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1) $v_1 = (2,1)$ and $v_2 = (3,0)$

$c_1v_1 + c_2v_2 = 0$ and $c_1v_1 + c_2v_2 = b$

Where, $b = (b_1, b_2)$

$2c_1 + 3c_2 = 0 \Rightarrow c_1 = 0$

$2c_1 + 3c_2 = b_1 \Rightarrow c_1 = b_2 \quad \ldots (1)$

The two systems have the same coefficient matrix

$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ $|A| = -3 \neq 0$ This implies that the matrix is invertible

Therefore the vectors are linearly independent and span the vector space R^2 and form basis.

2) $v_1 = (3,1,-4)$, $v_2 = (2,5,6)$ and $v_3 = (1,4,8)$

$c_1v_1 + c_2v_2 + c_3v_3 = 0$ and $c_1v_1 + c_2v_2 + c_3v_3 = b$

Where $b = (b_1, b_2, b_3)$

$3c_1 + 2c_2 + c_3 = 0$ And $= b_1$

$c_1 + 5c_2 + 4c_3 = 0$ $= b_2$

$-4c_1 + 6c_2 + 8c_3 = 0$ $= b_3$

Coefficient matrix

$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{bmatrix}$

$$|A| = 3\begin{vmatrix} 5 & 4 \\ 6 & 8 \end{vmatrix} - 2\begin{vmatrix} 1 & 4 \\ -4 & 8 \end{vmatrix} + 1\begin{vmatrix} 1 & 5 \\ -4 & 6 \end{vmatrix}$$

$= 3(40-24) - 2(8+16) + (6+20) = 48 - 48 + 26 = 26$

That is $|A| \neq 0$

This implies that the coefficient matrix is invertible. Therefore the vectors are linearly independent and span the vector space R^3

4) $P_1 = 1+x$, $P_2 = 1-x$, $P_3 = 1-x^2$, $P_4 = 1-x^3$

Let $S = (1,1,0,0), (1,-1,0,0), (1,0,-1,0), (1,0,0,-1)$. The standard basis for P_3 is $B = \{1, x, x^2, x^3\}$ and w.r.t B the coord. vector S can be expressed in the above form.

Augmented matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Expand the cofactors along row 3.

$$-1 \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1\left[-1 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}\right]$$ This can be simplified as,

$-1[-1(-1-1)] = 2$ The determinant is non-zero and hence, the given set in P_3 are linearly independent

$Ax = b$. On A $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -1 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$R_2 \to R_2/(-2)$.
$R_3 \to -R_3$
$R_4 \to -R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here $Ax = b$ has a sol. for every b. This implies the polynomials span P_3 and the given set is linearly independent.

5. Consider $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22}$

Suppose that,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = l \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + m \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + p \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix} + q \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

Here l, m, n and p are scalars

$$= \begin{bmatrix} 3l + 0m + 0p + q & 6l - 1m - 8p + 0q \\ 3l - 1m - 12 - 1q & -6l + 0m - 4p + 2q \end{bmatrix}.$$

Rewrite the equations:

$3l + q = a$ .... (1)

$6l - m - 8p = b$ .... (2)

$3l - m - 12p - q = c$ .... (3)

$-6l - 4p + 2q = d$ .... (4)

$(1) + (3) \Rightarrow 6l - m - 12p = a + c$ .... (5)

$(2)-(5) \Rightarrow -4p = c+a-b$

$p = \dfrac{c+a-b}{-4}$

Substitute $-4p = c+a-b$ in (4)

$-6l + 2q = d-c-a+b \quad \dots (6)$

Solve (6) and (1) then $q = \dfrac{d-c+b}{4}$

By (1), $3l+q = a$ then $3l + \dfrac{d-c+b}{4} = a$

$l = \dfrac{4a-d+c-b}{12}$

By (2), $6l - m - 8p = b$ then,

$\dfrac{4a-d+c-b}{2} + 2(c+a-b) - b = m$

$\dfrac{4a-d+c-b+4c+4a-4b-2b}{2} = m$

$\dfrac{8a-d+5c-7b}{2} = m$

Therefore the vector $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22}$ can be written as,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \left(\dfrac{4a-d+c-b}{12}\right)\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + \left(\dfrac{8a-d+5c-7b}{2}\right)\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + \left(\dfrac{c+a-b}{-4}\right)\begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix} + \left(\dfrac{d-c+b}{4}\right).$

$\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ This is the linear combination of the elements of S.

Therefore, $L(S) = M_{22}$

Thus, the set S ~~per~~ spans M_{22}. Assume that $aM_1 + bM_2 + cM_3 + dM_4 = 0$; Where a, b, c, d are scalars

$$\begin{bmatrix} 3a & 6a \\ 3a & -6a \end{bmatrix} + \begin{bmatrix} 0 & -b \\ -b & 0 \end{bmatrix} + \begin{bmatrix} 0 & -8c \\ -12c & -4c \end{bmatrix} + \begin{bmatrix} d & 0 \\ -d & 2d \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$3a + d = 0$$
$$6a - b - 8c = 0$$
$$3a - b - 12c - d = 0$$
$$-6a - 4c + 2d = 0$$

Matrix A with coefficients :

$$A = \begin{bmatrix} 3 & 0 & 0 & 1 \\ 6 & -1 & -8 & 0 \\ 3 & -1 & -12 & -1 \\ -6 & 0 & -4 & 2 \end{bmatrix}$$

Cofactor expansion along the first row.

$$= 3 \begin{vmatrix} -1 & -8 & 0 \\ -1 & -12 & -1 \\ 0 & -4 & 2 \end{vmatrix} - 0 + 0 - 1 \begin{vmatrix} 6 & -1 & -8 \\ 3 & -1 & -12 \\ -6 & 0 & -4 \end{vmatrix}$$

$= 3(-1(-12(2) - (-1)(-4)) - (-8)(-1(2) - 0(-1))$
$+ 0) - 6(((-1)(-4) - 0(-12)) - (-1)(3(-4) -$
$(-6)(-12)) + (-8)(3(0) - (-1)(-6)))$

$= 3(-1(-24-4)-(-8)(-2-0)) - 16(4-0) + (-12-72) - 8(0-6)$

$= 3(-1(-28)-(-8)(-2)) - (16(4) + (-84) - 8(-6))$

$= 3(28-16) - (24-84+48)$

$= 3(12) - (-12) = 36 + 12 = 48$

As $\det A = 48 \neq 0$, the matrix A is invertible. Therefore, the vectors in S are linearly independent and the set S is a basis for M_{22}

6. Let $c_1 M_1 + c_2 M_2 + c_3 M_3 + c_4 M_4 = 0$

$$\begin{bmatrix} c_1 & c_1 \\ c_1 & c_1 \end{bmatrix} + \begin{bmatrix} c_2 & -c_2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -c_3 \\ c_3 & 0 \end{bmatrix} + \begin{bmatrix} c_4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1+c_2+c_4 & c_1-c_2-c_3 \\ c_1+c_3 & c_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Solve the system of equations

$c_1 + c_2 + c_4 = 0$
$c_1 - c_2 - c_3 = 0$
$c_1 + c_3 = 0$
$c_1 = 0$

The system has the unique solution.
$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0$

The eq. has only the trivial solution.
Therefore, the matrices are linearly independent

Solve the system

$c_1 + c_2 + c_4 = a$

$c_1 - c_2 - c_3 = b$

$c_1 + c_3 = c$

$c_1 = d$

The system has the unique solution.

$c_1 = d$ $\quad c_2 = -b - c + 2d$

$c_3 = c - d$

$c_4 = a - b + b + c - 2d$

Therefore, every 2×2 matrix

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Therefore the matrices span M_{22}

Therefore, the matrices form a basis for M_{22}

7. (a) $v_1 = (2,-3,1)$ $\quad v_2 = (4,1,1)$ $\quad v_3 = (0,-7,1)$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ And $= b$ where

$b = (b_1, b_2, b_3)$

$2c_1 + 4c_2 = 0$

$-3c_1 + c_2 - 7c_3 = 0$

$c_1 + c_2 + c_3 = 0$

And

$2c_1 + 4c_2 = b_1$

$-3c_1 + c_2 - 7c_3 = b_2$

$c_1 + c_2 + c_3 = b_3$

$A = \begin{bmatrix} 2 & 4 & 0 \\ -3 & 1 & 7 \\ 1 & 1 & 1 \end{bmatrix}$ $\quad |A| = 2\begin{vmatrix} 1 & -7 \\ 1 & 1 \end{vmatrix} - 4\begin{vmatrix} -3 & -7 \\ 1 & 1 \end{vmatrix}$

$+ 0\begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} = 2(1+7) - 4(-3+7) + 0(-3-1)$

$= 16 - 16 + 0 = 0$

The coefficient matrix is not invertible.
The vectors are linearly dependent. Hence they do not form basis for R^3.

(b) $v_1 = (1,6,4)$, $v_2 = (2,4,-1)$ and $v_3(-1,2,5)$

$c_1v_1 + c_2v_2 + c_3v_3 = 0$ and $= b$ Where $b = (b_1, b_2, b_3)$

$c_1 + 2c_2 - c_3 = 0$
$6c_1 + 4c_2 + 2c_3 = 0$
$4c_1 - c_2 + 5c_3 = 0$

And

$c_1 + 2c_2 - c_3 = b_1$
$6c_1 + 4c_2 + 2c_3 = b_2$
$4c_1 - c_2 + 5c_3 = b_3$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 6 & 4 & 2 \\ 4 & -1 & 5 \end{bmatrix} \quad |A| = 1\begin{vmatrix} 4 & 2 \\ -1 & 5 \end{vmatrix} - 2\begin{vmatrix} 6 & 2 \\ 4 & 5 \end{vmatrix} - 1\begin{vmatrix} 6 & 4 \\ 4 & -1 \end{vmatrix}$$

$= 1(20+2) - 2(30-8) - 1(-6-16) = 22 - 44 + 22$
$= 0$

That is $|A| = 0$

This implies that the coefficient matrix is not invertible.
Therefore the vectors are linearly dependent and do not form basis for R^3.

8) (a) Let $p_0 = 1-3x+2x^2$, $p_1 = 1+x+4x^2$, $p_2 = 1-7x$

$a_0p_0 + a_1p_1 + a_2p_2 = 0$

$a_0(1-3x+2x^3) + a_1(1+x+4x^2) + a_2(1-7x) = 0$

$(a_0+a_1+a_2) + (-3a_0+a_1-7a_2)x + (2a_0+4a_1+0\cdot a_2)x^2 = 0$

Equate the coefficients

$a_0+a_1+a_2 = 0$
$-3a_0+a_1-7a_2 = 0$
$2a_0+4a_1+0\cdot a_2 = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ -3 & 1 & -7 \\ 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -3 & 1 & -7 \\ 2 & 4 & 0 \end{bmatrix}$$

$|A| = 1(0+28) - 1(0+14) + 1(-12-2)$
$= 28-14-14 = 0$

Thus the vectors are not linearly independent and the given set is not a basis for P_3.

9. $c_1M + c_2M_2 + c_3M_3 + c_4M_4 = 0$ ......(1)

$c_1M_1 + c_2M_2 + c_3M_3 + c_4M_4 = B$ ......(2)

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$c_1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} + c_3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + c_4\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ And

$$c_1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} + c_3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + c_4\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Further solve,

$$\begin{bmatrix} c_1+2c_2+c_3 & -2c_2-c_3-c_4 \\ c_1+3c_2+c_3+c_4 & c_1+2c_2+c_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ And

$$\begin{bmatrix} c_1+2c_2+c_3 & -2c_3-c_3-c_4 \\ c_1+3c_2+c_3+c_4 & c_1+2c_2+c_4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Therefore

$c_1+2c_2+c_3 = 0$ ...... (3)

$-2c_2-c_3-c_4 = 0$ ...... (4)

$c_1+3c_2+c_3+c_4 = 0$ ...... (5)

$c_1+2c_2+c_4 = 0$ ...... (6)

Add equations (3) and (4)

$$\begin{aligned} -2c_2-c_3-c_4 &= 0 \\ c_1+2c_2+c_3 &= 0 \\ \hline c_1 - c_4 &= 0 \end{aligned}$$

Hence $c_1 = c_4$

Sub. the values of $c_1 = c_4$ in (6)

$c_4+2c_2+c_4 = 0$

$c_2 = -c_4$

Sub. the values of $c_1 = c_4$ and $c_2 = -c_4$ in (3)

$c_4 + 2x - c_4 + c_3 = 0$ Hence, $c_1 = c_4$; $c_2 = -c_4$
$c_3 - c_4 = 0 \Rightarrow c_3 = c_4$. $c_3 = -c_4$

There are 4 variables but only 3 equations, hence atleast one of them must be 0.

Therefore, the matrix vectors are linearly dependent and do not form basis for M_{22}

11.(a) $u_1 = (2,-4)$, $u_2 = (3,8)$, $w = (1,1)$.

The basis of vector space R^2 is $\{u_1, u_2\}$.
Therefore w will be the linear combination of u_1 and u_2

$w = k_1 u_1 + k_2 u_2$

Substitute the value of $u_1 = (2,-4)$, $u_2 = (3,8)$ and $w = (1,1)$.

$(1,1) = k_1(2,-4) + k_2(3,8)$

Solve, $2k_1 + 3k_2 = 1$
$-4k_1 + 8k_2 = 1$ Apply Cramer rule

$$\frac{k_1}{\begin{bmatrix} 1 & 3 \\ 1 & 8 \end{bmatrix}} = \frac{k_2}{\begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix}} = \frac{1}{\begin{bmatrix} 2 & 3 \\ -4 & 8 \end{bmatrix}}$$ For k_1

$$k_1 = \frac{\begin{bmatrix} 1 & 3 \\ 1 & 8 \end{bmatrix}}{\begin{bmatrix} 2 & 3 \\ -4 & 8 \end{bmatrix}} = \frac{(8-3)}{(16+12)} = \frac{5}{28}$$

And for k_2

$$k_2 = \frac{\begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix}}{\begin{bmatrix} 2 & 3 \\ -4 & 8 \end{bmatrix}} = \frac{(2+4)}{(16+12)} = \frac{6}{28}$$

$= 3/14$ $\quad a = \left(\frac{5}{28}, \frac{3}{14}\right)$

(b) $u_1 = (1,1)$, $u_2 = (0,2)$, $w = (a,b)$

$w = k_1 u_1 + k_2 u_2$

Substitute the value of $u_1 = (1,1)$, $u_2 = (0,2)$ and $w = (a,b)$

$(a,b) = k_1(1,1) + k_2(0,2)$

Solve, $\quad$ Use Cramer rule

$k_1 = a$

$k_1 + 2k_2 = b$

$$\frac{k_1}{\begin{bmatrix} a & 0 \\ b & 2 \end{bmatrix}} = \frac{k_2}{\begin{bmatrix} 1 & a \\ 1 & b \end{bmatrix}} = \frac{1}{\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}}$$

For k_1

$$k_1 = \frac{\begin{bmatrix} a & 0 \\ b & 2 \end{bmatrix}}{\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}} = \frac{2a}{2} = a$$

For k_2

$$k_2 = \frac{\begin{bmatrix} 1 & a \\ 1 & b \end{bmatrix}}{\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}} = \frac{(b-a)}{2}$$

The required coordinate vector is $a = \left(a, \frac{b-a}{2}\right)$

14. (a) $p = ap_1 + bp_2 + cp_3$

Substitute the value of $p = 4 - 3x + x^2$;
$p_1 = 1$, $p_2 = x$ and $p_3 = x^2$

$4 - 3x + x^2 = a + bx + cx^2$

Equate the coefficients of x, x^2 and constant term

$a = 4$
$b = -3$
$c = 1$

The req. coord. vector is
$(p)S = (4, -3, 1)$

(b) $p = ap_1 + bp_2 + cp_3$

Sub. the value of $p = 2 - x + x^2$; $p_1 = 1 + x$,
$p_2 = 1 + x^2$ and $p_3 = x + x^2$

$2 - x + x^2 = a(1+x) + b(1+x^2) + c(x+x^2)$

Further solve,

$2 - x + x^2 = a + ax + b + bx^2 + cx + cx^2$
$= (a+b) + (a+c)x + (b+c)x^2$

$a + b = 2$
$a + c = -1$
$b + c = 1$

$2a + 2b + 2c = 2$
$a + b + c = 1$

Sub. the val. of $b + c = 1$ in eq. $a + b + c = 1$

$a=0$. Sub. the value of $a=0$ in eq.

$a+b=2 \Rightarrow 0+b=2 \Rightarrow b=2$

Sub. $a=0$ in $a+c=-1 \Rightarrow c=-1$

The req. coord. vector is $(p)_S=(0,2,-1)$

17. $ap_1+bp_2+cp_3=0$

$a(1+x+x^2)+b(x+x^2)+c(x^2)=0$

$a+(a+b)x+(a+b+c)x^2=0+0x+0x^2$

Compare on both sides, then;

$a=0,\ a+b=0,\ a+b+c=0$

It follows that $a=b=c=0$

Therefore, the set $S=\{p_1,p_2,p_3\}$ is linearly independent. ..... (1)

Let $p+qx+rx^2 \in S$

$p+qx+rx^2 = ap_1+bp_2+cp_3$

$p+qx+rx^2 = a+(a+b)x+(a+b+c)x^2$

Compare on both sides, then,

$a=p$

$a+b=q \Rightarrow b=q-a \Rightarrow b=q-p$

$a+b+c=r \Rightarrow c=r-a-b \Rightarrow c=r-p-(q-p)=r-q$

Therefore

$p+qx+rx^2 = p(1+x+x^2)+(q-p)(x+x^2)+(r-q)(x^2)$

Therefore, every vector in P_2 can be expressed as a linear combination of vectors in S.

Hence, S spans the vector space P_2 ..... (2)

Hence, S spans the vector space P_2 .... (2)

Therefore, the set S is a basis for P_2 (From (1) and (2))

$p = c_1p_1 + c_2p_2 + c_3p_3$

$7 - x + 2x^2 = c_1(1 + x + x^2) + c_2(x + x^2) + c_3(x^2)$

$7 - x + 2x^2 = c_1 + c_1x + c_1x^2 + c_2x + c_2x^2 + c_3x^2$

$7 - x + 2x^2 = c_1 + (c_1 + c_2)x + (c_1 + c_2 + c_3)x^2$

Equate on both sides, then

$c_1 = 7$; $c_1 + c_2 = -1$; $c_1 + c_2 + c_3 = 2$

Substitute $c_1 = 7$ in $c_1 + c_2 = -1$, then

$7 + c_2 = -1$; $c_2 = -8$

Now substitute $c_1 = 7$ and $c_2 = -8$ in $c_1 + c_2 + c_3 = 2$, then $7 - 8 + c_3 = 2 \Rightarrow c_3 = 3$

Therefore, $p = 7p_1 - 8p_2 + 3p_3$

Thus, the coord. vector of p relative to S is

$(p)_S = (7, -8, 3)$.

19. (c) $p_1 = 1 + x + x^2$, $p_2 = x$

The dimension of P_2 is 3

In n dimensional vector space, fewer than n elements cannot span the vector space.
So the two elements in 3-dimensional vector space P_2 cannot span P_2. Hence, the given sets of vectors are not the basis for P_2.

(d) For all the matrices A, B, C, D the element present in the first row and second column is ~~no~~ zero.

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22}$ and $b \neq 0$

The matrix X which has the first row and second column ~~[illegible]~~ as non-zero cannot express the linear combination of the vectors: A, B, C, D

For ex:- $\begin{bmatrix} 5 & 7 \\ 0 & -2 \end{bmatrix} \neq a \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} + b \begin{bmatrix} 6 & 0 \\ -1 & 4 \end{bmatrix} + c \begin{bmatrix} 3 & 0 \\ 1 & 7 \end{bmatrix} + d \begin{bmatrix} 5 & 0 \\ 4 & 2 \end{bmatrix}$ for any scala[r]

of a, b, c and d

So every vector in M_{22} cannot express the linear combination of vectors A, B, C, D

Hence the set of vectors $S = \{A, B, C, D\}$ is not the basis of the vector space M_{22}.