12/08/2021 QUIZ #4

MATH-245

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Note:- Answers not in order. Answering what I know best first.

2. Ans:- French = F
Business = B
Music = M

Total :- 191 students
=> 10 are taking French, business and music
=> 36 are taking French and business
=> 20 are taking French and music
=> 18 are taking business and music
=> 65 are taking French
=> 76 are taking business, &
=> 63 are taking music.

We know the addition rule:-

$$N(F \cup B \cup M) = N(F) + N(B) + N(M) - N(F \cap B) - N(F \cap M) - N(B \cap M) + N(F \cap B \cap M)$$

$$\Rightarrow N(F \cup B \cup M) = 65 + 76 + 63 - 36 - 20 - 18 + 10 = 140$$

∴ No of people not taking any classes $= 191 - N(F \cup B \cup M) = 191 - 140$

$= 51$

3. Ans. Step I:- lyrics not, byte and dog doesn't occur.

Step - 2:- Total permutation $= (26)!$

Total permutation of not occuring $= (26 - 3 + 1)!$

$= 24!$

$= N(A)$

Total permutation of bats occurring
$= 23! = N(B)$

Total permutation of dog occurring
$= 24! = N(C)$

$N(A \cup B) = (26 - 3 - 4 + 1 + 1) = 21!$
$N(B \cup C) = 21!$
$N(A \cup C) = (26 - 3 - 3 + 1 + 1) = 22!$
$N(A \cup B \cup C) = (26 - 3 - 3 - 4 + 3) = 18!$

Step - 3 :- The required permutation
$= N - (N(A) + N(B) + N(C)) +$
$(N(A \cup B) + N(A \cup C)) + N(B \cup C)$
$- N(A \cup B \cup C)$

$= 26! - (24! + 23! + 24!) +$
$(21! + 21! + 22!) - 18!$

$= 26! - 2 \times (24)! - 23! + 2 \times 21!$
$+ 22! - 19!$

Step 4:- Finally, from the above discussion, we conclude the number of ways :-

$$(26)! - 2\times 24! - 23! + 2(21)! + (22)! - (15)!$$

2. Ans $\{n \in Z, n \leq 700\}$ are divisible by 2, 3 or 5.

We have that :-

Z = {set of integers} which is countable and infinite.

$= \{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\}$

Numbers which are divisible by 2, 3 or 5 = $\{2, \ldots\ldots, 700\}$

= Countably infinite.

4. My BMW automobiles come in 7 models, 10 colours, 3 ranging sizes & 2 transmission types.

(a) Different types of BMW = $7 \times 10 \times 3 \times 2$
$= 420.$

(b) Here, colour is black.

So, different types of black color BMW $= 7 \times 3 \times 2 = 42$

5. An Integer solutions of $x_1 + x_2 + x_3 + x_4 = 32.$

(a) $x_i \geq 0 \quad i = 1, 2, 3, 4.$

No of solutions of $x_1 + x_2 + \ldots + x_k = n,$

$x_i \geq 0$

$\therefore$ a is given by $\binom{n+k-1}{k-1}$

(5)

Using the generating function for each $x_i = 1 + x + x^2 + \ldots\ldots$

$= \frac{1}{1-x}$ (exponent denote x_i).

$= \frac{1}{(1-x)^k} = \sum \binom{n+k-1}{k-1} x^n$

$n = 32$

$k = 4$

$\binom{32+4-1}{4-1}$

$= \binom{35}{3} = \frac{35!}{3!\,32!}$

$= \frac{6545}{2}$

(ii) If $x_1, x_2 \geq 5$

$x_3, x_4 > 7$

$\Rightarrow 7 \quad x_3, x_4 \geq 8$

$x_1 = y_1 + 5$ $\quad x_3 = y_3 + 8$

$y_2 = y_2 + 5$ $\quad x_4 = y_4 + 8$

$= y_1 + y_2 + y_3 + y_4 + 26 = 32$

$y_1 \geq 0$

$\Rightarrow$ no of solutions to $y_1 + y_2 + y_3 + y_4 = 6$

$$\binom{6+4-1}{4-1} = {}^9C_3$$

$$= \frac{9!}{3! \cdot 6!}$$

$$= 84$$

6. (a) ALGORITHM is a 9 character word. So, the number of ways is $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 362\,880$ ways.

(b) ${}^{9}P_6 = {}^{9}P_6 = \frac{9!}{(9-6)!}$

$= \frac{9\times8\times7\times6\times5\times4\times3!}{3!}$

$= 60480$ ways

(c) The first place can be filled with letter A. And the remaining 5 places are filled with the remaining 8 letters of ALGORITHM (except (A).

$\therefore {}^{8}P_6 = {}^{8}P_5 = \frac{8!}{(8-5)!}$

$= \frac{8\times7\times6\times5\times4\times3!}{3!}$

$= 6720$ ways