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Exercise 3.1

1.(a) $A(1,5)$ $B(4,1)$

$\overline{AB} = (x_2 - x_1, y_2 - y_1) = (3, -4)$

(b) $A(0,0,4)$ $B(2,3,0)$

$\overline{BA} = (0-2, 0-3, 4-0) = (-2, -3, 4)$

5.(a) $B(x,y)$

$\overline{AB} = (x_2 - x_1, y_2 - y_1) = (x-1, y-1)$

$\overline{AB} = u \Rightarrow (x-1, y-1) = (1,2)$

$x - 1 = 1 \Rightarrow x = 2$

$y - 1 = 2 \Rightarrow y = 3$

$B(x,y) = (2,3)$

(b) $\overline{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$= (-1-x, -1-y, 2-z)$

$u(1,1,3)$

$\therefore$ $-1-x=1 \Rightarrow x=-2$

$-1-y=1 \Rightarrow y=-2$

$2-z=3 \Rightarrow z=-1$

$A(x,y,z)=(-2,-2,-1)$

7. $P(x,y,z)$ $\quad u=\overline{PQ}$

$\overline{PQ}=(3-x,-y,-5-z)=(4,-2,-1)$

$3-x=4 \Rightarrow x=-1$

$-y=-2$

$-5-z=-1 \Rightarrow z=-4$

$\therefore$ $P(-1,2,-4)$

(b) $\overline{PQ}=(3-x,-y,-5-z)$

Since v is in the opp. direction as u,

$\overline{PQ}=-v$

$(3-x,-y,-5-z)=(-4,2,1)$

$3-x=-4$ $\qquad -y=2$

$x=3+4=7$ $\qquad y=-2$

$-5 - z = 1$

$z = -5 - 1 = -6$

$P(7, -2, -6)$

15. (a) $u = (-2, 1, 0, 3, 5, 1)$

Not Parallel

Since the proportional law is not satisfied by observing $4/-2 = -2$ but $2/1 = 2$ which are not equal

(b) Parallel

$4/-2 = -2/1 = -6/3 = -10/5 = -2/1 = -2$

~~Sa~~ All ratios are equal

(c) Parallel

Since null vector is parallel to every vector.

16. (a) $u = (4, -1)$

$(8t, -2) = 2(4t, -1) = 2u$

Using the parallel condition

$4 = 4t$ implies $t = 1$

Also $-1 = 1(-1)$

$\therefore t = 1$

(b) $u = (4,-1)$

Here is t is not possible because

$8t = 4 \Rightarrow t = 1/2$

But when substituting the second ordinate

$2t = 2(1/2) = 1$

(c) ~~ans~~ The value of t is not possible because there is no real value ~~which~~ is ~~equivalent~~ whose square is -1.

17. $u = (1,-1,3,5)$, $v = (2,1,0,-3)$

$au + bv = (1,-4,9,18)$

$= a(1,-1,3,5) + b(2,1,0,-3)$

$= (a,-a,3a,5a) + (2b,b,0,-3b)$

$= (a+2b; -a+b, 3a, 5a-3b) = (1,-4,9,18)$

$a+2b = 1$

$-a+b = 4$

$3a = 9$

$5a \cancel{+} -3b = 18$

$3a = 9 \Rightarrow a = 3$

$\Rightarrow 5(3) - 3b = 18$

$\Rightarrow 3b = 15 - 18$

$b = -1$

$\therefore a = 3, b = -1$

19. $(c_1, -c_1, 0) + (3c_2, 2c_2, c_2) + (0, c_3, 4c_3)$
$= (-1, 1, 19)$

$(c_1 + 3c_2, -c_1 + 2c_2 + c_3, c_2 + 4c_3)$
$= (-1, 1, 19)$

$c_1 + 3c_2 = -1 \quad \ldots (1)$

$-c_1 + 2c_2 + c_3 = 1 \quad \ldots (2)$

$c_2 + 4c_3 = 19 \quad \ldots (3)$

$(1) + (2)$

$c_1 + 3c_2 + (-c_1 + 2c_2 + c_3) = -1 + 1$
$5c_2 + c_3 = 0 \quad \ldots (4)$

$(5c_2 + c_3) - 5(c_2 + 4c_3) = 0 - 95$
$5c_2 + c_3 - 5c_2 - 20c_3 = -95$
$-19c_3 = -95 \Rightarrow c_3 = 5$

$c_3 = 5$ in (4)

$5c_2 + c_3 = 0$
$5c_2 + 5 = 0$
$c_2 = -1$
$c_3 = 5$

Subbing in eq.(2)

$$-c_1+2(-1)+5=1$$

$$3-1=c_1$$

$$c_1=2$$

$$c_1=2,\ c_2=-1,\ c_3=5$$

21. $c_1(-2,9,6)=(-2c_1,9c_1,6c_1)$

$c_2(-3,2,1)=(-3c_2,2c_2,c_2)$

$c_3(1,7,5)=(c_3,7c_3,5c_3)$

$v=(v_1,v_2,v_3)\quad w=(w_1,w_2,w_3)$

$v+w=(v_1+w_1,v_2+w_2,v_3+w_3)$

$c_1(-2,9,6)+c_2(-3,2,1)+c_3(1,7,5)$

$=(-2c_1-3c_2+c_3,\ 9c_1+2c_2+7c_3;\ 6c_1+c_2+5c_3)$

$=(0,5,4)$

$$-2c_1-3c_2+c_3=0$$
$$9c_1+2c_2+7c_3=5$$
$$6c_1+c_2+5c_3=4$$

$$\begin{bmatrix}-2 & -3 & 1 & 0\\ 9 & 2 & 7 & 5\\ 6 & 1 & 5 & 4\end{bmatrix}$$

Multiply the first row by $-1/2$

$$\Rightarrow \begin{bmatrix} 1 & 3/2 & -1/2 & 0 \\ 9 & 2 & 7 & 5 \\ 6 & 1 & 5 & 4 \end{bmatrix}$$

Add 9 times the first row to the second and -6 times the first row to the third.

$$\Rightarrow \begin{bmatrix} 1 & 3/2 & -1/2 & 0 \\ 0 & -23/2 & 23/2 & 5 \\ 0 & -8 & 8 & 4 \end{bmatrix}$$

Multiply the second row by $-2/23$

$$\Rightarrow \begin{bmatrix} 1 & 3/2 & -1/2 & 0 \\ 0 & 1 & -1 & -10/23 \\ 0 & -8 & 8 & \cancel{8}\,4 \end{bmatrix}$$

Add $-3/2$ times the second row to the first and 8 times the second row to the third

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 15/23 \\ 0 & 1 & -1 & -10/23 \\ 0 & 0 & 0 & 12/23 \end{bmatrix}$$

$$0c_1 + 0c_2 + 0c_3 = 12/23$$

Since this equation cannot be satisfied, there is no solution to the system. Therefore there are no such scalars such as c_1, c_2 and c_3

22. $(c_1, 0, c_1, 0) + (c_2, 0, -2c_2, c_2) + (2c_3, 0, c_3, 2c_3) = (1, -2, 2, 3)$

$\Rightarrow (c_1 + c_2 + 2c_3, 0, c_1 - 2c_2 + c_3; c_2 + 2c_3) = (1, -2, 2, 3)$

From the above equality we observe that in the second component $0 = 2$, which is wrong; thus the scalars do not exist in the given sense.