Food Model

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1 Crop Producer

The crop producer makes decisions on the land allotted for various crops. He has an "expectation" for the price of crops and their yield.

1.1 Problem

$$\text{Maximize} \quad : \quad \sum_{y\in Y} \mathsf{df}_y \left\{ \sum_{f\in C} \left(\pi^F_{ynf} \mathcal{Q}^F_{ynf} - \mathscr{C}^F_{ynf} A^F_{ynf} - \frac{1}{2} \mathscr{C}^{\text{change}}_{yn} \left(A^F_{ynf} - A^F_{(y-1)nf} \right)^2 \right) - \mathscr{C}^{\text{conv}}_{yn} \sum_{f\in C} \left(A^F_{ynf} - A^F_{(y-1)nf} \right) \right\} \tag{1.1}$$

such that for $f \in C$

$$\mathcal{Q}^F_{ynf}, A^F_{ynf} \geq 0 \tag{1.2a}$$

Total area used for crops is less than land available:

$$A_n \geq \sum_{f\in C} A^F_{ynf} \qquad (\delta^1_{yn}) \tag{1.2b}$$

Total Quantity is yield times area for that crop, both in real terms and expected terms:

$$\mathcal{Q}^F_{ynf} \leq \mathcal{Y}_{ynf} A^F_{ynf} \qquad (\delta^2_{ynf}) \tag{1.2c}$$

Linking yield with climate yield factor:

$$\mathcal{Y}_{ynf} = \mathsf{aCYF} \left(\pi^F_{ynf} \right)^{\mathsf{e}} \tag{1.2d}$$

Fallow constraint where ${}_{\text{Fal}}\mathscr{O}$ is a number between 0 and 1 indicating the fraction of land that gets fallowed. ${}_{\text{Fal}}\mathscr{O}^{\text{Dur}}$ is the duration of fallow cycle:

$$\sum_{f\in C} \sum_{y'=y}^{y+{}_{\text{Fal}}\mathscr{O}^{\text{Dur}}} A^F_{y'nf} \leq \sum_{y'=y}^{y+{}_{\text{Fal}}\mathscr{O}^{\text{Dur}}} A_n - {}_{\text{Fal}}\mathscr{O} A_n \tag{1.2e}$$

Crop rotation constraint where f is the primary crop rotated with f':

$${}_{\text{Rot}}\mathscr{O} \sum_{y'=y}^{y+{}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'}} A^F_{y'nf} \leq {}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'} \sum_{y'=y}^{y+{}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'}} A^F_{y'nf'} \tag{1.2f}$$

2 Livestock producer

2.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\ f\notin C}} \mathsf{df}_y \left(\pi^F_{ynf} \mathcal{Q}^F_{ynf} + p^H_{yn} \mathcal{Q}^H_{yn} - \sum_{i\in N} (\mathscr{C}^{\text{cow,trans}}_{yin} + \pi^{\text{cow}}_{yi}) \mathcal{B}^{\text{buy}}_{yin} - \mathcal{B}_{yn} \mathscr{C}^{\text{cow}}_{yn} \right) \tag{2.1}$$

such that for $f \notin C$

$$\begin{aligned}
\mathcal{B}_{yn}, \mathcal{B}^{\text{buy}}_{yin}, \mathcal{Q}^F_{ynf}, \mathcal{Q}^H_{yn}, \mathcal{B}^{\text{slg}}_{yn} &\geq 0 && && \text{(2.2a)}\\
\mathcal{Q}^F_{ynf} &\leq \mathcal{Y}_{ynf} \mathcal{B}_{yn} && (f = \text{Milk}) \quad (\delta^2_{ynf}) &&\\
\mathcal{Q}^F_{ynf} &\leq \mathcal{Y}_{ynf} \mathcal{B}^{\text{slg}}_{yn} && (f = \text{Beef}) \quad (\delta^2_{ynf}) &&\\
\mathcal{Q}^H_{yn} &\leq \mathcal{Y}^H_{yn} \mathcal{B}^{\text{slg}}_{yn} && (\delta^3_{yn}) && \text{(2.2b)}\\
\mathcal{B}^{\text{slg}}_{yn} &\leq \mathcal{B}_{yn} && (\delta^4_{yn}) && \text{(2.2c)}\\
\mathcal{B}_{yn} &\leq (1 + k - \kappa) \mathcal{B}_{(y-1)n} - \mathcal{B}^{\text{slg}}_{yn} + \sum_{i\in N} \left(\mathcal{B}^{\text{buy}}_{yin} - \mathcal{B}^{\text{buy}}_{yni} \right) && (\pi^{\text{cow}}_{yn}) && \text{(2.2d)}\\
\mathcal{B}^{\text{slg}}_{yn} &\geq \kappa^{\text{death}}_{yn} \mathcal{B}_{yn} && (\delta^9_{yn}) && \text{(2.2e)}\\
\mathcal{B}_{yn} &\geq \mathcal{B}^{\text{herd}}_n && (\delta^{10}_{yn}) && \text{(2.2f)}
\end{aligned}$$

3 Distribution

3.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\ f\in F}} \mathsf{df}_y \left\{ \sum_{n\in N} \left(\mathcal{Q}^{D_s}_{ynf} \pi^S_{ynf} - \mathcal{Q}^{D_b}_{ynf} \pi^F_{ynf} \right) - \sum_{r\in R} \mathscr{C}^R_{yrf} \mathcal{Q}^D_{yrf} \right\} \tag{3.1}$$

such that

$$\begin{aligned}
\mathcal{Q}^{D_b}_{ynf}, \mathcal{Q}^D_{yrf}, \mathcal{Q}^{D_s}_{ynf} &\geq 0 && && \text{(3.2a)}\\
\mathcal{Q}^{D_b}_{ynf} + \sum_{r\in R_{\text{in}}} \mathcal{Q}^D_{yrf} &\geq \mathcal{Q}^{D_s}_{ynf} + \sum_{r\in R_{\text{out}}} \mathcal{Q}^D_{yrf} && (\delta^6_{ynf}) && \text{(3.2b)}\\
\mathcal{Q}^D_{yrf} &\leq \mathcal{Q}^{R,\text{CAP}}_{yrf} && (\delta^7_{yrf}) && \text{(3.2c)}
\end{aligned}$$

4 Storage

4.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\ f\in F}} \left(\pi^U_{ynf} - \pi^S_{ynf} - \frac{1}{2} \mathscr{C}^{Sq}_{ynf} \mathcal{Q}^S_{ynf} - \mathscr{C}^{Sl}_{ynf} \right) \mathcal{Q}^S_{ynf} \tag{4.1}$$

such that

$$\begin{aligned}
\mathcal{Q}^S_{ynf} &\geq 0 && && \text{(4.2a)}\\
\mathcal{Q}^S_{ynf} &\leq \mathcal{Q}^{S,\text{CAP}}_{ynf} && (\delta^8_{ynf}) && \text{(4.2b)}
\end{aligned}$$

5 Market Clearing

$$\mathcal{Q}^F_{ynf} = \mathcal{Q}^{D_b}_{ynf} \qquad (\pi^F_{ynf}) \tag{5.1a}$$

$$\pi^U_{ynf} = \alpha_{ynf} - \beta_{ynf}\mathcal{Q}^S_{ynf} + \sum_{i \in F} \chi_{ynfi}\pi^U_{yni} \qquad (\pi^U_{ynf}) \tag{5.1b}$$

$$\mathcal{Q}^S_{ynf} = \mathcal{Q}^{D_s}_{ynf} \qquad (\pi^S_{ynf}) \tag{5.1c}$$

6 Electricity

6.1 Problem

We model electricity with a single operator. A subset of nodes can have production facilities. The production facilities have a minimum and maximum production capabilities. All nodes are connected by transmission lines, with a cost of transmission along each line. They have transmission capacities too. There is currently no possibility of expansion. The cost is minimized in the model.

$$\text{Minimize} \quad : \quad \sum_{\substack{y \in Y \\ n \in N}} \left(\mathscr{C}^{El}_{yn}\mathcal{Q}^{E-gen}_{yn} + \frac{1}{2}\mathscr{C}^{Eq}_{yn}\mathcal{Q}^{E-gen\,2}_{yn} \right) + \sum_{\substack{y \in Y \\ r \in R}} \mathscr{C}^{ER}_{yr}\mathcal{Q}^E_{yr} \tag{6.1}$$

such that

$$\mathcal{Q}^{E-gen}_{yn} \geq 0 \tag{6.2a}$$

Generator limits

$$\mathcal{Q}^{E-gen}_{yn} \geq \underline{\mathcal{Q}}^E_{yn} \tag{6.2b}$$

$$\mathcal{Q}^{E-gen}_{yn} \leq \overline{\mathcal{Q}}^E_{yn} \tag{6.2c}$$

The Kirchoff Current Law in each node. This also ensures that the demand is met in each node. $\mathscr{L}^E_{yr}$ is the transmission loss.

$$\mathcal{Q}^{E-gen}_{yn} + \sum_{r \in R_{\text{in}}} \mathcal{Q}^E_{yr}\left(1 - \mathscr{L}^E_{yr}\right) \geq \mathcal{Q}^{E-dem}_{yn} + \sum_{r \in R_{\text{out}}} \mathcal{Q}^E_{yr} \tag{6.2d}$$

Transmission capacity

$$\mathcal{Q}^E_{yr} \leq \overline{\mathcal{Q}}^E_{yr} \tag{6.2e}$$