

Section 2.2 (Evaluating Determinants by Row Reduction) Overview & To-Do List

Section 2.2 highlights **properties of $\det(A)$** for a square matrix A . Here are three such properties that can help in the evaluation of determinants:

- (a) If A has a row of zeros or a column of zeros, then $\det(A) = 0$. (Thm 2.2.1)
- (b) $\det(A) = \det(A^T)$. (Thm 2.2.2)
- (c) If A has two proportional rows (or columns), then $\det(A) = 0$. (Example 2)

Property (b) above suggests that any result about the rows of a determinant also holds for columns.

In this section, your author makes a case for evaluating a determinant by using row reduction to first change it to a matrix T in upper triangular form. There are pluses and minuses about doing this from a practical standpoint. On the minus side, one might be more prone to errors doing row reduction. On the plus side, if the row reduction needed is minimal it might be worth doing a little row reduction so as to eventually evaluate an easy determinant of T .

If we are planning to do some row reduction to evaluate a determinant, here are some **more determinant properties** that can help with this (See Thm 2.2.3) for an $n \times n$ matrix A .

- (a) $\det(B) = k \det(A)$ if B is obtained from A by multiplying a single row (or column) of A by k .
- (b) $\det(B) = -\det(A)$ if B is obtained from A by interchanging two rows (or columns) of A .
- (c) $\det(B) = \det(A)$ if B is obtained from A by adding a multiple of one row (column) of A to another row (column) of A .

When $A = I$ and $E = B$ in the above, where E then is an elementary matrix, we have

- (a) $\det(E) = k$.
- (b) $\det(E) = -1$.
- (c) $\det(E) = 1$.

Examples 3-5 show how row reduction or elementary column operations can be used in evaluating the determinant.

Read Section 2.2

Do problems 1, 2, 5-9, 11, 15, 17-18

Video: simpler 4 x 4 determinant (using row operations)

<https://youtu.be/QV0jsTiobU4>

Work Check Quiz 11

Do problems 21, 29, 30, 32