① MA-260

NAME :- SHREYAS SRINIVASA

Exercise 1.4

BLAZER ID :- SSRINIVA

2. (a) $a(BC) = (aB)C = B(aC)$

Ans. $4\left(\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right) = \left(4\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\right) \cdot \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\left(4\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right)$

$\Rightarrow 4\begin{bmatrix} 0-6 & 0-4 \\ 4+12 & 1+8 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ 4 & -16 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} 16 & 4 \\ -12 & -8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -24 & -16 \\ 64 & 36 \end{bmatrix} = \begin{bmatrix} 0-24 & 0-16 \\ 16+48 & 4+32 \end{bmatrix}$

$= \begin{bmatrix} 0-24 & 0-16 \\ 16+48 & 4+32 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -24 & -16 \\ 64 & 36 \end{bmatrix} = \begin{bmatrix} -24 & -16 \\ 64 & 36 \end{bmatrix} = \begin{bmatrix} -24 & -16 \\ 64 & 36 \end{bmatrix}$

∴ Properties are satisfied.

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(b) Ay $A(B-C) = AB - AC$

~~**Exercise 1.6** (bonus). Let X be a geometric random variable with parameter p. Denote $p_k = \mathbb{P}(X = k)$ for $k = 1, 2, \ldots$. Prove that~~

$$\mathbb{P}(X \geq m + n / X \geq m) = \mathbb{P}(X \geq n) \quad (1)$$

~~for any positive integers m and n. The property (1) is called the *memoryless property* of a discrete random variable (that takes values $1, 2, \ldots$).~~

~~Hint. Use the formula $\mathbb{P}(X = k) = pq^{k-1}$.~~

$$\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\left(\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right) = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -4 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 0-1 & 6+4 \\ 0+4 & 4-16 \end{bmatrix} - \begin{bmatrix} 12+3 & 3+2 \\ 8-12 & 2-8 \end{bmatrix}$$

$$= \begin{bmatrix} -12-4 & 3+2 \\ -8+16 & 2-8 \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ 4 & -12 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 5 \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} -16 & 5 \\ 8 & -6 \end{bmatrix}$$

$\therefore$ Properties are satisfied.

(c) Ans $(B+C)A = BA + CA$

$$\Rightarrow \left(\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \right) \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$+ \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 3 \\ -2 & -6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0+4 & 0+8 \\ 3-8 & -1-16 \end{bmatrix}$$

$$+ \begin{bmatrix} 12+2 & -4+4 \\ -9-4 & 3-8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 12+6 & -4+12 \\ -6-12 & 2-24 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ -5 & -17 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ -13 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 18 & 8 \\ -18 & -22 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ -18 & -22 \end{bmatrix}$$

$\therefore$ Properties are satisfied

4. (a) Ans $(A+B)^T = A^T + B^T$

$$A+B = \begin{bmatrix} 3 & 1 \\ 3 & 0 \end{bmatrix}$$

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$$(A+B)^T = \begin{bmatrix} 3 & 3 \\ 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3 & 3 \\ 1 & 0 \end{bmatrix}$$

$$\therefore (A+B)^T = A^T + B^T$$

Property is verified.

5. Ans $$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}$$

6. Ans $$B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

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15 Ans

$$(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$$

$$7A = \frac{1}{-1}\begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix}$$

$$7A = \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{2}{7} & 1 \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$$

16 Ans

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

$$5A^T = \frac{1}{-1}\begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$5A^T = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -\frac{2}{5} & -\frac{1}{5} \\ 1 & \frac{3}{5} \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{2}{5} & 1 \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

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19.(a) Ans:

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2 & 3+1 \\ 6+2 & 2+1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \rightarrow (1)$$

$$= \begin{bmatrix} 33+8 & 11+4 \\ 24+6 & 8+3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix}$$

(b) Ans

$$A^{-3} = \frac{1}{1} \begin{bmatrix} 11 & -15 \\ -30 & 41 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -15 \\ -30 & 41 \end{bmatrix}$$

(c) Ans From (1), $A^2 = \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix}$

$$A^2 - 2A + I$$

$$\begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - 2 \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 4 & 8 \end{bmatrix}$$

$=$

$$\begin{bmatrix} 5 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$$

$=$

20. (a)(i)

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 0+0 \\ 8+4 & 0+1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 0+0 \\ 24+4 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}$$

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(b) Ans $A^{-3} = \frac{1}{8}\begin{bmatrix} 1 & 0 \\ -28 & 8 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{8} & 0 \\ -\frac{\cancel{28}\,7}{\cancel{8}\,2} & 1 \end{bmatrix}$

$A^{-3} = \begin{bmatrix} \frac{1}{8} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix}$

(c) Ans $\begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}$

31. Ans (a) Ans Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\& \ B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$A + B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

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$$A-B = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

$$(A+B)(A-B) = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \cdot \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -24-32 & -24-32 \\ -40-48 & -40-48 \end{bmatrix}$$

$$= \begin{bmatrix} -56 & -56 \\ -88 & -88 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 25+42 & 30+48 \\ 35+56 & 42+64 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 78 \\ 91 & 106 \end{bmatrix}$$

$$\therefore A^2 - B^2 = \begin{bmatrix} -60 & -68 \\ -76 & -84 \end{bmatrix}$$

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$\therefore (A+B)(A-B) \neq A^2 - B^2$

(b) A. $A^2 - AB + BA - B^2$

(c) A. $AB = BA$

37. A. $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$\det(A) = 1(1-0) - 0(1-0) + 1(1-0)$

$= 1 - 0 + 1$

$= 2$

$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 = R_2 - R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$R_3 = \frac{R_3}{2}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$R_2 = R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$R_1 = -R_3 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

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$$\therefore A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

36. Ans

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(A) = 1(0-0) - 1(1-0) + 1(1-0)$$
$$= 0 - 1 + 1$$
$$= \underline{\underline{0}}$$

$\therefore$ A is not invertible

True- False Exercise

(a) Ans False because B is the inverse of A. So, $B = A^{-1}$ & $A = B^{-1}$

~~Let~~ $\therefore A.B = A.A^{-1} = I$

$\therefore$ The answer is not 0, but the identity matrix I.

(b) Ans: False. This would only be true if AB = BA.

(c) Ans False. It would only be ~~true~~ true if AB = BA.

(d) Ans False. It is $(AB)^{-1} = B^{-1}.A^{-1}$ because inverse of the product is the product ~~$(AB)(B^{-1}A^{-1}) = (B^{-1}A^{-1})(AB) = I$~~ of the inverses in reverse order.

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Exercise 1.5

9. (a) Ans $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$

Using theorem 1.4.5 :- $A^{-1} = \frac{1}{-1}\begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$

Using Inversion algorithm :-

$A^{-1} = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$

$R_2 = -2R_1 + R_2$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right]$

$R_2 = -R_2$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right]$

$R_1 = -4R_2 + R_1$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & 0 & -7 & 4 \\ 0 & 1 & 2 & -1 \end{array}\right]$

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$\therefore A^{-1} = \begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}$

using theorem 1,4,5 :-

$\det(A) = 16 - 16 = 0$

$\therefore$ A is not invertible.

using inversion algorithm:-

$A^{-1} = \left[\begin{array}{cc|cc} 2 & -4 & 1 & 0 \\ -4 & 8 & 0 & 1 \end{array}\right]$

$R_2 = 2R_1 + R_2$

$A^{-1} = \left[\begin{array}{cc|cc} 2 & -4 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right]$

We are getting a row of zeroes here, $\therefore$ A is not invertible

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10 (b) $A = \begin{bmatrix} 1 & -5 \\ 3 & -16 \end{bmatrix}$

Using theorem 1.4.5 :-

$\det(A) = -16 + 15$
$= -1$

$\therefore A^{-1} = \frac{1}{-1} \begin{bmatrix} -16 & 5 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 16 & -5 \\ 3 & -1 \end{bmatrix}$

Using inversion algorithm :-

$A^{-1} = \left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 3 & -16 & 0 & 1 \end{array}\right]$

$R_2 = -3R_1 + R_2$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right]$

$R_2 = -R_2$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & 1 & 3 & -1 \end{array}\right]$

$R_1 = 5R_2 + R_1$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & 0 & 16 & -5 \\ 0 & 1 & 3 & -1 \end{array}\right]$

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$\therefore A^{-1} = \begin{bmatrix} 16 & -5 \\ 3 & -1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}$

Using theorem 1.4.5:-

$\det(A) = -12 - (-12)$
$= -12 + 12$
$= 0$

$\therefore$ A is not invertible.

Using inversion algorithm:-

$A^{-1} = \left[\begin{array}{cc|cc} 6 & 4 & 1 & 0 \\ -3 & -2 & 0 & 1 \end{array}\right]$

$R_1 = R_1/6$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & \frac{4}{6}\frac{2}{3} & \frac{1}{6} & 0 \\ -3 & -2 & 0 & 1 \end{array}\right]$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & \frac{2}{3} & \frac{1}{6} & 0 \\ -3 & -2 & 0 & 1 \end{array}\right]$

$R_2 = 3R_1 + R_2$

$A^{-1} = \left[\begin{array}{cc|cc} 1 & \frac{2}{3} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{2} & 1 \end{array}\right]$

Since we have a row of zeroes here, A is not invertible.

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11.a) Ans. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = -2R_1 + R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = -R_1 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

~~[illegible]~~

$R_3 = 2R_2 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$R_3 = -R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

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$R_1 = -2R_2 + R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$R_2 = 3R_3 + R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$R_1 = -9R_3 + R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

1(b) Ans

$$A = \begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = -R_1$

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$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = -2R_1 + R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = 4R_1 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

$R_2 = R_2/10$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

$R_3 = 10R_2 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

Since we have a row of zeroes, A is not invertible.

12. (A) Ans

$$A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} & 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} & 0 & 1 & 0 \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} & 0 & 0 & 1 \end{array}\right]$$

$R_1 = 5R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} & 0 & 1 & 0 \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} & 0 & 0 & 1 \end{array}\right]$$

$R_2 = -R_3 + R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} & 0 & 0 & 1 \end{array}\right]$$

$R_3 = -\frac{1}{5}R_1 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & -1 & \frac{1}{2} & -1 & 0 & 1 \end{array}\right]$$

$R_3 = R_2 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & \frac{1}{2} & -1 & 1 & 0 \end{array}\right]$$

$R_3 = 2R_3$

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$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$R_1 = -R_2 + R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 5 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$R_1 = 2R_3 + R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$$

Q) A. $A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & -\frac{3}{5} & -\frac{3}{10} \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{ccc|ccc} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} & 1 & 0 & 0 \\ \frac{2}{5} & -\frac{3}{5} & -\frac{3}{10} & 0 & 1 & 0 \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} & 0 & 0 & 1 \end{array}\right]$$

$R_1 = 5R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ \frac{2}{5} & -\frac{3}{5} & -\frac{3}{10} & 0 & 1 & 0 \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} & 0 & 0 & 1 \end{array}\right]$$

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Date: 09/11/2022

$R_2 = \cancel{\ } -\frac{2}{5}R_1 + R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -1 & \frac{1}{2} & -2 & 1 & 0 \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} & 0 & 0 & 1 \end{array}\right]$$

$R_3 = -\frac{1}{5}R_1 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -1 & \frac{1}{2} & -2 & 1 & 0 \\ 0 & -1 & \frac{1}{2} & -1 & 0 & 1 \end{array}\right]$$

$R_2 = -R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 2 & -1 & 0 \\ 0 & -1 & \frac{1}{2} & -1 & 0 & 1 \end{array}\right]$$

$R_3 = R_2 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{array}\right]$$

Since we got a row of zeros here, the given matrix is not invertible.

15.14 $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 2 & 6 & 6 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

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$R_1 = R_1/2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = -2R_1 + R_2$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = -2R_1 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_3 = -R_2 + R_3$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$R_1 = -3R_2 + R_1$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & \frac{7}{2} & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$R_1 = -3R_3 + R_1$

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$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{2} & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{7}{2} & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$