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Chapter 2.3

1) Let $p :=\sqrt{2}$ is rational

$q := \sqrt{2} = a/b$ for some integers a and b

$p \to q$

$\sim q$

The conclusion is $\sim p : \sqrt{2}$ is not rational

3) Logic is not easy (From modus tollens)

6)

p	q	$p \to q$	$q \to p$	$p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

Therefor, the argument is invalid

9)

p	q	r	$\sim q$	$\sim r$	$p \wedge q$	$p \wedge q \rightarrow \sim r$	$p \vee \sim q$	$\sim q \rightarrow p$	$\sim r$	
T	T	T	F	F	T	F	T	T	F	
T	T	F	F	T	T	T	T	T	T	
T	F	T	T	F	F	T	T	T	F	→(*)
T	F	F	T	T	F	T	T	T	T	
F	T	T	F	F	F	T	F	T	F	
F	T	F	F	T	F	T	F	T	T	
F	F	T	T	F	F	T	T	F	F	
F	F	F	T	T	F	T	T	F	T	

The selected row (*) shows that is is possible for an argument of this form to have true premises and a false conclusion.

Hence, this argument form is invalid

11)

p	q	r	$\sim p$	$\sim q$	$\sim r$	$q \vee r$	$p \to q \vee r$	$\sim q \vee \sim r$	$\sim p \vee \sim$
T	T	T	F	F	F	T	T	F	F
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T

Row (*) shows that the argument has true premises and a false conclusion, this's implies the argument is invalid.

21)

p	q	r	p v q	p → r	q → r	r
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	T	F	T	T	F	F
F	[illegible] F	T	F	T	T	T
F	F	F	F	T	T	F

Premise Conclusion

In each situation where the premises are both true the conclusion is also true.

So, the argument is valid.

25) p : The real number is rational
q : The real number is irrational

p v q
~p
∴ q

∴ The argument is valid and is elimination

26). p: I go to the movies
q: I won't finish my homework
r: I won't do well on the exam tomorrow

$p \to q$
$q \to r$

$\therefore p \to r$
$\therefore$ The argument is valid and is transitive

27) p: "The number is larger than 2",
~~and q~~

q: "The square is larger than 4"

$p \to q$
$\sim q$
$\therefore \sim p$

Then the above argument has the form:-

$p \to q$

$\sim p$

$\therefore \sim q$

Hence the argument is invalid and is an inverse error.

29) p := At least one of these two numbers is divisible by 6.

q := The product of these two numbers is divisible by 6.

$p \to q$

$\sim p$

$\therefore \sim q$

p	q	$p \to q$	$\sim p$	$\sim q$	
T	T	T	F	F	
T	F	F	F	T	
F	T	T	T	F	(*)
F	F	T	T	T	

(*) This row shows that an argument of this form can have true premises and a false conclusion.

Hence, this is an invalid argument.

The conclusion of the argument would follow from the premises if the premise. $p \to q$ were replaced by its inverse. Thus the argument form is called the inverse error.

Chapter 3.1

1. a) There is no animal in the menagerie whose color is red and the colors are limited to brown, black, blue, yellow

   Therefore, the statement is false

   b) True

   c) False

   d) True

   e) False

   f) True

3) a) The statement m is a factor of n^2 is true for the values $m = 25$ and $n = 10$

Since,

$n^2 = 10^2 = 100$ and $m = 25$

$n^2 = 100 = 4 \cdot 25 = 4m$

But, the statement m is a factor of n is false. Since 10 is not a product of 25 and any other integer.

So, the statement as a whole is false

b) $R(m, n)$ is false when $m = 16$ and $n = 4$

Because $m = 16$ is a factor of $n^2 = 4^2 = 16$, but 16 is not a factor of 4.

c) The statement m is a factor of n^2 is true for the values $m = 5$ and $n = 10$

Since,

$n^2 = 10^2 = 100$ and $m = 5$

$n^2 = 100 = 20 \cdot 5 = 20m$

Also the statement m is a factor of n is true. Since 5 is a factor of 10.

So, the statement as a whole is true.

d) $R(m,n)$ is true when $m=4$ and $n=8$

$m=4$ is a factor of $n^2=8^2=64$ and 4 is also a factor of 8.

Also.

$R(m,n)$ is true when $m=3$ and $n=3$

$m=3$ is a factor of $n^2=3^2=9$ and 3 is also a factor of 8.

4) (a) $Q(-2,1)$ is the statement

If $-2<1$, then $(-2)^2<1^2$. The hypothesis of this statement is $-2<1$, which is true. The conclusion is $(-2)^2<1^2$, which is false because $(-2)^2<1^2$ and $4 \not< 1$. Thus $Q(-2,1)$ is false

(b) If $x=-2$ and $y=1$

$x=-4$ and $y=2$

$x=-6$ and $y=3$

$x=-7$ and $y=4$

(c) $Q(3,8)$ is the statement

If $3<8$, then $3^2<8^2$

The hypothesis of this statement is $3<8$, which is true. The conclusion is $3^2 < 8^2$ which is also true because $3^2 = 9$ and $8^2 = 64$. Thus $Q(3,8)$ is a conditional statement with a true hypothesis and a true conclusion.

(d) If $x = 4$ and $y = 7$

$x = 2$ and $y = 4$

$x = -2$ and $y = 6$

$x = -4$ and $y = 7$

5) (a) The value of d for which $6/d$ is an integer are $-6, -3, -2, -1, 1, 2, 3, 6$

The domain is the set of all integers.

Therefore, the truth set is. $\{-6, -3, -2, -1, 1, 2, 3, 6\}$

(b) The value of d for which $6/d$ is an integer are $-6, -3, -2, -1, 1, 2, 3, 6$

The domain is the set of all positive integers.

Therefore, the truth set is $\{1, 2, 3, 6\}$

(c) $1 \leq x^2 \leq 4$

$\pm 1 \leq x \leq \pm 2$

...based on a study conducted in school age children in 1999-2000 and published in a Journal of Nutrition...

$-1 \le x \le -2$ and $1 \le x \le 2$

The value of x for which $1 \le x^2 \le 4$ will lie between -2 and -1 inclusive together with those between 1 and 2 inclusive.

The domain is the set of all real numbers.

The truth set is $-2 \le x \le -1$ or $1 \le x \le 2$

(d) $1 \le x^2 \le 4$

$\pm 1 \le x \le \pm 2$

$-1 \le x \le -2$ and $1 \le x \le 2$

The value of x for which $1 \le x^2 \le 4$ will lie between -2 and -1 inclusive together with those between 1 and 2 inclusive.

The domain is the set of all integers.

Therefore, the truth set is $\{-2, -1, 1, 2\}$

9) Let $x = 1 : 1 \not> 1 \;/\!/ 1 = 1 = 1$

Therefore, the counter example is $1 \not> 1 \;/\!/$

11) Let $m = 2$, $n = 1$

Now $m \cdot n = 2$ But $m + n > m \cdot n$ since

$m + n = 3$ $3 > 2$

Therefore, the counter example is $2 \cdot 1 \not> 2+1$

13) The equivalent statements to the given statements are, a, e and f.

14) a) False

b) $x^2 = (\sqrt{2})^2 = 2$

True

c) True

d) False

e) True

f) True

19) (a) No

(b) Yes

(c) No

(d) Yes

(e) Yes

(f) No

Chapter 3.2

1) a) Negation

b) Not a negation

c) Not a negation

d) Not a negation

e) Negation

f) Not a negation

4) ~~a) No dogs are friendly~~

~~b) No people are happy~~

~~c)~~

~~a) Some dogs are friendly~~

~~b) Some peot~~

a) Only few dogs are friendly

b) Only few people are happy

c) All suspicions were substantiated

d) All estimates are accurate

a) $\exists$ a real number x such that $x > 3$ and $x^2 \leq 9$

13) The negation of a "for all" statement is a "there esxists" statement. Also, the negation of an "if - then" statement is a "such - that" statement.

The proposed negation is not correct.

The correct negation is:

There esxists an integer n such that n^2 is even and n is not even.

22) $\exists$ an integer n such that n^2 is odd and n is not odd.

Chapter 3.3

2) (a) $G(2,3)$ is true because $2^2 = 4 > 3$

(b) $G(1,1)$ is false because $1^2 = 1 \not> 1$

(c) $G(1/2, 1/2)$ is false because $(1/2)^2 = 1/4 \not> 1/2$

(d) $G(-2, 2)$ is true because $(-2)^2 = 4 > 2$

4) (a) $x = 15.83$

Let $n = 16$

Then, $n > x$ because $16 > 15.83$

(b) $x = 10^8$

Let $n = 10^9$

Then $n > x$ because $10^9 > 10^8$

(c) $x = 10^{10^{10}}$

Let $n = 10^{10^{12}}$

Then $n > x$ because $10^{10^{12}} > 10^{10^{10}}$

12) (a) Negation:- $\sim(\forall x \in D, \exists y \in E$, such that $x + y = 1) \equiv \exists x \in D, \forall y \in E$, such that $x + y \neq 1$

Let $x = -2 \in D$

If $y = 3$ then $x + y = 1$

But 3 is not in E

Thus, the statement is false and the negation of the statement is true.

(b) $\sim(\exists x \in D$, such that $\forall y \in E, x + y = -y)$
$\equiv \forall x \in D, \exists y \in E$, such that $x + y \neq -y$

The equation is equivalent to $x + 2y = 0$ and it is clear that no x can equal to $-2y$ for multiple values of y.

Thus, the statement is false.

Therefore, the negation of the statement is true

(c) $\sim(\forall x \in D, \exists y \in E$, such that $xy \geq y)$
$\equiv \exists x \in D, \forall y \in E$, such that $xy < y$.

Let $x = -2 \in D$
If $y = -1$ then $(-2)(-1) \geq -1$
Let $x = -1 \in D$
If $y = -2$ then $(-1)(-2) \geq -2$
Let $x = 0 \in D$
If $y = -2$ then $(0)(-2) \geq -2$
Let $x = 1 \in D$
If $y = 1$ then $(1)(1) \geq 1$
Let $x = 2 \in D$
If $y = 2$ then $(2)(2) \geq 2$
Thus, the statement is true.
Therefore, the negation of the statement is false.

(d) $\sim(\exists x \in D$, such that $\forall y \in E, x \leq y) \equiv$
$\forall x \in D, \exists y \in E$, such that $x > y$
To prove this is true, go through for a x and find all y's in E such that $x \leq y$

Let $x = -2 \in D$

If $y = -1$ then $-2 \leq -1$

Let $x = -1 \in D$

If $y = 0$ then $-1 \leq 0$

Let $x = 0 \in D$

If $y = 1$ then $0 \leq 1$

Let $x = 1 \in D$

If $y = 1$ then $1 \leq 1$

Let $x = 2 \in D$

If $y = 2$ then $2 \leq 2$

Thus, the statement is true.

Therefore, the negation of the statement is false.

15) (a) Every odd integer has the form $2k+1$ for some integer k.

(b) There is an odd integer that is not equal to $2k+1$ for any integer k.

16) (a) There is a real number whose product with any other real number will always equal to the second number.

(b) $\forall$ real numbers u, $\exists$ real number v, such that $uv \neq v$

For every real number, there is a real number such that their product is not equal to the second real number.

18) (a) For every real number x, there is a real number y such that $x+y=0$

(b) There is a real number x with the property $x+y \neq 0$ for any real number y.

21) (a)(1) If x is a real number, then $y = 7-2x$ is also a real number for which

$2x+y = 2x+(7-2x) = 2x+7-2x = 7$

Therefore, the statement (1) is true

(2) If $y=1$, then $x=3$ is the only real number such that $2x+y=7$

If $y=3$, then $x=2$ is the only real number such that $2x+y=7$

Hence, the statement (2) is false.

(b) (1) True, due to commutative property of addition

(2) True

(c) (1) $(x-y)^2 = 0$

$x - y = 0$

$x = y$

$\therefore$ Statement (1) is true

(2) The statement holds when $x = y$ and it requires different x-values for all y-values.

Therefore Statement (2) is false

(d) (1) $(x-5)(y-1) = 0$

$x = 5$ or $y = 1$

The statement holds when $y = 1$ for every x value, thus statement (1) is true.

(2) $(x-5)(y-1) = 0$

$x - 5 = 0$ or $y - 1 = 0$

$x = 5$ or $y = 1$

The statement holds when $x = 5$ for every y value.

Therefore, the statement (2) is true when $x = 5$

(e) (1) As the square of a real number is non negative, so the sum of square of two nonnegative is nonnegative.
Thus, the equation $x^2+y^2=-1$ does not hold for any real numbers x and y.
Therefore, statement (1) is false.

(2) For the same reason, statement (2) is false

41) (a) False

For $x=1$, the value of y would be $y=x-1$
$=1=1-1=0 \notin \mathbb{Z}^+$

(b) True

$y=x-1$

$y+1=x$

(c) False

If the number is one greater than 2, then it equals to 3. And it is not 1 greater than 3.

(d) True

For any integer x, take the value of y as $1/x$

$xy = x(1/x) = 1$

(e) False

For $x = 0$, $\nexists\, y \in R$ such that $xy = 1$

(f) True

For any $y \in R$, let $x = 0$

$x + y = 0 + y = y$

$x + y = y$

(g) True

For any integer x, take the value of y as $x/3$

$y = x/3 < x$

$y < x$

(h) False

The given statement is the negation of the statement given in part (g). The statement in part (g) is true. Then its negation would be false.

Chapter 4.1

11) Consider $2n^2-5n+2$

Constructive Proof of existence:

When $n=3$

$2n^2-5n+2 = 2(3)^2-5(3)+2 = 2(9)-15+2$

$= 18-13 = 5$

5 is a prime number

So, $2n^2-5n+2$ is also prime

Hence proved.

12) Let $a=-3$ and $b=-2$ then $a<b$,

because $-3<-2$

However $a^2 \not< b^2$

Because $(-3)^2=9$ and $(-2)^2=4$ and $9 \not< 4$

Hence there exist two real numbers $-3,-2 \in R$,

if $-3<-2$ then $(-3)^2 \not< (-2)^2$

Therefore, the statement is false.

13) Let $n = 5$ be an odd integer

Now, we calculate $\frac{n-1}{2} = \frac{5-1}{2} = 4/2 = 2$,

which is even. If n is odd, then $\frac{n-1}{2}$ is even

27) Assume that n is an odd number.

Thus $k \in \mathbb{Z}$ satisfying $n = 2k+1$

$n^2 = (2k+1)^2$

$n^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$

Assume $2k^2 + 2k = l$. l is also an integer.

$n^2 = 2l + l$

n^2 is also an odd integer

By the principle of generalizing from the generic particular, for every odd n, n^2 is odd integer.

(a) odd integer

(b) $2k+1$

(c) n

(d) generalizing from the generic particular.

28) (a) 1. For every integers a and b, if a and b are odd then a + b is even.

2. For every odd integers a and b, a + b is even.

3. If a and b are any odd integers, then a + b is even.

(b) Odd Number :- a is odd if it is of the form $a = 2r + 1$ for some integer r.
Even Number :- b is even if it is of the form $b = 2s$ for some integer s.

$m = 2r + 1 \quad n = 2s + 1$

$m + n = (2r + 1) + (2s + 1)$

$m + n = 2r + 2s + 2$

$m + n = 2(r + s + 1)$

Let $r + s + 1 = u$. Since $\mathbb{Z}$ is closed under addition, u is an integer

$m + n = 2u$

Thus, m+n is an even integer

(a) definition of odd integer
(b) Substitution (c) Sum of integers is again an integer (d) definition of even integer.

29) (a) 1. For every integers n, if n is even then -n is even

2. For every even integers x, -n is even

3. If x is any even integer, then -n is even.

(b) b is even if it is of the form $b = 2s$ for $s \in \mathbb{Z}$

$n = 2k$ for $k \in \mathbb{Z}$

$-n = -(2k)$

$= 2(-k)$

Let $-k = r$. Clearly $r \in \mathbb{Z}$

$-n = 2r$

Thus -n is an even integer.

(a) definition of even integer

(b) substitution

(c) multiplication of integers is again an integer.

(d) definition of even integer.

30) (a) 1. For every integers m and n, if m is even and n is odd, then $m+n$ is odd.

2. For every integer m and odd integer n, $m+n$ is odd.

3. If m is any even integer . and n is any odd integer, then $m+n$ is odd.

(b) $m = 2r$ and $n = 2s+1$ $r, s \in \mathbb{Z}$

$m + n = (2r) + (2s+1)$

$m + n = 2r + 2s + 1$

$m + n = 2(r+s) + 1$

Let $r+s = u$. $u \in \mathbb{Z}$

$m + n = 2u + 1$

Thus $m+n$ is an odd integer

(a) any odd integer.

(b) integer r

(c) $2r + (2s+1)$

(d) $m+n$ is odd integer

Chapter 4.7

3) Suppose that there is an integer n such that $3n+2$ is divisible by 3.

Then $3n+2 = 3m$ for some integer m.

Consider the equation, $3n+2 = 3m$

Subtract $3n$ from both sides.

$3n+2-3n = 3m-3n$

$2 = 3(m-n)$

$2/2 = \frac{3(m-n)}{2}$

$1 = 3/2\,(m-n)$

If a and b are +ve integers

and a/b, then $a \leq b$

3 and 2 are +ve integers and $3 > 2$

not $3 \leq 2$

Thus for all integers n, $3n+2$ is not divisible by 3.

4) Then,

$7m + 4 = 7k$, for some integer k

Subtracting $7m$ from both sides gives

$7m + 4 - 7m = 7k - 7m$

$\Rightarrow 4 = 7(k - m)$

So by the definition of divisibility $7 | 4$

However, we know that if a and b are +ve

then $a \leq b$

$7 | 4 \Rightarrow 7 \leq 4$ is a contradiction to $7 > 4$

Thus, for all $m \in \mathbb{Z}$, ~~4~~ $7m + 4$ is not divisible by 7.

5) X is an even integer and $X \geq x$, for every ~~even~~ even integer x.

Let $y = X + 2$

Then y is an even integer, ~~since~~ and $y > X$

Since $y = X + 2$, this contradicts the supposition that $X \geq x$ for every even integer x.

Thus there is no greatest even integer.

6) $X \geq x$ for every negative real number x, which means there is no negative real no. greater than x.

Let $y = X/2$. Then, y is a negative real number,

$y > X$ since $y = X/2$, and X is negative

But this contradicts the supposition that $X \geq x$ for every negative real number x, thus the original statement is correct.

Hence proved.

7) Let N be a least positive rational number

Then $0 < N < n$ for every rational number n.

Let $M = N/2$

So, M is a rational number as the denominator is non zero.

Implies that $0 < M < N < n$ which contradicts the fact N is the least +ve rational no. Implies supposition is false. Hence proved.

8) Suppose there exists a (a) a rational number x and (b) an irrational number y such that $x - y$ is rational.
By definitions of rational, there exists integers a, b, c and d with $d \neq 0$ so that
$x = $ (c) a/b and $x - y =$ (d) c/d. By substitution

$a/b - y = c/d$

Adding y and subtracting c/d on both sides gives.

$$y = a/b - c/d = \frac{ad}{bd} - \frac{bc}{dd} = \frac{ad - bc}{\cancel{d}bd}$$

~~Nota~~ Now both $ad - bc$ and bd are integers because products and differences of (f) integers are (g) integers. And ~~b~~$bd \neq 0$ by the (h) zero product property. Hence y is a ratio of Integers with a non - zero denominator and thus y is a (i) rational which is a contradiction

9) (a) The mistake in this proof occurs in the negation statement where the negation written by the student is incorrect. Instead of being existential it is a universal quantifier.

One example of a rational number and an irrational number whose difference is rational is sufficient to disprove the given statement.

(b) Suppose there exists an irrational number x and a rational number y, such that $x - y$ is rational. Derive a contradiction. Now substitute the value of y in $x - y = c/d$

$$x - y = c/d$$

$$x - a/b = c/d$$

$$x = c/d + a/b = \frac{ad + bc}{bd}$$

Now c/d and a/b are rational numbers, and the sum of two rational numbers is rational. Thus, x is a rational number. This contradicts the supposition that x is an irrational number. Thus for all real numbers, if x is irrational and y is rational, then $x - y$ is irrational.

11) The mistake is that the negation for T that was used in the proof is incorrect. Thus deducing a contradiction from it fails to prove that S is true. The actual negation is "There exists two rational numbers whose sum is not rational."

12) (a) There exists an irrational number whose square root is rational.

(b) Assume $x \in R - Q$ such that $\sqrt{x}$ is rational

By def. of rational, $\sqrt{x} = a/b$ for some integers a and b with $b \neq 0$. Then $(\sqrt{x})^2 = (a/b)^2$

$x = a^2/b^2$

Both a^2 and b^2 are integers and b^2 is non zero.

Thus x is rational and our assumption is false.

Statement R is true.

Chapter 4.8

6) Suppose $6 - 7\sqrt{2}$ is rational

Now $6 - 7\sqrt{2} = x/y$

$\Rightarrow -7\sqrt{2} = x/y - 6$

$\Rightarrow \sqrt{2} = -1/7\,[x/y - 6]$

$\Rightarrow \sqrt{2} = \dfrac{x - 6y}{-7y}$

$x - 6y$ and $-7y$ are integers

It follows that $\sqrt{2} = \dfrac{x - 6y}{-7y}$, with $-7y \neq 0$ is a ratio of two integers.

$\therefore$ $\sqrt{2}$ is rational ~~and 6 7~~ which is a contradiction. Thus $6 - 7\sqrt{2}$ is irrational.

8) This is false

Because $\sqrt{4} = 2 = 2/1$

Which is rational

$\therefore$ $\sqrt{4}$ is not irrational

10) This is false

Let $p = \sqrt{5}$ and $q = -\sqrt{5}$ be two irrational numbers.

Then, $p+q = \sqrt{5} + (-\sqrt{5})$

$= \sqrt{5} - \sqrt{5} = 0 = 0/1$, which is rational

11) The statement is false

For instance:

As $\sqrt{2}$ is an irrational number, then $\sqrt{2} - \sqrt{2} = 0$ is a rational number. Therefore, the difference of two irrational numbers is not an irrational.

13) Suppose that r/s is rational.

By the definitions of rational $r/s = p/q$, where p and q are integers with $q \neq 0$

Now, $r/s = p/q$

$r = s \cdot p/q$

Since p/q is rational and s is irrational the product is irrational.

Therefore r is irrational, which is a contradiction. Thus r/s is irrational

But $r = 0$ then irrespective of s, r/s is also 0. Zero is rational number. Hence, the given statement is false.

15) This is false

Since $\sqrt{2}$ is an irrational no.

$(\sqrt{2})(\sqrt{2}) = (\sqrt{2})^2 = 2$ is rational

$\therefore$ The product of two ~~Graph~~ irrational numbers is not necessarily irrational.

17) As $x = 4$ is a rational number, then $\sqrt{x} = 4 = \pm 2$ is also a rational number.

As $x = 2$ is a rational number, then $\sqrt{x} = \sqrt{2}$ is an irrational number.

Therefore, the statement if x is rational, then $\sqrt{x}$ is irrational is sometimes true and sometimes false

18) (a) Contraposition statement:

For all integers a, if a is odd, then a^3 is odd

Suppose a is odd

Then, there exists an integer k such that $a = 2k+1$

$a^3 = (2k+1)^3 = (2k)^3 + 3(2k)^2(1) + 3(2k)(1)^2 + (1)^3 = 8k^3 + 12k^2 + 6k + 1$

$= 2(4k^3 + 6k^2 + 3k) + 1$

$= 2q + 1$

Here, $q = 4k^3 + 6k^3 + 3k$ is an integer.

Therefore, a^3 is odd number.

Hence, for all integers, a if a^3 is even then a is even.

(b) Let $a = \sqrt[3]{2}$ be a rational number

Say $a = p/q$ where $p, q \in \mathbb{Z}$ and has only common factor 1.

$a^3 = p^3/q^3$ ~~(...)~~ ...... (1)

$a = \sqrt[3]{2}$

$a^3 = 2$ .... (2)

From equations (1) and (2)

$p^3/q^3 = 2 \Rightarrow p^3 = 2q^3$

Therefore p^3 is even and hence, p is even by part (a)

Let $p = 2k$ for some integer k

Then, substitute $p = 2k$ in $p^3/q^3 = 2$

$p^3/q^3 = 2 \Rightarrow p^3 = 2q^3 \Rightarrow (2k)^3 = 2q^3$

$4k^3 = q^3$

Hence q^3 is even

Therefore, q is even by part (a)

Then p and q are divisible by 2 and hence, p and q has common factor 2.

This is a contradiction.

Therefore, $a = \sqrt[3]{2}$ is an irrational number.

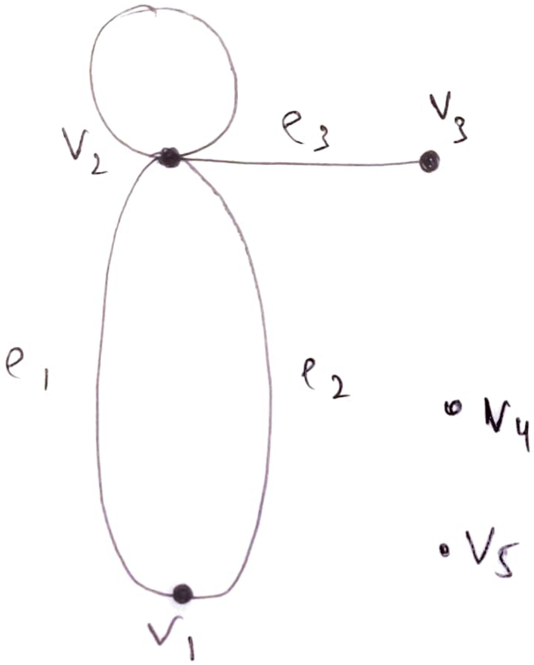
Chapter 1.4

1) $V(G) = \{v_1, v_2, v_3, v_4\}$

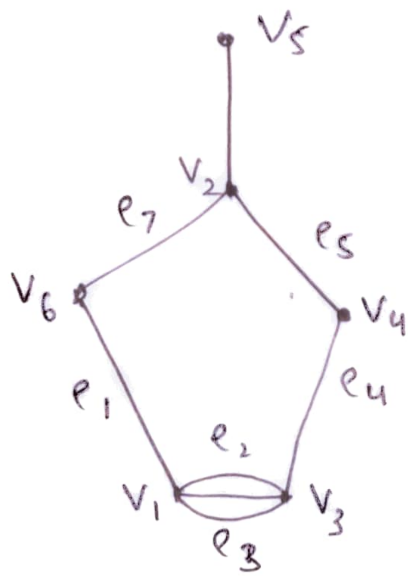
$E(G) = \{e_1, e_2, e_3\}$

Edge	Endpoints
e_1	$\{v_1, v_2\}$
e_2	$\{v_1, v_3\}$
e_3	$\{v_3\}$

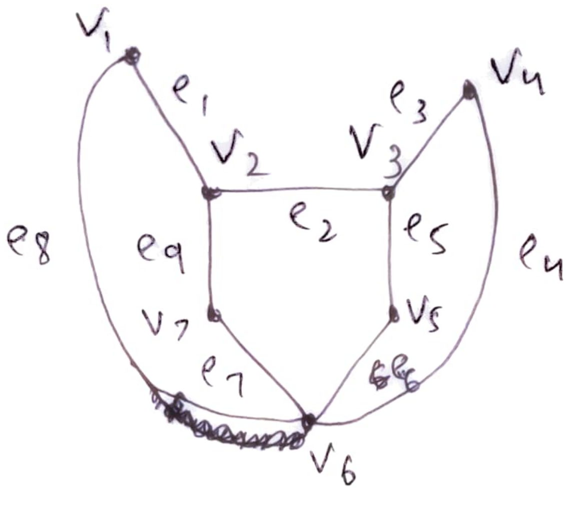
3)


V2
e3
V3
e1
e2
V4
V5
V1

5)


V5
V2
e7
e5
V6
V4
e1
e4
e2
V1
V3
e3

7)


V1
e1
V4
e3
V2
V3
e8
e9
e2
e5
e4
V7
V5
e7
e6
V6

a) (i) e_1, e_2 and e_7

(ii) V_1 and V_2

(iii) e_2 and e_1

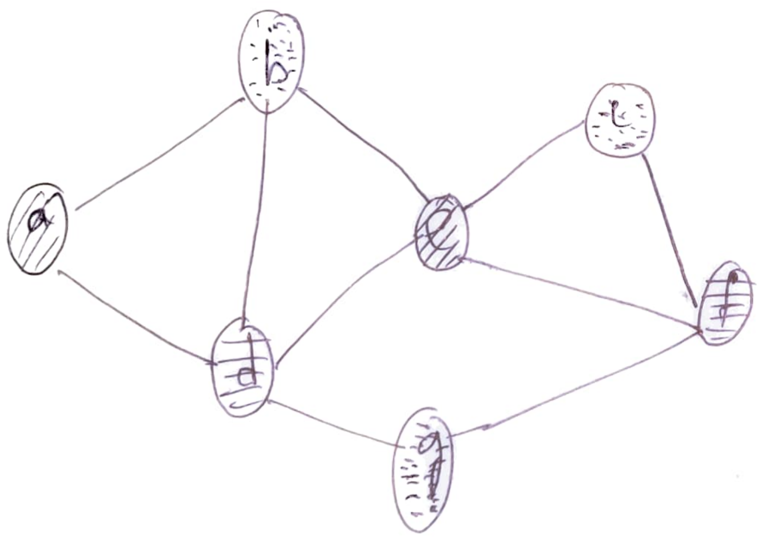
(iv) e_1 and e_3

(v) e_4 and e_5

(vi) V_4

(vii) ~~e_6 and e_7~~ 2.

15)



Chapter 4.9

1) Degrees:-

$V_1 = 3$

$V_2 = 2$

$V_3 = 4$

$V_4 = 2$

$V_5 = 1$

$V_6 = 0$

Total degree = 12

No. of edges = 6

The number of edges $= 6 = \frac{1}{2}(12) = \frac{1}{2}$ (Total degree)

Therefore, the number of edges is same as one-half of the total degree.

3) Total degree $= 0 + 2 + 2 + 3 + 9 = 16$

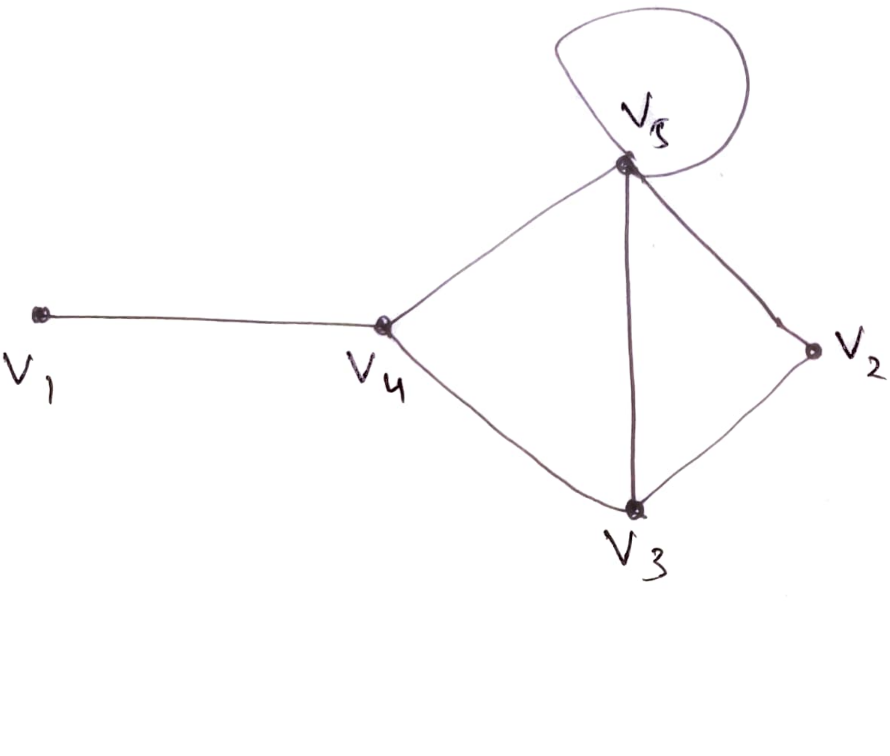
Handshake theorem states that the sum of degree of all the vertices equals twice the no. of edges.

$$2|E| = \sum \deg v$$

$$|E| = \frac{1}{2} \sum \deg v$$

Thus, the no. of edges $= \frac{1}{2}(16) = 8$

5)



7) Total degree = $1+1+1+4=7$

Handshake theorem:-

$$2|E| = \sum \deg v$$

$$|E| = \frac{1}{2} \sum \deg v$$

No. of edges = $\frac{1}{2}(7) = 3.5$

No. of edges should be a natural number.
Thus the graph does not exist.

9) If such a simple graph exists with $|V| = 4$ of the given degrees, then the vertex of degree 4 has to be connected to other 4 distinct vertices. This is impossible since it is given that the graph is simple and thus no loops or parallel edges are allowed. Thus 4 other vertices should be there to have a vertex of degree 4, which is also impossible. Since our assumption is $|V| = 4$

Thus there is no simple graph with $|V| = 4$ of degrees 1, 2, 3 and 4.

17) People = No. of vertices = 25
Handshake = Edge of graph
Total degrees = 75

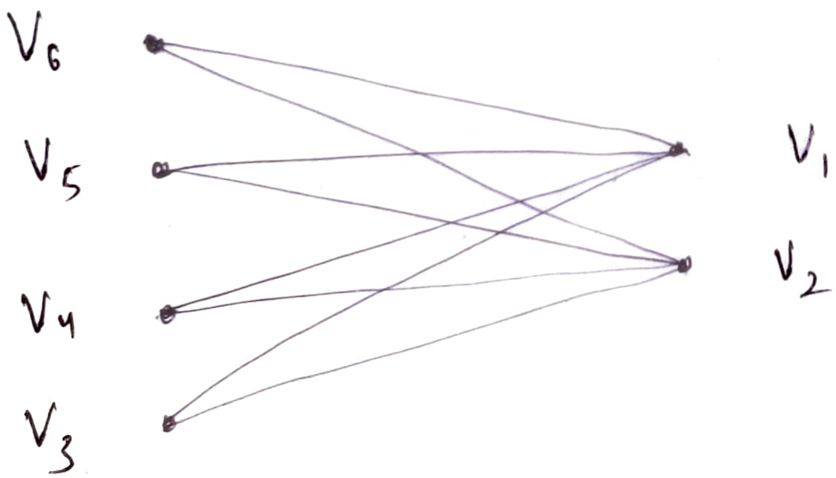
No. of edges = $\frac{1}{2}$ (total degree)

$= \frac{1}{2}(75) = 37.5$

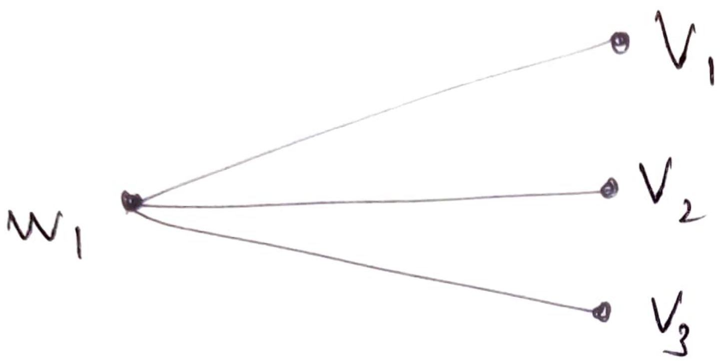
which is not possible, since $|E|$ should be a number not a decimal.

Thus, it is not possible for one to shake hands with exactly 3 other people in a group of 25 people.

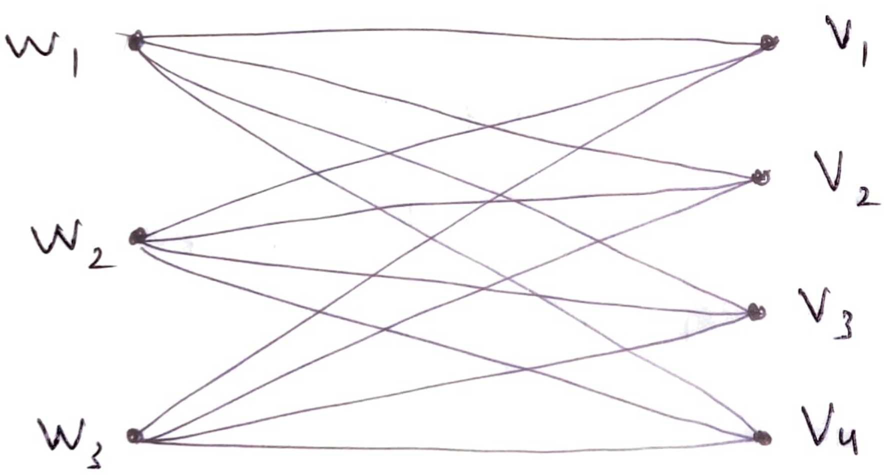
23) (a) $K_{4,2}$



(b) $K_{1,3}$



(c) $K_{3,4}$



(d) If $m = n$, then $n + m = 2m$

$K_{m,m}$ is divided into two groups of equal sizes m. All the vertices have the same degree say m.

If $m \neq n$, the vertices of $K_{m,n}$ is divided into groups: one of size m and the other of size n.

Every vertex in the group of size m has degree n because each is connected to every vertex in the group of size n. So $K_{m,n}$ has m vertices of degree n. Similarly, every vertex in the group of size n has degree m because each is connected to every vertex in the group of size m.

(e) From part (d) there are m vertices of degree n, a total of mn degree and n vertices of degree m another mn degree

Thus the total degree is $2mn$.

(f) From part (e), the total degree of $K_{m,n}$ is $2mn$. Thus the total no. of edges is given by $1/2(2mn) = mn$ edges.