

Food Model

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1 Crop Producer

1.1 Problem

$$\begin{aligned} \text{Maximize} \quad : \quad \sum_{y \in Y} \text{df}_y \left\{ \sum_{f \in C} \left(\pi_{ynf}^F \mathcal{Q}_{ynf}^F - \mathcal{C}_{ynf}^F A_{ynf}^F - \frac{1}{2} \mathcal{C}_{yn}^{\text{change}} \left(A_{ynf}^F - A_{(y-1)nf}^F \right) \right)^2 \right. \\ \left. - \mathcal{C}_{yn}^{\text{conv}} \sum_{f \in C} \left(A_{ynf}^F - A_{(y-1)nf}^F \right) \right\} \end{aligned} \quad (1.1)$$

such that for $f \in C$

$$\mathcal{Q}_{ynf}^F, A_{ynf}^F \geq 0 \quad (1.2a)$$

$$A_n \geq \sum_{f \in C} A_{ynf}^F \quad (\delta_{yn}^1) \quad (1.2b)$$

$$\mathcal{Q}_{ynf}^F \leq \mathcal{Y}_{ynf} A_{ynf}^F \quad (\delta_{ynf}^2) \quad (1.2c)$$

1.2 KKT Conditions

These KKT conditions hold for $f \in C$

$$\delta_{ynf}^2 - \text{df}_y \pi_{ynf}^F \geq 0 \quad (\mathcal{Q}_{ynf}^F) \quad (1.3a)$$

$$\begin{aligned} \delta_{yn}^1 + \text{df}_y \left(\mathcal{C}_{ynf}^F + \mathcal{C}_{yn}^{\text{conv}} - \mathcal{C}_{(y+1)n}^{\text{conv}} + \mathcal{C}_{yn}^{\text{change}} A_{ynf}^F + \mathcal{C}_{(y+1)n}^{\text{change}} A_{ynf}^F \right) \\ - \delta_{ynf}^2 \mathcal{Y}_{ynf} - \text{df}_y \left(\mathcal{C}_{yn}^{\text{change}} A_{(y-1)nf}^F + \mathcal{C}_{(y+1)n}^{\text{change}} A_{(y+1)nf}^F \right) \geq 0 \quad (A_{ynf}^F) \end{aligned} \quad (1.3b)$$

2 Livestock producer

2.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y \in Y \\ f \notin C}} \text{df}_y \left(\pi_{ynf}^F \mathcal{Q}_{ynf}^F + p_{yn}^H \mathcal{Q}_{yn}^H - \sum_{i \in N} (\mathcal{C}_{yin}^{\text{cow,trans}} + \pi_{yi}^{\text{cow}}) \mathcal{B}_{yin}^{\text{buy}} - \mathcal{B}_{yn} \mathcal{C}_{yn}^{\text{cow}} \right) \quad (2.1)$$

such that for $f \notin C$

$$\mathcal{B}_{yn}, \mathcal{B}_{yin}^{\text{buy}}, \mathcal{Q}_{ynf}^F, \mathcal{Q}_{yn}^H, \mathcal{B}_{yn}^{\text{slg}} \geq 0 \quad (2.2a)$$

$$\mathcal{Q}_{ynf}^F \leq \mathcal{Y}_{ynf} \mathcal{B}_{yn} \quad (f = \text{Milk}) \quad (\delta_{ynf}^2)$$

$$\mathcal{Q}_{ynf}^F \leq \mathcal{Y}_{ynf} \mathcal{B}_{yn}^{\text{slg}} \quad (f = \text{Beef}) \quad (\delta_{ynf}^2)$$

$$\mathcal{Q}_{yn}^H \leq \mathcal{Y}_{yn}^H \mathcal{B}_{yn}^{\text{slg}} \quad (\delta_{yn}^3) \quad (2.2b)$$

$$\mathcal{B}_{yn}^{\text{slg}} \leq \mathcal{B}_{yn} \quad (\delta_{yn}^4) \quad (2.2c)$$

$$\mathcal{B}_{yn} \leq (1 + k - \kappa) \mathcal{B}_{(y-1)n} - \mathcal{B}_{yn}^{\text{slg}} + \sum_{i \in N} (\mathcal{B}_{yin}^{\text{buy}} - \mathcal{B}_{yni}^{\text{buy}}) \quad (\pi_{yn}^{\text{cow}}) \quad (2.2d)$$

$$\mathcal{B}_{yn}^{\text{slg}} \geq \kappa_{yn}^{\text{death}} \mathcal{B}_{yn} \quad (\delta_{yn}^9) \quad (2.2e)$$

$$\mathcal{B}_{yn} \geq \mathcal{B}_n^{\text{herd}} \quad (\delta_{yn}^{10}) \quad (2.2f)$$

2.2 KKT Conditions

$$\left. \begin{aligned} & \text{df}_y \mathcal{C}_{yn}^{\text{cow}} - \delta_{ynf}^2 \mathcal{Y}_{ynf} - \delta_{yn}^4 + \kappa_{yn}^{\text{death}} \delta_{yn}^9 \\ & - \delta_{yn}^{10} + \pi_{yn}^{\text{cow}} - (1 + k - \kappa) \pi_{(y+1)n}^{\text{cow}} \end{aligned} \right\} \geq 0 \quad (f = \text{Milk}) \quad (\mathcal{B}_{yn}) \quad (2.3a)$$

$$\text{df}_y (\mathcal{C}_{yin}^{\text{cow,trans}} + \pi_{yi}^{\text{cow}}) + (\pi_{yi}^{\text{cow}} - \pi_{yn}^{\text{cow}}) \geq 0 \quad (\mathcal{B}_{yin}^{\text{buy}}) \quad (2.3b)$$

$$\delta_{ynf}^2 - \text{df}_y \pi_{ynf}^F \geq 0 \quad (\mathcal{Q}_{ynf}^F)$$

$$\delta_{yn}^3 - \text{df}_y p_{yn}^H \geq 0 \quad (\mathcal{Q}_{yn}^H) \quad (2.3c)$$

$$\delta_{yn}^4 - \delta_{yn}^{10} - \delta_{ynf}^2 \mathcal{Y}_{ynf} - \delta_{yn}^3 \mathcal{Y}_{yn}^H + \pi_{yn}^{\text{cow}} \geq 0 \quad (f = \text{Beef}) \quad (\mathcal{B}_{yn}^{\text{slg}}) \quad (2.3d)$$

3 Distribution

3.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y \in Y \\ f \in F}} \text{df}_y \left\{ \sum_{n \in N} \left(\mathcal{Q}_{ynf}^{D_s} \pi_{ynf}^S - \mathcal{Q}_{ynf}^{D_b} \pi_{ynf}^F \right) - \sum_{r \in R} \mathcal{C}_{yrf}^R \mathcal{Q}_{yrf}^D \right\} \quad (3.1)$$

such that

$$\mathcal{Q}_{ynf}^{D_b}, \mathcal{Q}_{yrf}^D, \mathcal{Q}_{ynf}^{D_s} \geq 0 \quad (3.2a)$$

$$\mathcal{Q}_{ynf}^{D_b} + \sum_{r \in R_{\text{in}}} \mathcal{Q}_{yrf}^D \geq \mathcal{Q}_{ynf}^{D_s} + \sum_{r \in R_{\text{out}}} \mathcal{Q}_{yrf}^D \quad (\delta_{ynf}^6) \quad (3.2b)$$

$$\mathcal{Q}_{yrf}^D \leq \mathcal{Q}_{yrf}^{R, \text{CAP}} \quad (\delta_{yrf}^7) \quad (3.2c)$$

3.2 KKT Conditions

Representing s_r and d_r as the source and destination nodes of the transport system $r \in R$, we have the following KKT conditions.

$$\text{df}_y \pi_{ynf}^F - \delta_{ynf}^6 \geq 0 \quad (\mathcal{Q}_{ynf}^{D_b}) \quad (3.3a)$$

$$\delta_{yrf}^7 + \text{df}_y \mathcal{C}_{yrf}^R + \delta_{ys_rf}^6 - \delta_{yd_rf}^6 \geq 0 \quad (\mathcal{Q}_{yrf}^D) \quad (3.3b)$$

$$\delta_{ynf}^6 - \text{df}_y \pi_{ynf}^S \geq 0 \quad (\mathcal{Q}_{ynf}^{D_s}) \quad (3.3c)$$

4 Storage

4.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y \in Y \\ f \in F}} \left(\pi_{ynf}^U - \pi_{ynf}^S - \frac{1}{2} \mathcal{C}_{ynf}^{Sq} Q_{ynf}^S - \mathcal{C}_{ynf}^{Sl} \right) Q_{ynf}^S \quad (4.1)$$

such that

$$Q_{ynf}^S \geq 0 \quad (4.2a)$$

$$Q_{ynf}^S \leq Q_{ynf}^{S, \text{CAP}} \quad (\delta_{ynf}^8) \quad (4.2b)$$

4.2 KKT Conditions

$$\pi_{ynf}^S - \pi_{ynf}^U + \mathcal{C}_{ynf}^{Sq} Q_{ynf}^S + \mathcal{C}_{ynf}^{Sl} + \delta_{ynf}^8 \geq 0 \quad (Q_{ynf}^S) \quad (4.3a)$$

5 Market Clearing

$$\mathcal{Q}_{ynf}^F = \mathcal{Q}_{ynf}^{D_b} \quad (\pi_{ynf}^F) \quad (5.1a)$$

$$\pi_{ynf}^U = \alpha_{ynf} - \beta_{ynf} \mathcal{Q}_{ynf}^S + \sum_{i \in F} \chi_{ynfi} \pi_{yni}^U \quad (\pi_{ynf}^U) \quad (5.1b)$$

$$\mathcal{Q}_{ynf}^S = \mathcal{Q}_{ynf}^{D_s} \quad (\pi_{ynf}^S) \quad (5.1c)$$