Name :- Shreyas Srinivasa

Palomar ID :- 012551187

Chapter 6.1

1) (a) (i) Let A and B be the two sets. A set A is said to be subset of B if it satisfies conditional statement,

$A \subseteq B \Leftrightarrow \forall x$, if $x \in A$ then $x \in B$

(ii) Let A and B be the two sets. A set A is said to be proper subset of B if it satisfies
(1) $A \subset B$, and
(2) There is at least one element in B that is not in A.

$A = \{2, \{2\}, 2\} = \{2, \{2\}\}$

Hence, A is proper subset of B

(b) $A = \{3, \sqrt{5^2 - 4^2}, 24 \bmod 7\}$

$A = \{3, \sqrt{9}, 3\} = \{3\}$

$B = \{8 \bmod 5\} = \{3\}$

Therefore the sets are $A = \{3\}$ and $B = \{3\}$

Hence $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$

Chapter 6.1

1) (a)(i) Let A and B be the two sets. A set A is said to be subset of B if it satisfies conditional statement,

$A \subseteq B \iff \forall x$, if $x \in A$ then $x \in B$

(ii) Let A and B be the two sets. A set A is said to be proper subset of B if it satisfies

(1) $A \subset B$, and

(2) There is at least one element in B that is not in A.

$A = \{2, \{2\}, 2\} = \{2, \{2\}\}$

Hence, A is proper subset of B

(b) $A = \{3, \sqrt{5^2 - 4^2}, 24 \bmod 7\}$

$A = \{3, \sqrt{9}, 3\} = \{3\}$

$B = \{8 \bmod 5\} = \{3\}$

Therefore the sets are $A = \{3\}$ and $B = \{3\}$

Hence $A = B \iff A \subseteq B$ and $B \subseteq A$

(c) The set A is not a subset of the set B since $\{1,2\} \in A$ but $\{1,2\} \notin B$, similarly the set B is not a subset of the set A since $2 \in B$ but $2 \notin A$. $A \not\subseteq B$ and $B \not\subseteq A$

(d) As the elements x and $\{x\}$ are distinct ~~for~~ for all $x \in A$ so the set A is not a subset of B. Similar argument for, the set B is not a subset of A. $A \not\subseteq B$ and $B \not\subseteq A$

(e) $A = \{4, \{4\}\}$

The set B is a subset of A

The element $\{4\}$ in the set A is not in the set B, so the set B is a proper subset of A.

(f) $A = \{x \in R \mid \cos x \in Z\}$

$= \{n\pi, (2n+1)\pi/2 \mid n \in Z\}$

Using $\cos(n\pi) = (-1)^n$ and $\cos\left(\frac{(2n+1)\pi}{2}\right) = 0$ for all $n \in Z$

$= \{0, \pi/2, \pi, \frac{3\pi}{2}, 2\pi, \dots\}$

$B = \{x \in \mathbb{R} \mid \sin x \in \mathbb{Z}\}$

$= \{n\pi, (4n-1)\pi/2, (4n+1)\pi/2 \mid n \in \mathbb{Z}\}$

Since $\sin(n\pi) = 0$, $\sin\left(\frac{(4n+1)\pi}{2}\right) = 1$

and $\sin\left(\frac{(4n-1)\pi}{2}\right) = -1$ for $n \in \mathbb{Z}$

$= \{-\pi/2, 0, \pi/2, \pi, 3\pi/2, 2\pi, \dots\}$

As the sets are equal so each element in the set A is contained in the set B and vice versa.

Hence. $A = B \iff A \subseteq B$ and $B \subseteq A$

10) (a) $A \cup B = \{1, 3, 5, 6, 7, 9\}$

(b) $A \cap B = \{3, 9\}$

(c) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(d) $A \cap C = \emptyset$

(e) $A - B = \{1, 5, 7\}$

(f) $B - A = \{6\}$

(g) $B \cup C = \{2, 3, 4, 6, 8, 9\}$

(h) $B \cap C = \{6\}$

27) (a) $A_1 \cup A_2 \cup A_3 = \{a, d, e\} \cup \{b, c\} \cup \{d, f\}$

$$= \{a, d, e, b, c\} \cup \{d, f\}$$

$$= \{a, d, e, b, c, f\} = A$$

$$A_1 \cap A_2 = \{a, d, e\} \cap \{b, c\} = \emptyset$$

$$A_1 \cap A_3 = \{a, d, e\} \cap \{d, f\} = \{d\}$$

$$A_2 \cap A_3 = \{b, c\} \cap \{d, f\} = \emptyset$$

The sets A_1, A_2, A_3 are not mutually disjoint because d is element in both A_1 and A_3. The set $\{A_1, A_2, A_3\}$ is not a partition of A. Therefore, the set $\{\{a, d, e\}, \{b, c\}, \{d, f\}\}$ is not a partition of the set $\{a, b, c, d, e, f\}$.

(b) $A_1 \cup A_2 \cup A_3 = \{w, x, v\} \cup \{u, y, q\} \cup \{p, z\}$

$$= \{w, x, v, u, y, q\} \cup \{p, z\}$$

$$= \{w, x, v, u, y, q, p, z\} = A$$

$$A_1 \cap A_2 = \emptyset$$

$$A_1 \cap A_3 = \emptyset$$

$$A_2 \cap A_3 = \emptyset$$

The sets A_1, A_2, A_3 are mutually disjoint sets. The set $\{A_1, A_2, A_3\}$ is a partition of A. Therefore the set $\{\{w, x, v\}, \{u, y, q\}, \{p, z\}\}$ is a partition of $\{p, q, u, v, w, x, y, z\}$

(c) $A_1 \cup A_2 \cup A_3 \cup A_4 = \{5,4\} \cup \{7,2\} \cup \{1,3,4\} \cup \{6,8\} = \{1,2,3,4,5,6,7,8\}$
$= A$

$A_1 \cap A_2 = \emptyset$
$A_1 \cap A_3 = 4$
$A_2 \cap A_3 = \emptyset$

The sets A_1, A_2, A_3 and A_4 are mutually disjoint because 4 is the element in both A_1 and A_3. By the definition of the partition of the sets $\{A_1, A_2, A_3, A_4\}$ is not a partition of A. Therefore, the set $\{\{5,4\},\{7,2\},\{1,3,4\},\{6,8\}\}$ is not a partition of $\{1,2,3,4,5,6,7,8\}$

(d) $A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,7,8,9\} \neq A$ since $6 \notin \{1,2,3,4,5,7,8,9\}$

The set $\{A_1, A_2, A_3\}$ is not a partition of A. Therefore, the set $\{\{3,7,8\},\{2,9\},\{1,4,5\}\}$ is a partition of $\{1,2,3,4,5,6,7,8,9\}$

(e) $A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,6,7,8\} = A$

$A_1 \cap A_2 = \emptyset \quad A_1 \cap A_3 = \emptyset \quad A_2 \cap A_3 = \emptyset$

The sets A_1, A_2, A_3 are mutually disjoint.
The set $\{A_1, A_2, A_3\}$ is a partition of A.

Therefore, the set $\{\{1,5\}, \{4,7\}, \{2,8,6,3\}\}$ is a partition of $\{1,2,3,4,5,6,7,8\}$

28) Zero is even because 2 divides 0.

Furthermore, every integer is either even or odd.

No even integer is odd and vice versa.

$\therefore$ $\{E, O\}$ make a partition over $\mathbb{Z}$.

30) All integers are exactly in one of the following forms $4k$ or $4k+1$ or $4k+2$ or $4k+3$ for some $k \in \mathbb{Z}$ by the quotient-remainder theorem

Thus observe the following.

$\mathbb{Z} = A_0 \cup A_1 \cup A_2 \cup A_3$

$A_0 \cap A_1 = \emptyset$; $A_1 \cap A_2 = \emptyset$; $A_2 \cap A_3 = \emptyset$;

$A_0 \cap A_2 = \emptyset$; $A_0 \cap A_3 = 0$; $A_1 \cap A_3 = 0$

Thus, $\{A_0, A_1, A_2, A_3\}$ forms a partion of $\mathbb{Z}$

31) (a) $P(A \cap B) = \{\emptyset, \{2\}\}$

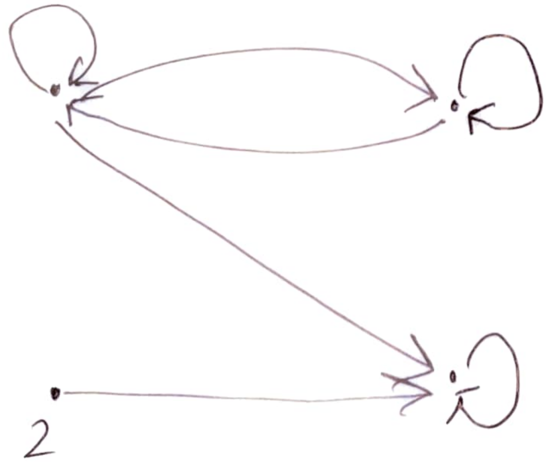
(b) $P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

(c) $P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

(d) $P(A \times B) = \{ \emptyset, \{(1,2)\}, \{(1,3)\}, \{(2,2)\}, \{(2,3)\},$
$\{(1,2),(1,3)\}, \{(1,2),(2,2)\},$
$\{(1,2),(2,3)\}, \{(1,3)\}, (2,2)\},$
$\{(1,3),(2,3)\}, \{(2,2),(2,3)\},$
$\{(1,2),(1,3),(2,2)\}, \{(1,2),(1,3),$
$(2,3)\}, \{(1,2),(2,2),(2,3)\},$
$\{(1,3),(2,2),(2,3)\}, \{(1,2),$
$(1,3),(2,2),(2,3)\} \}$

Chapter 8.2

(a)



(b) R_1 is not reflexive

Because there is no loop from 2 to ~~3~~ 2, that means $2 \in A$, but $(2,2) \notin R_1$ so R_1 is not reflexive.

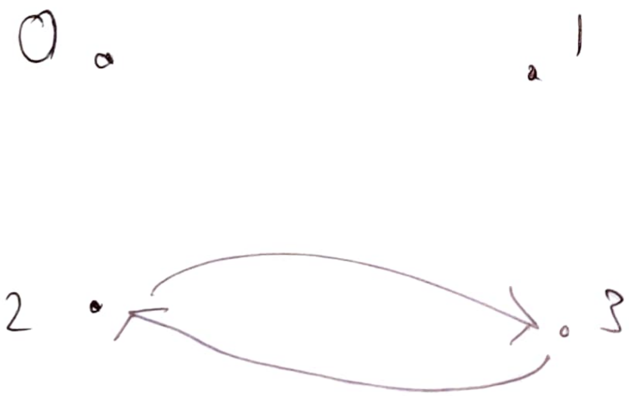
(c) R_1 is not symmetric

Because there is no anti arrow from 3 to 0 thus, $(0,3) \in R_1$, but $(3,0) \notin R_1$ similarly for $(2,3)$, so R_1 is not transitive

(d) R_1 is not transitive

Since there is an arrow from 1 to 0 and 0 to 3, but there is no arrow from 1 to 3 so R_1 is not transitive.

3) Directed graph of R_3
(a)



(b) R_3 is not reflexive

Since there is no loop around 2 and 3, also for that if $2 \in A$ but $(2,2) \in R_3$ and $3 \in A$ but $(3,3) \notin R_3$

So R_3 is not reflexive

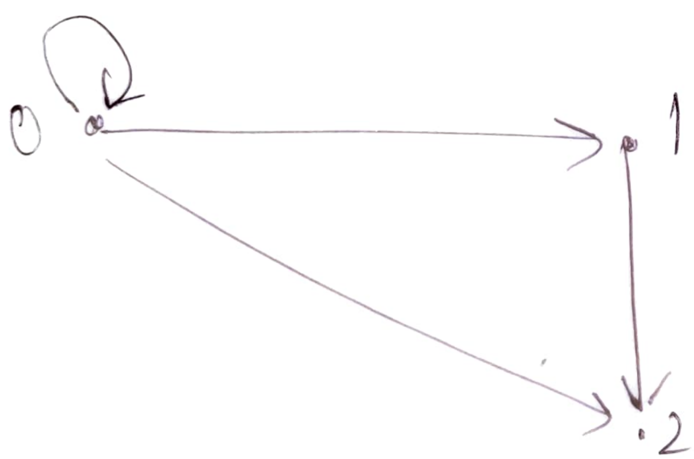
(c) R_3 is symmetric

Since there is ~~oo~~ an arrow from 2 to 3 and an arrow from 3 to 2 again.

(d) R_3 is not transitive

Since $(2,3) \in R_3$ and $(3,2) \in R_3$ but $(2,2) \notin R_3$

5) (a) Directed graph for R_3



(b) R_3 is not reflexive

Since there are no loops along 1 and 2 itself. That is if $1 \in A$, but $(1,1) \notin R_5$ similarly $2 \in A$, but $(2,2) \notin R_3$

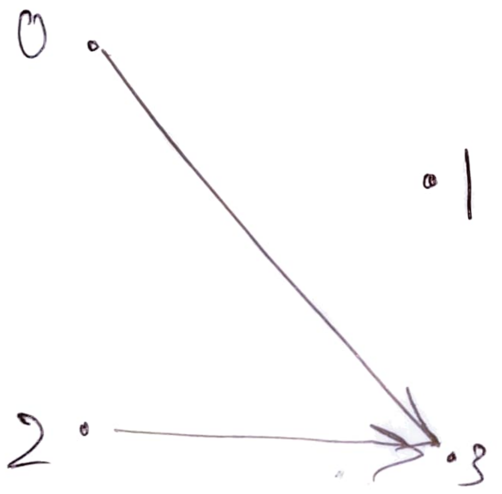
(c) R_5 is not symmetric

Because $(0,1) \in R_5$, but $(1,0) \notin R_5$, similarly for remains also. There are no anti-loops for any elements.

(d) R_5 is transitive

Since $(0,1) \in R_5$ and $(1,2) \in R_5 \Rightarrow (0,2) \in R_5$

7) (a)



(b) For x in a set A, $(x,x) \in R$ then R is called reflexive.

Here $(0,0) \notin R_7$, $(1,1) \notin R_7$, $(2,2) \notin R_7$ and $(3,3) \notin R_7$

Hence R_7 is not reflexive

(c) For all x and y in set A, if $(x,y) \in R$ then $(y,x) \in R$ then R is called symmetric

In this relation R_7, $(0,3) \in R_7$, but $(3,0) \notin R_7$

Hence R_7 is symmetric

(d) If R_7 were not transitive,

That is, there would have to be elements x, y and z in a set A such that $(x,y) \in R_7$, $(y,z) \in R_7$ and $(x,z) \notin R_7$

So it is clear by hypothesis no such elements exist in R_7. It follows that it is impossible for R_7 not to be transitive.

Thus R_7 is transitive.

9) · R is reflexive:

R is reflexive $\Leftrightarrow$ for all real numbers x, xRx.

By the definition of R, this means that for all real numbers, x, $x \geq x \Rightarrow x > x$ or $x = x$. This is true, so R is reflexive.

R is not Symmetric

R is Symmetric $\Leftrightarrow$ for all real numbers x and y, if $xRy \Leftrightarrow yRx$ by the definition of R. For all real numbers x and y., if $x \geq y$, then $y \geq x$, but this is false

R is transitive

By the definition of R for all ~~real~~ real nos. x, y and z, if $x \geq y$ and $y \geq z$, then $x \geq z$

This is true by the definition of $\geq$ and transitive property of order for the real numbers.

11) For all real numbers x, $x \cdot x = x^2 \geq 0$, which is true since by basic algebra, the square of any number is positive.

Thus, for all real numbers, x, xDx

Therefore, the relation D is reflexive

Suppose $xDy \Rightarrow xy \geq 0$

If $xy \geq 0$, then $yx \geq 0$ for all $x, y \in R$, which implies that yDx.

Thus if xDy then yDx.

Therefore, the relation D is Symmetric.

For all real numbers, x, y, z if $xy \geq 0$ and $yz \geq 0$, then $xz \ngeq 0$

Hence, the relation D is transitive.

13) F is reflexive since for $m \in \mathbb{Z} \Rightarrow 5 | m - m$

$\Rightarrow 5 | 0$, which is true. This implies for

$m \in \mathbb{Z} \Rightarrow m F m$

F is symmetric

If $m, n \in \mathbb{Z}$, then $5 | m - n \Rightarrow m - n = 5k$

where k is an integer.

$\Rightarrow n - m = -5k$

$\Rightarrow n - m = (-k) \cdot 5$ when $(-k)$ is an integer.

$\Rightarrow 5 | n - m$

$5 | m - n \Rightarrow 5 | n - m$

This shows that $mFn \Rightarrow nFm$,

F is transitive

If $5 | n - p \Rightarrow n - p = 5L$

Now consider $m - p = m - n + n - p$

$= (m - n) - (n - p)$

$= 5k - 5L = 5(k - L) = 5T$ where

$12 - L \supset T$ at length $\Rightarrow$ $5/m-p$ where it shows that mFn and nFp, then mFp for $m, n, p \in \mathbb{Z}$

15) For all +ve integers m, m/m, which is true, so D is reflexive.

D is not symmetric

$m|n$ but ~~m~~ $n \nmid m$. Let $m=2$ and $n=4$

then m/n because $2/4$ but $n \nmid m$ because $4/2$

D is transitive

If $m|n$ and n/P then m/P which is true by the divisibility rule.

17) $\exists$ a prime p such that p/m and p/n

But this is false for $n=1$

There is no prime number that divides 1.

Therefore, p is not reflexive.

Suppose mPn

By definition $P \exists p$ such that p/m and p/n.

This is logically equivalent to $p|n$ and p/m

Hence there exists a prime number P such that $p|n$ and $p|m$.

Therefore P is Symmmetric.

P is transitive

Suppose lPm and mPn.

Then need to show that lPn.

Take $l=2$, $m=6$. and $n=9$

Here the prime number 2 divides both 2 and 6 and prime number 3 divides 6 and 9. But the numbers 2 and 9 have no common prime factors.

That is $l \not P n$.

Therefore P is not transitive

Chapter 8.3

2. (a) Since $\{0,2\}$ is a subset of the partition.

$0 R 0$ since both 0 and 0 are in $\{0,2\}$

$0 R 2$ since both 0 and 2 are in $\{0,2\}$

$2 R 2$ since both 2 and 2 are in $\{0,2\}$

$2 R 0$ since both 2 and 0 are in $\{2,0\}$

Since $\{1\}$ is a subset of the partition.

$1 R 1$ since 1 and 1 are in $\{1\}$

Since $\{3,4\}$ is a partition 0 subset.

$3 R 3$ since both 3 and 3 are in $\{3,4\}$.

$3 R 4$ since both 3 and 4 are in $\{3,4\}$.

$4 R 4$ since both 4 and 4 are in $\{3,4\}$.

$4 R 3$ since both 4 and 3 are in $\{3,4\}$.

$\therefore R = \{(0,0)\ (0,2)\ (2,2)\ (2,0)\ (1,1)\ (3,3).$
$(3,4)\ (4,4)\ (4,3)\}$

(b) $R = \{(0,0)\ (1,1)\ (1,3)\ (1,4)\ (3,3)\ (3,1)\ (3,4)$
$(4,1)\ (4,3)\ (4,4)\ (2,2)\}$

Since $\{0\}$ is a subset of the partition

$0 R 0$ since 0 and 0 are in $\{0\}$

$\{1,3,4\}$ is a partition's subset.

$1 R 1$ since 1 and 1 are in $\{1,3,4\}$

$1 R 3$ since 1 and 3 are in $\{1,3,4\}$

$1 R 4$ since 1 and 4 are in $\{1,3,4\}$

Similarly, $3 R 3$, $3 R 1$ $R 3$ $R 4$

and $4 R 1$, $4 R 3$ and $4 R 4$ are belongs to

R{2}, which is a subset of partition as 2 R 2 since 2.

(c) R = { (0,0) (1,1) (1,2) (1,3) (1,4) (2,1) (2,2)
(2,3) (2,4) (3,1) (3,2) (3,3) (3,4)
(4,1) (4,2) (4,3) (4,4) }

Because {0} is a partitions subset 0 R 0
since 0 and 0 are in {0}

Since {1,2,3,4} is a subset of partition

1 R 1 since 1 and 1 are in {1,2,3,4}
1 R 2 since 1 and 2 are in {1,2,3,4}
1 R 3 since 1 and 3 are in {1,2,3,4}
1 R 4 since 1 and 4 are in {1,2,3,4}

3) $[0] = \{x \in A / x R 0\} = \{0,4\}$
$[1] = \{x \in A / x R 1\} = \{1,3\}$
$[2] = \{x \in A / x R 2\} = \{2\}$
$[3] = \{x \in A / x R 3\} = \{1,3\}$
$[4] = \{x \in A / x R 4\} = \{0,4\}$
Here $[0] = [4] = \{0,4\}$; $[1] = [3] = \{1,3\}$
and $[2] = \{2\}$

Therefore the classes are

$\{0,4\}, \{1,3\}, \{2\}$

4) $[a] = \{x \in A / x R a\} = \{a\}$

$[b] = \{x \in A / x R b\} = \{b, d\}$

$[c] = \{x \in A / x R c\} = \{c\}$

$[d] = \{x \in A / x R d\} = \{b, d\}$

$[b] = [d] = \{b, d\}$

$\{a\}, \{b, d\}$. and $\{c\}$

5) $[1] = \{x \in A \mid x R 1\}$

$= \{x \in A \mid 4 \mid x - 1\}$

$= \{1, 5, 9, 13, 17\}$

$[2] = \{x \in A \mid x R 2\}$

$= \{x \in A \mid 4 \mid x - 2\}$

$= \{2, 6, 10, 14, 18\}$

$[3] = \{x \in A \mid x R 3\}$

$= \{x \in A \mid 4 \mid x - 3\}$

$= \{3, 7, 11, 15, 19\}$

$[4] = \{x \in A \mid x R 4\} = \{x \in A \mid 4 \mid x - 4\}$

a) $= \{4, 8, 12, 16, 20\}$

$\{1, 5, 9, 13, 17\}, \{2, 6, 10, 14, 18\}, \{3, 7, 11, 15, 19\}, \{4, 8, 12, 16, 20\}$

b) $[a] = \{x \in A \mid xRa\}$

$= \{x \in A \mid 3 \mid x - a\}$

$= \{x \in A \mid x = 3k + a$, for some integer $k\}$

$[1] = \{x \in A : xR1\}$

$= \{x \in A : 3 \mid x - 1\}$

$= \{x \in A : x - 1 = 3k$, for some integer $k\}$

$= \{x \in A : x = 3k + 1$, for some integer $k\}$

The equivalence class is $\{-2, 1, 4\}$

$[2] = \{x \in A : xR2\}$

$= \{x \in A : 3 \mid x - 2\}$

$= \{x \in A : x - 2 = 3k$, for some int $k\}$

$= \{x \in A : x = 3k + 2$, for some int $k\}$

The equivalence class is $\{-4, -1, 2, 5\}$

$[3] = \{x \in A : x R 3\}$

$= \{x \in A : 3 | x-3\}$

$= \{x \in A : x-3 = 3k$, for some int $k\}$

$= \{x \in A : x = 3k+3$, for some int $k\}$

The equivalence class is $\{-3, 0, 3\}$

7) This implies that $ad = bc$

In particular,

$[(1,3)] = \{(a,b) \in A : (a,b) R (1,3)\}$

$= \{(a,b) \in A : 3a = b\}$

As, $(1,3) R (3,9)$

The equivalence class is $\{(1,3), (3,9)\}$

$[(2,4)] = \{(a,b) \in A : (a,b) R (2,4)\}$

$= \{(a,b) \in A : 4a = 2b\}$

As,

$(2,4) R (-4,-8)$

This implies that

$2 \times (-8) = 4 \times (-4)$

$-16 = -16$

And

$(2,4) R (3,6)$

$2 \times 6 = 4 \times 3$

$12 = 12$

The equivalence class is $\{(2,4),(-4,-8),(3,6)\}$

$$[(1,5)] = \{(a,b) \in A : (a,b) \, R \, (1,5)\}$$
$$= \{(a,b) \in A : 5a = b\}$$

As,

$(1,5) \, R \, (1,5)$

$1 \times 5 = 5 \times 1$

The equivalence class is

$\{(1,5)\}$

The classes are

$\{(1,3)\,(3,9)\}, \{(2,4)(-4,-8).(3,6)\}, \{(1,5)\}$

10) For $n = -5$

$$[-5] = \{m \in A : m R (-5)\}$$
$$= \{m \in A : 3 \mid m^2 - (-5)^2\}$$
$$= \{m \in A : 3 \mid m^2 - 25\}$$

The equivalence class is $\{-5,-4,-2,-1,1,2,4,5\}$

For $n = -4$

$$[-4] = \{m \in A : m R (-4)\}$$
$$= \{m \in A : 3 \mid m^2 - (-4)^2\}$$
$$= \{m \in A : 3 \mid m^2 - 16\}$$

The equivalence class is $\{-5,-4,-2,-1,1,2,4,5\}$

$[-3] = \{m \in A : m R (-3)\}$

$= \{m \in A : 3 \mid m^2 - (-3)^2\}$

$= \{m \in A : 3 \mid m^2 - 9\}$

The equivalence class is $\{-3,0,3\}$

For $n = -2$

$[-2] = \{m \in A, m R (-2)\}$

$= \{m \in A : 3 \mid m^2 - (-2)^2\}$

$= \{m \in A : 3 \mid m^2 - 4\}$

The equivalence class is $\{-5,-4,-2,-1,1,2,4,5\}$

For $n = 0$

$[0] = \{m \in A : m R 0\}$

$= \{m \in A : 3 \mid m^2 - 0^2\}$

$= \{m \in A : 3 \mid m^2\}$

The equivalence class is $\{-3,0,3\}$

The distinct classes of R are:

$\{-5,-4,-2,-1,1,2,4,5\}, \{-3,0,3\}$

11) For $n = -4$,

$[-4] = \{m \in A : mR(-4)\}$

$= \{m \in A : 4 \mid m^2 - (-4)^2\}$

$= \{m \in A : 4 \mid m^2 - 16\}$

The equivalence class is $\{-4, -2, 0, 2, 4\}$

For $n = -3$,

$[-3] = \{m \in A : mR(-3)\}$

$= \{m \in A : 4 \mid m^2 - (-3)^2\}$

$= \{m \in A : 4 \mid m^2 - 9\}$

The equivalence class is $\{-3, -1, 1, 3\}$

For $n = -2$,

$[-2] = \{m \in A : mR(-2)\}$

$= \{m \in A : 4 \mid m^2 - (-2)^2\}$

$= \{m \in A : 4 \mid m^2 - 4\}$

The equivalence class is $\{-4, -2, 0, 2, 4\}$

For $n = -1$,

$[-1] = \{m \in A : mR(-1)\}$

$= \{m \in A : 4 \mid m^2 - (-1)^2\}$

$= \{m \in A : 4 \mid m^2 - 1\}$

The equivalence class is $\{-3,-1,1,3\}$

For $n=0$

$[-1] = \{m \in A : mR0\}$

$= \{m \in A : 4 | m^2 - 0^2\}$

$= \{m \in A : 4 | m^2\}$

The equivalence class is $\{-4,-2,0,2,4\}$

The distinct classes are

$\{-4,-2,0,2,4\}, \{-3,-1,1,3\}$

12) For $n=-4$

$[-4] = \{m \in A : mR(-4)\}$

$= \{m \in A : 5 | m^2 - (-4)^2\}$

$= \{m \in A : 5 | m^2 - 16\}$

The equivalence class is $\{-4,-1,1,4\}$

For $n=-3$,

$[-3] = \{m \in A : mR(-3)\}$

$= \{m \in A : 5 | m^2 - (-3)^2\}$

$= \{m \in A : 5 | m^2 - 9\}$

The equivalence class is $\{-3,-2,2,3\}$

For $n = -2$

$$[-2] = \{m \in A : m R (-2)\}$$
$$= \{m \in A : 5 \mid m^2 - (-2)^2\}$$
$$= \{m \in A : 5 \mid m^2 - 4\}$$

The equivalence class is $\{-3, -2, 2, 3\}$

For $n = -1$

$$[-1] = \{m \in A : m R (-1)\}$$
$$= \{m \in A : 5 \mid m^2 - (-1)^2\}$$
$$= \{m \in A : 5 \mid m^2 - 1\}$$

The equivalence class is $\{-4, -1, 1, 4\}$

For $n = 0$,

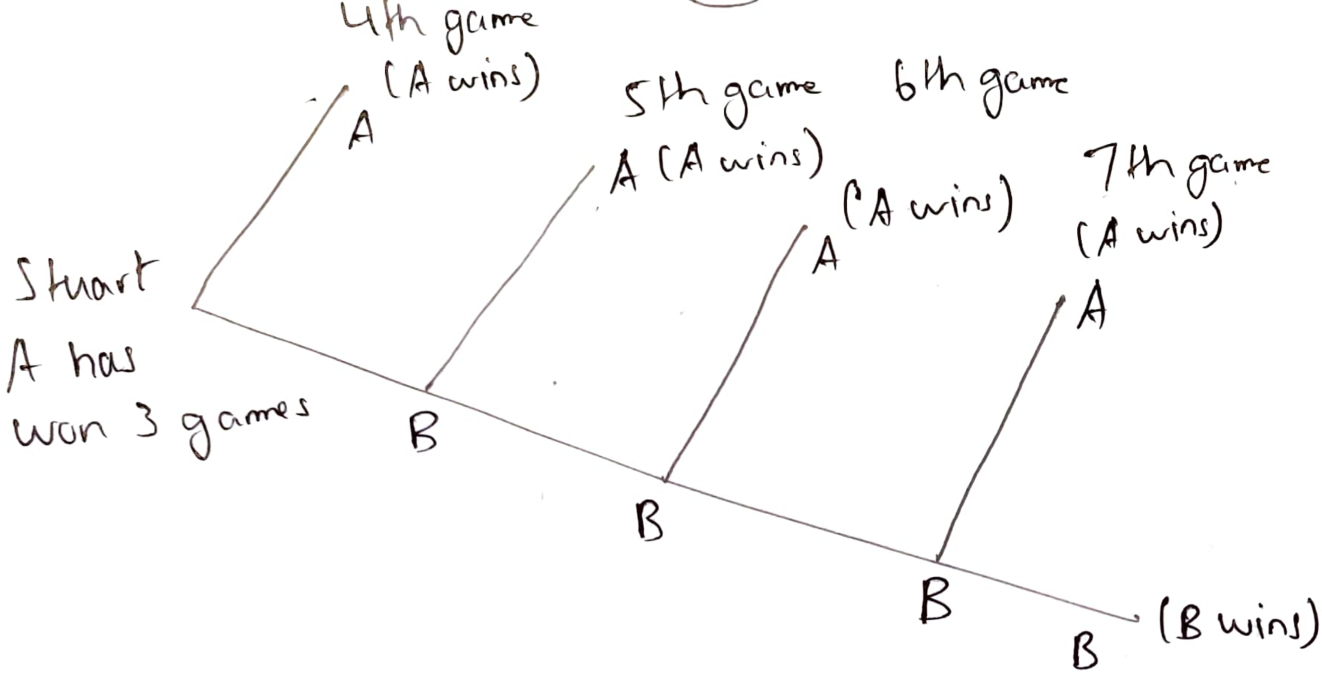
$$[0] = \{m \in A : m R 0\}$$
$$= \{m \in A : 5 \mid m^2 - 0^2\}$$
$$= \{m \in A : 5 \mid m^2\}$$

The equivalence class is $\{0\}$

The distinct classes are

$\{-4, 1, 1, 4\}, \{-3, -2, 2, 3\}, \{0\}$

1)



3) 1) A - A - A - A

2) B - A - A - A - A

3) B - B - A - A - A - A

4) B - B - B - A - A - A - A

Thus there are 4 ways in which A can win the game

8) The total number of ways in which the personal computer system can be purchased is $C(3,1) \times C(3,2) \times C(2,1) \times C(2,1)$

$$C(3,1) = \frac{3!}{2! \times 1!} = 3$$

$$C(2,1) = \frac{2!}{(2-1)! \times 1!} = 2$$

$\therefore$ The number of distinct systems purchased is $3 \times 2 \times 2 = 12$

9) (a) $C(5,1) \times C(3,1) = 5 \times 3 = 15$

(b) $C(3,1) \times C(5,1) \times C(5,1) \times C(3,1)$

$= 3 \times 5 \times 5 \times 3 = 225$

(c) $C(3,1) \times C(5,1) \times C(4,1) \times C(2,1) =$

$3 \times 5 \times 4 \times 2 = 120$

14) (a) $26 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 456,976,000$

(b) $1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1 = 1,757,600$

(c) $1 \times 10 \times 10 \times 10 = 1000$

(d) $26 \times 25 \times 24 \times 23 \times 10 \times 9 \times 8 = 258,336,000$

(e) $1 \times 1 \times 24 \times 23 \times 10 \times 9 \times 8 = 397,440$

32) (a) $P(9,9) = \dfrac{9!}{(9-9)!} = 9!/0! = 362,880$

(b) $8 \times 7! = 8! = 40,320$

(c) $7 \times 6! = 7! = 5040$

35) $P(4,2) = 4!/2! = \dfrac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$

$= \{WX, XW, WY, YW, WZ, ZW, XY, YZ, XZ, ZX, YX, ZY\}$

37) (a) $P(6,4) = \frac{6!}{(6-4)!} = 6!/2! = \frac{6\times5\times4\times3\times2\times1}{2\times1}$

$= 6\times5\times4\times3 = 360$

(b) $P(6,6) = \frac{6!}{(6-6)!} = 6!/0! = 720$

(c) $P(6,3) = \frac{6!}{(6-3)!} = 6!/3! = \frac{6\times5\times4\times3\times2\times1}{3\times2\times1}$

$= 6\times5\times4 = 120$

(d) $P(6,1) = \frac{6!}{(6-1)!} = 6!/5! =$

$\frac{6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1} = 6$

39) (a) $P(9,3) = \frac{9!}{(9-3)!} = 9!/6! = 504$

(b) $P(9,6) = \frac{9!}{(9-6)!} = 9!/3!$

$= 9\cdot8\cdot7\cdot6\cdot5\cdot4$

$= 60480$

(c) $P(8,5) = \dfrac{8!}{(8-5)!} = 8!/3! = 8\cdot7\cdot6\cdot5\cdot4$

$= 6720$

(d) $P(7,4) = \dfrac{7!}{(7-4)!} = 7!/3! = 7\cdot6\cdot5\cdot4$

$= 840$

40) $P(n+1,3) = \dfrac{(n+1)!}{(n+1-3)!} = \dfrac{(n+1)!}{(n-2)!}$

$= \dfrac{(n+1)(n)(n-1)(n-2)!}{(n-2)!} = (n+1)(n)(n-1)$

$= n(n^2-1) = n^3-n$

$P(n+1,3) = n^3-n$ for $n \geq 2$

41) $P(n+1,2) = \dfrac{(n+1)!}{(n+1-2)!} = \dfrac{(n+1)!}{(n-1)!}$

$= \dfrac{(n+1)(n)(n-1)!}{(n-1)!} = (n+1)n = n^2+n$

$P(n,2) = \dfrac{n!}{(n-2)!} = \dfrac{n(n-1)(n-2)!}{(n-2)!}$

$= n(n-1)$

$P(n+1,2) - P(n,2) = (n^2+n) - (n^2-n)$

$= n^2+n - n^2+n = 2n = 2P(n,1)$

Chapter 9.3

3) (a) The number of integers from 1 to 9 with no repeated digits is = 9

From 10 to 99 is $9 \cdot 9$

From 100 to 999 is $9 \cdot 9 \cdot 8$

Hence the numbers of integers from 1 through 999 with no repeated digits:

$9 + 9 \cdot 9 + 9 \cdot 9 \cdot 8 = 9 + 81 + 648 = 738$

(b) Number of integers from 1 through 999 with at least one repeated digits =
[total no. of integers from 1 through 999] -
[no. of integer with no repeated digits]
$= 999 - 738 = 261$

(c) $n(S) = 999 \quad n(E) = 261$

$P(E) = \dfrac{261}{999} = 0.261261$

6) (a) Case 1 :- There are 4 symbols

36^4 ways.

Case 2 :- There are 5 symbols

36^5 ways.

Case 3 :- There are 6 symbols

36^6 ways.

Total possible ways is equal to

$36^4 + 36^5 + 36^6 = 2,239,928,128$

(b) Case 1 :-

$36 \times 35 \times 34 \times 33 = 1413720$ ways

Case 2 :-

$36 \times 35 \times 34 \times 33 \times 32 = 45239040$ ways.

Case 3 :-

$36 \times 35 \times 34 \times 33 \times 32 \times 31 = 1402410240$ ways.

Total possible ways $= 1413720 + 45239040 + 1402410240 = 1449063000$

(c) The total no. of license plates if repitition is allowed is 2238928128.

The total license plates if repitition is not allowed is 1449063000

The total number of license plates is

$2238928128 - 1449063000 = 789865128.$

(d) $\text{Probability} = \dfrac{789865128}{2238928128} = 0.35278$

11) (a) $P(5,5) = 5! = 120$

(b) There are 4 ways in which 2 places can be blocked for Q and U.

Now in the remaining 3 places, the remaining 3 letters can be filled in $P(3,3)$ ways

$4 \times P(3,3) = 24$

(c) $4 \times P(2,2) \times P(3,3) = 4 \times 2 \times 6 = 48$

33) Let H be the set of students who checked Statement 1, C be the set of students who checked Statement 2 and D be the set of students who checked statement 3.

$N(H) = 28 \quad N(C) = 26 \quad N(D) = 14$

$N(H \cap C) = 8 \quad N(C \cap D) = 3 \quad N(H \cap D) = 4$

$N(H \cap C \cap D) = 2$

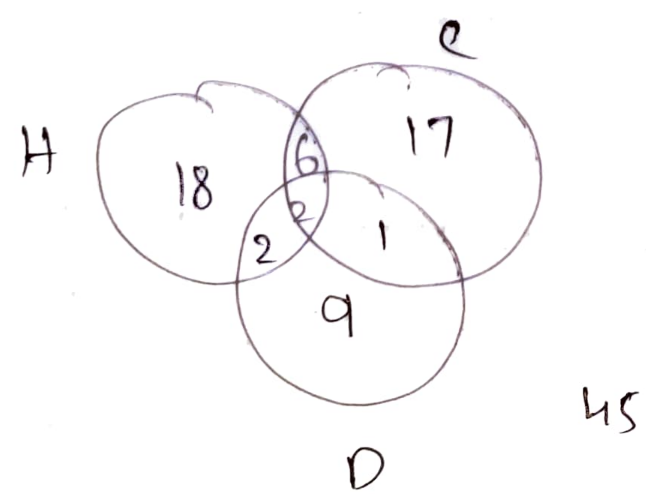
(a) $N(H \cup C \cup D) = N(H) + N(C) + N(D) - N(H \cap C) - N(C \cap D) - N(H \cap D) + N(H \cap C \cap D)$

$= 28 + 26 + 14 - 8 - 3 - 4 + 2 = 55$

(b) $100 - N(H \cup C \cup D) = 100 - 55$.

$100 - N(H \cup C \cup D) = 45$

(c)



(d) $N(H \cap C) - N(H \cap C \cap D) = 8 - 2 = 6$

(e) $N(C \cap D) - N(H \cap C \cap D) = 3 - 2 = 1$

(f) $N(H \cup D) = N(H) + N(D) - N(H \cap D)$

$= 28 + 14 - 4 = 38$

$N(H \cup C \cup D)' = 55 - N(H \cup D) = 55 - 38 = 17$

35) Let M represent the set of married people in the given sample.

Let Y represent the set of people between 20 and 30 in the sample.

Let F represent the set of females in the sample.

$N(M) = 675 \quad N(Y) = 682 \quad N(F) = 684$

$N(M \cap Y) = 195 \quad N(M \cap F) = 467 \quad N(F \cap Y) = 318$

$N(M \cap Y \cap F) = 165$

$N(M \cup Y \cup F) = N(M) + N(Y) + N(F) - N(M \cap Y) - N(M \cap F) - N(Y \cap F) + N(M \cap Y \cap F)$

$= 675 + 682 + 684 - 195 - 467 - 318 + 165 = 1226$

However it is given that the sample of 1200 adults $N(M \cup F \cup Y) \leq 1200$

Thus, the figures taken from the poll are inconsistent.

37) $1000 = 10 \times 10 \times 10 = 2^3 \times 5^3$

Let A be the set of all positive integers less than 1000, that are multiples of 2.

Let B be the set of all positive integers less than 1000 that are multiples of 5.

$n(A) = 499$ $n(B) = 199$ $n(A \cap B) = 99$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 499 + 199 - 99 = 599$

Number of positive integers, which are less than the number = 1000 that have no common factor with 1000

= Total no. of +ve ints < 100 −

Total no. of +ve ints < 100 and have a common factor with 1000 = 999 − 599

= 400

Chapter 9.5

1) (a) It is given that the set is $\{x_1, x_2, x_3, x_4\}$

All the possible combinations ~~are~~ of 2 are

$\{\{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$

Now, $\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3!}{2! \times 1!} = 3$

(b) The unordered selections of four elements from $\{a, b, c, d, e\}$ are

$\{\{a,b,c,d\}, \{a,b,c,e\}, \{a,b,d,e\}, \{a,c,d,e\}, \{b,c,d,e\}\}$

Now, we have to find $\binom{5}{4} = \frac{5!}{4!(5-4)!} =$

$\frac{5!}{4! \times 1!} = 5$

3) $\binom{n}{r} = \frac{P(n,r)}{r!}$

$\binom{7}{2} = \frac{7!}{(7-2)!\,2!} = \frac{7!}{5!\,2!} = \frac{7!}{5!} \cdot \frac{1}{2!}$

$= P(7,2) \cdot 1/2!$ Since $P(7,2) = \frac{7!}{(7-2)!} = \frac{7!}{5!}$

$P(7,2) = 2!\binom{7}{2}$

5) (a) $\binom{6}{0} = 6!/6! = 1$

(b) $\binom{6}{1} = 6!/1!5! = 6$

(c) $\binom{6}{2} = 6!/2!(6-2)! = \frac{6!}{2!4!} = 15$

(d) $\binom{6}{3} = \frac{6!}{3!(6-3)!} = 20$

(e) = $\binom{6}{4} = \frac{6!}{4!2!} = 15$

(f) $\binom{6}{5} = \frac{6!}{5!(6-5)!} = \frac{6!}{5!1!} = 6$

(g) $\binom{6}{6} = \frac{6!}{6!0!} = 1$

6) (a) $\binom{15}{6} = \frac{15!}{6!(15-6)!} = 5005$

(b) 1. Committes that contain A but not B

2. Committees that contain B but not A

3. Committees that do not contain A and B

$$\binom{13}{5} = \frac{13!}{5!(13-5)!} = \frac{13!}{5!8!} = 9 \cdot 11 \cdot 13 = 1287$$

$$\binom{13}{6} = \frac{13!}{6!7!} = 1716$$

No. of committees that do not contain both A and B = 1287 + 1287 + 1716 = 4290

(c) Number of committees that contain both A and B or neither A nor B = Number of committees that contain both A and B + Number of committees that contain neither A nor B

$$= \binom{13}{4} + \binom{13}{6} = 715 + 1716 = 2431$$

(d) (i) No. of ways selecting 3 men from 8 is

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = 56$$

No. of ways of selecting 3 women from 7 is

$$\binom{7}{3} = \frac{7!}{3!(7-3)!} = 35$$

So, by product rule the number of Committees of 6 members with 3 men and 3 women

$= (56)(35) = 1960$

(d) (ii) The number of committees which contain at least one woman = (Total number of 6 member committees) − (Number of 6 member committes with all men)

$$= \binom{15}{6} - \binom{8}{6} = 5005 - 28 = 4977$$

(e) 2 freshmen from 3 can be selected in $\binom{3}{2}$ ways, 2 sophmores from 4 in $\binom{4}{2}$ ways 2 juniors from 3 in $\binom{3}{2}$ ways and 2 seniors from 5 in $\binom{5}{2}$ ways.

By multiplication formula, the number of committees $= \binom{3}{2} \cdot \binom{4}{2} \cdot \binom{3}{2} \cdot \binom{5}{2}$

$= 3 \cdot 6 \cdot 3 \cdot 10 = 540$

Chapter 9.6

10) $\binom{20+2}{20} = \binom{22}{20} = 231$

11) $x_1 + x_2 + x_3 = 20$

Let $x_i = y_i + 1$

Then,

$y_1 + y_2 + y_3 = 20 - 3 = 17$

So. $\binom{17+3-1}{17} = \binom{19}{17} = 171$

13) Let $x_i = y_i - 2$ for each $i = 1, 2, 3, 4$

Then, for each $x_i \geq 0$

$y_1 + y_2 + y_3 + y_4 = 30$

$(x_i + 2) + (x_2 + 2) + (x_3 + 2) + (x_4 + 2) = 30$

$x_i + x_2 + x_3 + x_4 + 8 = 30$

$x_1 + x_2 + x_3 + x_4 = 22$

$\binom{22+4-1}{22} = \binom{25}{22} = 2300$

16) (a) $n = 15 - 7 = 8$

$K = 5$

$\binom{8+5-1}{8} = 495$

Total ways $= 3876 - 495 = 3381$

(b) $n(r_{\le 5} \cap I_{\le 6}) = n(T) - n(r_{\ge 6} \cup I_{\ge 7})$

By the inclusion/exclusion rule,

$n(r_{\ge 6} \cup I_{\ge 7}) = n(r_{\ge 6}) + n(I_{\ge 7}) - n(r_{\ge 6} \cap I_{\ge 7})$

From part a, $n(I_{\ge 7}) = 495$

$n(r_{\ge 6}) = 715$

$n(r_{\ge 6} \cap I_{\ge 7}) = \binom{2+3-1}{2} + \binom{1+3-1}{1} =$

$\binom{4}{2} + \binom{3}{1} = 6 + 3 = 9$

Substitute those values in $n(r_{\ge 6} \cup I_{\ge 7})$

$n(r_{\ge 6} \cup I_{\ge 7}) = 715 + 495 - 9 = 1201$

Now substitute those values in

$n(r_{\le 5} \cap I_{\le 6}) = 3876 - 1201 = 2675$