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Exercise 2.1

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix} = 29$$

$$C_{11} = (-1)^{1+1} M_{11} = M_{11} = 29$$

$$M_{12} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 6 & -1 \\ -3 & 4 \end{vmatrix} = 21$$

$$C_{12} = (-1)^{1+2} M_{12} = -21$$

$$M_{13} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 6 & 7 \\ -3 & 1 \end{vmatrix} = 27$$

$$C_{13} = (-1)^{1+3} M_{13} = M_{13} = 27$$

$$M_{21} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix} = -11$$

$$C_{21} = (-1)^{2+1} M_{21} = -M_{21} = 11$$

$$M_{22} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -3 & 4 \end{vmatrix} = 13$$

$$C_{22} = (-1)^{2+2} M_{22} = M_{22} = 13$$

$$M_{23} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} = -5$$

$$C_{23} = (-1)^{2+3} M_{23} = -M_{23} = -(-5) = 5$$

$$M_{31} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} -2 & 3 \\ 7 & -1 \end{vmatrix} = -19$$

$$C_{31} = (-1)^{3+1} M_{31} = M_{31} = -19$$

$$M_{32} = \begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix} = -19$$

$$C_{32} = (-1)^{3+2} M_{32} = -M_{32} = 19$$

$$M_{33} = \begin{vmatrix} 1 & -2 \\ 6 & 7 \end{vmatrix} = 19$$

$C_{33} = (-1)^{3+3} M_{33} = 19$

3. (a) $M_{13} = \begin{vmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = 0 - 0 + 3(4 \times 1 - 4 \times 1)$

$= 3(4-4) = 0$

$C_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13} = 0.$

(b) $M_{23} = \begin{vmatrix} 4 & -1 & 6 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix}$

$= 4[(1 \times 2) - (14 \times 1)] - (-1)(4 \times 2 - 4 \times 14)$
$+ 6(4 \times 1 - 4 \times 1) = 4(2-14) + 1(8-56) +$
$6(4-4) = 4(-12) + 1(-48) + 6(0) =$
$-48 - 48 = -96$

$C_{23} = (-1)^{2+3} M_{23} = (-1)^5 (-96) = 96$

(c) $M_{22} = \begin{vmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{vmatrix} = 4(0 \times 2 - 14 \times 3) -$
$1(4 \times 2 - 4 \times 14) +$
$6(4 \times 3 - 4 \times 0)$

$= 4(0-42) - 1(8-56) + 6(12-0)$

$= 4(-42) - 1(-48) + 6(12) = -168 + 48 + 72$

$= -48$

$$C_{22} = (-1)^{2+2} M_{22} = (-1)^4 (-48)$$
$$= (1)(-48) = -48$$

(d) $M_{21} = \begin{vmatrix} -1 & 1 & 6 \\ 1 & 0 & 14 \\ 1 & 3 & 2 \end{vmatrix}$

$$= -1 \begin{vmatrix} 0 & 14 \\ 3 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 14 \\ 1 & 2 \end{vmatrix} + 6 \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} = -1(0-42) - 1(2-14)$$
$$+ 6(3-0) = 42 + 12 + 18$$
$$= 72$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)(72) = -72$$

9. $\det(A) = \begin{bmatrix} a-3 & 5 \\ -3 & a-2 \end{bmatrix} = (a-3)(a-2)$
$$-5(-3)$$
$$= a^2 - 5a + 6 + 15 = a^2 - 5a + 21$$

11. $A = \left[\begin{array}{ccc} 2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{array}\right] \begin{array}{cc} 2 & 1 \\ 3 & 5 \\ 1 & 6 \end{array}$

$\text{sum } 1 = (-2)\cdot 5 \cdot 2 + 1\cdot(-7)\cdot 1 + 4\cdot 3\cdot 6$

$= -20 - 7 + 72 = 45$

$\text{sum } 2 = 4\cdot 5\cdot 1 + (-2)\cdot(-7)\cdot 6 + 1\cdot 3\cdot 2$

$= 20 + 84 + 6 = 110$

$|A| = 45 - 110$

$|A| = -65$

13. $\left|\begin{matrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{matrix}\right| \begin{matrix} 3 & 0 \\ 2 & -1 \\ 1 & 9 \end{matrix}$

$\text{sum } 1 = [3\,(-1)\,(-4)] + [0 \times 5 \times 1] + [0 \times 2 \times 9] = 12$

$\text{sum } 2 = [1 \times (-1) \times 0] + [9 \times 5 \times 3] + [(-4) \times 2 \times 0]$

$= 135$

$\det(A) = 12 - 135 = -123.$

21. $\det(A) = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$

$C_{11} = (-1)^{1+1} * \left|\begin{matrix} 5 & 1 \\ 0 & 5 \end{matrix}\right|$

$C_{12} = (-1)^{1+2}\ \cancel{\ } * \left|\begin{matrix} 2 & -1 \\ 1 & 5 \end{matrix}\right|$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix}$

$\Rightarrow \det(A) = -3(25-0) + 0 + 7(0-(-5))$

$= -40$

22. $A = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{bmatrix}$

$|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$C_{11} = (-1)^2 \begin{vmatrix} 0 & -4 \\ -3 & 5 \end{vmatrix}$

$= 1[0 \cdot 5 - (-4)(-3)]$

$= -12$

$C_{12} = (-1)^3 \begin{vmatrix} 1 & -4 \\ 1 & 5 \end{vmatrix} = -1[1 \cdot 5 - (-4) \cdot 1]$

$= -[5+4] = -9$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 1 & -3 \end{vmatrix}$

$= 1[1 \cdot (-3) - 0 \cdot 1] = [-3]$

$= -3$

$|A| = 3(-12) + 3(-9) + 1(-3) = -64+10+15$

$= -66$

22. $\det(A) = 1\begin{vmatrix} k & k^2 \\ 1 & k^2 \end{vmatrix} - k\begin{vmatrix} 1 & k^2 \\ 1 & k^2 \end{vmatrix} +$

$k^2\begin{vmatrix} 1 & k \\ 1 & k \end{vmatrix}$

$= 1(k(k^2) - k(k^2)) - k(1(k^2) - 1(k^2))$

$+ k^2(1(k) - 1(k))$

$= 1(k^3 - k^3) - k(k^2 - k^2) + k^2(k - k)$

$= \underline{0}$

23. $\det A = \det \begin{vmatrix} 1 & k & k^2 \\ 1 & k & k^2 \\ 1 & k & k^2 \end{vmatrix}$

$= 1 \begin{vmatrix} k & k^2 \\ k & k^2 \end{vmatrix} - k \begin{vmatrix} 1 & k^2 \\ 1 & k^2 \end{vmatrix} + k^2 \begin{vmatrix} 1 & k \\ 1 & k \end{vmatrix}$

$= 1(k(k^2) - k(k^2)) - k(1(k^2) - 1(k^2)) +$
$k^2(1(k) - 1(k))$

$= 1(k^3 - k^3) - k(k^2 - k^2) + k^2(k - k)$

$= 0$

25. $\det A = 0 \cdot (-1)^{1+3} M_{13} + 0 \cdot (-1)^{2+3} M_{23} + (-3) \cdot (-1)^{3+3}$
$M_{33} + 3 \cdot (-1)^{4+3} M_{43}$

$= -3M_{33} - 3M_{43}$

$= -3 \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}$

$= -3 \left[3 \begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix} \right]$

$-3 \left[3 \begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} - 3 \begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} \right]$

$= -3\left[3(4+20) - 3(4+4) + 5(20-4)\right]$

$= -3\left[3(0+2) - 3(0+8) + 5(2-8)\right]$

$= -3\left[72-24+80\right] - 3\left[6-24-30\right]$

$= -3(128) - 3(-48) = -384 + 144 = -240$

29. The given matrix is a triangular matrix. The determinant of a triangular matrix is the product of the diagonal elements. Since, the first entry of the diagonal is 0.

Therefore, the determinant of the matrix is,

$\det A = 0 \times 2 \times 3 \times 8 = 0$

30. A is a triangular matrix

Thus,

$\det(A) = a_{11} \cdot a_{22} \cdot a_{33} = (1)(2)(3)(4) = 24$

$\det(A) = 24$