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31.  $\theta = s/r$

(a) Substitute 3.0 m and 1.5 m for  $s$  and  $r$  respectively in the above mentioned expression of angular position and solve for  $\theta$

$$\theta = \frac{3.0 \text{ m}}{1.5 \text{ m}}$$

Hence, the particles rotate at 2.0 rad

$$\omega = \frac{d\theta}{dt}$$

(b) Substitute 2.0 rad for  $d\theta$  and 1.0 s for  $dt$  in the above mentioned exp. and solve for  $\omega$

$$\omega = \frac{2 \text{ rad}}{1.0 \text{ s}} = 2 \text{ rad/s}$$

Hence, the angular velocity of the particle in 1.0 s is 2.0 rad/s

Using  $a_c = v^2/r$  and  $v = s/t$

Substitute 3.0 m for  $s$  and 1.0 s for  $t$  in the exp. of linear speed as mentioned above and solve,

$$v = \frac{3.0 \text{ m}}{1.0 \text{ s}} = 3.0 \text{ m/s}$$

Thus, the linear speed of the particle is  $3.0 \text{ m/s}$

Substitute  $3.0 \text{ m/s}$  for  $v$  and  $1.5 \text{ m}$  for  $r$  in the above exp. of radial acc. and solve,

$$a_c = \frac{(3.0 \text{ m/s})^2}{1.5 \text{ m}} = 6.0 \text{ m/s}^2$$

Hence, the acceleration of the particle is  $6.0 \text{ m/s}^2$

35. 
$$\alpha = \frac{d\omega}{dt} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

(a) Using  $\omega = (25.0t) \text{ rad/s}$

Substitute  $2.0 \text{ s}$  for  $t$  in the above function and solve

$$\omega = (25.0)(2.0 \text{ s}) \text{ rad/s} = 50.0 \text{ rad/s}$$

Hence, the instantaneous angular velocity of the propeller at  $t = 2.0 \text{ s}$  is  $50.0 \text{ rad/s}$

(b) Substitute  $50.0 \text{ rad/s}$  for  $\omega_f$ ,  $0.0 \text{ rad/s}$  for  $\omega_i$ ,  $2.0 \text{ s}$  for  $t_f$  and  $0.0 \text{ s}$  for  $t_i$  in the above mentioned exp. for  $\alpha$  and solve,

$$\alpha = \frac{(50.0 \text{ rad/s}) - (0.0 \text{ rad/s})}{(2.0 \text{ s}) - (0.0 \text{ s})} = \frac{50.0 \text{ rad/s}}{2.0 \text{ s}} = 25.0 \text{ rad/s}^2$$

Hence, the angular acceleration of the propellers is  $25.0 \text{ rad/s}$

37.  $\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

(a) Substitute  $0 \text{ rad/s}$  for  $\omega_0$ ,  $5.0 \text{ rad/s}^2$  for  $\alpha$  and  $300 \text{ rad}$  for  $\Delta\theta$  in the above mentioned expression and solve,

$$\begin{aligned}\omega_f^2 &= (0 \text{ rad/s})^2 + 2(5.0 \text{ rad/s}^2)(300 \text{ rad}) \\ &= 3000 \text{ rad}^2/\text{s}^2\end{aligned}$$

Further solve for  $\omega_f$ ,

$$\omega_f = \sqrt{3000 \text{ rad}^2/\text{s}^2} = 54.8 \text{ rad/s}$$

Hence, the final angular velocity of the wheel is  $54.8 \text{ rad/s}$

$$\omega_f = \omega_0 + \alpha t$$

(b) Substitute  $54.8 \text{ rad/s}$  for  $\omega_f$ ,  $0 \text{ rad/s}$  for  $\omega_0$  and  $5.0 \text{ rad/s}^2$  for  $\alpha$  in the above mentioned expression and solve,

$$54.8 \text{ rad/s} = 0 \text{ rad/s} + (5.0 \text{ rad/s}^2)t$$

$$t = \frac{54.8 \text{ rad/s}}{5.0 \text{ rad/s}^2} = 11 \text{ s}$$

Hence, the time elapse for the wheel to turn through  $300 \text{ rad}$  is  $11 \text{ s}$ .

$$39. \quad \omega_f = \omega_0 + \alpha t$$

(a) Convert the units of initial and final angular velocity into rad/s

$$\omega_0 = 500 \left( \frac{\text{rev}}{\text{min}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 52.3 \text{ rad/s}$$

$$\omega_f = 1500 \left( \frac{\text{rev}}{\text{min}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 157 \text{ rad/s}$$

Substitute 157 rad/s for  $\omega_f$ , 52.3 rad/s for  $\omega_0$  and 120 s for  $t$  in the above mentioned equation and Solve,

$$157 \text{ rad/s} = 52.3 \text{ rad/s} + \alpha (120 \text{ s})$$

$$104.7 \text{ rad/s} = \alpha (120 \text{ s})$$

Further solve for  $\alpha$ ,

$$\alpha = \frac{104.7 \text{ rad/s}}{120 \text{ s}} = 0.87 \text{ rad/s}^2$$

Hence, the angular acceleration of the rotating body is  $0.87 \text{ rad/s}^2$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\theta_f - \theta_0)$$

Substitute  $157 \text{ rad/s}$  for  $\omega_f$ ,  $52.3 \text{ rad/s}$  for  $\omega_0$ ,  $0.87 \text{ rad/s}^2$  for  $\alpha$  and  $0 \text{ rad}$  for  $\theta_0$  in the above eq. and solve,

$$(157 \text{ rad/s})^2 = (52.3 \text{ rad/s})^2 + 2(0.87 \text{ rad/s}^2) (\theta_f - 0 \text{ rad})$$

$$24649 \text{ rad}^2/\text{s}^2 = 2735.29 \text{ rad}^2/\text{s}^2 + (1.74 \text{ rad/s}^2) \theta_f$$

$$21913.7 \text{ rad}^2/\text{s}^2 = (1.74 \text{ rad/s}^2) \theta_f$$

Further solve for  $\theta_f$ ,

$$21913.7 \text{ rad}^2/\text{s}^2 = (1.74 \text{ rad/s}^2) \theta_f$$

$$\theta_f = \frac{21913.7 \text{ rad}^2/\text{s}^2}{1.74 \text{ rad/s}^2}$$

$$= 12594.1 \text{ rad}$$

Hence, the angle the rigid body turns is  $12594.1 \text{ rad}$ .

44.  $\omega = \frac{d\theta}{dt}$

$$d\theta = \omega dt$$

(a)  $m = \tan \theta$

$$= \frac{(400 - 0) \left( \frac{\text{rev}}{\text{min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)}{(8 - 0) \text{ s}} = 0.83 \text{ rev/s}^2$$

$$\omega = (0.83 \text{ rev/s}^2)t + 0$$

$$= (0.83 \text{ rev/s}^2)t$$

Now, integrate the expression  $d\theta = \omega dt$  and solve,

$$\int_{\theta_1}^{\theta_2} d\theta = \int_{t_1}^{t_2} \omega dt$$

Substitute 0 rev for  $\theta_1$ ,  $(0.83 \text{ rev/s}^2)t$  for  $\omega$ , 0 s for  $t_1$  and 8 s for  $t_2$  in the expression

$$\int_{\theta_1}^{\theta_2} d\theta = \int_{t_1}^{t_2} \omega dt$$

$$\int_{\theta_1}^{\theta_2} d\theta = \int_{t_1}^{t_2} (0.83 \text{ rev/s}^2)t \cdot dt$$

$$\theta_2 - 0 = \left( (0.83 \text{ rev/s}^2) \frac{t^2}{2} \right)_0^{8s}$$

$$\theta_2 = (0.417 \text{ rev/s}^2) (64s^2 - 0s)$$

$$= 26.56 \text{ rev}$$

Now, change the final angular displacement into rad.

$$\theta_2 = 26.67 \text{ rev} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 167.4$$

Hence, the final angular displacement of the fan blades is 167.4

$$\omega_f = \omega_i + \alpha t \quad \text{--- (1)}$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \quad \text{--- (2)}$$



Substitute 400 rev/min for  $\omega_f$ , 0 rev/min for  $\omega_i$  and 8s for  $t$  in (1) and solve,

$$400 \text{ rev/min} = 0 \text{ rev/min} + \alpha(8\text{s})$$

$$400 (\text{rev}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = \alpha(8\text{s})$$

$$41.87 \text{ rad/s} = \alpha(8\text{s})$$

$$\alpha = \frac{41.87 \text{ rad/s}}{8\text{s}} = 5.23 \text{ rad/s}^2$$

Substitute 0 rad for  $\theta_i$ , 0 rad/s for  $\omega_0$ , 8s for  $t$  and 5.23 rad/s<sup>2</sup> for  $\alpha$  in (2) and solve,

$$\begin{aligned} \theta_f &= 0 \text{ rad} + (0 \text{ rad/s})(8\text{s}) + \frac{1}{2}(5.23 \text{ rad/s}^2)(8\text{s})^2 \\ &= \frac{1}{2}(5.23 \text{ rad/s}^2)(64\text{s}^2) = 167.4 \text{ rad} \end{aligned}$$

Hence, the angle through which the fan blades rotate in 8 s is 167.4 rad.

45.  $\omega_f = \omega_0 + \alpha t$  — (1)

$$v_f = r\omega \text{ — (2)}$$

Substitute 0 rad/s for  $\omega_0$  and 7s for  $t$  in the above equation (1) and solve,

$$\begin{aligned} \omega_f &= (0 \text{ rad/s}) + \alpha(7\text{s}) \\ &= \alpha(7\text{s}) \dots\dots (3) \end{aligned}$$

Substitute 20 m/s for  $v_t$ ,  $d(7s)$  for angular velocity of the bead in 7s from equation (3) and 10 cm for  $r$  in equation (2) and solve,

$$20 \text{ m/s} = (10 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) (d(7s))$$

$$20 \text{ m/s} = (0.1 \text{ m}) (d(7s))$$

Further solve for  $d$ ,

$$d = \frac{20 \text{ m/s}}{(0.1 \text{ m})(7s)} = 28.57 \text{ rad/s}^2$$

Hence, the angular acceleration of the rod to achieve the tangential speed of 20 m/s is  $28.57 \text{ rad/s}^2$

47.  $|\vec{a}| = \sqrt{(a_c)^2 + (a_t)^2} \quad - (1)$

$$a_c = \frac{v_t^2}{r} \quad - (2) \text{ and } a_t = r\alpha \quad - (3)$$

$$v_t = r\omega \quad - (4)$$

Substitute 2.5 rad/s for  $\omega$  in (4) and solve,

$$v_t = r(2.5 \text{ rad/s})$$

Substitute  $r(2.5 \text{ rad/s})$  for  $v_t$  in exp. (2) and solve,

$$a_c = \frac{(r(2.5 \text{ rad/s}))^2}{r} = (6.25)r$$

Therefore, the total acceleration of the merry-go-round is  $(6.25 \text{ s}^{-2})$  as the tangential acc. is 0.



$$F = ma$$

Substitute  $(6.25 \text{ s}^{-2})r$  for  $a$  in the above mentioned exp. of force and solve,

$$F = m((6.25)r)N$$

$$F_{\text{friction}} = \mu_s N$$

Substitute 0.5 for  $\mu_s$ ,  $mg$  for  $N$ ,  $9.81 \text{ m/s}^2$  for  $g$  in the above exp. of  $F_{\text{friction}}$  and solve,

$$\begin{aligned} F_{\text{friction}} &= (0.5)(mg) \\ &= (0.5)m(9.81 \text{ m/s}^2) \\ &= (4.9 \text{ m})N \end{aligned}$$

Equate both the forces,

$$(4.9 \text{ m}) = m(6.25)r$$

$$4.9 = (6.25)r$$

$$r = \frac{4.9}{6.25} = 0.78 \text{ m}$$

Hence, the man can stand 0.78 m from the axis of rotation without sliding.

49.

$$\omega_f = \omega_o + \alpha t$$

(a) Convert the units of initial angular velocity into rad/s

$$\omega_0 = 0.5 \left( \frac{\text{rev}}{\text{s}} \right) \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) = 3.14 \text{ rad/s}$$

Substitute 0 rad/s for  $\omega_f$ , 3.14 rad/s for  $\omega_0$  and 10 s for  $t$  in above equation and solve,

$$0 \text{ rad/s} = (3.14 \text{ rad/s}) + \alpha(10 \text{ s})$$

$$\alpha(10 \text{ s}) = -3.14 \text{ rad/s}$$

Further solve for  $\alpha$ ,

$$\alpha = \frac{-3.14 \text{ rad/s}}{10 \text{ s}} = -0.314 \text{ rad/s}^2$$

Hence, the angular acceleration of the turbine is  $-0.314 \text{ rad/s}^2$

$$a_c = v_t^2 / r \quad \text{and} \quad v_t = r\omega$$

(b) Substitute 20 m for  $r$  and 3.14 rad/s for  $\omega$  in the exp. for  $v_t$  and solve,

$$v_t = (20 \text{ m})(3.14 \text{ rad/s}) = 62.8 \text{ m/s}$$

Substitute 62.8 rad/s for  $v_t$  and 20 m for  $r$  in above exp. for  $a_c$  and solve,

$$a_c = \frac{(62.8 \text{ m/s})^2}{(20 \text{ m})} = \frac{3943.84 \text{ m}^2/\text{s}^2}{20 \text{ m}} = 197.2 \text{ m/s}^2$$

Hence, the centripetal acc. of the tip of the

blades at  $t=0$  is  $197.2 \text{ m/s}^2$

The magnitude of the total linear acc. on the tip of the blade is given by,

$$|\vec{a}| = \sqrt{a_c^2 + a_t^2}$$

$$a_t = r\alpha$$

(c) Substitute  $-0.314 \text{ rad/s}^2$  for  $\alpha$  and  $20 \text{ m}$  for  $r$  in above exp. for  $a_t$  and solve,

$$a_t = (20 \text{ m}) (-0.314 \text{ rad/s}^2) = -6.28 \text{ m/s}^2$$

Substitute  $197.2 \text{ m/s}^2$  for  $a_c$  and  $-6.28 \text{ m/s}^2$  for  $a_t$  in the above mentioned of total acc. and solve,

$$|\vec{a}| = \sqrt{(197.2 \text{ m/s}^2)^2 + (-6.28 \text{ m/s}^2)^2}$$

$$= \sqrt{38887.8 \text{ m}^2/\text{s}^4 + 39.44 \text{ m}^2/\text{s}^4}$$

$$= \sqrt{38927.24 \text{ m}^2/\text{s}^4} = 197.3 \text{ m/s}^2$$

Hence at  $t=0 \text{ s}$ , the total linear acc. of the tip of the blades is  $197.3 \text{ m/s}^2$

$$55. \text{ K.E. (ROT)} = \frac{1}{2} I \omega^2$$

$$(a) \quad I_{\text{(Earth)}} = \frac{2}{5} M r^2$$

Substitute,  $6 \times 10^{24} \text{ kg}$  for  $M$  and  $6.4 \times 10^6 \text{ m}$  for  $r$

$$I = \frac{2}{5} (6 \times 10^{24} \text{ kg}) (6.4 \times 10^6 \text{ m})^2$$

$$= 98.3 \times 10^{36} \text{ kg} \cdot \text{m}^2$$

$$\omega = \frac{2\pi}{24 \text{ h} \times \frac{3600 \text{ s}}{\text{h}}} \text{ rad/s} = 7.27 \times 10^{-5} \text{ rad/s}$$

$$\text{K.E. (ROT)} = \frac{1}{2} (98.3 \times 10^{36} \text{ kg} \cdot \text{m}^2) \left( 7.27 \times 10^{-5} \frac{\text{rad}}{\text{s}} \right)^2$$

$$= 2.59 \times 10^{29} \text{ J}$$

So, the rotational kinetic energy of the Earth is  $2.59 \times 10^{29} \text{ J}$ .

$$(b) \quad \text{K.E. (ROT/SUN)} = \frac{1}{2} I_{\text{(EARTH/SUN)}} \omega^2$$

$$I_{\text{(EARTH/SUN)}} = \frac{2}{5} M r^2 + M R^2$$

Substitute  $6 \times 10^{24} \text{ kg}$  of  $M$  and  $6.4 \times 10^6 \text{ m}$  for  $r$  and  $150 \times 10^9$  for  $R$

$$\begin{aligned}
 I_{(\text{EARTH/SUN})} &= \left( \left( \frac{2}{5} \right) (6 \times 10^{24} \text{ kg}) (6.4 \times 10^6 \text{ m})^2 \right) + \\
 &\quad \left( (6 \times 10^{24} \text{ kg}) (150 \times 10^9 \text{ m})^2 \right) \\
 &= (98.3 \times 10^{36} \text{ kg} \cdot \text{m}^2) + (13.5 \times 10^{46} \text{ kg} \cdot \text{m}^2) \\
 &= 1.35 \times 10^{47} \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

$$\omega = \frac{2\pi}{365 \times 24 \times 3600} = 1.99 \times 10^{-7} \text{ rad/sec}$$

$$\text{K.E. (ROT/SUN)} = \frac{1}{2} I_{(\text{EARTH/SUN})} \omega^2$$

Substitute, the values of  $I_{(\text{EARTH/SUN})}$  and  $\omega$  in,

$$\begin{aligned}
 \text{K.E. (ROT/SUN)} &= \left( \frac{1}{2} \right) (1.35 \times 10^{47} \text{ kg} \cdot \text{m}^2) \left( 1.99 \times 10^{-7} \frac{\text{rad}}{\text{s}} \right)^2 \\
 &= 2.67 \times 10^{33} \text{ J}
 \end{aligned}$$

So, the rotational kinetic energy of the Earth in its orbit around the Sun is  $2.67 \times 10^{33} \text{ J}$ .

57.

$$\omega (\text{forearm}) = \frac{v(\text{ball})}{r}$$

~~Where,  $v(\text{ball})$~~  Substitute  $20.0 \text{ m/s}$  for  $v(\text{ball})$  and  $0.480 \text{ m}$  for  $r$ .

$$\omega_{(\text{forearm})} = \frac{20}{0.48} \text{ rad/s} = 41.67 \text{ rad/s}$$

$$I_{(\text{forearm})} = 0.500 \text{ kg} \cdot \text{m}^2$$

$$\text{K.E. (ROT/FOREARM)} = \frac{1}{2} I_{(\text{forearm})} \omega^2_{(\text{forearm})}$$

Substitute  $0.500 \text{ kg} \cdot \text{m}^2$  for  $I_{(\text{forearm})}$  and  $41.67 \text{ rad/s}$  for  $\omega_{(\text{forearm})}$

$$\begin{aligned} \text{K.E. (ROT/FOREARM)} &= \frac{1}{2} (0.500 \text{ kg} \cdot \text{m}^2) \left( 41.67 \frac{\text{rad}}{\text{s}} \right)^2 \\ &= 434 \text{ J} \end{aligned}$$

So, the rotational kinetic energy of the forearm is  $434 \text{ J}$

67. Using

$$\text{PE} = mgh \quad \text{and} \quad \text{K.E. rotational} = \frac{1}{2} I \omega^2$$

$$I = \frac{mL^2}{3} \quad \text{and} \quad v_t = r\omega$$

Using

$$h = L/2 \sin \theta$$

Substitute  $2.0 \text{ m}$  for  $L$  and  $60^\circ$  for  $\theta$  in the above exp. of  $h$  and solve,

$$h = \frac{2.0 \text{ m}}{2} \sin 60^\circ = 0.866 \text{ m}$$



Substitute  $1.0 \text{ kg}$  for  $m$  and  $2.0 \text{ m}$  for  $L$  in  $I$  and solve,

$$I = \frac{(1.0 \text{ kg})(2.0 \text{ m})^2}{3} = \frac{4.0}{3} \text{ kg} \cdot \text{m}^2$$

$$PE = KE_{\text{rotational}}$$

Substitute  $mgh$  for  $PE$  and  $\frac{1}{2}I\omega^2$  for  $K.E._{\text{rotational}}$  in the above eq. and solve

$$mgh = \frac{1}{2}I\omega^2$$

Further substitute  $0.866 \text{ m}$  for  $h$ ,  $1.0 \text{ kg}$  for  $m$ ,  $9.8 \text{ m/s}^2$  for  $g$ ,  $\frac{4.0}{3} \text{ kg} \cdot \text{m}^2$  for  $I$  in the ~~xxx~~ exp.  $mgh = \frac{1}{2}I\omega^2$  and solve,

$$(1.0 \text{ kg})(9.8 \text{ m/s}^2)(0.866 \text{ m}) = \frac{1}{2} \left( \frac{4.0}{3} \text{ kg} \cdot \text{m}^2 \right) \omega^2$$

$$8.49 \text{ kg} \cdot \text{m}^2/\text{s}^2 = (0.67 \text{ kg} \cdot \text{m}^2) \omega^2$$

$$\omega^2 = \frac{8.49 \text{ kg} \cdot \text{m}^2/\text{s}^2}{0.67 \text{ kg} \cdot \text{m}^2}$$

Further solve for  $\omega$ ,

$$\omega = \sqrt{12.67 \text{ rad/s}^2} = 3.56 \text{ rad/s}$$

Substitute  $2.0 \text{ m}$  for  $r$  and  $3.56 \text{ rad/s}$  for  $\omega$  in  $V_t$  and solve,

$$V_t = (2.0 \text{ m})(3.56 \text{ rad/s})$$

$$= 7.12 \text{ m/s}$$

Hence, the speed of the tip of the rod as it passes the horizontal position is 7.12 m/s

71.  $|\vec{\tau}| = r_{\perp} F$

Substitute 50 cm for  $r$  and  $F$  for the force in the above mentioned exp. and solve,

$$|\vec{\tau}| = (50 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) F = (0.5 \text{ m}) F$$

Substitute 30 cm for  $r$  and 50 N for the force in the above mentioned exp. and solve,

$$|\vec{\tau}| = (30 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) (50 \text{ N})$$

$$= (0.3 \text{ m}) (50 \text{ N}) = 15 \text{ N} \cdot \text{m}$$

Equate both the torques

$$(0.5 \text{ m}) F = 15 \text{ N} \cdot \text{m}$$

$$F = \frac{15 \text{ N} \cdot \text{m}}{0.5 \text{ m}} = 30 \text{ N}$$

Hence, the pulling force applied to the cord connecting to the larger flywheel is 30 N.

75. Using  
 $|\vec{\tau}| = r_{\perp} F$

Substitute 20 cm for  $r_{\perp}$  and  $(5.0 \text{ kg})g \sin 30^\circ$  for  $F$  in the above exp. and solve,

$$|\vec{\tau}|_1 = (20 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) (5.0 \text{ kg})g \sin 30^\circ$$
$$= (0.2 \text{ m}) (24.5 \text{ N}) = 4.9 \text{ N}\cdot\text{m}$$

Substitute 30 cm for  $r_{\perp}$  and  $Mg$  for  $F$  in the above exp. and solve,

$$|\vec{\tau}|_2 = (30 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) (Mg) = 0.3 \text{ m} (Mg)$$

Equate the torques,

$$|\vec{\tau}|_1 = |\vec{\tau}|_2$$

Substitute  $4.9 \text{ N}\cdot\text{m}$  for  $|\vec{\tau}|_1$  and  $0.3 \text{ m} (Mg)$  for  $|\vec{\tau}|_2$  in the exp.  $|\vec{\tau}|_1 = |\vec{\tau}|_2$  and solve,

$$4.9 \text{ N}\cdot\text{m} = 0.3 \text{ m} (Mg)$$

$$M = \frac{4.9 \text{ kg} \cdot \text{m/s}^2 \cdot \text{m}}{(0.3 \text{ m}) (9.81 \text{ m/s}^2)}$$

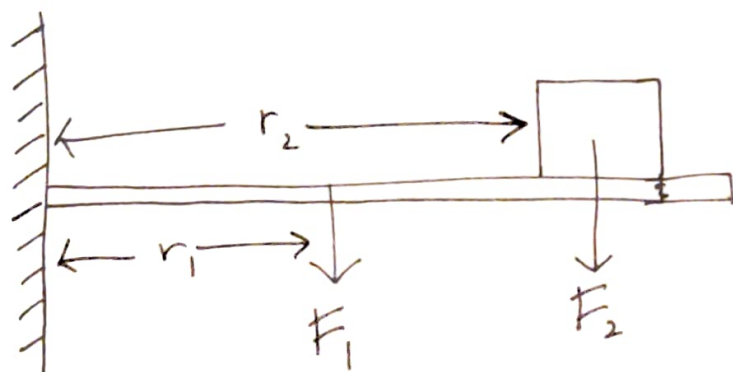
$$= \frac{4.9}{2.943} \text{ kg} = 1.67 \text{ kg}$$

Hence, the hanging mass is 1.67 kg.

$$81. \quad \vec{\tau}_{\text{net}} = \sum_i |\vec{\tau}_i|$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The following figure shows the different forces acting on the system.



$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \vec{r}_1 \times m_1 g$$

Calculate the value of  $\vec{r}_1$ .

$$\vec{r}_1 = 3/2 \text{ m} = 1.5 \text{ m}$$

Substitute  $1.5 \text{ m}$  for  $\vec{r}_1$ , and  $2.0 \text{ kg}$  for  $m_1$ , and  $9.8 \text{ m/s}^2$  for  $g$

$$\begin{aligned} \vec{\tau}_1 &= (1.5 \text{ m})(2.0 \text{ kg})(9.8 \text{ m/s}^2) \\ &= (29.4 \text{ N}\cdot\text{m}) \end{aligned}$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = \vec{r}_2 \times m_2 g$$

Calculate  $\vec{r}_2$

$$\vec{r}_2 = \left(3 - \frac{0.2}{2}\right) \text{ m} = 2.9 \text{ m}$$

Substitute  $2.9 \text{ m}$  for  $\vec{r}_2$  and  $1.0 \text{ kg}$  for  $m_2$  and  $9.8 \text{ m/s}^2$  for  $g$

$$\begin{aligned}\vec{\tau}_2 &= (2.9 \text{ m})(1.0 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 28.42 \text{ N}\cdot\text{m}\end{aligned}$$

$$\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2$$

Substitute  $29.4 \text{ N}\cdot\text{m}$  for  $\vec{\tau}_1$ , and  $28.42 \text{ N}\cdot\text{m}$  for  $\vec{\tau}_2$

$$\begin{aligned}\vec{\tau}_{\text{net}} &= (29.4 \text{ N}\cdot\text{m} + 28.42 \text{ N}\cdot\text{m}) \\ &= 57.82 \text{ N}\cdot\text{m}\end{aligned}$$

Thus, the torque of the system about the support of the wall is  $57.82 \text{ N}\cdot\text{m}$

83.  $\vec{\tau} = \vec{r} \times \vec{F}$

Substitute  $(-2\hat{i} + 4\hat{j}) \text{ N}$  for  $\vec{r}$  and  $(5\hat{i} - 2\hat{j} + 4\hat{k}) \text{ m}$  for  $\vec{F}$  in above torque equation.

$$\begin{aligned}\vec{\tau} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 4 & 0 \\ 5 & -2 & 1 \end{vmatrix} = (4-0)\hat{i} - (-2-0)\hat{j} + (-2-20)\hat{k} \\ &= (4\hat{i} + 2\hat{j} - 16\hat{k}) \text{ N}\cdot\text{m}\end{aligned}$$

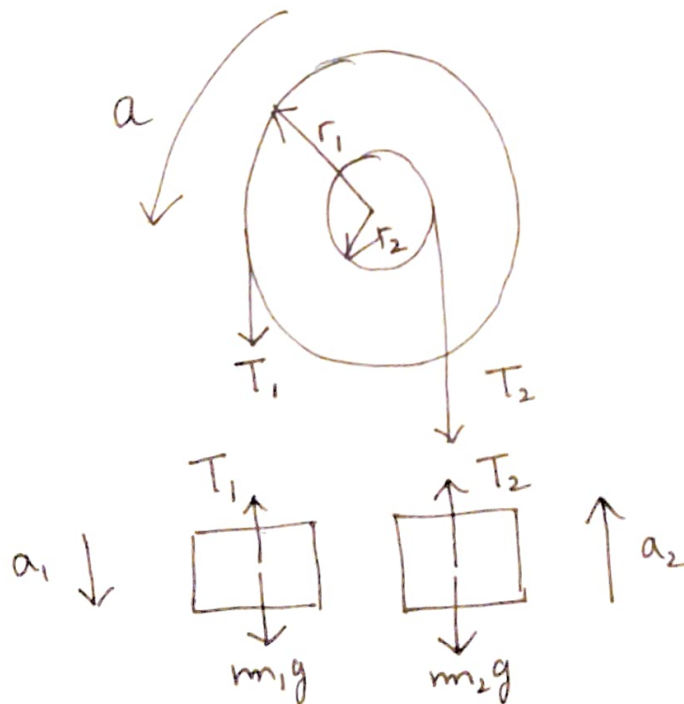
Therefore the torque about the origin of the force is  $(4\hat{i} + 2\hat{j} - 16\hat{k}) \text{ N}\cdot\text{m}$

92.

$$\vec{\tau}_{\text{net}} = \sum_{i=1}^n |\vec{\tau}_i|$$

$$a = r \times \alpha$$

The figure shows the various forces acting on the system along with its components



$$\tau_1 = r_1 \times T_1$$

$$\tau_2 = r_2 \times T_2$$

$$\tau = r_1 \times T_1 - r_2 \times T_2$$

$$\tau = I \alpha$$

Therefore,

$$\alpha = \frac{r_1 \times T_1 - r_2 \times T_2}{I}$$



$$a_1 = r_1 \times d$$

$$a_2 = r_2 \times d$$

From F.B.D of block 1

$$m_1 g - T_1 = m_1 a_1$$

$$T_1 = m_1 g - m_1 r_1 d$$

From F.B.D of block 2

$$T_2 - m_2 g = m_2 a_2$$

$$T_2 = m_2 g + m_2 r_2 d$$

Solve for  $d$ .

$$d = \frac{r_1 \times (m_1 g - m_1 r_1 a) - r_2 \times (m_2 g + m_2 r_2 d)}{I}$$

$$= \frac{m_1 r_1 g - m_2 r_2 g}{m_1 r_1 + m_2 r_2 + I}$$

(a) Convert cm to m for  $r_1$

$$r_1 = (50 \text{ cm}) \frac{(1 \text{ m})}{(100 \text{ cm})} = 0.5 \text{ m}$$

Convert cm to m for  $r_2$

$$r_2 = (20 \text{ cm}) \frac{(1 \text{ m})}{(100 \text{ cm})} = 0.2 \text{ m}$$

Substitute  $9.8 \text{ m/s}^2$  for  $g$ ,  $0.5 \text{ m}$  for  $r_1$ ,  $0.2 \text{ m}$  for  $r_2$ ,  $1.0 \text{ kg}$  for  $m_1$ ,  $2.0 \text{ kg}$  for  $m_2$ , and  $2.0 \text{ kg}\cdot\text{m}^2$  for  $I$

$$\alpha = \frac{(1 \text{ kg})(0.5 \text{ m})(9.8 \text{ m/s}^2) - (2 \text{ kg})(0.2 \text{ m})(9.8 \text{ m/s}^2)}{(2.0 \text{ kg}\cdot\text{m}^2) + (1 \text{ kg})(0.5 \text{ m})^2 + (2 \text{ kg})(0.2 \text{ m})^2}$$
$$= 0.42 \text{ rad/s}^2$$

Thus, the angular acceleration of the pulley is  $0.42 \text{ rad/s}^2$

(b) Solve for  $a_1$

Substitute  $0.42 \text{ rad/s}^2$  for  $\alpha$ , and  $0.5 \text{ m}$  for  $r_1$

$$a_1 = (0.42 \text{ rad/s}^2)(0.5 \text{ m}) = 0.21 \text{ m/s}^2$$

Solve for  $a_2$

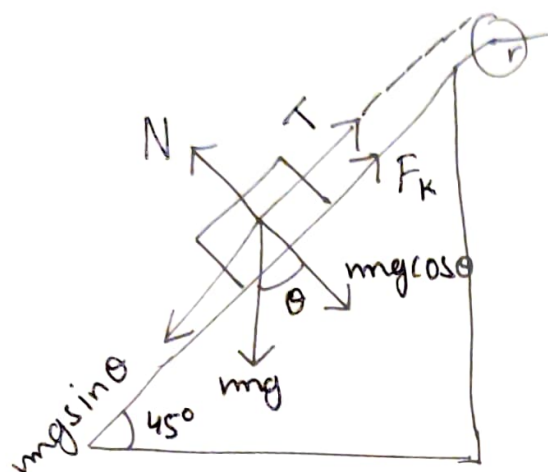
Substitute  $0.42 \text{ rad/s}^2$  for  $\alpha$ , and  $0.2 \text{ m}$  for  $r_2$

$$a_2 = (0.42 \text{ rad/s}^2)(0.2 \text{ m})$$

$$= 0.084 \text{ m/s}^2$$

Thus, the linear acc. of the weights 1 and 2 are  $0.21 \text{ m/s}^2$  and  $0.084 \text{ m/s}^2$  respectively.

93. The figure shows the various forces acting on the system along with its components.



From F.B.D,

$$N = mg \cos \theta$$

Using  $F_k = \mu_k N$ , substitute  $mg \cos \theta$  for  $N$

$$F_k = \mu_k mg \cos \theta$$

Substitute 0.4 for  $\mu_k$ , 3kg for  $m$ ,  $9.8 \text{ m/s}^2$  for  $g$  and  $45^\circ$  for  $\theta$ .

$$F_k = 0.4 (3 \text{ kg}) (9.8 \text{ m/s}^2) \cos 45^\circ = 8.32 \text{ N}$$

From F.B.D of the block,

$$T = mg \sin \theta - F_k - ma$$

Substitute 8.32 N for  $F_k$ , 3kg for  $m$ ,  $9.8 \text{ m/s}^2$  for  $g$  and  $45^\circ$  for  $\theta$ .

$$\begin{aligned} T &= (3 \text{ kg}) (9.8 \text{ m/s}^2) \sin 45^\circ - (8.32 \text{ N}) - (3 \text{ kg}) a \\ &= 12.47 - 3a \text{ N} \cdot \text{m} \end{aligned}$$

Using  $I = \frac{Mr^2}{2}$  and  $d = a/r$  and  $\tau = Id$

$$\tau = \left( \frac{Mr^2}{2} \right) \left( \frac{a}{r} \right) \quad \text{--- (1)}$$

$$= \frac{Mrxa}{2}$$

Also  $\tau = r \times T$ . Substitute in eq. (1)

$$r \times T = \frac{Mrxa}{2}$$

$$a = \frac{2T}{M}$$

Substitute  $12.47 - 3a \text{ N}\cdot\text{m}$  for  $T$  and  $1 \text{ kg}$  for  $M$ .

$$a = \frac{2(12.47 - 3a \text{ N}\cdot\text{m})}{(1 \text{ kg})} = 24.94 - 6a$$

$$a + 6a = 24.94$$

Further solve,

$$7a = 24.94$$

$$a = \frac{24.97}{7} = 3.6 \text{ m/s}^2$$

Thus, the acceleration of the block is  $3.6 \text{ m/s}^2$