05/18/2021

PHYS-230
TEST - 3

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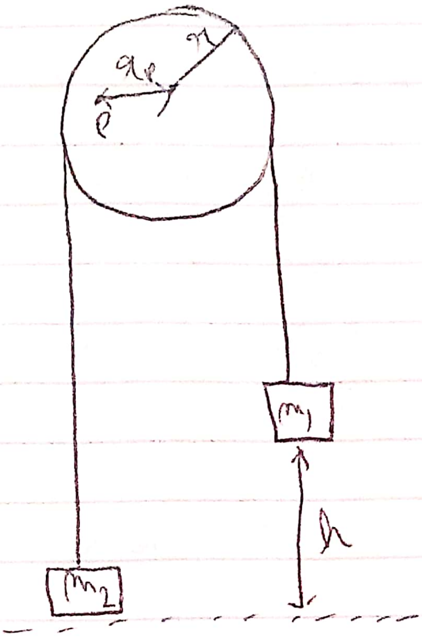
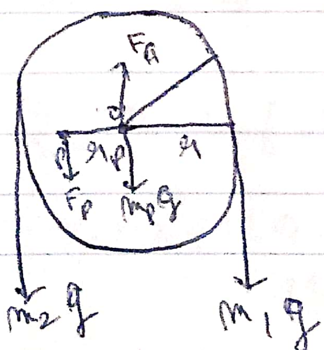
Q1. Ans



Given :- $m_1 = 5.3\,kg$
$m_2 = 2.5\,kg$
$d = 2.0\,m$
$m_p = 0.75\,kg$
$r = 0.65\,m$
$r_p = 0.45\,m$

Free - body diagram of pulley



P.T.O

(a) As system is in equilibrium,

$$\therefore \sum \vec{\tau} = 0$$

Let F_p be the force due to the pin.

$$\therefore \sum \vec{\tau}_o = 0,$$

$$m_1 g r - m_2 g r - F_p r_p = 0$$

$$\therefore F_p = \frac{(m_1 - m_2) g r}{r_p}$$

$$\therefore F_p = \frac{(5.3 - 2.5) \times 9.8 \times 0.65}{0.45}$$

$$= \underline{39.6356 \text{ N}}$$

$$\therefore \underline{F_p \approx 40\text{N}}$$

$\therefore$ The magnitude of the force that the pin applies to the pulley is approximately 40N

(b) Since, the system is in equilibrium,

$$\therefore \sum \vec{F} = 0.$$

Let F_A be the force due to the axel.

$$\therefore F_A = F_p + m_p g + m_2 g + m_1 g$$

$$F_A = 39.64 + 0.75 \times 9.8 + 5.3 \times 9.8 + 2.5 \times 9.8$$

$$= \underline{123.426 \text{ N}}$$

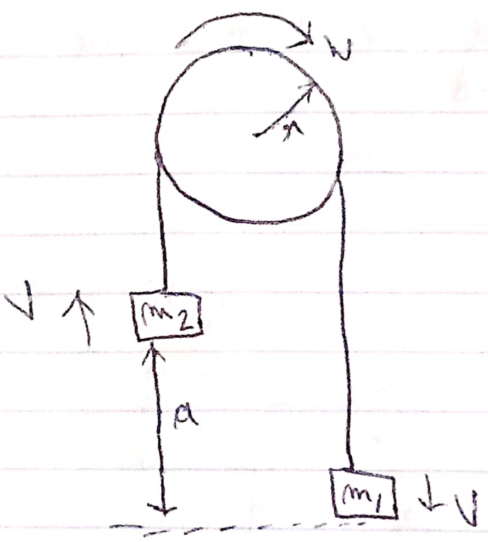
P.T.O

$\therefore \quad F_a \approx 123\,N$

$\therefore$ The magnitude of the force that the axle applies to the pulley is approximately 123N.

(c) When the pin is removed,

let the speed of the block just before it hits the ground be v.



Now, $v = w r$

Moment of inertia of disc (pulley) is,

$$I = \frac{m_p r^2}{2}$$

From conservation of mechanical energy,

$$m_1 g d = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I w^2 + m_2 g d$$

$$d(m_1 - m_2) g = \frac{1}{2}(m_1 + m_2) v^2 + \frac{1}{2} \frac{m_p r^2}{2} \times \frac{v^2}{r^2}$$

$$d(m_1 - m_2) g = \frac{(2m_1 + 2m_2 + m_p) v^2}{4}$$

$$\therefore \; V = \sqrt{\frac{4d(m_1 - m_2)g}{2m_1 + 2m_2 + m_p}}$$

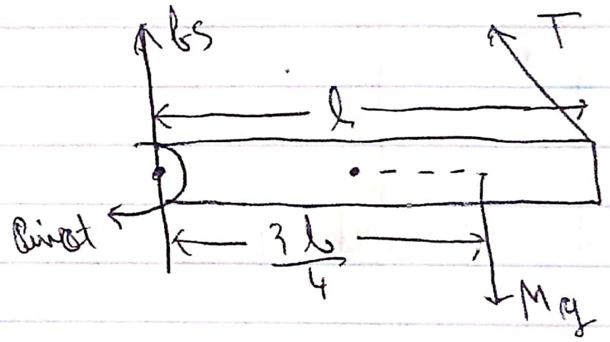
$$V = \sqrt{\frac{4 \times 2\,(5.3 - 2.5) \times 9.8}{2 \times 5.3 + 2 \times 2.5 + 0.75}}$$

$$= \underline{3.664192}$$

$$\therefore \; V = \underline{3.7\,m/s}$$

$\therefore$ The speed of block one just before it hits the ground $= 3.7\,m/s$

Q2. Ans: (a)



(a) $\lambda = Dx^2$

$$\Rightarrow \frac{dm}{dx} = Dx^2$$

$$\Rightarrow dm = Dx^2\,dx$$

$$\Rightarrow \int_0^M dm = \int_0^l Dx^2\,dx$$

$$\therefore \; M = \underline{\left(\frac{Dl^3}{3}\right)}$$

(4)

P.T.O

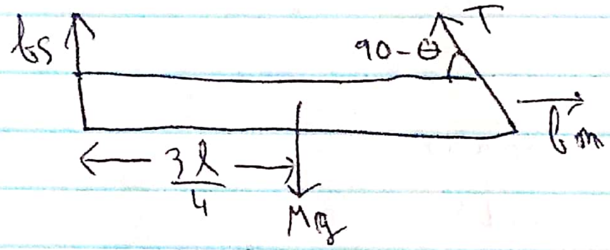
$$X_{cm} = \frac{\int x\,dm}{\int dm}$$

$$= \frac{\int_0^l x \cdot b x^2\,dx}{\int_0^l b x^2\,dx}$$

$$= \frac{\frac{\cancel{b}\,\cancel{l^4}\,l}{4}}{\frac{\cancel{b}\,\cancel{l^3}}{3}}$$

$$= \frac{3l}{4}$$

(d)



By Newton's law along vertical direction,

$$T \sin(90-\theta) + f_s = Mg$$

$$T \cos\theta + f_s = Mg \longrightarrow (1)$$

Along horizontal direction,

$$T \sin\theta = f_m \longrightarrow (2)$$

(e) Rotational law

$$-Mg\left(\frac{3l}{4}\right) + T\cos\theta\, l = 0 \longrightarrow (3)$$

P.T.O

(b) $T\cos\theta + \mu_s (T\sin\theta) = Mg \rightarrow$ from (1)

Also, $T\cos\theta = \frac{3Mg}{4} \rightarrow$ from (3)

solving (1) and (3), we get :-

$$\frac{3\cancel{Mg}}{4\cos\theta}(\cos\theta + \mu_s\sin\theta) = \cancel{Mg}$$

$$\frac{3}{4}(1 + \mu_s \tan\theta) = 1$$

$$\therefore \mu_s = \left(\frac{4}{3} - 1\right) \times \frac{1}{\tan\theta} = \frac{1}{3\tan\theta}$$

$$= \frac{1}{3\tan 55^\circ}$$

$$\mu_s = 0.233$$

Q3. Ans:- (a) $Mg = kx$

$$\therefore k = \frac{Mg}{x}$$

$$= \frac{0.65 \times 9.8}{12 \times 10^{-2}}$$

$$k = 53.08 \text{ N/m}$$

$\therefore$ The spring constant, $k = 53.08$ N/m

P.T.O

(c) Total energy $= \frac{1}{2} mv^2 + \frac{1}{2} k x^2$

$= \frac{1}{2} \times 0.65 \times (7.5)^2 + \frac{1}{2} \times 53.08 \times (0.0525)^2$

$= 18.28125 + 0.07315$

$= 18.3544 J$

$\therefore \frac{1}{2} k A^2 = 18.3544$

$A^2 = \frac{18.3544 \times 2}{53.08}$

$A^2 = 0.6916$

$\therefore$ Amplitude, $A = 0.8316 m$

At $t=0$, $y(0) = 0.0525$

Angular frequency, $\omega = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{53.08}{0.65}}$

$= 9.04 rad/s$

$y = A \sin(\omega t + \phi)$

At $t=0$, $0.0525 = 0.8316 \times \sin\phi$

$\sin\phi = \frac{0.0525}{0.8316} = 0.063 rad$

P.T.O

$$\therefore\ y(t) = 0.8316 \times \sin(9.04t + 0.063)$$

(c) $$\frac{1}{2} m v_{max}^2 = 18.3544$$

$$v_{max}^2 = \frac{2 \times 18.3544}{0.65}$$

$$= 56.475$$

$$\therefore\ v_{max} = 7.515 \text{ m/s}$$

when $$v = \frac{3}{4} v_{max}$$

$$= \frac{3 \times 7.515}{4}$$

$$= 5.636 \text{ m/s}$$

$$\therefore\ \frac{1}{2} k x^2 = 18.3544 - \frac{1}{2} \times 0.65 \times (5.636)^2$$

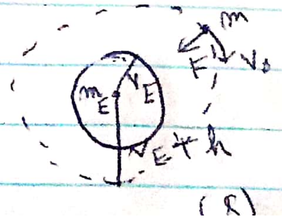
$$= 8.03$$

$$x^2 = \frac{8.03 \times 2}{53.08}$$

$$= 0.30256$$

$$x = 0.55 \text{ m}$$

Q4. Ans. (a)



P.T.O

(a) Along radial direction,

$$F = \frac{G m_E m}{(r_E + h)^2}$$

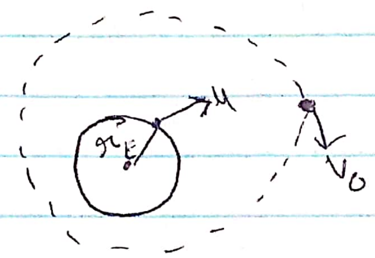
$$= \frac{m v_0^2}{r_E + h}$$

$$v_0 = \sqrt{\frac{G m_E}{r_E + h}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6380 + 11500) \times 10^3}}$$

Orbital speed, $v_0 = 4723.128 \ m/s$

(b)



By energy conservation at the surface & in orbit, we get :-

$$U_i + K_i = U_f + K_f$$

$$-\frac{G M_E m}{r_E} + \frac{1}{2} m v^2 = -\frac{G M_E m}{r_E + h} + \frac{1}{2} m v_0^2$$

$$\frac{u^2}{2} = \frac{v_0^2}{2} - G m_E \left[\frac{1}{r_E + h} - \frac{1}{r_E} \right]$$

(9)

P.T.O

Substituting all available values, we get :-

$$\frac{u^2}{2} = \frac{(4723.128)^2}{2} - 6.67\times10^{-11}\times5.98\times10^{24}\times\left[\frac{1}{17880}-\frac{1}{6380}\right]\times10^{-3}$$

$\therefore$ $u =$ $\underline{\underline{10134.49 \text{ m/s}}}$

$\therefore$ Launch speed of the spacecraft must be $\underline{\underline{10134.49 \text{ m/s}}}$