Section 2.3 Overview and To-Do List

Section 2.3 is all about properties of the determinant and adding an important statement to our Summing Up Theorem (Equivalence Theorem).

Here are some important ones:

$(a)\ \det(kA) = k^n \det(A)$

$(b)\ \det(AB) = \det(A)\det(B)$

$(c)\ \det(A) \neq 0\ iff\ A\, is\, invertible$

Property (a) follows from factoring k out of each row of A and a related result from the last section, Theorem 2.2.3 (a) which had a related determinant result when A just had k as a factor of each term in just one row or column. Property (b) is first proved when A is an elementary matrix and then extended using the fact that an invertible matrix is a product of elementary matrices (Summing-up Theorem). Property (b) is then used to prove (c).

One Non-Property: $\det(A+B) = \det(A) + \det(B)$

Since $AA^{-1} = I$, Property (b) above gives us:

$$(d)\ \det(A^{-1}) = \frac{1}{\det(A)}.$$

ADJOINT OF A, Adj A. You need to know the definition of this matrix as the transpose of the matrix of cofactors of A for one problem on the next Check Quiz. **BUT we will not use it to find the inverse of matrix. You can use it (and some of you already seem to be doing that); but it won't be necessary to know how to do this.** Hence, you can skip Thm 2.3.6 and Example 2—they are optional.

We won't be seeing the adjoint after this section. It's primary purpose is to give us Cramer's Rule in this section. Pay particular attention to **Cramer's Rule (Thm 2.3.7).** It is often the easiest way to solve a system of linear equations of n equations in n unknowns.

See how our **Summing-up Theorem (Thm 2.3.8)** looks now with the addition of Property (c) from above.

Read Section 2.3 (Thm 2.3.6 and Example 2 are optional—see note above on Adj A)
Do Problems 1, 5, 7-8, 10, 13, 24, 27, 30, 33-35
Work Check Quiz 12
Do Problems 33-35.