Section 4.1 Vector Spaces Overview and To-Do List

Back in Calculus II, we were introduced to the set of two component vectors with two particular operations which we call vector addition and scalar multiplication. This set of vectors R^2 with these two operations had a lot of nice properties numbering more than ten. These properties can be culled to a minimal list of ten properties where those properties left off the list can be derived from the basic minimal ten ones.

In Section 4.1 we become acquainted with a vector space which is a collection of objects called vectors and two operations called vector addition and scalar multiplication and which satisfy the same minimal list of properties (called **axioms)** as with R^2 mentioned above. **See Definition 1, p. 184 of Section 4.1 for the definition of a vector space including the 10 axioms.**

There are two main goals of Section 4.1.

Goal #1 is to create a small portfolio of known vector spaces through examples given in the text. These vector spaces that we must know include:

- R^n
- The trivial vector space
- $P_n = \{a_0 + a_1x + a_2x^2 + ... + a_nx^n\}$, the set of all polynomials with real coeficients and **of degree n or less** with the usual addition and scalar multiplication.
- F = the set of all real-valued functions on $(-\infty, \infty)$ with the usual function addition and multiplication by a scalar (real number).
- M_{mn} = the set of all *m x n* matrices with real entries and with the usual addition and scalar multiplication of matrices.

It is a daunting task to do this since it technically must be done by showing all ten axioms hold (are satisfied) for each of the above. But the work here will help us find other vector spaces in the future with a lot less work. Note that to show that a possible vector space candidate is **not actually a vector space**, we need only find one axiom that fails to hold. (However, we might have to go through all 10 axioms to find it.)

Goal #2 is to become acquainted with what the axioms are really saying and what they do not say. **Example 8** shows us that the zero vector of Axiom 4 need not have any zeros appearing in it. Some do, some don't, depending on the vector space.

Some Comments on Axioms 1 and 6. Axiom 1 states that when you add two objects in your set, you must always obtain a like object of the same type. **Axiom 6** states that whenever you multiply an object in your set by a scalar (real number) you must always obtain a like object, not just for *some* k but for *all* k.

Example A. The set S of all polynomials of degree 1 with the usual addition and scalar multiplication violates Axiom 1 since when p(x) = x is added to q(x) = - x + 1, you do not obtain a like polynomial of degree 1. Thus we do not have a vector space here.

Example B. The set S with operations above in Example A also violates Example 6 since there is a scalar k, namely k = 0, where multiplication of q(x)=- x + 1 by 0 does not yield a like polynomial of degree 1. Axiom 6 fails to hold.

Thm 4.1.1 lists some additional properties of vector spaces that are not in the minimal list of ten axioms, but which we might wish to make use of from time to time.

Read Section 4.1. Check out the vector spaces and non-vector spaces in the Examples of Section 4.`.
Do Problems 1 (a – e), 2(a – 3), 3-5, 7-8, 10, 13
Videos:

What makes R^n a vector space? Part 1 https://www.youtube.com/watch?v=TcekNmQ1IrI

What makes R^n a vector space? Part 2 https://youtu.be/ZghibySynqM

Definition of a vector space. https://youtu.be/Rf1poVnKTS8

Take Check Quiz 16