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Assignment :- 1

Chapter 1.2

1) The set A and C both contain element a, b, c, d and arrange in different sequence. Therefore $A = C$

The set B and D both contain element a, c, d, e and are arranged in different sequence. Therefore $B = D$

3) (a) No, 4 is an element and {4} is a set.

(b) The set {3, 4, 3, 5} contains three elements 3, 4, 5.

(c) The set contains 3 elements

5) The set A and D are equivalent because they contain the same no. of elements. Hence A is equal to D

7) (a) $\{-1, +1\}$

(b) $\{0, 2\}$

(c) $\emptyset$

(d) $\{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$

(e) $\emptyset$

(f) $\{\dots; -2, -1, 0, 1, 2, 3\}$

9) (a) $3 \in \{1, 2, 3\}$

(b) $1 \not\subseteq \{1\}$

(c) $\{2\} \not\subseteq \{1, 2\}$

(d) $\{3\} \in \{1, \{2\}, \{3\}\}$

(e) $1 \in \{1\}$

(f) $\{2\} \not\subseteq \{1, \{2\}, \{3\}\}$

(g) $\{1\} \subseteq \{1, 2\}$

(h) $1 \not\subseteq \{\{1\}, 2\}$

(i) $\{1\} \subseteq \{1, \{2\}\}$

(j) $\{1\} \subseteq \{1\}$

11) (a) 8 elements $\{(w,a),(w,b),(x,a),(x,b),$
(b) 8 elements $(y,a),(y,b),(z,a),(z,b),$
$\}$
(c) 16 elements $\{(a,a),(a,b),(b,a),(b,b)\}$
(d) 4 elements

13) (a) $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$
$A \times (B \times C) = \{(a,b) \mid a \in A \text{ and } b \in B \times C\}$
$(A \times B) \times C = \{(a,b) \mid a \in A \times B \text{ and } b \in C\}$
$A \times B \times C = \{(a,b,c) \mid a \in A,\ b \in B \text{ and }$
$c \in C\}$

$A \times (B \times C) = \{(1,(u,m)),(1,(u,n)),$
$(2,(u,m)),(2,(u,n)),(3,(u,m)),$
$(3,(u,n))\}$

(b) $(A \times B) \times C = \{((1,u),m),((1,u),n),$
$((2,u),m),((2,u),n),$
$((3,u),m),((3,u),n)\}$

(c) $A \times B \times C = \{(1,u,m), (1,u,n), (2,u,m), (2,u,n), (3,u,m), (3,u,n)\}$

15) $S = \{0,1\}$

$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

$S \times S \times S \times S = \{(a,b,c,d) \mid a \in S, b \in S, c \in S, d \in S\}$

$= \{(0,0,0,0), (0,0,0,1), (0,0,1,0), (0,1,0,0), (1,0,0,0), (0,0,1,1), (0,1,1,0), (1,1,0,0), (0,1,0,1), (1,0,1,0), (1,0,0,1), (0,1,1,1), (1,1,1,0), (1,0,1,1), (1,1,0,1), (1,1,1,1)\}$

Thus, the strings of length 4 over S that contain three or more 0's are

0000, 0001, 0010, 0100, 1000

Chapter 1.3

1) (a) No, because $6/4 = 3/2$ which is not an integer.

Yes because $8/4 = 2$ is an integer

No because $8/3$ is not an integer

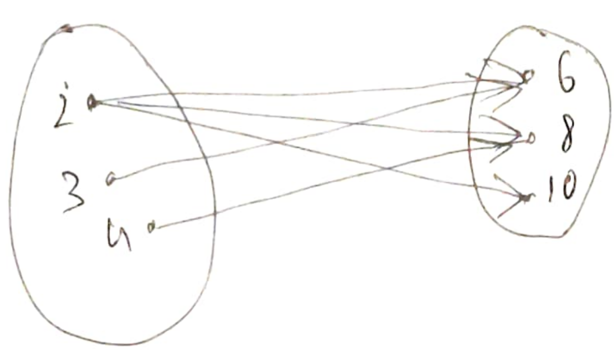
Yes because $10/2 = 5$ is an integer

(b) $R = \{(2,6), (2,8), (2,10), (3,6), (4,8)\}$

(c) Domain $= \{2, 3, 4\}$

Codomain $= \{6, 8, 10\}$

(d)



3) (a) For $(3,0) \in E \times F$

$$\frac{3-0}{3} = 1 \in \mathbb{Z}$$

Thus, $(3,0) \in T$

For $(1,-1) \in E \times F$

$$\frac{1-(-1)}{3} = 2/3 \notin \mathbb{Z}$$

Thus, $(1,-1) \notin T$

For $(2,-1) \in E \times F$,

$$\frac{2-(-1)}{3} = 1 \in \mathbb{Z}$$

Thus, $(2,-1) \in T$

For $(3,-2) \in E \times F$

$$\frac{3-(-2)}{3} = 5/3 \notin \mathbb{Z}$$

Thus, $(3,-2) \notin T$

(b) $E \times F = \{(1,-2), (2,-2), (3,-2),$
$(1,-1), (2,-1), (3,-1), (1,0), (2,0),$
$(3,0)\}$

For $(1,-2) \in E \times F$

$$\frac{1-(-2)}{3} = 1 \in Z$$

For $(2,-2) \in E \times F$

$$\frac{2-(-2)}{3} = 4/3 \notin Z$$

For $(3,-2) \in E \times F$

$$\frac{3-(-2)}{3} = 5/3 \notin Z$$

For $(1,-1) \in E \times F$

$$\frac{1-(-1)}{3} = 2/3 \notin Z$$

For $(2,-1) \in E \times F$

$$\frac{2-(-1)}{3} = 1 \in Z$$

For $(3,-1) \in E \times F$,

$$\frac{3-(-1)}{3} = 4/3 \notin \mathbb{Z}$$

For $(1,0) \in E \times F$

$$\frac{1-0}{3} = 1/3 \notin \mathbb{Z}$$ For $(2,0) \in E \times F$,

$$\frac{2-0}{3} = 2/3 \notin \mathbb{Z}$$

For $(3,0) \in E \times F$

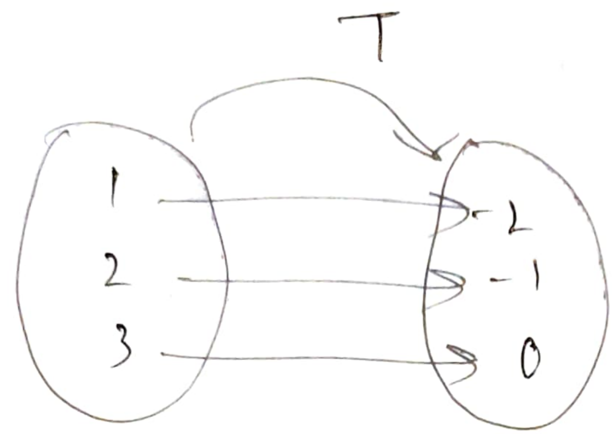
$$\frac{3-0}{3} = 1 \in \mathbb{Z}$$

Thus, there are only three ordered pairs which satisfy the condition for T.

$T = \{(1,-2),(2,-1),(3,0)\}$

(c) Domain of $T = \{1,2,3\}$

Co-domain of $T = \{-2,-1,0\}$



5) (a) $(x,y) \in S$ if $x \geq y$

$(2,1)$. Since $2 \geq 1$ is true, $(2,1) \in S$

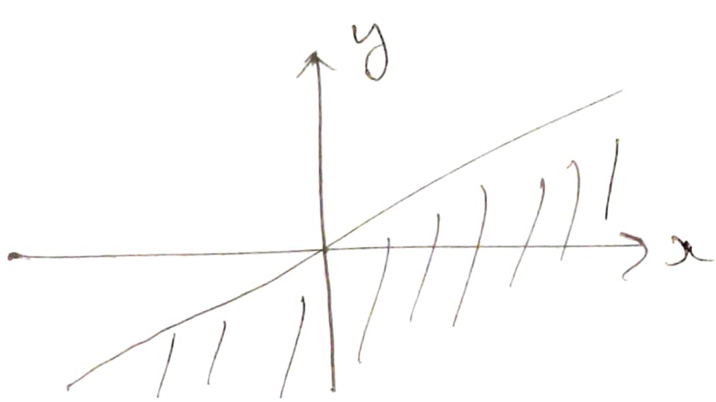
$(2,2)$. Since $2 \geq 2$ is true, $(2,2) \in S$

$(2,3)$. Since $2 \geq 3$ is false, $(2,3) \notin S$

$(-1,-2)$. Since $-1 \geq -2$ is true

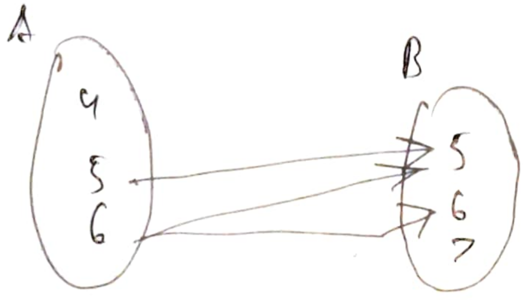
$(-1,-2) \in S$ or

(x,y) such that $x \geq y$

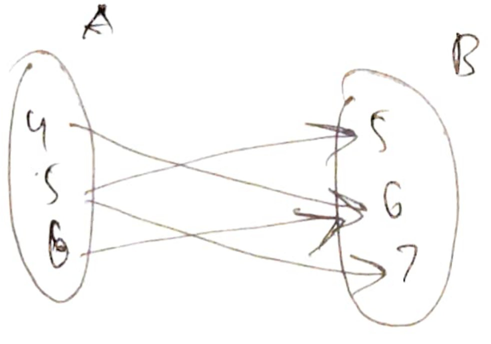


7) (a) $A \times B = \{(4,5), (4,6), (4,7), (5,5), (5,6), (5,7), (6,5), (6,6), (6,7)\}$

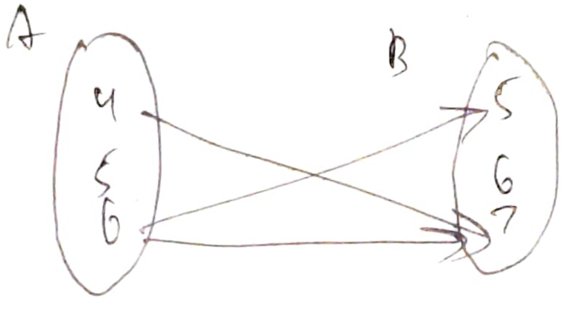
$R = \{(6,5), (5,5), (6,6)\}$



$S = \{(4,6), (5,7), (6,6), (5,5)\}$



$T = \{(4,7), (6,5), (6,7)\}$



(b) R is not a function because in domain the element 4 has no image

S in not a function because $(5,5) \in S$ and $(5,7) \in S$ but $5 \neq 7$

T is not a function because in domain the element 5 has no image

11. $A = \{0,1,2\}$

$(s,n) \in L$ means that the length of s is n.

(i.e,) $L(s) = n$

$L(0201) = 4$, since $(0201, 4) \in L$

$L(12) = 2$, since $(12, 2 \in L)$

Thus, $L(0201) = 4$, $L(12) = 2$

Chapter 2.1

5) (a) The sentence is a statement because it fact stated that 1024 is the smallest 4 digit no. which is a perfect square is true.

(b) This sentence is not a statement as its truthfulness or falseness depends on the person it is being referred to which cannot be known.

(c) This sentence is a statement because it can be easily verified that this is true

(d) This sentence is not a statement because it cannot be verified.

6) (a) $s \wedge i$

(b) $\sim s \wedge \sim i$

(a) $(h \wedge w) \wedge \sim s$

(b) $\sim w \wedge (h \wedge s)$

(c) $\sim w \wedge \sim h \wedge \sim s$

(d) $(\sim w \wedge \sim s) \wedge h$

(e) $w \wedge \sim (h \wedge s)$

12)

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

(14)

13)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee q$	$\sim(p \wedge q) \vee (p\vee$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

16)

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

17)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T

From the above truth table, the statement forms $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values.
Hence, the statement forms $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent

18)

p	t	$p \vee t$
T	T	T
F	T	T

From the last column, either true or false $p \vee t$ is true
From the truth table, $p \vee t$ and t have the same truth values.
So. they are logically equivalent.

19)

p	t	$p \wedge t$
T	T	T
F	T	F

From the truth table, $p \wedge t$ and p have the same truth values; so they are logically equivalent.

20)

p	c	$p \wedge c$	$p \vee c$
T	F	F	T
F	F	F	F

$(p \wedge c) \not\equiv (p \vee c)$

So they have different truth values and hence they are not logically equivalent.

24).

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	F	F	F

$(p \vee q) \wedge r$
T
F
T
F
T
F
F
F

$(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ have different truth values. Hence they are not logically equivalent.

26) Sam is not an ~~orange~~ orange belt, or Kate is not a red belt

27) p : the connector is loose

q : the machine is unplugged.

$\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$

The connection is not loose, and the machine is not unplugged.

30) The dollar is not at an all time a high, or the stock market is not at a record low.

31) ~~(a)~~ a = "the first character of s is 0"

b = "the first character of s is 1"

c = "the second character of s is 1"

d = "the second character of s is 2"

(a) $a \vee b$ means that the first character of s can be any of 0 or 1

$c \vee d$ means that the second character of s can be any of 1 or 2.

$\Rightarrow$ Set of strings = {01, 02, 11, 12}

(b) $a \vee b$ means that the first character of s can be any of 0 or 1.

$\sim(a \vee b)$ means that the first

character of s will be 2.

$c \vee d$ means that the second character of s can be any of 1 or 2.

$\Rightarrow$ Set of strings = {21, 22}

(c) $\sim a$ means that the first character of s can be any of 1 or 2.

$(\sim a) \vee b$ means that the first character of s can be any of 1 or 2.

$\sim d$ means that the second character of s can be any of 0 or 1.

$c \vee (\sim d)$ means that the second character of s can be any of 0 or 1.

$\Rightarrow$ Set of strings = {10, 11, 20, 21}

32) $-2 < x$ and $x < 7$

$p : -2 < x$
$q : x < 7$

Thus the negation of the statement: $(p \wedge q)$ is $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim p : -2 \geq x$

$\sim q : x \geq 7$

Negation of the given statement is,

$-2 \geq x$ or $x \geq 7$

35) $p : x \leq -1$

$q : x > 1$

The statement is equivalent to $p \vee q$

$\sim(p \vee q) = \sim p \wedge \sim q$

$\sim p : x > -1$

$\sim q : x \leq 1$

Negation :- $-1 < x \leq 1$

41)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

$(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

43) ~~Clay~~

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

$(\sim p \vee q) \vee (p \wedge \sim q)$ is a tautology.

44) (a) $2 < x < 0$

$\Rightarrow 2 < x$ and $x \leq 0$

$\Rightarrow 2 < x$ and $(x < 0$ or $x = 0)$

$\Rightarrow x > 2$ and $(x < 0$ or $x = 0)$

No real numbers satisfy the inequality.

(b) $1 \leq x < -1$

$1 \leq x$ and $x < -1$

$(1 < x$ or $x = 1)$ and $x < -1$

$(x > 1$ or $x = 1)$ and $x < -1$

No real numbers satisfy the inequality.