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Exercise 1.6 (continued)

19. (b)

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & 7 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1}$$

Finding the inverse first :-

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \quad R_2 = -2R_1 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 = \frac{R_2}{5}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 = -2R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{5} & \frac{4}{5} & -\frac{2}{5} & 1 \end{array}\right]$$

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$R_3 = -5R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$R_2 = \frac{2}{5}R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$R_1 = R_2 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$R_1 = -R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$$\therefore X = \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 6-4+9 & -3-0+15 & 15+3-21 & 21-0+6 & 24-1+3 \\ -4+4-6 & 2+0-10 & -10-3+14 & -14+0-4 & -16+1-2 \\ -8+8-15 & 4+0-25 & -20-6+35 & -28+0-10 & -32+2-5 \end{bmatrix}$$

$$X = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$$

20 Ans.

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & -4 \end{bmatrix} X = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

Finding the inverse first :-

$R_1 = \dfrac{-R_1}{2}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & -4 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = -R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & \frac{7}{2} & -\frac{1}{2} & 0 & -1 \end{array}\right]$$

-1+3
+1-2
2+2-5

$R_2 = -R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & -1 & \frac{7}{2} & -\frac{1}{2} & 0 & -1 \end{array}\right]$$

$R_3 = R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & \frac{9}{2} & -\frac{1}{2} & -1 & -1 \end{array}\right]$$

$R_3 = \frac{2}{9} R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{9} & -\frac{2}{9} & -\frac{2}{9} \end{array}\right]$$

$R_2 = -R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{9} & -\frac{7}{9} & \frac{2}{9} \\ 0 & 0 & 1 & -\frac{1}{9} & -\frac{2}{9} & -\frac{2}{9} \end{array}\right]$$

$R_1 = \frac{1}{2} R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{9} & -\frac{1}{9} & -\frac{1}{9} \\ 0 & 1 & 0 & \frac{1}{9} & -\frac{7}{9} & \frac{2}{9} \\ 0 & 0 & 1 & -\frac{1}{9} & -\frac{2}{9} & -\frac{2}{9} \end{array}\right]$$

$$X = \begin{bmatrix} -\frac{5}{9} & -\frac{1}{9} & -\frac{1}{9} \\ \frac{1}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & -\frac{2}{9} & -\frac{2}{9} \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{20}{9} - \frac{6}{9} - \frac{1}{9} & -\frac{15}{9} - \frac{7}{9} - \frac{3}{9} & -\frac{10}{9} - \frac{8}{9} - \frac{7}{9} & -\frac{5}{9} - \frac{9}{9} - \frac{9}{9} \\ \frac{4}{9} - \frac{49}{9} + \frac{2}{9} & \frac{3}{9} - \frac{49}{9} + \frac{6}{9} & \frac{2}{9} - \frac{56}{9} + \frac{14}{9} & \frac{1}{9} - \frac{63}{9} + \frac{18}{9} \\ -\frac{4}{9} - \frac{12}{9} - \frac{2}{9} & -\frac{3}{9} - \frac{14}{9} - \frac{6}{9} & -\frac{2}{9} - \frac{16}{9} - \frac{14}{9} & -\frac{1}{9} - \frac{18}{9} - \frac{18}{9} \end{bmatrix}$$

Assume A & B are matrices of the same size

Is $C = A \pm B$ of the same type?

Is $C = AB$ of the same type?

Is $C = A^T$ of the same type?

eg

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$$X = \begin{bmatrix} \frac{-27}{9} & \frac{-25}{9} & \frac{-25}{9} & \frac{-23}{9} \\ \frac{-43}{9} & \frac{-40}{9} & \frac{-40}{9} & \frac{-44}{9} \\ \frac{-18}{9} & \frac{-23}{9} & \frac{-32}{9} & \frac{-37}{9} \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & -\frac{25}{9} & -\frac{25}{9} & -\frac{23}{9} \\ -\frac{43}{9} & -\frac{40}{9} & -\frac{40}{9} & -\frac{44}{9} \\ -2 & -\frac{23}{9} & -\frac{32}{9} & -\frac{37}{9} \end{bmatrix}$$

Section 1.7

Assume A & B are matrices of the same size	Matrices A & B are diagonal	Matrices A & B are upper (lower) triangular	Matrices A & B are symmetric
Is C = A ± B of the same type?	Yes	No, only addition of two upper (lower) triangular matrix is upper (lower) triangular.	Yes
Is C = AB of the same type?	Yes	Yes	No, Product of two symmetric matrices may not be symmetric
Is C = A^T of the same type?	Yes	No, Transpose of an upper triangular matrix is lower triangular and vice versa.	Yes

$$\begin{matrix} -\frac{5}{9} & -\frac{9}{9} & -\frac{9}{9} \\ \frac{1}{9} & -\frac{63}{9} & +\frac{18}{9} \\ -\frac{1}{9} & -\frac{18}{9} & -\frac{18}{9} \end{matrix}$$

A − B may not give the same type of matrix.

eg:- $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ $\therefore A-B = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

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Assume A & B are matrices of the same size	Matrices A & B are diagonal	Matrices A & B are upper (lower) triangular	Matrices A & B are symmetric
Does $C = A^{-1}$ exist?	No, not always	Yes	No, not always. If the diagonal entries of the symmetric matrix is zero, then the inverse does not exist. e.g. $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$
If $C = A^{-1}$ exists, is it of the same type?	Yes	Yes	Yes

If the main diagonal contains a zero, then the inverse will not exist.

e.g. $D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 32 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

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Exercise 1.7

1.(a) Ans Upper triangular. It is invertible since all its diagonal entries are non-zero

(b) Ans Lower triangular. Not invertible - se the diagonal entries ~~...~~ have

(c) Ans Diagonal matrix, upper and lower. It is invertible

(d) Ans Upper triangular matrix. Not invertible because there is a zero diagonal entries.

2.(a) Ans Lower triangular matrix. It is since all its diagonal entries non-zero.

(b) Ans Upper triangular matrix invertible since it has diagonal entries.

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riant of
the symmetric matrix is zero, then the inverse does not exist. eg. $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

Exercise 1.7

1. (a) Ans. Upper triangular. It is invertible since all its diagonal entries are non-zero.

(b) Ans. Lower triangular. Not invertible because the diagonal entries ~~are~~ have zeros.

(c) Ans. Diagonal matrix, upper and lower triangular. It is invertible.

(d) Ans. Upper triangular matrix. Not invertible because there is a zero in its diagonal entries.

2. (a) Ans. Lower triangular matrix. It is invertible since all its diagonal entries are non-zero.

(b) Ans. Upper triangular matrix. It is not invertible since it has zeros in its diagonal entries.

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(c) Ans. Diagonal matrix, upper and lower triangular. It is invertible.

(d) Ans. Lower triangular matrix, It is not invertible because it has a zero in the diagonal entries.

7. Ans.
$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

$$A^{-k} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{(-2)^k} \end{bmatrix}$$

8. Ans.
$$A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 36 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} \frac{1}{36} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{25} \end{bmatrix}$$

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$$A^{-k} = \begin{bmatrix} \frac{1}{(-6)^k} & 0 & 0 \\ 0 & \frac{1}{(3)^k} & 0 \\ 0 & 0 & \frac{1}{(4)^k} \end{bmatrix}$$

11. Ans.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying only the diagonal entries because these are diagonal matrices.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

12. Ans.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Multiplying only diagonal entries because these are diagonal matrices.

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 28 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -15 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & 84 \end{bmatrix}$$

17. Ans (a) Ans

$$\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

(b) Ans

$$\begin{bmatrix} 1 & 3 & 7 & 2 \\ 3 & 1 & -8 & -3 \\ 7 & -8 & 0 & 9 \\ 2 & -3 & 9 & 0 \end{bmatrix}$$

18. Ans :- (a) Ans :-

$$\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

(b) Ans

$$\begin{bmatrix} 1 & 7 & -3 & -2 \\ 7 & 4 & 5 & -7 \\ -3 & 5 & 1 & -6 \\ -2 & -7 & -6 & 3 \end{bmatrix}$$

19. Ans :- Not invertible because it is an upper triangular matrix with a zero in its diagonal

20. Ans Invertible because it is an upper triangular matrix with no zeroes in its diagonal.

23. Ans Diagonal entries of AB will be -3, 5 & -6.

24. Ans Diagonal entries of AB will be 24, 0 & 42.

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25 Ans: $A = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$

Since A is symmetric, $a+5 = -3$
$a = -3-5$
$a = -8$

26 Ans $a - 2b + 2c = 3 \rightarrow (1)$
$2a + b + c = 0 \rightarrow (2)$
$a + c = -2 \rightarrow (3)$

We ~~get~~ know these values because matrix A is symmetric.

Solving (3) :-

$c = -2 - a \rightarrow (4)$

Substituting (c) in (2) :-

$2a + b + (-2-a) = 0$
$2a + b - 2 - a = 0$
$a + b = 2$
$b = 2 - a \rightarrow (4)$

Substituting (b) & (c) in (1) :-

$a - 2(2-a) + 2(-2-a) = 3$
$a - 4 + \cancel{2a} - 4 - \cancel{2a} = 3$
$a - 8 = 3$
$a = 3 + 8$
$a = 11$

Substituting a in (4) :-

$b = 2 - 11$
$b = -9$

Substituting (a) in (5) :-

$c = -2 - 11$
$= -13$

$\therefore a = 11,\ b = -9\ \&\ c = -13$

27. Ans. A is an upper triangular matrix it will only be invertible if its diagonal values are not equal to 0.

$\therefore x - 1 \neq 0$
$x \neq 1$

$x + 2 \neq 0$
$x \neq -2$

$x - 4 \neq 0$
$x \neq 4$

$\therefore$ A is invertible for all values of x, other than 1, -2 & 4.

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28. Ans

$$A = \begin{bmatrix} x-\frac{1}{2} & 0 & 0 \\ x & x-\frac{1}{3} & 0 \\ x^2 & x^3 & x+\frac{1}{4} \end{bmatrix}$$

A is a lower triangular matrix & is invertible only if all its diagonal entries are $\neq 0$.

$\therefore \quad x - \frac{1}{2} \neq 0$

$x \neq \frac{1}{2}$

$x - \frac{1}{3} \neq 0$

$x \neq \frac{1}{3}$

$x + \frac{1}{4} \neq 0$

$x \neq -\frac{1}{4}$

$\therefore$ A is invertible for all values of x, other than $\frac{1}{2}, \frac{1}{3}$ & $-\frac{1}{4}$.

36 Ans. To find :- All 3×3 diagonal matrices A that satisfy $A^2 - 3A - 4I = 0$.

Let A be $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3a & 0 & 0 \\ 0 & 3b & 0 \\ 0 & 0 & 3c \end{bmatrix}$$

$$4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} - \begin{bmatrix} 3a & 0 & 0 \\ 0 & 3b & 0 \\ 0 & 0 & 3c \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 0$$

$$\begin{bmatrix} a^2-3a-4 & 0 & 0 \\ 0 & b^2-3b-4 & 0 \\ 0 & 0 & c^2-3c-4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ We have :- $a^2-3a-4=0$

$b^2-3b-4=0$

$c^2-3c-4=0$

Solving for a :-

$(a^2-4a)+(1a-4)=0$

$a(a-4)+1(a-4)=0$

$\therefore (a-4)(a+1)=0$

$\therefore a=4$ or -1

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Solving for b

$(b^2-4b)+(1b \ldots$

$b(b-4)+1(\ldots$

$(b-4)(\ldots$

$\therefore b=4 \ldots$

Solving fo…

$(c \ldots$

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Solving for b :-

$(b^2 - 4b) + (1b - 4) = 0$

$b(b-4) + 1(b-4) = 0$

$(b-4)(b+1) = 0$

$\therefore b = 4$ or $b = -1$

Solving for c :-

$(c^2 - 4c) + (1c - 4) = 0$

$c(c-4) + 1(c-4) = 0$

$(c-4)(c+1) = 0$

$\therefore c = 4$ or $c = -1$

$\begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = 0$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ The 3x3 diagonal matrices that satisfy $A^2 - 3A - 4I = 0$ are :-

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \& \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

4(i) Matrix $A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 1 \\ -4 & -1 & 0 \end{bmatrix}$

(ii) Ans $A = \begin{bmatrix} 0 & 0 & -8 \\ 0 & 0 & -4 \\ 8 & 4 & 0 \end{bmatrix}$

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42 Ans $A = \begin{bmatrix} 0 & 2a-3b+c & 3a-5b+5c \\ -2 & 0 & 5a-8b+6c \\ -3 & -5 & d \end{bmatrix}$

$2a - 3b + c = 2 \quad -(1)$

$3a - 5b + 5c = 3 \quad -(2)$

$5a - 8b + 6c = 5 \quad -(3)$

$d = 0$ (most likely).

subtracting 2 times equation (2) from 3 times equation (1), we get:

$$6a - 9b + 3c = 6$$
$$\underset{(-)}{6a} \; \underset{(+)}{-10b} \; \underset{(-)}{+10c} = \underset{(-)}{6}$$

$$b - 7c = 0 \quad -(4)$$

$$b = 7c$$

~~substituting b in equation (3), we get~~

$$5a - 56c + 6c = 5$$
$$5a - 50c = 5$$
$$5a = 5 + 50c$$
$$a = \frac{5 + 50c}{5}$$

$$a = \frac{5(1 + 10c)}{5}$$

$$a = 1 + 10c$$

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Subtracting 3 times equation (2) from 5 times equation (1), we get :-

$$10a - 15b + 5c = 10$$
$$\underline{-9a \mp 15b \pm 15c = 9}$$

$$a \qquad -10c = 1$$
$$a = 1 + 10c$$

Substituting a in (3) :-

$$5(1+10c) - 8b + 6c = 5$$
$$5 + 50c - 8b + 6c = 5$$
$$55c - 8b + 6c = 5$$
$$-8b + 61c = 5 \quad - (5)$$

Solving (4) & (5) :-

$$b - 7c = 0 \quad - (4)$$
$$-8b + 61c = 5 \quad - (5)$$

Adding ~~Subtracting~~ 8 times equation (4) from (5), we get :-

$$-8b + 61c = 5$$
$$\underline{8b - 49c = 0}$$
$$12c = 5$$
$$c = \frac{5}{12}$$

Substituting (c) in (4) :-

$$b - 7c = 0$$
$$b - 7 \times \frac{5}{12} = 0$$

$b - \frac{35}{12} = 0$

$b = \frac{35}{12}$

~~Substituting b and c in (5), we get:~~

$$\cancel{-8 \times \frac{35}{12} + 61 \times \frac{5}{12} = 5}$$

$$\cancel{\frac{-280 + 305}{12} = 5}$$

Substituting b and c in (1), we get:-

$$2a - 3 \times \frac{35}{12} + \frac{5}{12} = 2$$

$$\frac{2a}{1} - \frac{35}{4} + \frac{5}{12} = 2$$

$$\frac{24a - 105 + 5}{12} = 2$$

$$\frac{24a - 100}{12} = 2$$

$$24a = 24 + 100$$

$$a = \frac{124}{24}$$

$$a = \frac{31}{6}$$

$\therefore a = \frac{31}{6}$, $b = \frac{35}{12}$ & $c = \frac{5}{12}$

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Substituting these values and checking if it is skew-symmetric.

$$A = \begin{bmatrix} 0 & \cancel{2}^{1}\times\frac{31}{\cancel{6}_{3}} - \cancel{3}^{1}\times\frac{35}{\cancel{12}_{4}} + \frac{5}{12} & 3\times\frac{31}{6} - 5\times\frac{35}{12} + 5\times\frac{5}{12} \\ -2 & 0 & 5\times\frac{31}{6} - 8\times\frac{35}{12} + 6\times\frac{5}{12} \\ -3 & -5 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{31}{3}^{\times 4} - \frac{35}{4}^{\times 3} + \frac{5}{12} & \frac{31}{2}^{\times 6} - \frac{175}{12} + \frac{25}{12} \\ -2 & 0 & \frac{155}{6} - \frac{70}{3} + \frac{5}{2} \\ -3 & -5 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{124-105+5}{12} & \frac{186-175+25}{12} \\ -2 & 0 & \frac{155-140+15}{6} \\ -3 & -5 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{\cancel{24}^{2}}{\cancel{12}_{1}} & \frac{\cancel{36}^{3}}{\cancel{12}_{1}} \\ -2 & 0 & \frac{\cancel{30}^{5}}{\cancel{6}_{1}} \\ -3 & -5 & 0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$$ which is skew-symmetric.