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MATH-245
TEST #2

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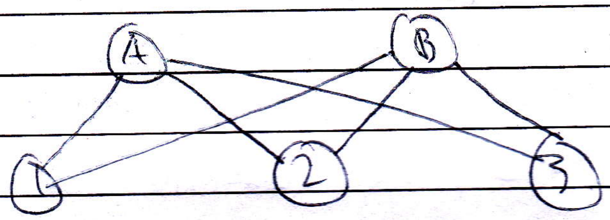
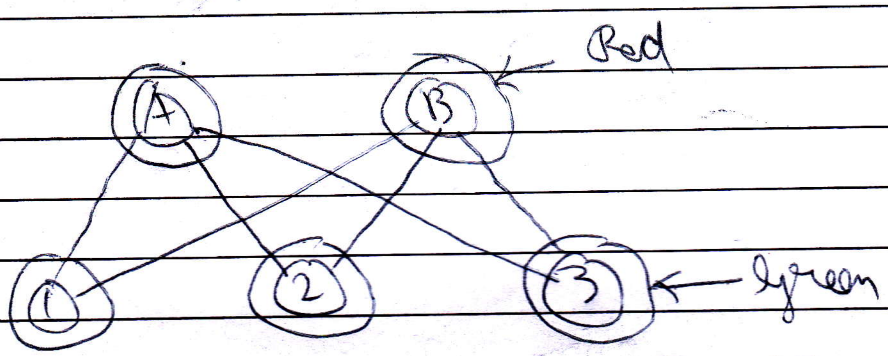
1. Ans (a) $K_{2,3}$ GRAPH



Chromatic number



(b) 9 vertices have degree 5, that have each of them getting connected to other 5 in the graph.

5 vertices have degree 9, that have each of them getting connected to other 9 in the graph.

Total number of edges of $K_{5,9}$ are 45, because there are edges between 5 vertices to other set of 9 vertices, i.e. $5 \times 9 = 45$.

2\. Ans (a) Let for some natural number 'n', n^2-2 is divisible by 4., i.e. $n^2-2 = 4q$, for some $q \in Z$.

$\Rightarrow n^2 = 4q+2$

$\Rightarrow n^2 = 2(2q+1)$, which is not possible.

Thus we have a contradiction.

$\therefore$ Our assumption was wrong & (n^2-2) is not divisible by 4 for any natural number 'n'.

(b) Let $(5+7\sqrt{2})$ is not irrational, let $(5+7\sqrt{2})$ be rational.

$\Rightarrow (5+7\sqrt{2}) = \frac{p}{q}$, for some co-prime integers 'p' and 'q'.

$\Rightarrow 7\sqrt{2} = \frac{p}{q} - 5 = \frac{p-5q}{q}$

$\Rightarrow \sqrt{2} = \frac{p-5q}{7q}$

R.H.S is a rational number because the sum, product, quotient of two rational numbers is always rational;

but, L.H.S is an irrational number.

P.T.O

This is a contradiction, as a rational and irrational number can never be equal.

$\therefore$ Our assumption was wrong and $(\sqrt{5}+7\sqrt{2})$ is an irrational number.

3\. Ans. (a) If the square of a number is an even integer, then the number is an even integer.

<u>Proof</u> :- Let the number a is not even.

Since a is not even $\Rightarrow$ a is odd

$\Rightarrow a = 2n+1 \; ; \; n \in \mathbb{Z}$

$$\Rightarrow a^2 = (2n+1)^2 = 4n^2+1+4n$$
$$= 4n^2+4n+1$$
$$= \underbrace{2(2n^2+2n)}_{\in \mathbb{Z}}+1$$

Let $k = 2n^2+2n \in \mathbb{Z}$

$\Rightarrow 2k+1 \; ; \; k \in \mathbb{Z}$

$\Rightarrow a^2$ is odd.

a is even.

(b) If $x+y<20$ then $x<10$ and $y<10$.

P.T.O

Proof :- Let $x \geq 10$ and $y \geq 10$

$= x + y \geq 20$

$= x < 10$ and $y < 10$

4. Ans :- Will come back later. Takes time to think.

5. (a) Ans. $a_n = 2^n - 1,\ n \geq 1$

First five terms of this sequence is for $n = 1, 2, 3, 4, 5$.

$a_1 = 2^1 - 1 = 1$
$a_2 = 2^2 - 1 = 3$
$a_3 = 2^3 - 1 = 7$
$a_4 = 2^4 - 1 = 15$
$a_5 = 2^5 - 1 = 31$

First five terms are :- 1, 3, 7, 15 & 31

(b) $b_n = b_{n-1} + b_{n-2},\ b_1 = 1,\ b_2 = 2,\ n \geq 3$

$b_3 = b_2 + b_1$

$= 3$

$b_4 = b_3 + b_2$
$= 3 + 2 = 5$

$b_5 = b_4 + b_3$

$= 8$

$b_6 = b_5 + b_4$

$= 13$

$b_7 = b_6 + b_5$

$= 21$

Given terms are :- $1, 2, 3, 5, 8, 13, 21, \dots$

6. Ans $9^3 + 9^4 + \dots\dots + 9^n = \frac{9(9^n - 81)}{8}, n \geq 3$

for $n = 3$

$9^3 = \frac{9(9^3 - 81)}{8}$

$= \frac{9 \cdot (81 \times 8)}{8}$

$9^3 = 9 \cdot 81$

Let it be true for $n = K$

$9^3 + 9^4 + \dots\dots + 9^k = \frac{9(9^k - 81)}{8}$ — (1)

Now for $n = k+1$

$9^3 + 9^4 + \dots\dots + 9^{k+1} = 9^3 + 9^4 + \dots + 9^k + 9^{k+1}$

From eqn (1) :-

$9^3 + 9^4 + \dots\dots + 9^k = \frac{9(9^k - 81)}{8}$

$$9^3 + 9^4 + \dots + 9^{k+1} = \frac{9(9^k - 81)}{8} + 9^{k+1}$$

$$= \frac{9 \cdot 9^k - 9^3}{8} + 9^{k+1}$$

$$= \frac{9^{k+1} - 9^3 + 8 \cdot 9^{k+1}}{8}$$

$$9^3 + 9^4 + \dots + 9^{k+1} = \frac{9(9^k - 9^2 + 8 \cdot 9^k)}{8}$$

$$9^3 + 9^4 + \dots + 9^n = \frac{9(9^k - 9^2 + 8 \cdot 9^k)}{8}$$

$$= \frac{9(9^k(1+8) - 9^2)}{8}$$

$$= \frac{9(9 \cdot 9^k - 9^2)}{8}$$

$$9^3 + 9^4 + \dots + 9^n = \frac{9(9^{k+1} - 9^2)}{8}$$

Hence, it is also true for $n = k+1$

(1) Hence, by induction :-

$$9^3 + 9^4 + \dots + 9^n = \frac{9(9^n - 81)}{8}$$

$\boxed{+ 9^{k+1}}$

2.(b) $a_n = 2a_{n-1} + 5$, $a_0 = 1$

If $n = 1$, $a_1 = 2a_0 + 5 = 7$

If $n = 2$, $a_2 = 2a_1 + 5 = 19$

 P.T.O.

Claim $a_n = 6(2^n) - 5$

Proof by induction :-

If $n=1$, then L.H.S $= a_1 = 7$

R.H.S $= 6(2^1) - 5 = 7$

L.H.S = R.H.S for $n=1$.

$\therefore$ Result holds for case $n=1$.

By induction hypothesis, assume result is true.

For $n=k$, i.e. $a_k = 6(2^k) - 5$ —— (2)

Now, we prove result for $n=k+1$

If we put $n=k+1$ in given data :-

We get :-

$$a_{k+1} = 2a_k + 5$$

Now, eqn. (2) (i.e values of a_k)

$$\therefore a_{k+1} = 2(6(2^k) - 5) + 5$$

$$a_{k+1} = 6(2^{k+1}) - 10 + 5$$

$$a_{k+1} = 6(2^{k+1}) - 5$$

Which is the desired result.

P.T.O

Hence, result is true for $n = k+1$.

$\therefore$ By induction hypothesis:-

$$a_n = 6(2^n) - 5 \quad \forall\, n \geq 0$$

8 Ans Let $P(n) = 7^n - 1$

To prove:- $P(n)$ is divisible by 6.

$P(1) = 7^1 - 1 = 6$, which is divisible ~~by~~ by 6

As $P(1)$ is true.

(2) Assumption:-

$P(m)$ is true, i.e.

$P(m) = 7^m - 1 = 6k$ (where k is some positive integer)

To prove $P(m+1)$ is also true for the valid assumption.

$P(m+1) = 7^{m+1} - 1 = 7^m \times 7 - 1$

$\Rightarrow (6k+1) \times 7 - 1$, from $P(m)$ using $7^m = 6k+1$

$\Rightarrow 6 \times 7k + 7 - 1 = 6 \times 7k + 6$

$\Rightarrow 6(7k+1) = 6k'$ (where k' is some positive integer)

P.T.O

Since we can show that $P(m+1)$ is also a multiple of 6 or divisible by 6.

$\therefore$ Our assumption is true.

This implies that $P(n) = 7^n - 1$ is always divisible by 6, for n belongs to natural numbers.

9. Ans The statement is true for $n=1$, because

$$1^2 = 1 = \frac{1(1+1)(2.1+1)}{6}$$

Let the statement be true for some natural number k.

Then, $1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Now, $1^2 + 2^2 + \dots + k^2 + (k+1)^2$

$$\Rightarrow \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$\Rightarrow k(k+1)(2k+1) + 6(k+1)^2$$

$$\Rightarrow \frac{(k+1)\{2k^2+7k+6\}}{6}$$

$$\Rightarrow \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\Rightarrow \frac{(k+1)\{(k+1)+1\}\{2(k+1)+1\}}{6}$$

P.T.O

This shows that the statement is true for the natural number $(k+1)$, if it is true for k.

Also, the statement is true for $n = 1$.

By the principal of mathematical induction, the statement is true for all $n \geq 1$.

7. Ans (a) This statement always holds true without 2.

If $P=2$, then $P^2-1 = 2^2-1$
$= 4-1$
$= 3$ which is an odd number.

(b) If we choose two irrational numbers, say $\sqrt{8}$ & $\sqrt{2}$.

Then, their quotient is $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$

which is a rational number.

(c) If we choose any ~~___~~ decimal number between 0 and 1, the answer will always come out to be greater than the number itself.

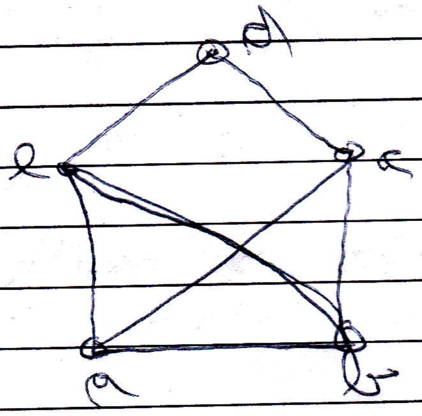
$\frac{1}{25}$

$\Rightarrow \sqrt{\frac{1}{25}} = \frac{1}{5}$

$\therefore \frac{1}{5} > \frac{1}{25}$

 P.T.O

(d) The shown graph has 7 vertices.



$\deg(a) = \deg(b) = \deg(c) = \deg(e) = 3$

but $\deg(d) = 2 \neq 3$