Section 4.6B Overview and To-Do List

In Example 2 of Section 4.3 we saw that if we had an independent set of *two* non-parallel vectors in R^3, the vectors in the set spanned a plane, and did not span R^3 itself which has a standard basis of *three* vectors.
In Thm 4.4.3 of Section 4.4 we found that if we have a set S of ***r*** vectors in R^n where $r > n$, **n** = the number of vectors in the standard basis, the set S must be dependent.
We have similar results for an arbitrary finite-dimensional vector space as shown in the following theorem.

Theorem 1. Suppose V has a basis $\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_n\}$ with *n* vectors in it.
(a) If $S \subseteq V$ and S has **more than *n*** vectors in it, *S* is a **dependent** set.
(b) If $S \subseteq V$ and *S* contains **fewer than n** vectors, *S* does not span *V.*
To make use of this, it helps to know the standard bases for the vector spaces found in the last section.

Example of (a) above. The set $S = \{x,\ x^2 + 2x - 1,\ 2x - 5,\ 3x^2 - x + 9\}$ in P_2 is dependent since S has 4 vectors in it while the standard basis for P_2 has three vectors.
Example of (b) above. The set $S = \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 7 \\ -2 & -5 \end{bmatrix} \right\}$ will not span M_{22} since *S* only contains 3 vectors while the standard basis for M_{22} contains 4 matrices. (The standard basis for M_{22} is similar to the one for M_{23} which we developed in the last overview.)

From Theorem 1, it follows that we have
Theorem 2. All bases for a single finite-dimensional vector space V have the same number of vectors.
This common number is called the **dimension** of the vector space and is denoted $\dim(V)$.

Dimension Examples. $\dim(R^n) = n,\ \dim(P_n) = n + 1,\ \dim(M_{23}) = 6$

Sometimes we have a set $S \subseteq V$ that is too big to be a basis for V (so S is dependent) or too small (S will not span). And, we would like to modify *S* to create a basis by dropping out some vectors or by adding some. The following theorem indicates how we might do this by adding/deleting vectors one by one.

Theorem 3. The Plus/Minus Theorem. Let S be a nonempty finite set of vectors in a vector space V.
(a) If *S* is a linearly independent set, and if $\vec{v}$ is a vector in *V* that is outside of span(*S*), then $S \cup (\vec{v})$ is still linearly independent.
(b) If $\vec{v}$ is a vector in *S* that is expressible as a linear combination of other vectors in *S,* then $S - (\vec{v})$, i.e. the set *S* with the vector $\vec{v}$ removed, then $S - (\vec{v})$ spans the same space as does *S.*
See **Example 4** in the text as an example of the Plus/Minus Theorem.

We now find that if we know the dimension of a vector space *V* is, say *m* , and we have a subset *B* of *V* of exactly *m* vectors, then we don't have to check both independence and span requirements to see if *B* is a basis. If we have the **right number** of vectors for a basis, we need only check that **one** of independence and span holds to know *B* is a basis. This is the subject of **Theorem 4** in the text.

Read Section 4.6 Include Theorem 4.6.5.6 as well as the theorems mentioned above.
Work Text Problems 1, 2, 3, 7(a-d), 9(a-c), 10, 12 (a,b),13, 15-17, 19
Work Check Quiz 20.