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Exercise 3.2

1. (a) $v = (2,2,2)$

$$\|v\| = \sqrt{2^2+2^2+2^2} = \sqrt{12} = 2\sqrt{3}$$

$$u = -\frac{v}{\|v\|} = \frac{(2,2,2)}{2\sqrt{3}} = -\left(\frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}\right)$$

$$= -\left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right)$$

(b) $v = (1,0,2,1,3)$

$$\|v\| = \sqrt{1+0+4+1+9} = \sqrt{15}$$

$$u = \frac{-v}{\|v\|} = \left(-\frac{1}{\sqrt{15}}, 0, \frac{-2}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{-3}{\sqrt{15}}\right)$$

2. (a) $v = (1,-1,2)$

$$\|v\| = \sqrt{1+1+4} = \sqrt{6}$$

$$u = \frac{v}{\|v\|} = \left(1/\sqrt{6}, -1/\sqrt{6}, 2/\sqrt{6}\right)$$

$u = -\frac{v}{\|v\|} = \left(-1/\sqrt{6}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$

(b) $v = (-2, 3, 3, -1)$

$\|v\| = \sqrt{4+9+9+1} = \sqrt{23}$

$u = \frac{v}{\|v\|} = \frac{1}{\sqrt{23}}(-2, 3, 3, -1)$

$= \left(-\frac{2}{\sqrt{23}}, \frac{3}{\sqrt{23}}, \frac{3}{\sqrt{23}}, -1/\sqrt{23}\right)$

$u = -v/\|v\| \cdot \left(\frac{2}{\sqrt{23}}, -3/\sqrt{23}, -3/\sqrt{23}, \frac{1}{\sqrt{23}}\right)$

3. (a) $u+v = (2,-2,3) + (1,-3,4) = (2+1, -2+(-3), 3+4)$

$= (3, -5, 7)$

$\|u+v\| = \sqrt{3^2 + (-8)^2 + 7^2} = \sqrt{9+25+49}$

$= \sqrt{83}$

(b) $u = (2, -2, 3)$

$\|u\| = \sqrt{2^2 + (-2)^2 + 3^2} = \sqrt{4+9+9} = \sqrt{17}$

$v = (1, -3, 4)$

$\|v\| = \sqrt{1^2 + (-3)^2 + 4^2} = \sqrt{1+9+16} = \sqrt{26}$

$\|u\| + \|v\| = \sqrt{17} + \sqrt{26}$

(c) $\|-2u+2v\|$

$-2u = (-4,4,-6)$

$2v = (2,-6,8)$

$-2u+2v = (-2,-2,2)$

$\|-2u+2v\| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

(d) $u = (2,-2,3)$

$3u = (6,-6,9)$

$v = (1,-3,4)$

$-5v = (-5,15,-20)$

$w = (3,6,-4)$

$3u-5v+w = (6,-6,9)+(-5,15,-20)+(3,6,-4)$
$= (4,15,-15)$

$\|3u-5v+w\| = \sqrt{4^2+15^2+(-15)^2}$

$= \sqrt{16+225+225} = \sqrt{466}$

7. $|k|\,\|v\| = 5$

$\Rightarrow |k|\sqrt{(-2)^2+(3)^2+(0)^2+6^2} = 5$

$\Rightarrow |k|\sqrt{4+9+0+36} = 5$

$\Rightarrow |k|(7) = 5 \Rightarrow |k| = \frac{\pm 5}{7}$ $k = \frac{\pm 5}{7}$

$k = 5/7$ or $k = -5/7$

8. $\|v\| = \sqrt{1^2+1^2+2^2+(-3)^2+1^2}$

$= \sqrt{1+1+4+9+1} = \sqrt{16} = 4$

$\|kv\| = |k|\|v\|$

$\Rightarrow |k|(4) = 4$

$|k| = 1$

$k = 1$ or $k = -1$

10. (a) $u \cdot v = 1\cdot(-1) + 1\cdot 0 + (-2)\cdot 5 + 3\cdot 1$

$= -1 + 0 - 10 + 3 = -8$

$u \cdot u = 1\cdot 1 + 1 + (-2)(-2) + 3\cdot 3$

$= 1 + 1 + 4 + 9 = 15$

$v \cdot v = 1 + 0 + 25 + 1 = 27$

(b) $u = (2,-1,1,0,-2)$ $v = (1,2,2,2,1)$

$u \cdot v = 2\cdot 1 + (-1)\cdot 2 + 1\cdot 2 + 0\cdot 2 + (-2)\cdot 1$

$= 2 - 2 + 2 + 0 + -2 = 0$

$u \cdot u = 2 \cdot 2 + (-1) \cdot (-1) + 1 \cdot 1 + 0 \cdot 0 + (-2) \cdot (-2) = 4 + 1 + 1 + 0 + 4 = 10$

$v \cdot v = 1 \cdot 1 + (2) \cdot 2 + 2 \cdot 2 + 2 \cdot 2 + (1) \cdot 1$

$= 1 + 4 + 4 + 4 + 1 = 14$

12. (a) $u = (1, 2, -3, 0)$ $v = (5, 1, 2, -2)$

$d(u, v) = \sqrt{(1-5)^2 + (2-1)^2 + (-3-2)^2 + (0-(-2))^2}$

$= \sqrt{16 + 1 + 25 + 4} = \sqrt{46}$

$u \cdot v = \|u\| \|v\| \cos\theta$

$u \cdot v = 1 \cdot 5 + 2 \cdot 1 + -3 \cdot 2 + 0 \cdot -2$

$= 5 + 2 - 6 = 7 - 6 = 1$

$\|u\| = \sqrt{1^2 + 2^2 + (-3)^2 + 0} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$\|v\| = \sqrt{5^2 + 1^2 + 2^2 + (-2)^2} = \sqrt{25 + 1 + 4 + 4}$

$= \sqrt{25 + 1 + 8} = \sqrt{34}$

$u \cdot v = \|u\| \|v\| \cos\theta$

$1 = \sqrt{14} \cdot \sqrt{34} \cos\theta$

$$\cos\theta = \frac{1}{\sqrt{476}} \approx 0.04583$$

$$\theta = \cos^{-1}(0.04583) = 87.37^\circ$$

Hence, this is an acute angle.

The angle between u and v is $\cos\theta = \frac{1}{\sqrt{476}}$

(b) $u = (0,1,1,1,2)$ $v = (2,1,0,-1,3)$

$$d(u,v) = \sqrt{(0-2)^2 + (1-1)^2 + (1-0)^2 + (1+1)^2 + (2-3)^2} = \sqrt{4+0+1+4+1} = \sqrt{10}$$

$u \cdot v = \|u\| \|v\| \cos\theta$

$= 0\cdot2 + 1\cdot1 + 1\cdot0 + 1\cdot-1 + 2\cdot3$

$= 1 - 1 + 6 = 6$

$$\|u\| = \sqrt{0+1^2+1^2+1^2+2^2} = \sqrt{0+1+1+1+4}$$

$= \sqrt{7}$

$$\|v\| = \sqrt{2^2+1^2+0+(-1)^2+3^2} = \sqrt{4+1+0+1+9}$$

$= \sqrt{15}$

$u \cdot v = \|u\| \|v\| \cos\theta$

$6 = \sqrt{7} \cdot \sqrt{15} \cos\theta$

$$\cos\theta = \frac{6}{\sqrt{105}} \approx 0.5855$$

$$\theta = \cos^{-1}(0.5855) = 54.16^\circ$$

Hence this is an acute angle

$$\cos\theta = \frac{6}{\sqrt{105}}$$

13. $\|a\| = 9$

$\|b\| = 5$

The angle between a and b is $120^\circ - 90^\circ = 30^\circ$

$$a \cdot b = \|a\|\|b\| \cos\theta$$

$$a \cdot b = 9 \times 5 \times \cos(30^\circ)$$

$$= \frac{45\sqrt{3}}{2}$$

$$\therefore a \cdot b = \frac{45\sqrt{3}}{2}$$

14. ~~From the figure~~ We can observe that the angle between the given two vectors is 90°. Hence they are perpendicular and their dot product is 0. $a \cdot b = 0$

15. (a) The expression $u \cdot (v \cdot w)$ is equivalent to $u \cdot$ (some scalar)

Thus, the given expression $u \cdot (v \cdot w)$ does not make sense mathematically.

(b) The expression $u \cdot (v + w)$ makes sense mathematically because it satisfies distributive law, that is $u \cdot (v + w) = u \cdot v + u \cdot w$.

(c) The dot product $u \cdot v$ yields a scalar. The norm of a scalar is not possible. Therefore the expression $\|u \cdot v\|$ does not make sense mathematically.

(d) $u \cdot v$ yields a scalar

$\|u\|$ is a scalar

The subtraction of two scalars is possible.

There the expression $(u \cdot v) - \|u\|$ makes sense mathematically.

18.(a) $u = (4,1,1)$ $v = (1,2,3)$

$u \cdot v = (4,1,1)\cdot(1,2,3)$
$= 4(1) + 1(2) + 1(3)$
$= 4 + 2 + 3 = 9$

$\|u\| = \sqrt{4^2+1^2+1^2} = \sqrt{18} = 3\sqrt{2}$

$\|v\| = \sqrt{1+4+9} = \sqrt{14}$

$|u \cdot v| \leq \|u\|\|v\|$
$|9| \leq (3\sqrt{2})(\sqrt{14})$
$9 \leq 3\sqrt{28} \approx 15.87$

This is true. Hence it proves Cauchy-Schwarz inequality.

(b) $u = (1,2,1,2,3)$ $v = (0,1,1,5,-2)$

$u \cdot v = 1(0) + 2(1) + 1(1) + 2(5) + 3(-2)$
$= 0 + 2 + 1 + 10 - 6 = 7$

$\|u\| = \sqrt{1^2+2^2+1^2+2^2+3^2} = \sqrt{1+4+1+4+9}$
$= \sqrt{19}$

$\|v\| = \sqrt{0^2+1^2+1^2+5^2+(-2)^2}$
$= \sqrt{0+1+1+25+4} = \sqrt{31}$

$|u \cdot v| \leq \|u\| \|v\|$

$|7| \leq (\sqrt{19})(\sqrt{31})$

$|7| \leq \sqrt{584} \approx 24.27$

This is true. Hence, it proves Cauchy-Schwarz inequality.