Date: 10/9/2021

NAME :- SHREYAS SRINIVASA

PALOMAR ID :- 012551187

MATH-245

QUIZ #2

Note :- Answers will not be in order because I am answering whatever comes to me easily first!

2) In (a) Let $x = 2$, $y = 7$.

then x & y are prime numbers, but $x + y = 2 + 7 = 9$.

But, 9 is not a prime number.

(b) Let $x = y = \sqrt{2}$

then x & y are irrational numbers.

but, $x - y = \sqrt{2} - \sqrt{2} = 0$

$\therefore$ $x - y$ is not irrational.

(c) Let $x = \frac{1}{2}$ then $x^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow x^2 < x \quad \because \frac{1}{4} < \frac{1}{2}$

$\Rightarrow$ $\therefore$ Square of x is less than x.

(A) Let $n = 4$

$\Rightarrow n^2 - 4 = 16 - 4 = 12$

$\Rightarrow$ 12 is divisible by 4.

$\Rightarrow$

1. (a) Let x be an even integer. Then we have to show that $(x^2 + x + 1)$ is an odd integer.

x^2 is also an even integer.

Sum of two even integers $(x^2 + x)$, is an even integer.

Now adding 1 in any even integer makes the integer odd, and $(x^2 + x + 1)$ is obtained by adding 1.

$\therefore x^2 + x + 1$ is an odd integer.

$\Rightarrow$

(b) Let $x > 6$ and $y < 6$.

We have to show that $x + y > 12$

$\because x > 6 \Rightarrow x - 6 > 0$

& $y < 6 \Rightarrow 6 - y > 0$

By subtracting, we get :-

$(x - 6) - (6 - y) > 0$

$\Rightarrow x - 6 - 6 + y > 0$

$\Rightarrow x + y > 12$

$\Rightarrow$

8.7.0

(iii) Let $x < 6$ and $y > 6$.

$\therefore x < 6 \Rightarrow 6 - x > 0$
& $y > 6 \Rightarrow y - 6 > 0$

By subtracting, we get :-
$(y-6) - (6-x) > 0$

$\Rightarrow y - 6 - 6 + x > 0$

$\Rightarrow x + y > 12$

~~(iv) Let x is an integer~~

4. (a) Let x is an integer & x^2 is even.

To prove $\rightarrow$ x is even

Let us assume that x is not even.

$\Rightarrow x$ is odd
$\Rightarrow x = 2m+1$
$\Rightarrow$ Squaring both sides, we get :-
$\Rightarrow x^2 = (2m+1)^2$

$\Rightarrow x^2 = 4m^2 + 1 + 4m$
$\Rightarrow x^2 = 4(m^2+m) + 1$
$\Rightarrow x^2 = 4y + 1$, where $y = m^2 + m$

$\Rightarrow x^2$ is odd, which is not true.
(∵ we are given that x^2 is even).
∴ Our supposition is wrong.
∴ x is even.

P.T.O

(b) Let us assume that n is an integer satisfies in both given 2 cases.

By definition of given, $y = 2m, m \in \mathbb{N}$
& $y = 2k+1, k \in \mathbb{N}$

$\therefore 2m = 2k+1$

$\Rightarrow 1 = 2(k-m)$

As $k \in \mathbb{N}$, $m \in \mathbb{N}$

$\Rightarrow k - m \in \mathbb{N}$

Let's say $n = k - m$.

$\Rightarrow 1 = 2n$, $n \in \mathbb{N}$

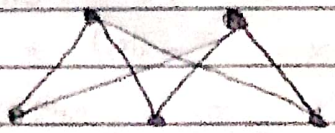
As $2n$ is an integer.

Hence does not exist any values in integers which makes $2n = 1$

$\therefore$ Our supposition is wrong.

$\therefore$ An integer can't be both given & odd.

(c) $K_{2,3}$



Smallest no of colors needed to color $K_{2,3}$ is 2.

$\therefore$ Its chromatic number is 2.

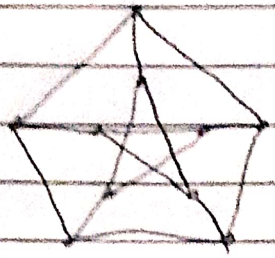
Date: 15/6/2021

K_4

Here each vertex vertex needs a new color since it is connected to all other vertices & they are connected to each other.

∴ Chromatic number of K_4 is 4.

5.4a



outer and inner vertices are different colors.