Section 4.2, Subspaces Overview and To-Do List

Once we have a portfolio of known vector spaces from Section 4.1, we can find other vector spaces that are subspaces of these.

Definition. A subspace W of a vector space V is called a **subspace** of V if W is itself a vector space under the same addition and scalar multiplication defined on V.

To show W, a subset of V is a *subspace* of the vector space V, we do not need to verify that ten vector spaces axioms hold. We only need to verify that the two *closure* axioms # 1 and #6 hold.
WHY?
Axioms 2, 3, 7, 8, 9, 10 hold automatically (i.e., are inherited from V) since W is a subset of V. Then if Axiom 6 holds, Axioms 4 and 5 will also hold by making use of Thm 4.1.1 (the extra properties of a vector space).

Theorem 4.2.3 shows that the solution set of a homogeneous system $A\vec{x} = \vec{0}$ *m* equations in *n* unknowns is a subspace of R^n.
Example 13 shows that if the solution is in R^3, then the *solution space* must turn out to be one of the subspaces of R^3 as listed in Table 1 of this section.

Part 1: Read Section 4.2, and the examples therein.
Work through Problems 1 (a-e), 2(a-g), 3(a-d) and 4(a-d).
These problems ask you to determine if a given *subset* W of a vector space V is a *subspace* of W. These are not just yes/no problems.

If you answer "yes,"you must be able to verify *in general* that Axioms 1 and 6 hold. A single example won't work in this case. Also, you cannot assume, say, that a matrix is *2x2* if V is a vector space of ***nxn*** matrices.

If you answer "no," you need to come up with a counterexample to ***one*** of Axioms 1 and 6 as was done in the text examples 6 and 10 or come up with a known result that states one of the two axioms fails to hold.

Later, Part 2:
Work through Problems 6 a, b; 7 a ,b; 11 a-c, 13, 15-16, 20-21.