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Exercise 2.3

1. $k^n|A| = 2^2 \begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix} = 2^2[4(-1) - 2(3)]$

$= 2^2(-4-6) =$

$= 2^2(-10)$

$k^n|A| = -40 \quad \ldots \text{(i)}$

$KA = 2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 6 & 8 \end{bmatrix}$

$\det(KA) = \begin{vmatrix} -2 & 4 \\ 6 & 8 \end{vmatrix} = (-2)(8) - (4)(6)$

$= -16 - 24 = 40$

$\det(KA) = -40 \quad \ldots \text{(ii)}$

From (i) and (ii), we have

$\det(kA) = k^n|A|$

5. $AB = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2\cdot 1+1\cdot 7+0\cdot 5 & 2(-1)+1\cdot 1+0\cdot 0 & 2\cdot 3+1\cdot 2+0\cdot 1 \\ 3\cdot 1+4\cdot 7+0\cdot 5 & 3(-1)+4\cdot 1+0\cdot 0 & 3\cdot 3+4\cdot 2+0\cdot 1 \\ 0\cdot 1+0\cdot 7+2\cdot 5 & 0(-1)+0\cdot 1+2\cdot 0 & 0\cdot 3+0\cdot 2+2\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -1 & 8 \\ 31 & 1 & 17 \\ 10 & 0 & 2 \end{bmatrix}$$

$$\det(AB) = 9(2-0) + 1(62-170) + 8(0-10)$$
$$= 18 - 108 - 80 = -170$$

$$BA = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 2-3+0 & 1-4+0 & 0+0+6 \\ 14+3+0 & 7+4+0 & 0+0+4 \\ 10+0+0 & 5+0+0 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -3 & 6 \\ 17 & 11 & 4 \\ 10 & 5 & 2 \end{bmatrix}$$

det(BA).

$$\det(BA) = -1(22-20) + 3(34-40) + 6(85-110)$$
$$= 2 - 18 - 150 = \cancel{8} - 170$$

$$\det(AB) = \det(BA)$$

$$\det(A+B) = \det(A) + \det(B)$$

$$A + B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 3 \\ 10 & 5 & 2 \\ 5 & 0 & 3 \end{bmatrix}$$

$$\det(A+B) = 3\,\cancel{\{5}\,(15-0) - 0 + 3(0+25)$$
$$= 45 - 75 = -30$$

$$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2(8-0) - 1(6-0) + 0$$
$$= 16 - 6 = 10$$

$$|B| = \begin{vmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{vmatrix} = 1(1-0)+1(7-10)+3(0-5)$$
$$= 1-3-15$$

$\det(B) = -17$

$\det(A)+\det(B) = 10-17 = -7 \neq \det(A+B)$

Therefore $\det(A+B) \neq \det(A)+\det(B)$

7. $$|A| = \begin{vmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{vmatrix} = 2(-3+0)-5(-3+0)+5(-4+2)$$
$$= -6+15-10$$

$|A| = -1 \neq 0$ $\therefore$ A is invertible

8. $$\det(A) = \begin{vmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{vmatrix} = 2(-12)+3(6)$$
$$= -24+18 = -6 \neq 0$$

Thus A is invertible

10. $\det(A) = \begin{vmatrix} -3 & 0 & 1 \\ 5 & 0 & 6 \\ 8 & 0 & 3 \end{vmatrix} = -3(0) + 0(-43) + 1(0)$

$= 0$

Thus A is not invertible

13. $\det(A) = \begin{vmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{vmatrix} = 2(6) = 12 \neq 0$

Thus A is invertible

24. $7x_1 + 2x_2 = 3$

$3x_1 + x_2 = 5$

$A = \begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$

$\det(A) = |A| = \begin{vmatrix} 7 & -2 \\ 3 & \cancel{4}1 \end{vmatrix} = (7)(1) - (-2)(3)$

$= 13$

$\det(A_1) = |A_1| = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} = (3)(1) - (-2)(5)$

$= 13$

$\det(A_2) = |A_2| = 35 - 9 = 26$

By Cramer's rule

$$x_1 = \frac{|A_1|}{|A|} = 1$$

$$x_2 = \frac{|A_2|}{|A|} = 2$$

$x_1 = 1,\ x_2 = 2$

27. $A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{bmatrix}$, $A_1 = \begin{bmatrix} 4 & -3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

$A_2 = \begin{bmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & -3 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} -1 & 0 \\ 0 & -3 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 0 \\ 4 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix}$$

$$= 3 + 3(-6) + 4 = -11$$

$$|A_1| = 4\begin{vmatrix} -1 & 0 \\ 0 & -3 \end{vmatrix} - (-3)\begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} -2 & -1 \\ 0 & 0 \end{vmatrix}$$

$$= 4(3) + 3(6) = 30$$

$$|A_2| = \begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} - 4 \begin{vmatrix} 2 & 0 \\ 4 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix}$$

$$= (6) - 4(-6) + 8 = 38$$

$$|A_3| = \begin{vmatrix} -1 & -2 \\ 0 & 0 \end{vmatrix} - (-3) \begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} +$$

$$4 \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix} = 3(8) + 4(4) = 40$$

By Cramer's Rule,

$x_1 = |A_1| / |A| = 30/-11$

$x_2 = |A_2| / |A| = 38/-11$

$x_3 = |A_3| / |A| = 40/-11$

30. $|A| = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta(\cos\theta) - \sin\theta(-\sin\theta)$

$= \cos^2\theta + \sin^2\theta = 1 \neq 0$

This means that A is invertible for all values of θ.

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

The cofactors of A are

$C_{11} = \cos\theta \quad C_{12} = \sin\theta \quad C_{13} = 0$

$C_{21} = -\sin\theta \quad C_{22} = \cos\theta \quad C_{23} = 0$

$C_{31} = 0 \quad C_{32} \quad C_{33} = 1$

The matrix of cofactors is

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \operatorname{adj}(A) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A) = 1/1 \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

33. (a) $|A| = -7$

$$\det(3A) = 3^3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 27 \det(A)$$

$$= -189$$

(b) Since $A \cdot A^{-1} = I$, taking determinant on both sides we get,

$$\det(A) \cdot \det(A^{-1}) = \det(I)$$

$$\det(A^{-1}) = \frac{1}{\det(A)} = -1/7$$

(c) $\det(2A^{-1}) = 2^3 \det(A^{-1}) = -8/7$

(d) $\det((2A)^{-1}) = \dfrac{1}{\det(2A)}$

$\det(2A) = 8 \times -7$

$\therefore \det((2A)^{-1}) = -1/56$

(e) The matrix $\begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix}$ is obtained from the matrix A by interchanging row 2 and 3.

When we interchange rows we have to multiply the determinant by -1.

Therefore $\begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix} = (-1)\det(A) = 7$

34. (a) $\det(kA) = k^n \det(A)$

$\det(A) = -2$

$\det(-A) = \det(-1 \cdot A)$

$= (-1)^4 \det(A) = (-1)^4 \cdot (-2) = \underline{\underline{-2}}$

(b) $\det(A^{-1}) = \dfrac{1}{\det(A)} = 1/-2 = -1/2$

(c) $\det(2A^T) = 2^n \cdot \det(A^T)$

$= 2^4 \cdot \det(A) = 16(-2) = -32$

(d) $\det(A^3) = (\det A)^3 = (-2)^3 = -8$

35. (a) $\det(3A) = 3^3 \det(A) = 27(7) = 189$

(b) $\det(A^{-1}) = 1/7$

(c) $\det(2A^{-1}) = 2^3 \cdot \det(A^{-1}) = 8 \cdot \dfrac{1}{\det(A)}$

$= 8 \cdot 1/7 = 8/7$

(d) $\det((2A)^{-1}) = \dfrac{1}{\det(2A)} = \dfrac{1}{2^3 \det(A)} = \dfrac{1}{8(7)}$

$= 1/56$