MA 260 SECTIONS 3.1-3.2 OVERVIEW and TO-DO LIST

There is a lot of review in these sections of what was seen Calculus II with 2- and 3-component vectors. All this is generalized to cover n-component vectors in R^n. Vector addition, scalar multiplication, dot or scalar product, distance between vectors, and equality of vectors all work like they did when n was 2 or 3. Even the angle R^n R^n R^n R^n between two vectors is extended to R^n using a result that worked for n = 2 and 3.

There is a bit of **new notation and terminology**. The magnitude or length of a vector $\vec{v}$ is denoted $\|\vec{v}\|$ with double bars instead of single ones. It is also referred to as the **Euclidean norm** of the vector. The distance **between two vectors** $\vec{u}$ **and** $\vec{v}$ is denoted $d(\vec{u},\vec{v})$. The dot product is now referred to as the **Euclidean inner product.**

Check out some **new vector results** that are expressed as inequalities in Thm 3.2.4. and Thm 3.2.5 and as equalities in Thm 3.2.6 and Thm 3.2.7.

A new concept that is introduced in Section 3.1 is that of a vector being a **linear combination** of a set of vectors in R^n. Since the section is missing examples on this, some examples follow the definition below..

DEFINITION. If $\vec{w}$ is a vector in R^n, then $\vec{w}$ is said to be a **linear combination** of the vectors $\vec{v}_1, \vec{v}_2, ...\vec{v}_r$ in R^n If it can be expressed in the form $\vec{w} = k_1\vec{v}_1 + k_2\vec{v}_2 + ... + k_r\vec{v}_r$ where $k_1, k_2, ..., k_r$ are scalars and referred to as the **coefficients** of the linear combination. When $r = 1$, $\vec{w} = k_1\vec{v}_1$, so that a linear combination of a single vector is just a scalar multiple of that vector.

Example 1. We learned in Calculus II, that a vector $\vec{v}$ in R^2 can be written as $\vec{v} = (x, y) = x(1,0) + y(0,1) = x\vec{i} + y\vec{j}$ and, thus, is a linear combination of vectors $\vec{i}$ and $\vec{j}$.

Example 2. Is the vector (6, 30, 8) a linear combination of the vectors (1, -3, -1) and (0, 4, -2)?

That is, is $(6,30,8) = k_1(1,-3,1) + k_2(0,4,2) + k_3(2,6,3)$ for some k_1, k_2, k_3?

If so, then $(6,30,8) = (1k_1+0k_2+3k_3, -3k_1+4k_2+6k_3, 1k_1+2k_2+3k_3)$

this yields

$$\left.\begin{array}{l} 1k_1+0k_2+2k_3=6 \\ -3k_1+4k_2+6k_3=30 \\ -1k_1-2k_2+3k_3=8 \end{array}\right\} \quad A\vec{k}=\vec{b} \text{ where } \vec{k}=\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \text{ and } \vec{b}=\begin{bmatrix} 6 \\ 30 \\ 8 \end{bmatrix}$$

If we take $\det(A) = \begin{vmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{vmatrix}$, we find $\det(A) = 44 \neq 0$

By the latest summing-up (Equivalence) theorem, thm 2.3.8, the linear system has a solution. So, yes, (6,30,8) is a linear combination of the other two vectors.

Example 3. Is $\vec{u} = (1,2,3)$ a linear combination of the vectors $\vec{v}_1 = (-1,0,3)$ and $\vec{v}_2 = (4,1,1)$?

Let $\vec{u} = (1,2,3) = k_1(-1,0,2) + k_2(4,1,1)$. Equating corresponding components of the left and right sides of the above equation,

$$\begin{aligned} -k_1 + 4k_2 &= 1 \\ 0k_1 + 1k_2 &= 2 \\ 2k_1 + 1k_2 &= 3 \end{aligned} \Rightarrow k_2 = 2. \text{ Then } \left.\begin{aligned} &-k_1 + 8 = 1,\ k_1 = 7 \\ &\text{and} \\ &2k_1 + 2 = 3,\ k = 1/2 \end{aligned}\right\} \not= \text{. Contradiction,}$$

so no solution.

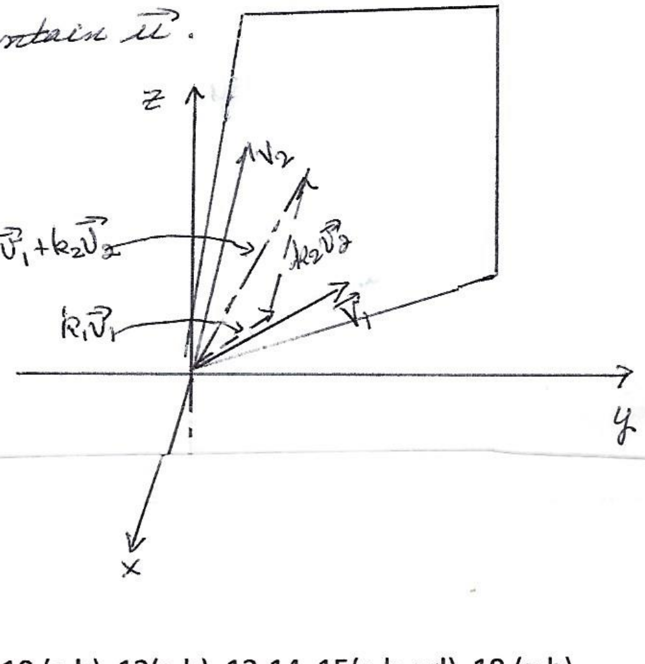
Thus, here $\vec{u}$ is not a linear combination of $\vec{v}_1, \vec{v}_2$.

Geometric Interpretation of Example 3: $\vec{u} = (1,2,3)$ is not in the plane through the origin determined by $\vec{v}_1$ and $\vec{v}_2$.

Why?

Consider the vectors as position vectors (initial points at (0,0,0)). $\vec{v}_1$ is not parallel to $\vec{v}_2$ and vice versa since neither vector is a scalar multiple of the other.

Hence $\vec{v}_1$ and $\vec{v}_2$ determine a plane thru the origin. This plane contains all position vectors that are linear combinations of $\vec{v}_1$ and $\vec{v}_2$. It does not contain $\vec{u}$.



Read Section 3.1. Do Problems 1, 5, 7, 15-17, 19, 21-22
Take Check Quiz 13

Read Section 3.2. Do Problems 1(a,b), 2(a,b), 3(a,b,c,d), 7, 8, 10 (a,b), 12(a,b), 13-14, 15(a,b,c,d), 18 (a,b)
Video: https://youtu.be/WNulhXo39_k

Take Check Quiz 14