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Date: 09/22/2021

MATH-245

TEST #1

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1. Ans: $X = \{5, 6\}$, $Y = \{a, b\}$, $Z = \{c\}$

(a) $X \times Y = \{(5, a), (5, b), (6, a), (6, b)\}$

(b) $Y \times Y = \{(a, a), (a, b), (b, a), (b, b)\}$

(c) $Y \times Z = \{(a, c), (b, c)\}$

(d) $X \times Y \times Z = \{(5, a, c), (5, b, c), (6, a, c), (6, b, c)\}$

2. Ans: P : You run 10 laps daily
Q : You are healthy
R : You take multi-vitamins.

$\neg P$: You don't run 10 laps daily.
$\neg Q$: You are not healthy
$\neg R$: You don't take multi-vitamins

(a) $P \wedge \neg Q$

Ans: You run 10 laps daily and you are not healthy.

(b) $\neg P \vee \neg R$

Ans: You don't run 10 laps daily or you don't take multi-vitamins.

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(c) $p \to (\neg R \wedge Q)$

Ans. If you run 10 laps daily, then you don't take multi-vitamins and you are not healthy.

4. Ans. (a) Let the statement "The function is differentiable" be p.

And the statement "The function is continuous" be q.

The given conditional statement, i.e. "If a function is differentiable then it is continuous" is if-then statement, which can be written as $p \to q$.

~~To form~~ The converse of the conditional statement, can be written as :- $q \to p$

Converse of the given statement is "If a function is continuous then it is differentiable".

The inverse of the conditional statement is given by $\sim p \to \sim q$

So the inverse of the given statement is "If a function is not differentiable then it is not continuous".

The contrapositive of the conditional statement is given by $\sim q \to \sim p$

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So, the contrapositive of the given statement is "If a function is not continuous then it is not differentiable".

(b) Contrapositive : $\sim q \Rightarrow \sim p$

~~[illegible]~~ $x \leq 2$ and $x \geq -2$ then,

$|x| \leq 2$

Inverse statement :- if $|x| \leq 2$, then

$-2 \leq x \leq 2$

Contrapositive statement :-

if $-2 \leq x \leq 2$ then $|x| \leq 2$

5. Ans. (a) Given statement :- $(p \wedge q) \wedge \sim(p \vee q)$

Truth table :-

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since all the entries of the last column are false, the given statement is a contradiction.

(b) Given statement :- $(p \vee \sim q) \rightarrow (p \wedge q)$

Truth table :-

p	q	$\sim q$	$p \vee \sim q$	$p \wedge q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Since entries of the last column contain both True & False, the given statement is contingency.

3. Ans (a) $3 < x \leq 5$

Negation of the statement will be :-
$3 \geq x > 5$

(b) John will not use the treadmill to lift weights.

(c) You will fix my computer and I will not pay you \$50.

6. Ans (a) If the sky is red at morning it will not rain.

This is a statement of type :- $p \Rightarrow q$

where p is the sky is red at morning & q is It will rain.

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Contrapositive of this is :-
$\sim q \Rightarrow \sim p$

∴ It rained therefore the sky may not red at morning.

∴ It is a valid argument.

(b) It is not a valid argument because $p \Rightarrow q$ is true.

Then $\sim p \Rightarrow \sim q$ may not be true.
where p is you will fail the final & q is you will fail the course.

7. Ans:- (a) It is a valid argument.

Let the statements be A & B. If any of the two get satisfied, the result can be the same.

∴ The argument is correct.

(b) It is a valid argument.

The first statement says that the event of Fred's singing can only happen if the cat doesn't bark.

And the second statement follows the first.

∴ If the cat won't bark, Fred will sing. ∴ Correct!

(5)

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8. (a) Ans Statement is true.

Negation of given statement is :-

$\exists x \forall y (3x + y \neq 4)$.

(b) Statement is true.

Negation of given statement is :-
$\forall y \forall x (x - y \neq 2$ or $xy \neq 36)$

(c) The statement is false.

The negation of the given statement is :-
$\exists x (x^2 = 1$ and $x^3 \neq 1)$.

9. Ans. (a) Some dogs are not friendly.
(b) No estimate is accurate.
(c) All discrete mathematics students are athletes.

10. Ans In domain of all people,
$P(x) =$ x is a professional athlete.
$Q(x) =$ x plays soccer.

(a) $\forall x (Q(x))$

=> Every person plays soccer.

(b) $\exists x (\neg P(x))$

There are some people who are not professional athletes.

(c)

(c) $\exists x (P(x) \land \neg Q(x))$

There are some people who are professional athletes but do not play soccer.

(d) $\forall x (Q(x) \to P(x))$

Every person who plays soccer is a professional athlete.