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Homework 3 :- 2D Motion

21. Using formulas

$\vec{d} = a\hat{i} + b\hat{j}$

$a = |\vec{d}| \cos\theta$

$b = |\vec{d}| \sin\theta$

Substitute 5.0 km for $|\vec{d}|$ and $0°$ for θ in above equations,

$a = (5.0 \text{ km}) \cos 0° = 5 \text{ Km}$

And,

$b = (5.0 \text{ km}) \sin 0° = 0 \text{ Km}$

Substitute above values of a and b in equation of displacement vector.

$\vec{d}_1 = (5 \text{ km})\hat{i} + (0 \text{ km})\hat{j}$

$= (5 \text{ km})\hat{i}$

$\theta = 90° + 20°$

$= 110°$

Substitute 10 km for $|\vec{d}|$ and 110° for θ in above equations of combonents

$a = (10.0\ km)\cos 110°$

$= 3.42\ km$

$b = (10.0\ km)\sin 110°$

$= 9.40\ km$

Substitute above values of a and b in above equation of displacement vector.

$\vec{d}_2 = (-3.42\ km)\hat{i} + (9.40\ km)\hat{j}$

$\theta = 180°$

Substitute 8.0 km for $|\vec{d}|$ and 180° for θ in the above equations of components.

$a = (8.0\ km)\cos 180° = -8\ km$

$b = (8.0\ km)\sin 180° = 0\ km$

Substitute above values of a and b in above equation of displacement vector.

$\vec{d}_3 = (-8\ km)\hat{i} + (0\ km)\hat{j}$

$= (-8\ km)\hat{i}$

Now the net displacement is,

$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$

Substitute values of $\vec{d}_1$, $\vec{d}_2$ and $\vec{d}_3$

$\vec{d} = (5 \text{ km})\hat{i} + (-3.42 \text{ km})\hat{i} + (9.40 \text{ km})\hat{j} + (-8 \text{ km})\hat{i}$

$= (-6.42 \text{ km})\hat{i} + (9.40 \text{km})\hat{j}$

Hence the final displacement of cyclist from where he started is $(-6.42 \text{ km})\hat{i} + (9.40 \text{ km})\hat{j}$.

23. (a) Using $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt}$

Substitute $4.0t^2\hat{i} - 3.0\hat{j} + 2.0t^3\hat{k}$ m for $\vec{r}(t)$ in the above eq.

$\vec{v}(t) = \dfrac{d(4.0t^2\hat{i} - 3.0\hat{j} + 2.0t^3\hat{k} \text{ m})}{dt}$ $\Rightarrow$ Sub. 0 for t,

$\vec{v}(0) = 8.0t\hat{i} + 6t^2\hat{k}$ m/s

$\vec{v}(0) = 0$ m/s

Thus, velocity at time ~~t~~ at $t = 0$ is 0 m/s

(b) Using

$V_{av} = \dfrac{\vec{r}_f(t) - \vec{r}_i(t)}{t_f - t_i}$

Substitute 0 for t in the above equation of $\vec{r}(t)$

$$\vec{r}(0) = 4.0(0)^2\hat{i} - 3.0\hat{j} + 2.0(0)^3\hat{k}\ \text{m}$$

$$= -3.0\hat{j}\ \text{m}$$

Substitute 1s for t in above equation of $\vec{r}(t)$

$$\vec{r}(1) = 4.0(1)^2\hat{i} - 3.0\hat{j} + 2.0(1)^3\hat{k}\ \text{m}$$

$$= 4.0\hat{i} - 3.0\hat{j} + 2.0\hat{k}\ \text{m}$$

Substitute $4.0\hat{i} - 3.0\hat{j} + 2.0\hat{k}$ m for $\vec{r}_f(t)$, $-3.0\hat{j}$ m for $\vec{r}_i(t)$, 1s for t_f and 0s for t_i in the above equation of V_{av},

$$V_{av} = \frac{(4.0\hat{i} - 3.0\hat{j} + 2.0\hat{k})\,\text{m} - (-3.0\hat{j})\,\text{m}}{(1-0)\,\text{s}}$$

$$= 4.0\hat{i} + 2.0\hat{k}\ \text{m/s}$$

Thus, average velocity between 0s and 1s is

$V_{av} = 4.0\hat{i} + 2.0\hat{k}$ m/s.

26. (a) Using equation,

$$\vec{r}(t) = 3.0t^2\hat{i} + 5.0\hat{j} - 6.0\hat{k}$$

Differentiate with respect to time t

$$\frac{d\vec{r}(t)}{dt} = \frac{d\vec{r}\,(3.0t^2\hat{i} + 5.0\hat{j} - 6.0\hat{k})}{dt}$$

$$= \frac{d(3.0t^2\hat{i})}{dt} + \frac{d(5.0\hat{j})}{dt} + \frac{d(6.0+\hat{k})}{dt}$$

$$= 3.0(2t)\hat{i} + 5.0(0) - 6.0\hat{k}$$

$$= 6.0t\hat{i} - 6.0\hat{k}$$

Using $\frac{d\vec{r}(t)}{dt} = \vec{v}(t)$

$$\vec{v}(t) = 6.0t\hat{i} - 6.0\hat{k}$$

Hence, the velocity of the particle as a function of time is $\vec{v}(t) = 6.0t\hat{i} - 6.0\hat{k}$

Differentiate velocity vector w.r.t. time.

$$\frac{d\vec{v}(t)}{dt} = \frac{d(6.0t\hat{i} - 6\hat{k})}{dt} = \frac{d(6t\hat{i})}{dt} - \frac{d(6\hat{k})}{dt}$$

$$= 6\hat{i}$$

Again from definition of acceleration vector,

$$\frac{d\vec{v}(t)}{dt} = \vec{a}(t)$$

Therefore $\vec{a}(t) = 6\hat{i}$

(b) Use the equation for velocity vector as function of time from part (a)

$$\vec{v}(t) = 6.0t\hat{i} - 6.0\hat{k}$$

Substitute 0 for time and solve for velocity.

$$\vec{v}(t) = 6.0t\hat{i} - 6.0\hat{k}$$

$$= 6.0(0)\hat{i} - 6.0\hat{k}$$

$$= -6.0\hat{k}$$

Using $\vec{a}(t) = 6\hat{i}$

The velocity of the particle at $t=0$ is $6.0\hat{k}$, at time $t=0$ the velocity has a magnitude 6.0 and is in the negative z-axis direction.

The acceleration of the particle at $t=0$ is $6\hat{i}$. This means it has a constant value of 6 and is in the positive x-axis direction.

31. (a) Differentiate the position function w.r.t t,

$$\frac{d(\vec{r}(t))}{dt} = \frac{d(\cos(1.0t)\hat{i} + \sin(1.0t)\hat{j} + t\hat{k})}{dt}$$

$$= \frac{d(\cos(1.0t))}{dt}\hat{i} + \frac{d(\sin(1.0t))}{dt}\hat{j} + \frac{d(t)}{dt}\hat{k}$$

$$= -\sin(1.0t)\hat{i} + \cos(1.0t)\hat{j} + \hat{k}$$

But, $\frac{d(\vec{r}(t))}{dt} = \vec{v}(t)$

Substitute $\vec{v}(t)$ for $\frac{d(\vec{r}(t))}{dt}$

$$\vec{v}(t) = -\sin(1.0t)\hat{i} + \cos(1.0t)\hat{j} + \hat{k}$$

Hence, the particle's velocity is $\vec{v}(t) = -\sin(1.0t)\hat{i} + \cos(1.0t)\hat{j} + \hat{k}$.

(b) Now, differentiate velocity vector w.r.t time t.

$$\frac{d(\vec{v}(t))}{dt} = \frac{d(-\sin(1.0t)\hat{i} + \cos(1.0t)\hat{j} + \hat{k}}{dt}$$

$$= \frac{d(-\sin(1.0t))}{dt}\hat{i} + \frac{d(\cos(1.0t))}{dt}\hat{j} + \frac{d(1)}{dt}\hat{k}$$

$$= \cos(1.0t)\hat{i} - \sin(1.0t)\hat{j}$$

But $\frac{d(\vec{v}(t))}{dt} = \vec{a}(t)$

Substitute $\vec{a}(t)$ for $\frac{d(\vec{v}(t))}{dt}$

$\vec{a}(t) = -\cos(1.0t)\hat{i} - \sin(1.0t)\hat{j}$

Hence, the particle's acceleration as function of time is $\vec{a}(t) = -\cos(1.0t)\hat{i} - \sin(1.0t)\hat{j}$

33. (a) Using $y = y_0 + v_{y_0}t + \frac{1}{2}a_y t^2$

Substitute 1.5 m for y_0, 0 for v_0, 0 for v_{y_0} and $-9.81\ m/s^2$ for acceleration due to gravity

This gives,

$0 = 1.5\ m + (0)t + \frac{1}{2}(-9.81\ m/s^2)t^2$

$-1.5\ m = (-4.905\ m/s^2)t^2$

Solve for time t,

$t = \sqrt{\frac{1.5\ m}{4.905\ m/s^2}} = 0.553$

Therefore, the time elapsed before the bullet hits the ground is 0.553 s

(b) $x = x_0 + v_{y_0} t$

Substitute 0 for x_0, 200 m/s for v_{x_0} and 0.553 s for t,

$x = 0 + (200\ ms^{-2})(0.553 s)$

$= 110.6\ m$

Therefore, the distance covered by the bullet in horizontal direction before it hit the ground is 110.6 m

36. Using $y = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$

Substitute 800 m for y_0, 0 for y, 0 for v_{y_0} and $-9.81\ m.s^{-2}$ for a_y

$0\ m = 800\ m + (0)t + \frac{1}{2}(-9.81\ m.s^{-2})(t^2)$

$-800\ m = -4.905\ m.s^{-2} t^2$

Solve for time t

$$t = \sqrt{\frac{800 m}{4.905\ m.s^{-2}}} = 12.77 s$$

Converting initial velocity from km/h to m/s

$$V_{x_0} = (500\ km/h)\left(\frac{5/18\ m/s}{1\ km/h}\right) = 138.89\ m/s$$

Using $x = x_0 + V_{x_0} t$

Substitute 0 for x_0, 138.89 m/s for V_{x_0} and 12.77s for t. Solve for x

$$x = 0 + 138.89\ m/s\ (12.77s) = 1773.625\ m$$

Therefore, the crate when released from a flying airplane falls on the ground 1773.625 m away from the release point of the crate.

39. Using $y = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$

Re-write the expression in terms of V_{y_0}

$$V_{y_0} = \frac{y - y_0 - \frac{1}{2} a_y t^2}{t}$$

Substitute 0 m for y_0, 0 for y, 20s for t and -9.81 m/s² for a_y,

$$V_{y_0} = \frac{0 - 0 - \frac{1}{2}(-9.81\,m/s^2)(20\,s)^2}{20\,s}$$

$$= 98.1\ m/s$$

Using $V_{y_0} = V_0 \sin\theta$

Re-write in terms of V_0

$$V_0 = \frac{V_{y_0}}{\sin\theta}$$

Substitute 98.1 m/s for V_{y_0} and 30° for θ

$$V_0 = \frac{98.1\ m/s}{\sin 30^\circ} = 196.2\ m/s$$

Hence, the projectile will have an initial velocity of 196.2 m/s when it is thrown at an angle of 30° from the ground.

(b) Using $H = \frac{V_0^2 \sin^2\theta}{2g}$

Substitute 196.2 m/s for V_0, 30° for θ and 9.81 m/s² for a_y.

$$H = \frac{(196.2\ m/s)^2 \sin^2 30^\circ}{2(9.81\ m/s^2)} = 490.5\ m$$

Therefore, the maximum altitude reached by the projectile is 490.5 m

(c) Using $R = \dfrac{v_0^2 \sin 2\theta}{g}$

Substitute 196.2 m/s for v_0, 30° for angle of projectile and 9.81 m/s² for a_y,

$$R = \frac{(196.2\ m/s)^2 \sin(2(30^\circ))}{9.81\ m/s^2} = 3398.284\ m$$

Therefore, the range of the projectile is 3398.284 m when the projectile reaches the same height from where it was released.

(d) Using $x = x_0 + (v_0 \cos\theta)t$

Substitute 0 for x_0, 196.2 m/s for v_0, 30° for θ and 15 s for t,

$$x = 0 + (196.2\ m/s\ (\cos 30^\circ))(15\ s)$$

$$= 2548.713\ m$$

Using $y = y_0 + v_{y_0} t + 1/2 a_y t^2$

Substitute 0 for y_0, 98.1 m/s for v_{y_0}, 15 s for t and -9.81 m/s^2 for a_y,

$y = 0 + (98.1 \text{ m/s})(15 \text{ s}) + 1/2(-9.81 \text{ m/s}^2)((15 \text{ s})^2) = 367.875 \text{ m}$

The displacement at 15 s is given by,

displacement $= x\hat{i} + y\hat{j}$

Substitute 2548.713 m for x and 367.875 m for y,

displacement $= (2548.713\hat{i} + 367.875\hat{j})$ m

Therefore, total displacement is $(2548.713\hat{i} + 367.875\hat{j})$ m

45. Using $R = \dfrac{u^2 \sin 2\theta}{g}$ and $T = \dfrac{2\sin\theta}{g}$

Using $u_x = u\cos\theta$

Solve for u_x by substituting 30 m/s for u and 53° for θ in the equation $u_x = u\cos\theta$

$u_x = (30\ m/s)\cos 53^\circ$

$= 18.05\ m/s$

Using $u_y = u\sin\theta$

Solve for u_y by substituting 30 m/s for u and 53° for θ in the equation $u_x = u\cos\theta$

$u_y = (30\ m/s)\sin 53^\circ$

$= 23.96\ m/s$

(a) The expression for the third equation of motion is as follows:

$v_y^2 = u_y^2 + 2a_y h$

Rearrange the equation for h

$$h = \frac{v_y^2 - u_y^2}{2a_y}$$

Solve for h by substituting 0 m/s for v_y, 23.96 m/s for u_y, and $-9.8\ m/s^2$ for a_y in the equation

$$h = \frac{v_y^2 - u_y^2}{2a_y}$$

$$h = \frac{(0\ m/s)^2 - (23.96\ m/s)^2}{2(-9.8\ m/s^2)} = 29.28\ m$$

Hence, the height above the edge of the cliff is 29.28 m

(b) Using $v_y = u_y + a_y t$

Rearrange the equation for t

$$t = \frac{v_y - u_y}{a_y}$$

Solve for t by substituting 0 m/s for v_y, 23.96 m/s for u_y, and $-9.8\ m/s^2$ for a_y in the equation

$$t = \frac{v_y - u_y}{a_y} = \frac{(0\ m/s) - (23.96\ m/s)}{-9.8\ m/s^2} = 2.44\ s$$

Using $x_{om} = u_x t$

Solve for x_{om} by substituting 18.05 m/s for u_x and 2.44 s for t in the equation $x_{om} = u_x t$

$$x_{om} = (18.05\ m/s)(2.44\ s) = 44.042\ m$$

$$= 44.04\ m$$

Hence, the horizontal distance covered when the rock is at maximum height is 44.04 m

(c) Using $H = h_{cliff} + h$

Now, substitute 100 m for h_{cliff} and 29.28 m for h in the equation $H = h_{cliff} + h$.

$H = 100\ m + 29.28\ m = 129.28\ m$

Using $H = u_y t + \frac{1}{2} a_y t_1^2$

Now substitute 129.28 m for H, 0 m/s for u_y, and $-9.8\ m/s^2$ for a_y in the equation

$129.28\ m = (24\ m/s)t + \frac{1}{2}(-9.8\ m/s^2)t_1^2$

$129.28\ m = (24\ m/s)(2.44\ s) + \frac{1}{2}(-9.8\ m/s^2)t_1^2$

$129.28\ m = (58.56\ m) + (-4.9\ m/s^2)t_1^2$

$70.72\ m = (-4.9\ m/s^2)t_1^2$

Solve for t_1

$$t_1 = \sqrt{\frac{70.72\ m}{(4.9\ m/s^2)}} = 3.79\ s$$

Using $T = t + t_1$

$= 2.44\ s + 3.79\ s = 6.23\ s$

Hence, the total time taken by the rock from release to hit the ground is 6.23 s

(d) Using $R = u_x t$

Substitute 18.05 m/s for u_x and 6.23s for t in the equation $R = u_x t$

$R = (18.05 \text{ m/s})(6.23 \text{ s})$

$= 112.61 \text{ m} \approx 113 \text{ m}$

Hence, the range of the projectile motion is 113 m

(e) Using $x_t = x_0 + u_x t$

Substitute 0 m/s for x_0 and 18.05 m/s for u_x in the equation $x_t = x_0 + u_x t$

$x_t = 0 \text{ m} + (18.05 \text{ m/s}) t$

$= (18.05) t$ — (1)

Substitute 2s for t in (1)

$x_{t=2s} = (18.05 \text{ m/s})(2 \text{ s}) = 36.1 \text{ m}$

Substitute 4s for t in (1)

$x_{t=4s} = (18.05 \text{ m/s})(4 \text{ s})$

$= 72.2 \text{ m}$

Substitute 6s for t in the equation (1)

$x_{t=6} = (18.05 \text{ m/s})(6 \text{ s}) = 108.3 \text{ m}$

Hence, the horizontal position of the rock to the edge of the cliff at 2s, 4s and 6s are 36.1 m, 72.2 m and 108.3 m respectively.

Using $y_t = y_0 + u_y t - \frac{1}{2} g t^2$

Substitute 0 m for y_0, 23.96 m/s for u_y, and 9.8 m/s² for g in the equation

$$y_t = y_0 + u_y t - \frac{1}{2} g t^2$$

$$y_t = 0\,m + (23.96\,m/s)t - \frac{1}{2}(9.8\,m/s^2)t^2$$

$$= (23.96\,m/s)t - \frac{1}{2}(9.8\,m/s^2)t^2$$

$$= (23.96\,m/s)t - (4.9\,m/s^2)t^2$$

$$y_t = (23.96\,m/s)t - (4.9\,m/s^2)t^2 \quad \text{---(2)}$$

Substitute 2s for t in equation (2)

$$y_{t=2} = (23.96\,m/s)(2s) - (4.9\,m/s^2)(2s)^2$$

$$= 28.32\,m$$

Substitute 4s for t in the equation (2)

$$y_{t=4} = (23.96\,m/s)(4s) - (4.9\,m/s^2)(4s)^2$$

$$= 17.44\,m$$

Substitute 6s for t in the equation (2)

$$y_{t=6} = (23.96\text{ m/s})(6\text{ s}) - (4.9\text{ m/s}^2)(6\text{ s})^2$$

$$= 32.64\text{ m}$$

Hence, the vertical positions of the rock relative to the edge of the cliff at 2s, 4s and 6s are 28.32 m, 17.44 m and -32.64 m respectively.

61. Using $a_c = v^2/r$

Substitute (20 m/s) for v and (10 m) for r in the expression for a_c.

$$a_c = v^2/r = \frac{(20\text{ m/s})^2}{(10\text{ m})} = 40\text{ m/s}^2$$

Thus, the particle's centripetal acceleration has a magnitude 40 m/s²

63. Using $a_c = v^2/r$

$$= \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \left(\frac{2\pi}{T}\right)^2 r = \omega^2 r$$

Therefore $a_c = \omega^2 r$

$$\omega = \sqrt{\frac{a_c}{r}}$$

Substitute 9.8 m/s² for a_c and 8 m for r

$$\omega = \sqrt{\frac{9.8\ m/s^2}{8\ m}} = (1.1\ rad/s)\left(\frac{1\ rev}{2\pi\ rad}\right)\left(\frac{60\ s}{1\ min}\right)$$

$$= 10.6\ rev/min$$

Therefore, the angular speed of riders is 10.6 rev/min