Name :- Shreyas Srinivasa
ID Number :- 012551187

Homework 8 :- Linear Momentum and Collisions

21. Using $v = \frac{2\pi R}{T}$ and $p = Mv$

Substitute $\frac{2\pi R}{T}$ for v.

$$p = \frac{2\pi RM}{T} \quad \dots (1)$$

The Earth takes 365.25 days to complete one revolution around the sun.

Express T in seconds.

$$T = (365.25\,d)\left(\frac{24\,h}{1\,d}\right)\left(\frac{3600\,s}{1\,h}\right) = 3.156 \times 10^7 s$$

Substitute (3.14) for π, $(1.50 \times 10^{11}\,m)$ for R, $(5.97 \times 10^{24}\,kg)$ for M and $(3.156 \times 10^7 s)$ for T in equation (1) to calc. avg. linear momentum

$$p = \frac{2\pi RM}{T} = \frac{2(3.14)(1.50 \times 10^{11}\,m)(5.97 \times 10^{24}\,kg)}{(3.156 \times 10^7 s)}$$

$$= 1.78 \times 10^{29}\ kg\ m/s$$

The Earth's average linear momentum is 1.78×10^{29} kg m/s

25. Using $v^2 = u^2 + 2as$

Rearrange the above equation in terms of a.

$$a = \frac{v^2 - u^2}{2s}$$

Substitute 0 m/s for v, 20 m/s for u and 1 cm for s.

$$a = \frac{(0\ \text{m/s})^2 - (20\ \text{m/s})^2}{2(1\ \text{cm})} = \frac{-400\ \text{m}^2/\text{s}^2}{2(1\ \text{cm})\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)}$$

$$= \frac{-400\ \text{m}^2/\text{s}^2}{2(10^{-2}\ \text{m})} = -2 \times 10^4\ \text{m/s}^2$$

Hence, the deceleration of the person is $2 \times 10^4\ \text{m/s}^2$

Using $F = ma$

Substitute 75 kg for m and $2 \times 10^4\ \text{m/s}^2$ for a

$$F = 75 \times 2 \times 10^4\ \text{N}$$
$$= 1.5 \times 10^6\ \text{N}$$

Hence, the force exerted on the man if he is stopped by the padded dashboard is 1.5×10^6 N

(b) Using $v^2 - u^2 = 2aS$

Rearrange the above equation in terms of a and Solve,

$$a = \frac{v^2 - u^2}{2s}$$

Substitute 0 m/s for v, 20 m/s for u and 15 cm for s,

$$a = \frac{(0\text{ m/s})^2 - (20\text{ m/s})^2}{2(15\text{ cm})} = \frac{-400\text{ m}^2/\text{s}^2}{2(15\text{ cm})\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)}$$

$$= \frac{-400\text{ m}^2/\text{s}^2}{2(15 \times 10^{-2}\text{ m})} = -1.33 \times 10^3\text{ m/s}^2$$

Hence the deceleration of the person is 1.33×10^3 m/s²

$F = ma$

Substitute 75 kg for m and 1.33×10^3 m/s² for a

$F = 75 \times 1.33 \times 10^3$ N

$= 10 \times 10^4$ N

$= 1 \times 10^5$ N

Hence, the force exerted on the man if he is stopped by the padded dashboard is 1×10^5 N

28. $F_{avg} = \dfrac{m(v_f - v_i)}{t}$ .....(1)

Substitute 110 kg for m, 8 m/s for v_i, 5.50×10^{-2} s for t and -1.76×10^4 N for F_{avg} (as the force is in opposite direction of the motion) in eq (1)

$$(-1.76 \times 10^4 \text{ N}) = \frac{(110 \text{ kg})(v_f - 8 \text{ m/s})}{(5.50 \times 10^{-2} \text{ s})}$$

Rearrange the above equation in terms of v_f and solve,

$$(v_f - 8 \text{ m/s}) = \frac{(-1.76 \times 10^4 \text{ N})(5.50 \times 10^{-2} \text{ s})}{110 \text{ kg}}$$

$$v_f = -8.8 \text{ m/s} + 8 \text{ m/s} = -0.8 \text{ m/s}$$

Hence, the final speed of the rugby player will be in the opposite direction of its initial motion with a magnitude of 0.8 m/s.

33. Using $p_x = mv_x$

Substitute 150 g for m and 10 m/s for v_x,

$$p_x = (150\,g)(10\,m/s)$$

$$= (150\,g)\left(\frac{10^{-3}\,kg}{1\,g}\right)(10\,m/s)$$

$$= 15 \times 10^{-1}\,kg \cdot m/s = 1.5\,kg \cdot m/s$$

Hence the momentum of the hockey puck along east direction is 1.5 kg·m/s.

$$J = F_{avg}\Delta t = \Delta P$$

$$\Rightarrow F_{avg}\Delta t = \Delta P$$

$$\Delta P = F_{avg}\Delta t$$

$$\Delta P_y = F_{avg}\Delta t$$

Also, $\Delta P_y = P_{fy} - P_{iy}$

The initial momentum along the north direction is 0 kg·m/s

$$\Delta P_y = P_{fy}$$

Substitute P_{fy} for ΔP_y, 5 N for F_{avg} and 1.5 s for Δt in eq. $\Delta P_y = F_{avg}\Delta t$ and solve,

$$P_{fy} = (5\,N)(1.5\,s) = 7.5\,kg \cdot m/s$$

Hence the momentum of the hockey pluck along north direction is 7.5 kg · m/s

35. $m_1v_1 + m_2v_2 = (m_1 + m_2)v$ ...... (1)

Substitute 150,000 kg for m_1, 110,000 kg for m_2, 0.300 m/s for v_1 and −0.120 m/s for v_2 in equation (1) and solve as,

$$(150{,}000\ \text{kg})(0.300\ \text{m/s}) + (110{,}000\ \text{kg})(-0.120\ \text{m/s}) = (150{,}000\ \text{kg} + 110{,}000\ \text{kg})v$$

$$(45{,}000\ \text{kg}\cdot\text{m/s} - 13{,}200\ \text{kg}\cdot\text{m/s}) = (260{,}000\ \text{kg})v$$

Rearrange and solve,

$$v = \frac{(48{,}000\ \text{kg}\cdot\text{m/s} - 13{,}200\ \text{kg}\cdot\text{m/s})}{(260{,}000\ \text{kg})}$$

$$= \frac{31{,}800\ \text{kg}\cdot\text{m/s}}{260{,}000\ \text{kg}} = 0.12\ \text{m/s}$$

Therefore, the final velocity is 0.12 m/s

39. When the truck is at loaded condition, the total mass of the paving truck is,

$m_i = m_{truck} + m_{gravel}$

Substitute 5000 kg for m_{truck} and 1000 kg for m_{gravel}.

$m_i = 5000\,kg + 1000\,kg = 6000\,kg$

Using $p_i = m_i v_i$

Substitute 6000 kg for m_i and 2.5 m/s for v_i

$p_i = (6000\,kg)(2.5\,m/s) = 15000\,N\cancel{fo}m/s$

$$m_i = m_{truck} - m_{gravel}$$
$$= 5000\,kg - 1000\,kg$$
$$= 4000\,kg$$

The final momentum of the system is calculated as:

$p_f = m_f v_f$

Substitute 4000kg for m_f in the above equation.

$p_f = (4000\,kg)\,v_f$

Applying the law of conservation of linear momentum.

$P_i = P_f$

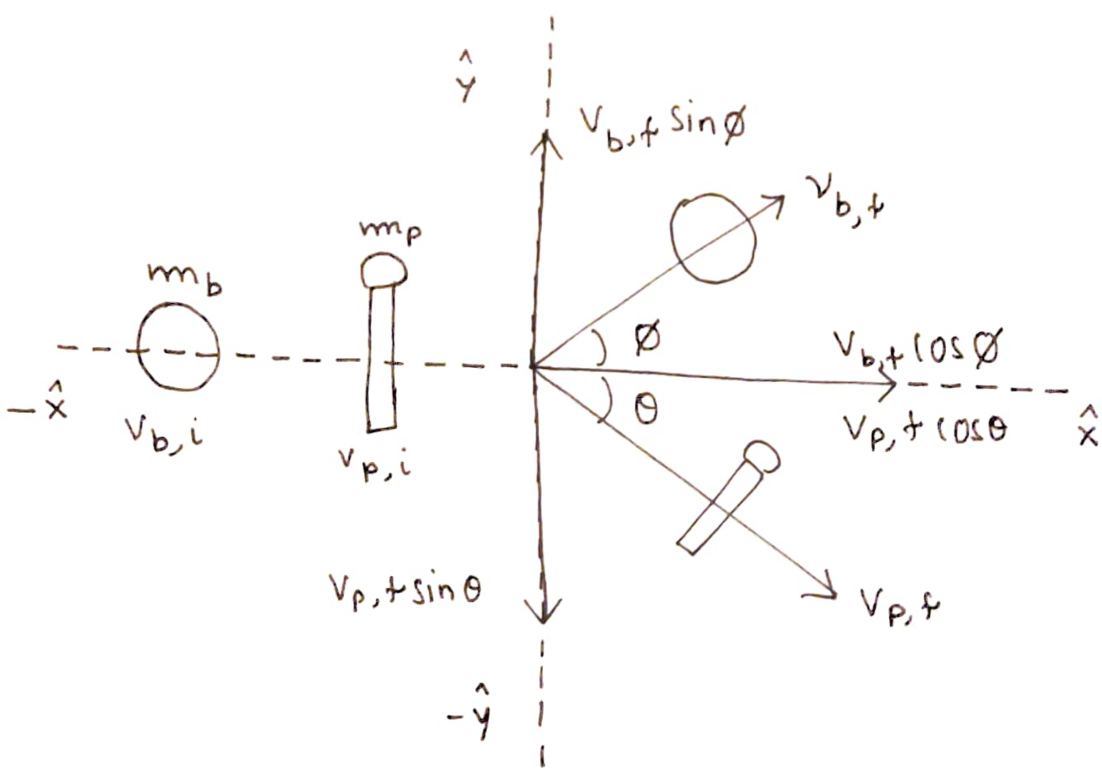
Substitute 15000 N m/s for p_i and $(4000 kg) v_f$ for P_f in the above equation. Also rearrange ~~the~~ the equation for v_f.

$$15000 \text{ N} \cdot \text{m/s} = (4000 \text{ kg}) \times V_f$$

$$V_f = \frac{15000 \text{N} \cdot \text{m/s}}{4000 \text{ kg}} = 3.75 \text{ m/s}$$

Hence, the final velocity after dumping is 3.75 m/s

43. The direction of the velocities of the bowling ball



(a) Scattering angle value :- 15.8°

The expression of the law of conservation of momentum:-

$$p_{b,i} + p_{p,i} = p_{b,f} + p_{p,f} \quad — (1)$$

The expression for the initial momentum of the bowling ball is,

$$p_{b,i} = (m_b v_{b,i})\hat{x} \quad — (2)$$

The expression for the initial momentum of the bowling pin :-

$$p_{p,i} = (m_p v_{p,i})\hat{x} \quad — (3)$$

The expression for the final momentum of the bowling ball :-

$$p_{b,f} = m_b((v_{b,f}\cos\phi)\hat{x} + (v_{b,f}\sin\phi)\hat{y}) \quad — \cancel{(3)}\ (4)$$

The expression for final momentum of the bowling pin is,

$$p_{p,f} = m_p((v_{p,f}\cos\theta)\hat{x} - (v_{p,f}\sin\theta)\hat{y}) \quad — (5)$$

Substitute (2), (3), (4), (5) in (1)

$$(m_b v_{b,i})\hat{x} + (m_p v_{p,i})\hat{x} = m_b((v_{b,f}\cos\phi)\hat{x} + (v_{b,f}\sin\phi)\hat{y}) + m_p((v_{p,f}\cos\theta)\hat{x} - (v_{p,f}\sin\theta)\hat{y})$$

Substitute 0 for $v_{p,i}$ in the above equation

$$(m_b v_{b,i})\hat{x} + (m_p(0))\hat{x} = m_b((v_{b,f}\cos\phi)\hat{x} + (v_{b,f}\sin\phi)\hat{y}) + m_p((v_{p,f}\cos\theta)\hat{x} - (v_{p,f}\sin\theta)\hat{y})$$

$$(m_b v_{b,i})\hat{x} + (0)\hat{y} = (m_b v_{b,f}\cos\phi + m_p v_{p,f}\cos\theta)\hat{x} + (m_b v_{b,f}\sin\phi - m_p v_{p,f}\sin\theta)\hat{y}$$

Compare the coefficients of unit vector $\hat{x}$

$$m_b v_{b,i} = m_b v_{b,f}\cos\phi + m_p v_{p,f}\cos\theta$$

Multiply the above equation with $\tan\phi$.

$$(m_b v_{b,i})(\tan\phi) = (m_b v_{b,f}\cos\phi + m_p v_{p,f}\cos\theta)(\tan\phi)$$

$$m_b v_{b,i}\tan\phi = m_b v_{b,f}\cos\phi\tan\phi + m_p v_{p,f}\cos\theta\tan\phi$$

$$m_b v_{b,i}\tan\phi = m_b v_{b,f}\cos\phi\left(\frac{\sin\phi}{\cos\phi}\right) + m_p v_{p,f}\cos\theta\tan\phi$$

$$m_b v_{b,i}\tan\phi = m_b v_{b,f}\sin\phi + m_p v_{p,f}\cos\theta\tan\phi \quad \ldots\ldots (1)$$

Compare the coefficients of unit vector $\hat{y}$.

$$0 = m_b v_{b,f}\sin\phi - m_p v_{p,f}\sin\theta$$

$$0 = -m_b v_{b,f}\sin\phi = m_p v_{p,f}\sin\theta \quad \ldots\ldots (2)$$

Solve the equations (1) and (2) for θ

$$m_b v_{b,i}\tan\phi = m_b v_{b,f}\sin\phi + m_p v_{p,f}\cos\theta\tan\phi$$

$$0 = -m_b v_{b,f}\sin\phi + m_p v_{p,f}\sin\theta$$

$m_b v_{b,i} \tan\phi - m_p v_{p,f} \cos\theta \tan\phi = m_p v_{p,f} \sin\theta$

$$\phi = \tan^{-1}\left(\frac{m_p v_{p,f} \sin\theta}{m_b v_{b,i} - m_p v_{p,f} \cos\theta}\right)$$

Substitute 5.50 kg for m_b, 0.850 kg for m_p, 15.0 m/s for $v_{p,f}$, 9.00 m/s for $v_{b,i}$ and 15.8° and solve for ϕ

$$\phi = \tan^{-1}\left(\frac{(0.850\text{ kg})(15.0\text{ m/s})(\sin 15.8°)}{(5.50\text{ kg})(9.00\text{ m/s}) - (0.850\text{kg})(15.0\text{m/s})(\cos 15.8°)}\right)$$

$= 5.33°$

Rewrite the equation $m_b v_{b,i} = m_b v_{b,f} \cos\phi + m_p v_{p,f} \cos\theta$ for $v_{b,f}$ as follows:

$$v_{b,f} = \frac{m_b v_{b,i} - m_p v_{p,f} \cos\theta}{m_b \cos\phi}$$

Substitute 5.50 kg for m_b, 0.850 kg for m_p, 15.0 m/s for $v_{p,f}$, 9.00 m/s for $v_{b,i}$, 15.8° for θ, and 5.33° for ϕ in the equation $v_{b,f} = \dfrac{m_b v_{b,i} - m_p v_{p,f} \cos\theta}{m_b \cos\phi}$ and solve for $v_{b,f}$

$$v_{b,f} = \frac{(5.50\text{kg})(9.00\text{ m/s}) - (0.850\text{ kg})(15.0\text{ m/s})(\cos 15.8°)}{(5.50\text{ kg})(\cos 5.33°)}$$

$= 6.8$ m/s

Therefore, the final velocity of the bowling ball is 6.8 m/s and its direction is 5.33° with respect to the positive x-axis

(b) KE_i of the bowling ball-bowling pin system is,

$$\frac{1}{2} m_b v_{b,i}^2 = \frac{1}{2} m_b v_{b,f}^2 + m_p v_{p,f}^2$$

$$KE_i = \frac{1}{2} m_b v_{b,i}^2$$

Substitute 5.50 kg for m_b and 9.00 m/s for $v_{b,i}$ in the equation $KE_i = \frac{1}{2}(m_b v_{b,i}^2)$ and solve,

$$KE_i = \frac{1}{2}(5.50\,kg)(9.00\,m/s)^2 = 222.8\,J$$

$$KE_f = \frac{1}{2} m_b v_{b,f}^2 + m_p v_{p,f}^2$$

Substitute 5.50 kg for m_b, 0.850 kg for m_p, 15.0 m/s for $v_{p,f}$, and 6.8 m/s for $v_{b,f}$ in the equation and solve.

$$KE_f = \left(\frac{1}{2}\right)(5.50\,kg)(6.8\,m/s)^2 + \left(\frac{1}{2}\right)(0.850\,kg)(15.0\,m/s)^2$$
$$= 222.8\,J$$

KE_i is equal to KE_f. Here, the momentum and the kinetic energy of the bowling-ball-~~po~~ bowling pin system are conserved.

Therefore, yes, the collision between the bowling ball and the bowling pin is an elastic collision.

47. According to the principal of conservation of momentum and the principal of conservation of energy,

$$mv_1 + mv_2 = mv_1' + mv_2' \quad \text{—— (1)}$$

~~Therefore~~,

$$\cancel{v_1^2 + v_2^2 = v_1'^2 + v_2'^2}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$$

Therefore,

$$v_1^2 + v_2^2 = v_1'^2 + v_2'^2 \quad \text{—— (2)}$$

Substitute 5.6 m/s for v_1 and 6 m/s for v_2 in equation (1)

$$5.6 \text{ m/s} + 6 \text{ m/s} = v_1' + v_2'$$

$$v_1' + v_2' = 5.6 \text{ m/s} + 6 \text{ m/s}$$

Therefore,

$$v_1' + v_2' = 11.6 \text{ m/s} \quad \ldots\ldots (3)$$

Substitute 5.6 m/s for v_1 and 6 m/s for v_2 in equation (2)

$$v_1'^2 + v_2'^2 = (5.6 \text{ m/s})^2 + (6 \text{ m/s})^2$$

Therefore,

$$v_1'^2 + v_2'^2 = 67.36 \text{ m}^2/\text{s}^2 \quad \ldots\ldots (4)$$

Solve the equation (3) and (4) further and substitute $v_2 = 11.6 \text{ m/s} - v_1$

$$v_1'^2 + (11.6 \text{ m/s} - v_1')^2 = 67.36 \text{ m}^2/\text{s}^2$$

$v_1'^2 + v_1'^2 + 11.6^2 - 23.2v_1' = 0$

$v_1' = 6$ m/s

$v_2 = 11.6$ m/s $- 6$ m/s $= 5.6$ m/s

Therefore the final velocity of leading bumper is $v_1' = 6$ m/s and the final velocity of trailing bumper is $v_2' = 5.6$ m/s

53. The momentum of hawk and dove before impact is,

$\vec{p} = m_H \vec{v}_{H,i} + m_D \vec{v}_{D,i}$

The momentum after impact is,

$\vec{p} = (m_H + m_D)\vec{v}_f$

Equate initial and final momentum

$(m_H + m_D)\vec{v}_f = m_H \vec{v}_{H,i} + m_D \vec{v}_{D,i}$

The value of $\vec{v}_{H,i}$ is $(28.0 \text{ m/s})\cos 35^\circ \hat{i} - (28.0 \text{ m/s}) \sin 35^\circ \hat{j}$

The value of $\vec{v}_{D,i}$ is 7.00 m/s $\hat{i}$

The expression of Pythagoras theorem

$v_f = \sqrt{v_{f,x}^2 + v_{f,y}^2}$

$\theta = \tan^{-1}\left(\frac{v_{f,x}}{v_{f,y}}\right)$

The conservation of momentum in x direction is,

$$(m_H + m_D) v_{f,x} = m_H v_{H,i,x} + m_D v_{D,i,x}$$

Rearrange for $v_{f,x}$

$$v_{f,x} = \frac{m_H v_{H,i,x} + m_D v_{D,i,x}}{m_H + m_D}$$

Substitute 1.80 kg for m_H, $(28.0\ m/s)\cos 35°$ for $v_{H,i,x}$, 0.650 kg for m_D and 7.00 m/s for $v_{D,i,x}$

$$v_{f,x} = \frac{(1.80\ kg)((28.0\ m/s)\cos 35°) + (0.650\ kg)(7.00\ m/s)}{(1.80\ kg + 0.650\ kg)}$$

$$= 18.71\ m/s$$

The conservation of momentum in y direction is,

$$(m_H + m_D) v_{f,y} = m_H + v_{H,i,y} + m_D v_{D,i,y}$$

Rearrange for $v_{f,y}$

$$v_{f,y} = \frac{m_H v_{H,i,y} + m_D v_{D,i,y}}{m_H + m_D}$$

Substitute 1.80 kg for m_H $-(28.0\ m/s)\sin 35°$ for $v_{H,i,y}$, 0.650 kg for m_D and 0 for $v_{D,i,y}$

$$v_{f,y} = \frac{(1.80\ kg)(-(28.0\ m/s)\sin 35°) + (0.650\ kg)(0\ m/s)}{1.80\ kg + 0.650\ kg}$$

$= -12.13$ m/s

Substitute 18.71 m/s for $v_{f,x}$ and -12.13 m/s for $v_{f,y}$ in $v_f = \sqrt{v_{f,x}^2 + v_{f,y}^2}$

$$v_f = \sqrt{(18.71\ \text{m/s})^2 + (-12.13\ \text{m/s})^2} = 22.30\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{18.71\ \text{m/s}}{12.13\ \text{m/s}}\right) \approx 57.1$$

The direction of velocity is 57.1° from negative y-direction or 32.9° below the horizon.

57. $\vec{P}_i = \vec{P}_f$

$\vec{p} = m\vec{v}$

$\vec{P}_i = m\vec{v}_i$

Substitute $200\text{kg}((121\ \text{m/s})\hat{i} + (38.0\ \text{m/s})\hat{j})$ for $\vec{v}_i$.

$$\vec{P}_i = 200\text{kg}((121\ \text{m/s})\hat{i} + (38.0\ \text{m/s})\hat{j})$$

$m_3 = m_i - m_1 - m_2$

Substitute 200 kg for m_i, 78 kg for m_1 and 56 kg for m_2.

$m_3 = 200\ \text{kg} - 78\ \text{kg} - 56\ \text{kg} = 66\ \text{kg}$

$\vec{P}_f = \vec{P}_{1f} + \vec{P}_{2f} + \vec{P}_{3f}$

$= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} + m_3\vec{v}_{3f}$

Substitute 78 kg for m_1, $-(321\ m/s)\hat{i} + (228\ m/s)\hat{j}$ for $\vec{v}_{1f}$, 56 kg for m_2, $(16\ m/s)\hat{i} - (88\ m/s)\hat{j}$ and 66 kg for m_3

$\vec{P}_f = 78\ kg((-321\ m/s)\hat{i} + (228\ m/s)\hat{j}) + 56\ kg((16\ m/s)\hat{i} - (88\ m/s)\hat{j}) + 66\ kg(\vec{V}_{3f})$

$= -25038\hat{i} + 17784\hat{j} + 896\hat{i} - 4928\hat{j} + 66\ kg(\vec{V}_{3f})$

$= -24142\hat{i} + 12856\hat{j} + 66\ kg(\vec{V}_{3f})$

Using

$\vec{P}_i = \vec{P}_f$

Substitute $200((121)\hat{i} + (38.0)\hat{j})$ for $\vec{P}_i$ and $-24142\hat{i} + 12856\hat{j} + 66\ kg(\vec{V}_{3f})$ for $\vec{P}_f$

$200((121)\hat{i} + (38.0)\hat{j}) = -24142\hat{i} + 12856\hat{j} + 66(\vec{V}_{3f})$

$$\vec{V}_{3f} = \frac{200((121)\hat{i} + (38.0)\hat{j}) + 24142\hat{i} - 12856\hat{j}}{66}$$

$= (732.4\ m/s)\hat{i} - (79.6\ m/s)\hat{j}$

Hence, velocity of 3^{rd} piece is $(732.4\ m/s)\hat{i} - (79.6\ m/s)\hat{j}$

63.

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

Take point with 150 g mass at origin.

So,

$m_1 = 100\,g$; $\vec{r}_1 = -4\ cm\,\hat{i}$

$m_2 = 150\,g$; $\vec{r}_2 = 0$

$m_3 = 75\,g$; $\vec{r}_3 = 3\ cm\,\hat{j}$

Substitute in the expression of $\vec{R}_{CM}$

$$\vec{R}_{CM} = \frac{(100g)(-4\,cm\,\hat{i}) + (75g)(3\,cm\,\hat{j})}{(100g + 150g + 75g)}$$

$$= \frac{(-400 g\cdot cm\,\hat{i}) + (225 g\cdot cm\,\hat{j})}{325}$$

$$= -1.23\ cm\,\hat{i} + 0.69\ cm\,\hat{j}$$

Hence, center of mass if 150 g is considered at origin is at $x_{CM} = -1.23$ cm and $y_{CM} = 0.69$ cm

69.

$$\vec{r}_{CM} = 1/M \sum_{j=1}^{N} m_j \vec{r}_j$$

$$\vec{V}_{CM} = 1/M \sum_{j=1}^{N} m_j \vec{v}_j$$

$$r_{CM} = \frac{\int_0^L x \, dm}{\int_0^L dm} = \frac{\int_0^L x \rho(x) S \, dx}{\int_0^L \rho(x) S \, dx}$$

Solve $\int_0^L x \rho(x) S \, dx$

$$\int_0^L x \rho(x) S \, dx = \int_0^L \rho_0 + (\rho_1 - \rho_0)\left(\left(x/L\right)^2\right) S \, dx$$

$$= S \int_0^L \left(\rho_0 x + \left(\rho_1 \left(x^3/L^2\right) - \rho_0\left(\frac{x^3}{L^2}\right)\right)\right) dx$$

$$= S \left(\rho_0 x^2/2 + \left(\rho_1 \left(\frac{x^4}{4L^2}\right) - \rho_0 \left(\frac{x^4}{4L^2}\right)\right)\right)\Bigg|_0^L$$

$$= S\left(\rho_0 L^2/2 + \left(\rho_1 \left(\frac{L^4}{4L^2}\right) - \rho_0 \left(\frac{L^4}{4L^2}\right)\right)\right)$$

$$= S\left[\rho_0 L^2/2 + \left(\rho_1 \left(L^2/4 \right) - \rho_0 \left(L^2/4 \right)\right)\right]$$

$$= S\left[\rho_0 \left(\frac{L^2}{4}\right) + \rho_1 L^2/4\right)$$

Therefore;

$$r_{CM} = \frac{S\left(\rho_0 \left(L^2/4\right) + \rho_1 L^2/4\right)}{SL\left(\frac{2\rho_0 + \rho_1}{3}\right)}$$

The density of the rod varies as;

$$\rho(x) = \rho_0 + (\rho_1 - \rho_0)(x/L)^2$$

The expression for the mass of the infinitesmal element is,

$$dm = \rho(x) S\, dx$$

Total mass M $\Rightarrow$ $M = \int dm = \int_0^L \rho(x) S\, dx$

Therefore;

$$\int_0^L \rho(x) S\, dx = \int_0^L \rho_0 + ((\rho_1 - \rho_0)(x/L)^2)\, S\, dx$$

$$= S\int_0^L \rho_0 + ((\rho_1 - \rho_0)(x/L)^2)\, dx$$

Solving as follows;

$$\int_0^L \rho(x) S\, dx = S\left(\rho_0 L + \rho_1 \left(\frac{x^3}{3L^2}\right)_0^L - \rho_0 \left(\frac{x^3}{3L^2}\right)_0^L\right)$$

$$= S\left(\rho_0 L + \rho_1 (L/3) - \rho_0 (L/3)\right)$$

$$= SL\left(\rho_0 + \rho_1 (1/3) - \rho_0 (1/3)\right)$$

$$= SL\left(\frac{2\rho_0 + \rho_1}{3}\right)$$

Using $\vec{r}_{CM}$,

$$= \frac{\rho_0 (L/4) + \rho_1 L/4}{\frac{2\rho_0 + \rho_1}{3}}$$

$$= \frac{L(\rho_0 (1/4) + \rho_1 1/4)}{\frac{2\rho_0 + \rho_1}{3}}$$

Therefore, the radius of center of mass is.

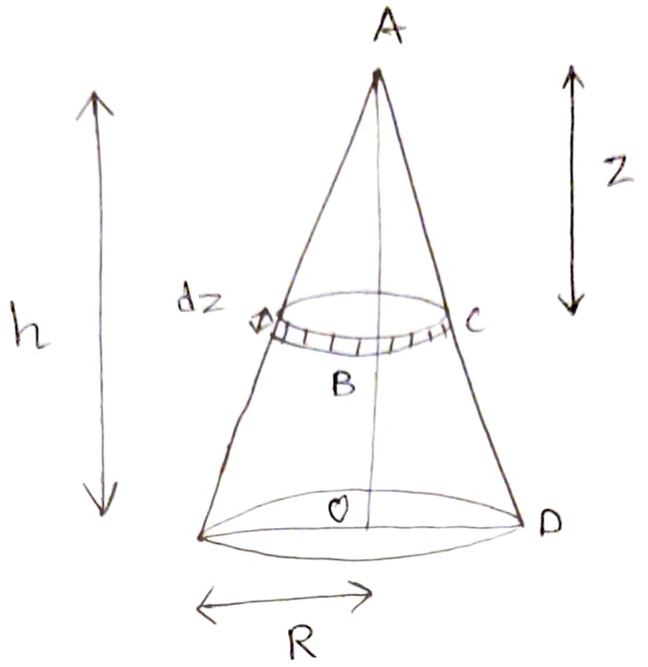
$$r_{CM} = 3L/4 \left(\frac{\rho_0 + \rho_1}{2\rho_0 + \rho_1} \right)$$

73. $x_{CM} = 1/M \int x \, dm_x$

$y_{CM} = 1/M \int y \, dm_y$

$z_{CM} = 1/M \int z \, dm_z$

Figure of the cone:-



The mass of the elemental disc:-

$$dm = \left(\frac{\text{Total mass}}{\text{Total volume}}\right)(\text{Volume Occupied by the disc})$$

Volume occupied by the disc = (Area of disc)(thickness of disc)

Substitute πr^2 for Area of disc, dz for thickness of the disc and solve,

Volume occupied = $\pi r^2 dz$

Substitute M for total mass, $1/3\,\pi R^2 h$ for volume of cone, $\pi r^2 dz$ for vol. of disc and solve for dm.

$$dm = \frac{M}{1/3\,\pi R^2 h}(\pi r^2 dz)$$

Since, the ΔABC and ΔAOD are two similar triangles, It can be said from the properties of similar triangles that,

$$R/h = r/z$$

$$r = \frac{zR}{h}$$

Substitute $\frac{zR}{h}$ for r in the equation to calculate dm

$$dm = \frac{M}{1/3 \pi R^2 h}\left(\pi \left(\frac{zR}{h}\right)^2 dz\right)$$

$$= \frac{3M}{h^3}(z^2 dz)$$

$$z_{CM} = 1/M \, (z) \, dm_z$$

Substitute, $\frac{3M}{h^3}(z^2 dz)$ for dm and solve for z_{CM}

$$z_{CM} = 1/M \int \frac{3Mz^2}{h^3} z \, dz$$

$$= 3/h^3 \int z^3 dz$$

Integrate z from 0 to h,

$$z_{CM} = 3/h^3 \left[\frac{x^4}{4}\right]_0^h = \frac{3h}{4}$$

The centre of mass from origin O :-

$$z_{CM} = h - \frac{3h}{4} = h/4$$

Therefore, the centre of mass of the cone is

$(0, 0, h/4)$