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Homework 7 - Potential Energy and Conservation of Energy

19. $$n = \frac{E_{electron}}{E_{DNA}} \quad \ldots\ldots (1)$$

Substitute 4.0×10^{-15} J for E_{elect} (from table) and 10^{-19} J for E_{DNA} (from table) in equation (1)

$$n = \frac{4.0 \times 10^{-15} \text{ J}}{10^{-19} \text{ J}}$$

$$= 40{,}000$$

Hence, the number of DNA molecules that can be broken by the energy of one electron is 40,000

21. Using

$$\Delta U_{AB} = U_B - U_A$$

$$= mg(y_B - y_A)$$

(a) The weight (mg) of camera is 10 N, the initial position of camera is 20 m and final position is 0 m.

$$\Delta U_{AB} = mg(y_B - y_A)$$

$$= (10\ N)(0-20)\ m$$

$$= -200\ N\,m\left(\frac{1\ J}{N\,m}\right)$$

$$= -200\ J$$

Thus, the potential energy change is $-200\ J$

(b) The final position of camera is 0 m and initial position is 20 m.

$$\Delta U_{AB} = mg(y_B - y_A)$$

$$= (10\ N)(0-20)\ m$$

$$= -200\ N\left(\frac{1\ J}{N\,m}\right)$$

$$= -200\ J$$

(c) The camera's energy before falling down from 30 m to 20 m is:

$$\Delta U_{AB} = (10\ N)(20-30)\ m$$

$$= -100\ N\,m\,(1\ J/N\,m) = -100\ J$$

Thus, the potential energy change of camera before it falls from the drone is $-100\ J$

(d) The potential energy of camera after falling down from 30 m to 0 m is evaluated as follows:

$$\Delta U_{AB} = mg(y_B - y_A)$$

$$= (10\ N)(0-30)\ m$$

$$= -300\ Nm\left(1\ J/Nm\right)$$

$$= -300\ J$$

Thus, the potential energy change of camera after it falls on the ground is $-300\ J$

25. Using

$$W = \int_{x_1}^{x_2} F dx$$

(a) The particle moves from 2.0 m to 5.0 m and the force $F(x)$ is $(-5.0x^2 + 7.0x)$ N.

$$W = \int_{x_1}^{x_2} F dx = \int_{2.0}^{5.0} (-5.0x^2 + 7.0x)\, dx$$

$$= \left[\frac{-5.0x^3}{3} + \frac{7.0x^2}{2} \right]_{2.0}^{5.0}$$

$$= -120\,J$$

Thus, the work done on the particle is $-120\,J$

(b) $W_{AB} = U_B - U_A$

The potential energy of a particle moving from $\infty \to x$ is determined as follows:

$$W = U_\infty - U_x$$

$$-120\,J = 0 - U_x$$

$$U_x = 120\,J$$

Thus, the potential energy for this force is $120\,J$

27. $F_x = -\frac{dU(x)}{dx}$

(a) Acc. to the problem,

$$U(x) = -a/x^{12} - b/x^6$$

$$\frac{dU(x)}{dx} = \frac{d(-a/x^{12} - b/x^6)}{dx}$$

$$= \frac{d(a/x^{12})}{dx} - \frac{d(b/x^6)}{dx}$$

$$= \frac{12ax^{11}}{x^{14}} + \frac{6bx^5}{x^{12}} = \frac{12a}{x^{13}} + \frac{6b}{x^7}$$

Substitute $\frac{12a}{x^{13}} + \frac{6b}{x^7}$ for $\frac{dU(x)}{dx}$ in the above mentioned expression of F_x and solve,

$$F_x = -\left(\frac{12a}{x^{13}} + \frac{6b}{x^7}\right) = -\frac{12a}{x^{13}} - \frac{6b}{x^7}$$

$F_x = 0$

Substitute $-\frac{12a}{x^{13}} - \frac{6b}{x^7}$ for F_x

$$-\frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$-\frac{12a}{x^{13}} = \frac{6b}{x^7}$$

$$-\frac{12a}{x^6} = 6b$$

$$x^6 = -\frac{12a}{6b}$$

Further solve for x,

$$x^6 = -\frac{12a}{6b}$$

$$x = \left(-\frac{2a}{b}\right)^{1/6}$$

Hence, the distance of separation where the potential energy has a local minimum is $\left(-\frac{2a}{b}\right)^{1/6}$

(b) At equilibrium the conservative force is zero. Hence, from the above section (a), it is clear that the force on atom at this separation is 0 N

(c) The expression of the separation of distance is calculated in the section (a) and is given as,

$$x^6 = -\frac{12a}{6b}$$

Hence, this means that the force varies with the separation of distance of $\sim x^6$

29. Using $W = \int_{x_1}^{x_2} F(x)\,dx$

Substitute $(-5x^2 + 7x)$ for F, −4 for x_1 and 4 for x_2

$$W = \int_{-4}^{4} (-5x^2 + 7x)\,dx = \left(-5\left(\frac{x^3}{3}\right) + 7\left(\frac{x^2}{2}\right)\right)\Big|_{-4}^{4}$$

$$= \frac{-5\times4^3}{3} + \frac{7\times(4^2)}{3} - \left[\frac{-5\times4^3}{3} + \frac{7\times(4^2)}{3}\right]$$

$$= -213.33\ \mathrm{J}$$

Using $KE = 1/2\, mv^2$

To get K_i, substitute 2kg for m and 20 m/s for v,

$$KE_i = 1/2\,(2\,kg)\,(20\,(m/s))^2 = 400\,J$$

Also K_f is,

$$KE_f = 1/2\, m v_f^2$$

Substitute 2 kg for m,

$$KE_f = 1/2\,(2\,kg)\,v_f^2$$

Using $W = KE_f - KE_i$

Substitute values of W, KE_f and KE_i from above

$$-213.33\,J = 1/2\,(2\,kg)\,v_f^2 - 400\,J$$

Rearrange to get.

$$1/2\,(2\,kg)\,v_f^2 = 400\,J - 213.33\,J$$

$$v_f = \sqrt{\frac{186.33\,J}{1\,kg}} = 13.66\,m/s$$

Hence the magnitude of velocity ~~of~~ or speed at $x = 4\,m$ is 13.66 m/s

31. $W_g + W_a = KE_f - KE_i$ ......(1)

Using $KE = \frac{1}{2}mv^2$

For KE_f:

Substitute 17 m/s for v and 0.25 kg for m

$KE_f = \frac{1}{2}(0.25\,kg)(17(m/s))^2$

$= 36.125\,J$

For KE_i,

Substitute 20 m/s for v and 0.25 kg for m.

$KE_i = \frac{1}{2}(0.25\,kg)(20\,(m/s))^2 = 50\,J$

Now, substitute 0 J for W_g, 36.125 J for KE_f and 50 J for KE_i in (1)

$0J + W_a = 36.125\,J - 50\,J$

$W_a = -13.875\,J$

$W_a \approx -14\,J$

Hence, the work done by the force of resistance is 14 J

33. $\Delta PE_{rock} = mgh$

$\Delta KE_{rock} = 1/2 mv_f^2 - 1/2 mv_o^2$

$\Delta KE_{rock} = \Delta PE_{rock}$

$\left(1/2 mv_f^2 - 1/2 mv_0^2\right) = mgh$

Therefore,

$1/2 mv_f^2 = 1/2 mv_0^2 + mgh$

Or

$v_f = (v_0^2 + gh)^{1/2}$

Substitute, 2.00 m/s for v_0 and 20.0 m for h in equation $v_f = (v_0^2 + 2gh)^{1/2}$

$v_f = [(15.0\ m/s)^2 + 2(9.80\ m/s^2)(20.0\ m)]^{1/2}$

$= 24.84\ m/s$

Hence the speed of the rock when it strikes the water is 24.8 m/s

37. Convert 50 cm into m

$(50\ cm) = (50\ cm)\left(\frac{10^{-2}\ m}{1\ cm}\right) = 0.5\ m$

Using $F = -kx$

Rearranging the equation in terms of k

$k = F/x$

Substitute 150 N for F and 0.5 m for x

$$k = \frac{150\ N}{0.5\ m} = 300\ N/m$$

Hence, the value of spring constant is 300 N/m

Using $P.E. = \frac{1}{2} kx^2$

Substitute 300 N/m for k and 0.5 m for x and Solve for P.E.

$$P.E. = \frac{1}{2}(300\ N/m)(0.5\ m)^2 = 37.5\ J$$

$$K.E. = \frac{1}{2}mv^2$$

$$K.E. = P.E.$$

$$\frac{1}{2}mv^2 = P.E$$

Substitute 50 g for m and 37.5 J for P.E.

$$\frac{1}{2}(50 g)\left(\frac{10^{-3} kg}{1 g}\right)v^2 = 37.5 J$$

$$\frac{1}{2}(0.050 kg)v^2 = 37.5$$

Rearrange the equation in terms of v.

$$V = \sqrt{\frac{2(37.5J)}{0.05\,kg}} = 38.7\ m/s$$

Hence, the speed with which the arrow immediately leaves the bow is 38.7 m/s

41. $T_i = T_f$

$T_i = K_i + U_i$

and $T_f = K_f + U_f$

According to the conservation of energy,

$K_i + U_i = W + K_f + U_f$

$K_i = \frac{1}{2} m v_i^2$

$K_f = \frac{1}{2} m v_f^2$

$U_i = mgh_i$

$U_f = mgh_f$

Substitute U_f, U_i, K_f, K_i in the equation

$K_i + U_i = W + K_f + U_f$

$\frac{1}{2} m v_i^2 + mgh_i = W + \frac{1}{2} m v_f^2 + mgh_f$

Rearrange the equation for W.

$W = \frac{1}{2} m v_i^2 + mgh_i - \left(\frac{1}{2} m v_f^2 + mgh_f\right)$

Now, substitute $(h_i + 20)$ m for h_f and h for h_i in the equation

$$W = \frac{1}{2} m v_i^2 + m g h_i - \left(\frac{1}{2} m v_f^2 + m g h_f\right)$$

$$W = \frac{1}{2} m v_i^2 + m g h_i - \left(\frac{1}{2} m v_f^2 + m g (h_i + 20)\right)$$

$$= \frac{1}{2} m v_i^2 - \left(\frac{1}{2} m v_f^2 + m g (20\text{ m})\right)$$

Solve for W by substituting 0.25 kg for m, 40 m/s for v_i, 30 m/s for v_f, 9.8 m/s² for g in the equation $W = \frac{1}{2} m v_i^2 - \left(\frac{1}{2} m v_f^2 + m g (20\text{ m})\right)$

$$W = \frac{1}{2} (0.25\text{ kg})(40\text{ m/s})^2 - \left(\frac{1}{2}(0.25\text{ kg})(30\text{ m/s})^2 + (0.25\text{ kg})(9.8\text{ m/s}^2)(20\text{ m})\right) = -38.45\text{ J} \approx -38.5\text{ J}$$

Hence, the work done by the air resistance is -38.5 J

47. $$F = -\frac{dU}{dx}$$

Re-arrange the equation in terms of potential energy.

$dU = -F dx$

Integrate both sides within limits U_1 and U_2 for dU, x_A and x_B for dx $\int_{U_1}^{U_2} dU = -\int_{x_A}^{x_B} F dx$

Substitute $-cx^3$ N for F.

$$\int_{U_1}^{U_2} dU = -\int_{x_A}^{x_B} (-cx^3 \text{ N})\, dx$$

$$U_2 - U_1 = c/4 \,(x_B^4 - x_A^4) \text{ J}$$

Substitute -2 m for x_B, 1 m for x_A and 8 N/m^3 for c.

$$U_2 - U_1 = \frac{8 \text{ N/m}^3}{4}\left((-2 \text{ m})^4 - (1 \text{ m})^4\right) = 30 \text{ J}$$

Therefore, change in potential energy is 30 J

Change in kinetic energy is,

$$\Delta K.E. = 1/2\, m(v_i^2 - v_2^2)$$

From the law of conservation of energy,

$$\Delta K.E. = \Delta P.E.$$

$$1/2\, m(v_i^2 - v_2^2) = U_2 - U_1$$

Re-write in terms of v_2.

$$v_2 = \sqrt{v_1^2 - \frac{2(U_2 - U_1)}{m}}$$

Substitute 30 J for $(U_2 - U_1)$, 4 kg for m and 6 m/s for v_1.

$$v_2 = \sqrt{(6 \text{ m/s})^2 - \frac{2(30 \text{ J})}{4 \text{ kg}}} = 4.6 \text{ m/s}$$

Therefore, the velocity of the body is 4.6 m/s when x_B is -2 m.

61. At the maximum height,

$$\frac{1}{2} m v_0^2 = mgh$$

(a) Use the above formula,

$$h = \frac{v_0^2}{2g}$$

Substitute 110 km/h for v_0 and 9.80 m/s² for g

$$h = \frac{v_0^2}{2g} = \frac{\left((110 \text{ km/h}) \times \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \times \left(\frac{1 \text{h}}{3600 \text{ s}}\right)\right)^2}{2(9.80 \text{ m/s}^2)}$$

$$= 47.6 \text{ m}$$

Therefore, the maximum height attained by the car is 47.6 m

(b) $\frac{1}{2} m v_0^2 = mgh + E_f$

$$E_f = m\left(\frac{1}{2} v_0^2 - gh\right)$$

Substitute 110 km/h for v_0, 9.80 m/s² for g and 22 m for h

$$E_f = m\left(\tfrac{1}{2}v_0^2 - gh\right)$$

$$= 750\ kg\left[\tfrac{1}{2}\left(\left(110\ km/h\right)\times\left(\frac{1000\ m}{1\ km}\right)\times\left(\frac{1h}{3600\ s}\right)\right)^2\right.$$

$$\left. - \left(9.80\ m/s^2\right)\left(22\ m\right)\right]$$

$$= 1.89 \times 10^5\ J$$

Therefore, the energy lost due to friction is $1.89 \times 10^5 J$

(c) $W = F_f \cdot d$

$W = F_f\left(\frac{h}{\sin\theta}\right)$

$F_f = \frac{W\sin\theta}{h}$

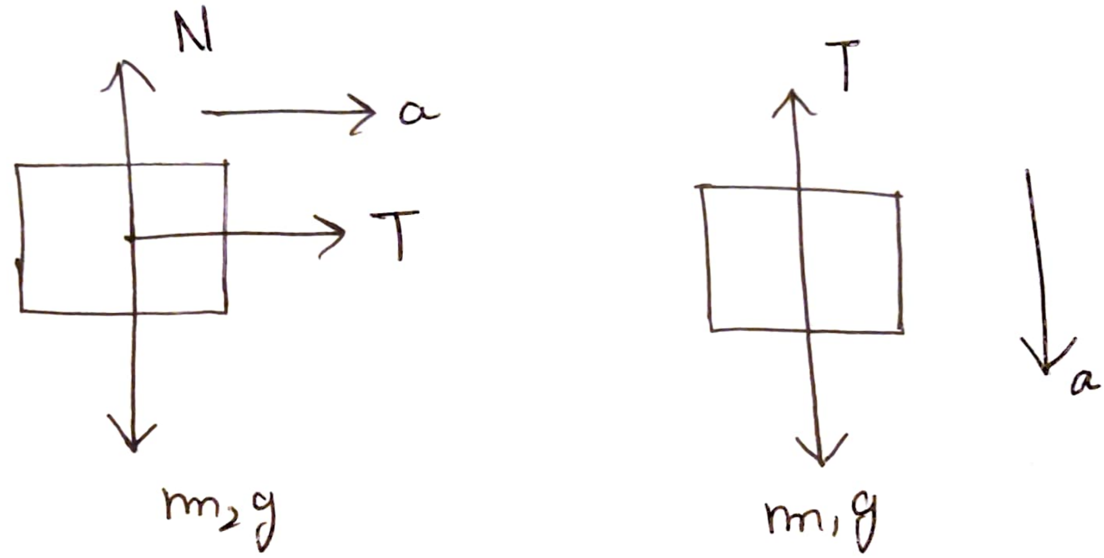
$F_f = \frac{W\sin\theta}{h}$

$$F_f = \frac{(1.89 \times 10^5\ J)\sin 2.5°}{22\ m}$$

$= 374\ N$

Therefore, the average friction force is 374 N

71. The free body diagram of an object placed on the plane is,



Using Newton's second law for block 1

$$m_1 g - T = m_1 a$$

Using Newton's second law for block 2

$$T = m_2 a$$

Combining both,

$$m_1 g = m_2 a + m_2 a$$

$$a = \left(\frac{m_1}{m_1 + m_2}\right) g$$

Substitute 2.0 kg for m_1, 4.0 kg for m_2 and $9.81\ m/s^2$ for g

$$a = \left(\frac{2.0\,kg}{(2.0+4.0)\,kg}\right)(9.8\,m/s^2) = 3.27\,m/s^2$$

Therefore, the acceleration of the blocks is $3.27\,m/s^2$

Using $v^2 = u^2 + 2aS$

Taking square root both side,

$$v = \sqrt{u^2 + 2aS}$$

Substitute $0\,m/s$ for u, $3.27\,m/s^2$ for a and $2.0\,m$ for S

$$v = \sqrt{2(3.27\,m/s^2)(2.0\,m)} = 3.6\,m/s$$

Therefore, the speed of the blocks after they have each moved $2.0\,m$ is $v = 3.6\,m/s$

75. Using $F_c = \frac{mv^2}{r}$

The initial potential energy of the object is =

mga

As object swing around the peg, the radius of the circle is $a-h$

So,

$$mg = \frac{mv^2}{a-h}$$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mg(a-h) \Rightarrow K.E.$

P.E. of object at max. height of circle =
$mg(2)(a-h)$ because height from lowest point
is $2(a-h)$

$mga = KE + PE$

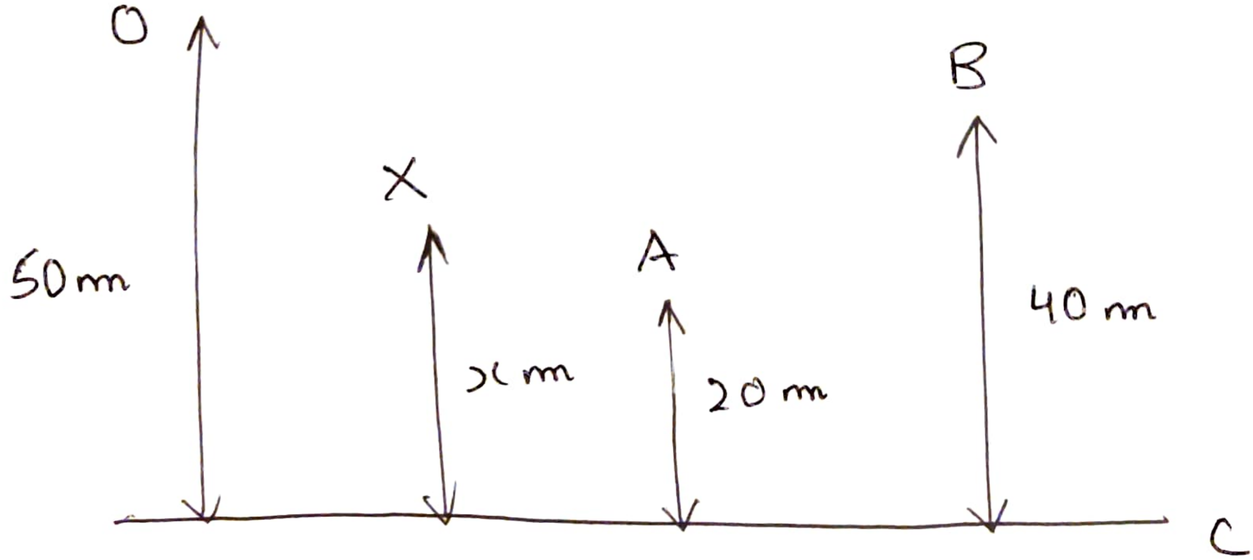
$mga = \frac{1}{2}mg(a-h) + mg(2)(a-h)$

Rearrange for h

$h = 3/5\, a$

Hence, h must be greater than $3/5\, a$ if the
ball is swung around the peg.

83. The figure show the position of the car at different
points when it rolls in a track,



At point O $U = mgh$

Substitute $9.81\ m/s^2$ for g and $50\ m$ for h.

$U = m(50\ m)(9.81\ m/s^2) = 490.5m$

K.E. at point O is 0.

E at point O is

$E = 0 + 490.5\ m = 490.5\ m$

Total energy at point X is,

$E_x = KE_x + U_x = \frac{1}{2} m v_x^2 + mgh_x$

$E_x = E_0$

Substitute $\frac{1}{2} m v_x^2 + mgh_x$ for E_x and $490.5m$ for E_0

$\frac{1}{2} m v_x^2 + mgh_x = (490.5)(m)$

Rewrite the above equation in terms of v_x.

$v_x = \sqrt{2(490.5 - gh_x}$

Substitute $9.81\ m/s^2$ for g and x for h_x and solve for v_x.

$v_x = \sqrt{2((490.5) - (9.81\ m/s^2 (x))}$ ...... (1)

$= \sqrt{981 - 19.62x}$ m/s

Substitute 20 m for x in eq.(1)

$v_A = \sqrt{981 - 19.62(20)}$ m/s $= 24.26$ m/s

Substitute 40 m for x for v_B

$v_B = \sqrt{981 - 19.62(40)}$ m/s $= 14.007$ m/s

$\simeq 14$ m/s

Substitute 0 m for x for v_c

$v_c = \sqrt{981}$ m/s $= 31.32$ m/s

Hence, the speed of the car at a point C is 31.32 m/s.

Hence, the speed of the car at points A, B and C are 24.26 m/s, 14.0 m/s and 31.32 m/s respectively.