\
 &= -15.0 \text{ m/s} - 14.715 \text{ m/s} \\
 &= -28.7 \text{ m/s}
 \end{aligned}$$

Hence, for $t = 1.50 \text{ s}$, the displacement is -32.0 m and its final velocity is -28.7 m/s

(d) Substitute -14.0 m/s for v_0 , 2.0 s for t_4 and -9.81 m/s^2 for g in the above mentioned equation and solve,

$$y_4 = (-14.0 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(2.00 \text{ s})^2$$

$$y_4 = -28.0 \text{ m} - 19.62 \text{ m} \\ = -47.6 \text{ m}$$

Substitute -14.0 m/s for v_0 , -9.81 m/s^2 for g and 2.0 s for t_4 in the equation of kinematic motion for velocity and acceleration and solve,

$$v_4 = (-14.0 \text{ m/s}) + (-9.81 \text{ m/s}^2)(2.00 \text{ s}) \\ = -14.0 \text{ m/s} - 19.62 \text{ m/s} = -33.6 \text{ m/s}$$

Hence, for $t = 2.00 \text{ s}$, the displacement is -47.6 m and final velocity is -33.6 m/s

(e) $y_5 = (-14.0 \text{ m/s})(2.50 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(2.50 \text{ s})^2$

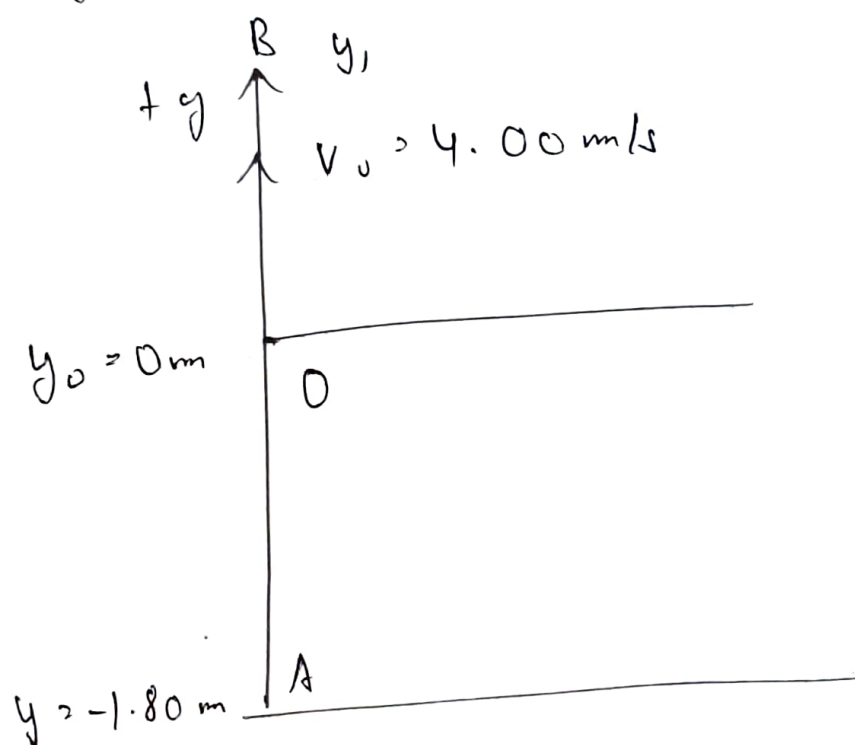
$$y_5 = -35.0 \text{ m} - 30.66 \text{ m} = -65.6 \text{ m}$$

$$v_5 = (-14.0 \text{ m/s}) + (-9.81 \text{ m/s}^2)(2.50 \text{ s})$$

$$= -14.0 \text{ m/s} - 24.525 \text{ m/s} = -38.5 \text{ m/s}$$

Hence, for $t = 2.50 \text{ s}$, the displacement is -65.6 m and its final velocity is -38.5 m/s

71. The diver jumps up with a velocity v_0 from a diving board. Assume the origin of the coordinate system to be located on the diving board, with the $+$ y direction directed upwards.



The diving board located at O has coordinate y_0 . The surface of the pool has the coordinate y . The diver moves upwards and reaches the point B with coordinate y_1 , when her speed v_1 becomes zero.

$$v_1^2 = v_0^2 - 2g(y_1 - y_0)$$

Substitute (0 m/s) for v_1 , (4.00 m/s) for v_0 , (9.80 m/s^2) for g and (0 m) for y_0

$$v_1^2 = v_0^2 - 2g(y_1 - y_0)$$

$$(0 \text{ m/s}^2)^2 = (4.00 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)[y_1 - (0 \text{ m})]$$

Solve for y_1

$$y_1 = \frac{(4.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.816 \text{ m}$$

The highest point reached by the diver above the board is 0.816 m

$$(b) \quad y = y_0 + v_0 t - \frac{1}{2} g t^2$$

Substitute (-1.8 m) for y , (4.00 m/s) for v_0 , (9.80 m/s^2) for g and (0 m) for y_0 .

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$(-1.8 \text{ m}) = (0 \text{ m}) + (4.00 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

$$(4.9 \text{ m/s}^2)t^2 - (4.00 \text{ m/s})t - (1.8 \text{ m}) = 0$$

$$t = \frac{(4 \text{ m/s}) \pm \sqrt{(-4 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(-1.8 \text{ m})}}{2(4.9 \text{ m/s}^2)}$$

$$= \frac{(4 \text{ m/s}) \pm (7.16 \text{ m/s})}{(9.8 \text{ m/s}^2)}$$

Take the positive root since the negative root would give a -ve value for t .

$$t = \frac{11.16 \text{ m/s}}{9.8 \text{ m/s}^2} = 1.14 \text{ s}$$

The diver's feet would be in air for a time 1.14 s

$$v_2^2 = v_0^2 - 2g(y - y_0)$$

Substitute (-1.8 m) for y , (4 m/s) for v_0 , (9.8 m/s^2) for g and (0 m) for y_0 .

$$\begin{aligned} v_2^2 &= v_0^2 - 2g(y - y_0) \\ &= (4 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)[(-1.8 \text{ m}) - (0 \text{ m})] \\ &= 51.28 \text{ (m/s)}^2 \end{aligned}$$

$$\begin{aligned} v_2 &= \sqrt{51.28 \text{ (m/s)}^2} \\ &= \pm 7.16 \text{ m/s} \end{aligned}$$

The velocity of the diver is directed along the -y direction, hence the velocity is -ve. Hence, the diver hits the surface of the water with a velocity - 7.16 m/s

77. From the kinematic third equation of motion,

$$v^2 = u^2 + 2as$$

From the second kinematic equation of motion,

$$s = ut + \frac{1}{2}at^2$$

(a) Substitute 0 for u , 9.8 m/s^2 for a , 250 m for s

$$v^2 = u^2 + 2as$$

$$v^2 = (0)^2 + 2(9.8 \text{ m/s}^2)(250 \text{ m})$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(250 \text{ m})} = 70 \text{ m/s}$$

Hence, the required velocity is 70 m/s

(b) Substitute 0 for u , 9.8 m/s^2 for a , 250 m for s

$$s = ut + \frac{1}{2}at^2$$

$$(250 \text{ m}) = (0)t + \left(\frac{1}{2}\right)(9.8 \text{ m/s}^2)t^2$$

$$t^2 = \frac{2(250 \text{ m})}{9.8 \text{ m/s}^2}$$

$$t = 7.143 \text{ s}$$

The required time is,

$$t_c = t - t_{\text{reaction}}$$

Substitute 7.143 s for t and 0.3 s for t_{reaction} in the above equation.

$$t = (7.143 \text{ s}) - (0.3 \text{ s}) = 6.84 \text{ s}$$

Hence, the total time a tourist at the bottom have to get out of the way after hearing the sound of the rock is 6.84 s

78. The velocity from the acceleration is given by,

$$v(t) = \int a(t) dt + C_1$$

The position from velocity is given by

$$x(t) = \int v(t) dt + C_2$$

$$(a) \quad a(t) = pt^2 - qt^3$$

Substitute $pt^2 - qt^3$ for $a(t)$ and solve for $v(t)$

$$v(t) = \int (pt^2 - qt^3) dt + C_1$$

$$v(t) = \left(p \frac{t^{2+1}}{2+1} - q \frac{t^{3+1}}{3+1} \right) + C_1$$

$$= p \frac{t^3}{3} - q \frac{t^4}{4} + C_1$$

$$v(0 \text{ s}) = p \frac{(0 \text{ s})^3}{3} - q \frac{(0 \text{ s})^4}{4} + C_1$$

$$0 = 0 - 0 + C_1$$

$$C_1 = 0$$

Substitute 0 for C_1 ,

$$\begin{aligned}v(t) &= pt^3/3 - qt^4/4 + 0 \\&= pt^3/3 - qt^4/4\end{aligned}$$

Hence, the velocity as a function of time is given by $v(t) = pt^3/3 - qt^4/4$

$$(b) \quad v(t) = pt^3/3 - qt^4/4$$

Substitute $pt^3/3 - qt^4/4$ for $v(t)$ and solve for $x(t)$

$$x(t) = \int (pt^3/3 - qt^4/4) dt + C_2$$

$$x(t) = \left(\frac{pt^4}{12} - \frac{qt^5}{20} \right) + C_2$$

$$0 = 0 + 0 + C_2$$

$$C_2 = 0$$

Substitute 0 for C_2 and solve for $x(t)$

$$x(t) = \left(\frac{pt^4}{12} - \frac{qt^5}{20} \right) + 0 = \frac{pt^4}{12} - \frac{qt^5}{20}$$

Hence, position as a function of time is

$$x(t) = pt^4/12 - \frac{qt^5}{20}$$

79. $v(t) = \int a(t) dt$

$$x(t) = \int v(t) dt$$

(a) The acceleration of the rocket between $t = 0$ and $t = t_0$ is provided in the problem as

$$a(t) = A - Bt^{1/2}$$

The units of A is similar to the units of $a(t)$ and the units of $Bt^{1/2}$ is similar to ~~all~~ the units of $a(t)$

The unit of A is m/s^2

The unit of B is calculated as

$$B(1/t^2) = m/s^2$$

Substitute s for t

$$B(1/s^{1/2}) = m/s^2$$

Compare both and solving:

$$B = m/s^{5/2}$$

Hence the units of A is m/s^2 and the units of B is $m/s^{5/2}$

$$b) \quad v(t) = \int a(t) dt$$

Substitute $A - Bt^{1/2}$ for $a(t)$ in the above expression.

$$v(t) = \int (A - Bt^{1/2}) dt$$

$$v(t) = \int (A - Bt^{1/2}) dt + C_1$$

$$= At - \frac{Bt^{1/2+1}}{3/2} + C_1$$

$$v(t) = At - \frac{2}{3}Bt^{3/2} + C_1 \quad \text{--- (1)}$$

Substitute $0s$ for t , 0 m/s for v in (1)

$$0 \text{ m/s} = A(0s) - \frac{2}{3}B(0s)^{3/2} + C_1$$

$$C_1 = 0$$

Put the value of C_1 in (1)

$$v(t) = At - \frac{2}{3}Bt^{3/2} \quad \text{--- (2)}$$

For the velocity in time $t=0$, substitute $0s$ for t in the equation (2) and solve,

$$v(0s) = A(0s) - \frac{2}{3}B(0s)^{3/2} = 0 \text{ m/s}$$

For the velocity in time $t=t_0$, substitute $t=t_0$ in (2) and solve,

$$v(t_0) = A(t_0) - \frac{2}{3} B(t_0)^{3/2}$$

Hence the velocity of the rocket varies between times $t=0$ and $t=t_0$ as $A(t_0) - \frac{2}{3} B(t_0)^{3/2}$

$$(c) \quad x(t) = \int v(t) dt$$

$$\text{Substitute } v(t) = At - \frac{2}{3} Bt^{3/2}$$

$$x(t) = \int (At - \frac{2}{3} Bt^{3/2}) dt$$

$$x(t) = \frac{A t^{1+1}}{1+1} - \frac{2}{3} B \frac{t^{3/2+1}}{3/2+1} + C_2$$

$$= At^2/2 - \frac{2}{3} B \frac{t^{5/2}}{5/2} + C_2$$

$$= At^2/2 - \frac{2}{3} \left(\frac{2}{5} \right) Bt^{5/2} + C_2$$

$$x(t) = A/2 t^2 - 4/15 Bt^{5/2} + C_2 \quad \text{--- (3)}$$

Substitute 0s for t and 0m for $x(t)$ in (3)

$$0 = A/2 (0s)^2 - 4/15 B(0s)^{5/2} + C_2$$

$$C_2 = 0$$

Put the value of C_2 in (3)

$$x(t) = A/2 t^2 - 4/15 B t^{5/2} \quad \text{--- (4)}$$

For the position in time $t=0$, substitute 0s for t in equation (4)

$$\begin{aligned} x(0s) &= A/2 (0s)^2 - 4/15 B (0s)^{5/2} \\ &= 0 \text{ m} \end{aligned}$$

For the position in time $t=t_0$ substitute t_0 for t in (4)

$$x(t_0) = A/2 (t_0)^2 - 4/15 B (t_0)^{5/2}$$

Hence the position of the rocket varies between times $t=0$ and $t=t_0$ as

$$A/2 (t_0)^2 = 4/15 B (t_0)^{5/2}$$

93. Distance between both the trains when engine see is that means distance cover by each train before colliding is,

$$S = \frac{1000 \text{ m}}{2} = 500 \text{ m}$$

The required acceleration of train to stop just short of colliding is calculated as follows,

Rearrange the linear motion of equation in one dimension,

$$a = \frac{v^2 - u^2}{2s}$$

Substitute 0 m/s for v , 30 m/s for u and 500 m for s in rearrange equation.

$$a = \frac{(0 \text{ m/s})^2 - (30 \text{ m/s})^2}{2(500 \text{ m})}$$

$$= \frac{-900 \text{ m}^2/\text{s}^2}{1000 \text{ m}} = -0.9 \text{ m/s}^2$$

Hence the required acceleration of train to stop just short of colliding is -0.9 m/s^2

95. From the kinematic equation of motion, the relation between time and position is given by

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

The initial position of both the police and speeding cars become zero.

When the police car catches the speeding car, their final position will be the same.

Now, consider the situation of the speeding car and the police car separately and finally equate both the equation obtained to get the time.

For speeding car:

Substitute 0 m for x_0 , 40 m/s for v_0 , 0 m/s^2 for a

$$x = (0 \text{ m}) + (40 \text{ m/s})t + \frac{1}{2}(0 \text{ m/s}^2)t^2 \\ = (40 \text{ m/s})t \quad \text{--- (1)}$$

For police car:

Substitute 0 m for x_0 , 0 m/s for v_0 , 4 m/s^2 for a in the kinematic equation of motion for time and position and solve,

$$x = (0 \text{ m}) + (0 \text{ m/s})t + \frac{1}{2}(4 \text{ m/s}^2)t^2 \quad \text{--- (2)} \\ = (2 \text{ m/s}^2)t^2$$

Further equate equation (1) and (2) and solve,

$$(40 \text{ m/s})t = (2 \text{ m/s}^2)t^2$$

$$t = \frac{40 \text{ m/s}}{2 \text{ m/s}^2} = 20 \text{ s}$$

Hence, it takes 20 s for the police car to catch the speeding car.