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Homework 10:- Angular Momentum

19. $$\omega = \frac{V_{CM}}{R}$$

Substitute 90 km/h for V_{CM} and 37.5 cm for R in above exp.

$$\omega = \frac{(90\ km/h)\left(\frac{10^3\ m}{1\ km}\right)\left(\frac{1\ h}{3.6\times10^3\ s}\right)}{(37.5\ cm)\left(\frac{1\ m}{100\ cm}\right)} = 66.7\ s^{-1} \text{ or } 66.7\ rad/s$$

Hence, the angular velocity of the tire is 66.7 rad/s

25. $$x - x_0 = V_0 t + \tfrac{1}{2} a_{CM} t^2$$

Rearrange the exp. of distance for acceleration.

$$a_{CM} = \frac{2(x - x_0 - V_0 t)}{t^2}$$

Substitute 10 m for x, 0 m for x_0, 0 m/s for V_0 and 2.6 s for t.

$$a_{CM} = \frac{2(10\ m - 0 - 0\cdot 2.6\ s)}{(2.6\ s)^2} = 2.96\ m/s^2$$

$$a_{CM} = \frac{mg\sin\theta}{m + (I_{CM}/r^2)}$$

Rearrange expression of moment of inertia off the body.

$$I_{CM} = \left(\frac{g\sin\theta}{a_{CM}} - 1\right) mr^2$$

Substitute $9.8\ m/s^2$ for g, $30°$ for θ and $2.96\ m/s^2$ for a_{CM}

$$I_{CM} = \left(\frac{(9.8\ m/s^2)(\sin 30°)}{2.96\ m/s^2} - 1\right) mr^2 = 0.66 mr^2$$

Hence, the moment of inertia of the object is $0.66 mr^2$

29. $W = KE_f - KE_i$

$$KE = 1/_2\, mv_{CM}^2 + 1/_2\, I_{CM}\omega^2$$

$$I_{CM} = 1/_2\, mr^2$$

$$KE_f = 0$$

Therefore, $W = -KE_i$

$$= -\left(1/_2\, mv_{CM}^2 + 1/_2\, I_{CM}\omega^2\right)$$

$$= -\left(1/_2\, mv_{CM}^2 + 1/_2\left(1/_2\, mr^2\right)\left(\frac{v_{CM}}{r}\right)^2\right)$$

$$= -3/_4\, mv_{CM}^2$$

Substitute 40 kg for m and 6.0 m/s for v_{cm}

$$W = -3/4\,(40\,kg)(6.0\,m/s)^2 = -1080.0\,J$$

Hence, -1080.J work is required to stop the rolling cylinder.

37. Using,

$$\vec{p} = m\vec{v} \quad -(1)$$

$$\vec{l} = \vec{r} \times \vec{p} \quad -(2)$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad -(3)$$

$$\vec{A} \times \vec{B} = AB\sin\theta(\hat{n})$$

Substitute 5.0 kg for m and $(3.0\hat{i})$ m/s for $\vec{v}$ in (1)

$$\vec{p} = (5.0\,kg)(3.0\hat{i})\,m/s$$
$$= (15\hat{i})\,kg\cdot m/s$$

(a) Substitute $(2.0\hat{i} - 3.0\hat{j})$ m for $\vec{r}$ and $(15\hat{i})$ kg·m/s for $\vec{p}$ in (2)

$$\vec{l} = (2.0\hat{i} - 3.0\hat{j})\,m \times (15\hat{i})\,kg\cdot m/s$$

$$= \left[(2.0\hat{i}) \times (15\hat{i}) - (3.0\hat{j}) \times (15\hat{i})\right] kg\cdot m^2/s$$

$$= \left[0 - 45(-\hat{k})\right] kg\cdot m^2/s$$

$$= (45.0\hat{k})\,kg\cdot m^2/s$$

Hence, angular momentum of the particle is $(45.0\hat{k})$ kg·m^2/s

(b) Substitute $(2.0\hat{i} - 3.0\hat{j})\,m$ for $\vec{r}$ and $5.0\hat{j}\,N$ for $\vec{F}$ in (3)

$$\vec{\tau} = (2.0\hat{i} - 3.0\hat{j})\,m \times (5.0\hat{j}\,N) = 10.0\,N\cdot m\,\hat{k}$$

Hence, the torque about the origin is $10.0\,N\cdot m\,\hat{k}$

42. $\vec{I} = \vec{r} \times \vec{p}$ —(1) | $\vec{\tau} = \vec{r} \times F$ —(3)

$\vec{p} = m\vec{v}$ —(2) | $\dfrac{d\vec{I}}{dt} = \sum \vec{\tau}$ —(4)

(a) Substitute 1.0 kg for m and $(-1.0\hat{i} + 4.0\hat{j} + 1.0\hat{k})$ m/s for $\vec{v}$ in (2)

$$\vec{p} = 1.0\,kg\,(-1.0\hat{i} + 4.0\hat{j} + 1.0\hat{k})\,m/s$$
$$= (-1.0\hat{i} + 4.0\hat{j} + 1.0\hat{k})\,kg\cdot m/s$$

Substitute $(2.0\hat{i} - 4.0\hat{j} + 6.0\hat{k})\,m$ for $\vec{r}$ and $(-1.0\hat{i} + 4.0\hat{j} + 1.0\hat{k})\,kg\cdot m/s$ for $\vec{p}$ in ($\cancel{2}$1)

$$\vec{I} = (2.0\hat{i} - 4.0\hat{j} + 6.0\hat{k})\,m \times (-1.0\hat{i} + 4.0\hat{j} + 1.0\hat{k})$$

kg · m/s ~~for $\vec{p}$~~

$$= (-28\hat{i} - 8.0\hat{j} + 4.0\hat{k})\,kg\cdot m^2/s$$

Hence, the angular momentum of the particle about the origin will be $(-28\hat{i} - 8.0\hat{j} + 4.0\hat{k})\,kg\cdot m^2/s$

(b) Substitute $(2.0\hat{i} - 4.0\hat{j} + 6.0\hat{k})$ m for $\vec{r}$ and $(10.0\hat{i} + 15.0\hat{j})$ N for $\vec{F}$ in (3)

$$\vec{\tau} = (2.0\hat{i} - 4.0\hat{j} + 6.0\hat{k})\,\text{m} \times (10.0\hat{i} + 15.0\hat{j})\,\text{N}$$
$$= (-90.0\hat{i} + 60.0\hat{j} + 70.0\hat{k})\,\text{N}\cdot\text{m}$$

Hence, the torque acting on the particle about the origin will be $(-90.0\hat{i} + 60.0\hat{j} + 70.0\hat{k})$ N·m

(c) Substitute $(-90.0\hat{i} + 60.0\hat{j} + 70.0\hat{k})$ N·m for $\sum \vec{\tau}$ in (4)

$$\frac{d\vec{I}}{dt} = (-90.0\hat{i} + 60.0\hat{j} + 70.0\hat{k})\,\text{N}\cdot\text{m}$$

Hence, the time rate of change of angular momentum of the particle will be $(-90.0\hat{i} + 60.0\hat{j} + 70.0\hat{k})$ N·m

47. Using,

$L = I\omega$ — (1) $\qquad$ $\omega(t) = \omega(0) + \alpha t$ — (3)

$I_{Propeller} = \frac{1}{12} ML^2$ — (2) $\qquad$ $\tau = I\alpha$ — (4)

$$I_{Propeller} = \frac{1}{12}(240\,\text{kg})(6.0\,\text{m})^2 = 720.0\,\text{kg}\cdot\text{m}^2$$

Substitute 1200 rpm for $\omega(t)$ at time 30s and 0 for $\omega(0)$ in exp. (3)

$$(1200 \text{ rpm})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 0 + \alpha(30 \text{ s})$$

Rearrange for angular acceleration.

$$\alpha = \frac{(1200 \text{ rpm})}{(30 \text{ s})}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 4.20 \text{ rad/s}^2$$

(a) At $t = 10$ s

$$\omega(10 \text{ s}) = 0 + (4.20 \text{ rad/s}^2)(10 \text{ s})$$
$$= 42.0 \text{ rad/s}$$

Substitute $720.0 \text{ kg}\cdot\text{m}^2$ for $I_{Propellor}$ and 42.0 rad/s for $\omega(10 \text{ s})$ in (1)

$$L(10 \text{ s}) = (720.0 \text{ kg}\cdot\text{m}^2)(42.0 \text{ rad/s})$$
$$= 3.02 \times 10^4 \text{ kg}\cdot\text{m}^2/\text{s}$$

Hence, the angular momentum at $t = 10$ s is $3.02 \times 10^4 \text{ kg}\cdot\text{m}^2/\text{s}$

At $t = 20$ s

$$\omega(20 \text{ s}) = 0 + (4.20 \text{ rad/s}^2)(20 \text{ s})$$
$$= 84.0 \text{ rad/s}$$

Substitute $720.0 \text{ kg}\cdot\text{m}^2$ for $I_{Propellor}$ and 84.0 rad/s for $\omega(20 \text{ s})$ in (1)

$$L(20\,s) = (720.0\ kg\cdot m^2)(84.0\ rad/s)$$
$$= 6.04\times10^4\ kg\cdot m^2/s$$

Hence, the angular momentum at $t = 20\ s$ is $6.04\times10^4\ kg\cdot m^2/s$

(b) Substitute $720.0\ kg\cdot m^2$ for $I_{Propellor}$ and $4.20\ rad/s^2$ for α in exp. (4)

$$\tau = (720.0\ kg\cdot m^2)(4.20\ rad/s^2)$$
$$= 3.02\times10^3\ N\cdot m$$

Hence, the torque on the propellor is $3.02\times10^3\ N\cdot m$

53. Using

$$I\omega = I\omega' \quad \text{and} \quad I = 2/5\ mr^2$$

According to conservation of angular momentum

$$2/5\ mr^2\left(\frac{2\pi\ rad}{t}\right) = 2/5\ mr'^2\left(\frac{2\pi\ rad}{t'}\right)$$

Rearrange for final period of sun t'

$$t' = (r'/r)^2 t$$

Substitute $7.0\times10^5\ km$ for r, $3.5\times10^3\ km$ for r' and 28 days for t

$$t' = \left(\frac{3.5\times10^3\ km}{7.0\times10^5\ km}\right)^2(28\ days)$$

$$= 7.0\times10^{-4}\ \text{day}\left(\frac{24\ h}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ h}\right)\left(\frac{60\ s}{1\ \text{min}}\right)$$

$$= 60.5\ s$$

Hence, the final period of the sun is 60.5 s

55. Using $I_o\omega_o = I_f\omega_f$ and $\omega_f = \theta/t$

Substitute 3.0 rev for θ, 1.4 s for t for ω_f

$$\omega_f = \frac{3.0\ \text{rev}}{1.4\ s} = 2.1\ \text{rev/s}$$

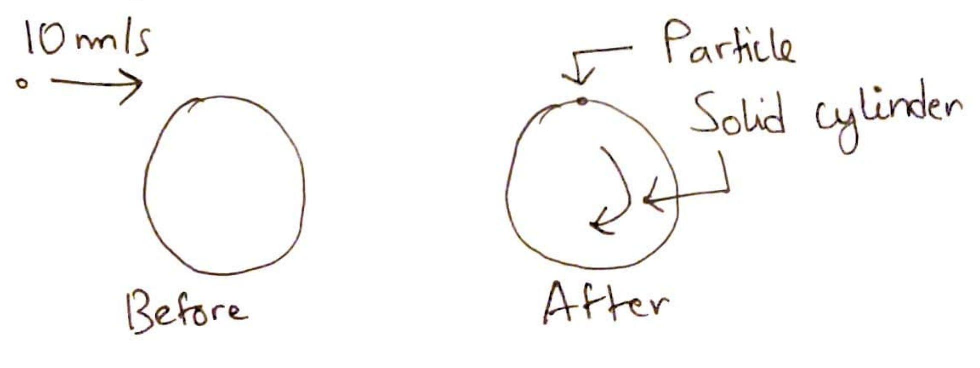
Now, Substitute 16.9 $kg\cdot m^2$ for I_o, 4.2 $kg\cdot m^2$ for I_f, 2.1 rev/s for ω_f in exp. of conservation

$$(16.9\ kg\cdot m^2)\omega_o = (4.2\ kg\cdot m^2)(2.1\ \text{rev/s})$$

$$\omega_o = \frac{(4.2\ kg\cdot m^2)(2.1\ \text{rev/s})}{(16.9\ kg\cdot m^2)} = 0.5\ \text{rev/s}$$

Hence, rotational rate imparts to its body is 0.5 rev/s

58. The figure is shown below.



(a) $L_i = mvR$

$L_f = I\omega_f = \left(m + \frac{M}{2}\right)R^2\omega_f$

Using the conservation of angular momentum.

$L_i = L_f$

Substitute mvR for L_i and $\left(m + M/2\right)R^2\omega_f$ for L_f in the above equation and solve for ω_f

$$mvR = \left(m + M/2\right)R^2\omega_f$$

$$\omega_f = \frac{mv}{\left(m + M/2\right)R}$$

Substitute 20g for m, 10.0 m/s for v and 10 cm for R, 0.5 kg for M in the above eq.

$$\omega_f = \frac{(20\,g)\left(\frac{1\,kg}{1000\,g}\right)(10.0\,m/s)}{\left[(20g)\left(\frac{1\,kg}{1000\,g}\right) + \frac{0.5\,kg}{2}\right](10\,cm)\left(\frac{1\,m}{100\,cm}\right)}$$

$\omega_f = 7.4$ rad/s

Hence, the angular velocity of the particle - cylinder system after the collision is 7.4 rad/s

(b) $\Delta K = K_p + K_s$ .... (1)

Estimate K_p and K_s as follows.

$K_p = \frac{1}{2}mv^2$ $\quad K_s = \frac{1}{2}I\omega_f^2$

Substitute for K_s in eq. (1)

$$\Delta K = 1/_2 mv^2 - 1/_2 I\omega^2 \ldots\ldots (2)$$

$$I = \left(m + \frac{M}{2}\right) R^2$$

Substitute 20g for m, 0.5 kg for M and 10 cm for R in the above equation.

$$I = \left(20g\left(\frac{10^{-3} kg}{1g}\right) + \left(\frac{0.5 kg}{2}\right)\right)\left(10 cm\left(\frac{10^{-2} m}{1 cm}\right)\right)^2$$

$$= 2.7 \times 10^{-3} kg m^2$$

Substitute 20g for m, 10 m/s for v, 2.7×10^{-3} kg/m^2 for I and 7.4 rad/s for ω_f in eq. (2)

$$\Delta K = 1/_2 (20g)\left(\frac{10^{-3} kg}{1g}\right)(10.0 m/s)^2 - 1/_2 (2.7\times10^{-3} kg\cdot m^2)$$
$$(7.4 rad/s)^2 = (1.0 - 0.073) J = 0.927 J$$

Hence, the energy lost in the collision is 0.927 J

65. $L = Mvr = I\omega$. Using

$I\omega = 2mvr$ — (1)

$I = 2mr^2$ — (2)

Substitute $2mr^2$ for I in (1)

$$2mr^2\omega = 2mvr$$

$$\omega = \frac{2mvr}{2mr^2} = v/r$$

Substitute 2.50 m/s for v and 0.800 m for r.

$$\omega = \frac{2.50 \text{ m/s}}{0.800 \text{ m}} = 3.125 \text{ rad/s}$$

The angular speed is 3.125 rad/s

(b) $KE = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

Substitute 70.0 kg for m and 2.50 m/s for v.

$KE = (70.0 \text{ kg})(2.50 \text{ m/s})^2 = 438 \text{ J}$

$KE' = \frac{1}{2}I'\omega'^2 = \frac{1}{2}(2mr^2)\left(\frac{v}{r}\right)^2 = mv^2 = 438 \text{ J}$

Hence, the initial and final kinetic energy are the Same and is equal to 438 J.

73. $m_i = m_s + n(m_p)$

Re-arrange the exp. for the mass of the spaceship

$m_s = m_i - n(m_p)$

Substitute 10^6 kg for m_i, 100 persons for n and 65.00 kg/person for m_p:

$m_s = 10^6 \text{ kg} - 100 \text{ persons} (65.00 \text{ kg per person})$

$= 993500 \text{ kg}$

$I_i = m_i r^2$

Substitute 10^6 kg for m_i and 100.0 m for r

$I_i = (10^6 \text{ kg})(100.00 \text{ m})^2 = 10^{10} \text{ kg}\cdot\text{m}^2$

$I_f = m_s r^2$

Substitute 993500 kg for m_s and 100.00 m for r:

$I_f = (993500 \text{ kg})(100.00 \text{ m})^2$

$= 0.9935 \times 10^{10} \text{ kg} \cdot \text{m}^2$

$$N_f = \frac{I_i N_i}{I_f}$$

Substitute 10^{10} kg · m² for I_i, 3.30 rev/min for N_i and 0.9935×10^{10} kg · m² for I_f

$$N_f = \frac{(10^{10} \text{ kg} \cdot \text{m}^2)(3.30 \text{ rev/min})}{(0.9935 \times 10^{10} \text{ kg} \cdot \text{m}^2)} = 3.32 \text{ rev/min}$$

Hence, the new rotation rate when all the people are off the station is 3.32 rev/s.