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Exercise 1.8

1. (a) Ans Domain :- R^2
Co-domain :- R^3

(b) Ans Domain :- R^3
Co-domain :- R^2

(c) Ans Domain :- R^3
Co-domain :- R^3

(d) Ans Domain :- R^6
Co-domain :- R

2. (a) Ans: Domain :- R^5
Co-domain :- R^4

(b) Ans: Domain :- R^4
Co-domain :- R^5

(c) Ans Domain :- R^4
Co-domain :- R^4

(d) Ans: Domain :- R
Co-domain :- R^3

3. (a) Ans Domain :- R^2
Co-domain :- R^2

(b) Ans. Domain :- R^2
Co-Domain :- R^3

4. (a) Ans Domain :- R^3
Co-Domain :- R^3

4. (b) Ans. Domain :- R^3
Co-domain :- R^2

5. (a) Ans Domain :- R^3
Co-domain :- R^2

(b) Ans Domain :- R^2
Co-Domain :- R^3

6. (a) Ans Domain :- R^2
Co-Domain :- R^2

(b) Ans Domain :- R^3
Co-Domain :- R^3

11. Ans (a) Ans :- $w_1 = 2x_1 - 3x_2 + x_3$

$w_2 = 3x_1 + 5x_2 - x_3$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$w = Ax$.

$\therefore A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix}$

i.e. the standard matrix.

(b) Ans.

$$w_1 = 7x_1 + 2x_2 - 8x_3$$
$$w_2 = -x_2 + 5x_3$$
$$w_3 = 4x_1 + 7x_2 - x_3$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$w = A.x$$

$$A = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix}$$, i.e. the standard matrix.

12. (a) Ans:

$$w_1 = -x_1 + x_2$$
$$w_2 = 3x_1 - 2x_2$$
$$w_3 = 5x_1 - 7x_2$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$w = Ax$$

$$A = \begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{bmatrix}$$, i.e. the standard matrix.

(b) Ans.

$$w_1 = x_1$$
$$w_2 = x_1 + x_2$$
$$w_3 = x_1 + x_2 + x_3$$
$$w_4 = x_1 + x_2 + x_3 + x_4$$

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$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$w = Ax$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$, i.e. the standard matrix.

9. Ans: $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 4x_1 \\ x_1 - x_2 \\ 3x_2 \end{bmatrix}$

Domain :- R^2
Co-Domain :- R^3

10. Ans: $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 - x_3 \\ 0 \end{bmatrix}$

Domain :- R^3
Co-Domain :- R^4

13 Ans: (a) Ans $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -x_1 \\ x_1 + 3x_2 \\ x_1 - x_2 \end{bmatrix}$

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$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\-1\\1+3\times 0\\1-0\end{bmatrix}$$

$$= \begin{bmatrix}0\\-1\\1\\1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\3\\-1\end{bmatrix}$$

$\therefore$ Standard matrix for T is :-

$$\begin{bmatrix}0 & 1\\-1 & 0\\1 & 3\\1 & -1\end{bmatrix}$$

(b) Ans

$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}7x_1+2x_2-x_3+x_4\\x_2+x_3\\-x_1\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right) = \begin{bmatrix}7+0-0+0\\0+0\\-1\end{bmatrix}$$

$$= \begin{bmatrix}7\\0\\-1\end{bmatrix}$$

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$$T\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}0+2-0+0\\1+0\\-0\end{bmatrix}$$

$$= \begin{bmatrix}2\\1\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0+0-1+0\\0+1\\-0\end{bmatrix}$$

$$= \begin{bmatrix}-1\\1\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0+0-0+1\\0+0\\-0\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$\therefore$ Standard matrix for $T = \begin{bmatrix}7 & 2 & -1 & 1\\0 & 1 & 1 & 0\\-1 & 0 & 0 & 0\end{bmatrix}$

(c) 4) $T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$

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$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$\therefore$ Standard matrix for $T = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

(d) Ans $T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right)=\begin{bmatrix}x_4\\x_1\\x_3\\x_2\\x_1-x_3\end{bmatrix}$

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$$T\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\0\\1\\0\\-1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$$

$\therefore$ Standard matrix for $T = \begin{bmatrix}0 & 0 & 0 & 1\\1 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\1 & 0 & -1 & 0\end{bmatrix}$

Q4. (a) $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}2x_1 - x_2\\x_1 + x_2\end{bmatrix}$

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$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\1\end{bmatrix}$$

$\therefore$ Standard matrix for $T = \begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}$

(d) Ans, $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_1\\x_2\end{bmatrix}$

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}$$

$\therefore$ Standard matrix for $T = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(e) Ans $T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix}x_1+2x_2+x_3\\x_1+5x_2\\x_3\end{bmatrix}$

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}2\\5\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$\therefore$ Standard matrix for $T=\begin{bmatrix}1&2&1\\1&5&0\\0&0&1\end{bmatrix}$

(d) Ans:- $T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right)=\begin{bmatrix}4x_1\\7x_2\\-8x_3\end{bmatrix}$

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}4\\0\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}0\\7\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\\-8\end{bmatrix}$$

$\therefore$ Standard matrix for $T=\begin{bmatrix}4&0&0\\0&7&0\\0&0&-8\end{bmatrix}$

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21. (a) Ans: $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x+y \\ x-y \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore$ Standard matrix for $T = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$

which is of the form:-

$T = Ax$ or $T = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

21. (a) Ans: $T(x, y) = (2x+y, x-y)$

Let $\vec{u} = (x_1, x_2)$ & $\vec{v} = (y_1, y_2) \in \mathbb{R}^2$

(i) $T(\vec{u}+\vec{v}) = T((x_1, x_2) + (y_1, y_2))$

$= T((x_1+y_1), (x_2+y_2))$

$= (2(x_1+y_1) + (x_2+y_2),$
$(x_1+y_1) - (x_2+y_2))$

$= (2x_1 + 2y_1 + x_2 + y_2, x_1 - x_2 + y_1 - y_2)$

$= ((2x_1 + x_2) + (2y_1 + y_2), (x_1 - x_2) + (y_1 - y_2)$

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$= (2x_1+x_2, x_1-x_2) + (2y_1+y_2, y_1-y_2)$

$= T(x_1, x_2) + T(y_1, y_2)$

$T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})\ ;\ \vec{u}, \vec{v} \in R^2$

(ii) Let k be any scalar

$T(k\vec{u}) = T(k(x_1, x_2))$

$= T((kx_1, kx_2))$

$= (2kx_1 + kx_2, kx_1 - kx_2)$

$= (k(2x_1+x_2), k(x_1-x_2))$

$= k(2x_1+x_2, x_1-x_2)$

$= k\,T(x_1, x_2)$

$\therefore T(k\vec{u}) = k\,T(\vec{u})$, $\forall$ scalar k.

$\therefore$ T is a matrix transformation.

(b) Ans. $T(x_1, x_2, x_3) = (x_1, x_3, x_1+x_2)$

For $\vec{u} = (a_1, a_2, a_3)$, $\vec{v} = (b_1, b_2, b_3)$ $\in R^3$

(i) $T(\vec{u}+\vec{v}) = T((a_1, a_2, a_3) + (b_1, b_2, b_3))$

$= T(a_1+b_1, a_2+b_2, a_3+b_3)$

$= (a_1+b_1, a_3+b_3, a_1+b_1+a_2+b_2)$

$= (a_1+b_1, a_3+b_3, (a_1+a_2)+(b_1+b_2))$

$= (a_1, a_3, a_1+a_2) + (b_1, b_3, b_1+b_2)$

$= T(a_1, a_2, a_3) + T(b_1, b_2, b_3)$

$\therefore T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v}), \forall \vec{u}, \vec{v} \in R^3$

(ii) Let k be any scalar.

$T(k\vec{u}) = T(k(a_1, a_2, a_3))$
$= T(ka_1, ka_2, ka_3)$
$= (ka_1, ka_3, ka_1+ka_2)$
$= (ka_1, ka_3, k(a_1+a_2))$
$= k(a_1, a_3, a_1+a_2)$
$= kT(a_1, a_2, a_3)$

$T(k\vec{u}) = kT(\vec{u})$, $\forall$ scalar k

$\therefore T$ is a matrix transformation.

22.(a) Ans: $T(x,y,z) = (x+y, y+z, x)$

(i) Let $\vec{u} = (x_1, y_1, z_1)$ & $\vec{v} = (x_2, y_2, z_2)$

By vector addition law:-

$\vec{u}+\vec{v} = (x_1+x_2, y_1+y_2, z_1+z_2)$

$T(\vec{u}+\vec{v}) = T(x_1+x_2, y_1+y_2, z_1+z_2)$

$T(\vec{u}+\vec{v}) = ((x_1+x_2)+(y_1+y_2), (y_1+y_2)+(z_1+z_2), (x_1+x_2))$

$T(\vec{u}+\vec{v}) = ((x_1+y_1)+(x_2+y_2), (y_1+z_1)+(y_2+z_2), (x_1+x_2))$

$T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v}), \forall \vec{u}, \vec{v} \in R^3$

(ii) $T(k\vec{u}) = T(kx_1, ky_1, kz_1)$

$T(k\vec{u}) = ((kx_1 + ky_1), (ky_1 + kz_1), (kx_1))$

$T(k\vec{u}) = (k(x_1+y_1), k(y_1+z_1), k(x_1))$

$T(k\vec{u}) = kT(\vec{u})$, $\forall$ scalars k.

$\therefore$ T is a matrix transformation.

(2) Ans $T(x_1, x_2) = (x_2, x_1)$

(i) Let $\vec{u} = (a_1, a_2)$ & $\vec{v} = (b_1, b_2)$, $\forall \vec{u}, \vec{v} \in R^2$.

$$\begin{aligned} T(\vec{u}+\vec{v}) &= T((a_1, a_2)+(b_1, b_2)) \\ &= T(a_1+b_1, a_2+b_2) \\ &= (a_2+b_2, a_1+b_1) \\ &= (a_2, a_1) + (b_2, b_1) \\ &= T(a_1, a_2) + T(b_1, b_2) \end{aligned}$$

$\therefore T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$, $\forall \vec{u}, \vec{v} \in R^2$

(ii) Let k be any scalar.

$$T(k\vec{u}) = T(k(a_1, a_2))$$
$$= T(ka_1, ka_2)$$
$$= (ka_2, ka_1)$$
$$= k(a_2, a_1)$$
$$= kT(a_1, a_2)$$

$\therefore T(k\vec{u}) = kT(\vec{u})$, $\forall$ scalar k

$\therefore$ T is a matrix transformation since it satisfies both (i) & (ii).