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MATH-245

QUIZ-#3

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1. Ans. $S(0) = 2000$

$S(n+1) = S(n) + S(n) \times \frac{6}{100} + 150$ 5

is the required recurrence relation

We have $S(n+1) = 1.06\, S(n) + 150$

let $S(n) = T(n) + b$,

then we get :-

$T(n+1) + b = 1.06\,T(n) + 1.06\,b + 150$

Choosing values so that $b = 1.06\,b + 150$

$b = \frac{-150}{0.6} = -250$

$T(n+1) = 1.06\,T(n)$

$T(n) = (1.06)^n \cdot T(0)$, where

$T(0) = S(0) - b = 2250$

$T(n) = S(n) - b$

$= S(n) - 250$

$\therefore S(n) + 250 = (1.06)^n (2250)$

$S(n) = 2250\,(1.06)^n - 250$ is the required solution.

P.T.O

$\therefore$ At the end of 5 years we will have $S(5) \simeq 2761$ dollars in the account.

NOTE :- Answers not in order. Answering what I know well first.

5 Ans The given linear homogenous recurrence relation is :-

$$a_n = 8a_{n-1} - 15a_{n-2} \quad -(1)$$

with initial conditions $a_0 = 0$ & $a_1 = 4$

It is a linear recurrence relation of degree 2.

Comparing (1) with

$$a_n = A a_{n-1} + B a_{n-2},$$

we get :-

$$A = 8 \quad \& \quad B = -15$$

Then the characteristic equation is given by :-

$$t^2 - At - B = 0$$

$$\Rightarrow t^2 - 8t + 15 = 0$$
$$\Rightarrow t^2 - 5t - 3t + 15 = 0$$
$$\Rightarrow (t-5)(t-3) = 0$$
$$\Rightarrow t = 3, 5 \text{ are real \& distinct roots.}$$

P.T.O

Hence, the solution to this recurrence relation are of the form :-

$$a_n = C_1 3^n + C_2 5^n$$

To find the constants C_1 & C_2, lets use the initial conditions :-

$a_0 = 0$

$\Rightarrow \quad C_1 + C_2 = 0 \quad - \quad (2)$

$a_1 = 4$

$\Rightarrow \quad 3C_1 + 5C_2 = 4 \quad - \quad (3)$

On solving (2) & (3), we get :-

$C_2 = -C_1$

Substituting, $3C_1 - 5C_1 = 4$

$-2C_1 = 4$

$C_1 = -2$

$C_2 = 2$

$C_1 = -2$ & $C_2 = 2$

Hence, the unique solution to this recurrence relation & the given initial conditions is the sequence ~~[illegible]~~ $\{a_n\}$ with :-

$$a_n = -2(3^n) + 2(5^n)$$

P.T.O

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3 Ans The given equation is:-

$a_n = 7 a_{n-1}$, $a_1 = 3$

for $n=2$, $a_2 = 7a_1 = 7.3$

for $n=3$, $a_3 = 7a_2 = 7.7.3 = (7)^2.3$

for $n=4$, $a_4 = 7a_3 = 7.(7)^2.3 = (7)^3.3$

$\therefore a_n = (7)^{n-1}.3$

$\therefore \boxed{a_n = 7^{n-1}.3}$