Section 4.4B Linear Independence Overview and To-Do List

Let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, ...\vec{v}_r\}$ be a set of a finite number of vectors from a vector space V.

There are two definitions commonly used for what is meant by saying that **S is an independent set of vectors**.. Some books use what your textbook author calls Definition 1 as the primary definition, and other books use an equivalent Definition 2 which appears in Thm 4.4.1. We will want the flexibility to switch back and forth between the two and use whichever is most convenient.

Definition 1. If the set S mentioned above has at least two vectors, we say S is an independent set of vectors if no vector of S can be expressed as a linear combination of the other vectors. We will assume that if S contains just a single vector, then S is independent if and only if that vector is non-zero.

Definition 2. (Comes from Thm. 4.4.1) The non-empty S mentioned above is linearly independent if and only if whenever

$$k_1\vec{v}_1 + k_2\vec{v}_2 + ... + k_r\vec{v}_r = 0,$$

it follows that $k_1 = k_2 = k_3 = ... = k_r = 0$.

If S is not independent, it is said to be a **dependent** set.

The work in Thm 4.3.1 shows the equivalence of the two definitions.

Examples 1, 2, 3, 4, and 5 all take the approach of Definition 2 in sorting out whether sets of vectors are independent or not. Work through all of these examples. However, one should keep one's eyes open to the possibility that Definition 1 might apply. It might save you some work. See the example below.

Example. Suppose S is the set of polynomials whose entries are the polynomials below:

$$p(x) = x^2 - x + 1, \quad q(x) = x^2 - 2, \quad r(x) = -x + 3$$

If one spends a moment looking to see if these polynomials are related in some way, one might happen on the fact that we have

$$q(x) = p(x) - h(x) = 1p(x) + (-1)h(x).$$

This shows that q(x) is a linear combination of $p(x)$ and $r(x)$; and, thus, S is a **dependent** set.

Read Section 4.4 up to and including Example 8. Be sure to look at Thm 4.4.3. We will not be dealing with the Wronskian which appears at the end of Section 4.3.

Work Text Problems 1(a,b,c,d), 2 (a,b), 4 (a,b), 5 (a,b), 9(a,b), 13, and 16(a,b, d and e); T/F

Work Check Quiz #18