Food Model

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1 Crop Producer

1.1 Problem

$$\text{Maximize} \quad : \quad \sum_{y \in Y} \mathsf{df}_y \left\{ \sum_{f \in C} \left(\pi^F_{ynf} \mathcal{Q}^F_{ynf} - \mathscr{C}^F_{ynf} A^F_{ynf} - \frac{1}{2} \mathscr{C}^{\text{change}}_{yn} \left(A^F_{ynf} - A^F_{(y-1)nf} \right) \right)^2 - \mathscr{C}^{\text{conv}}_{yn} \sum_{f \in C} \left(A^F_{ynf} - A^F_{(y-1)nf} \right) \right\} \tag{1.1}$$

such that for $f \in C$

$$\mathcal{Q}^F_{ynf},\ A^F_{ynf} \geq 0 \tag{1.2a}$$

$$A_n \geq \sum_{f \in C} A^F_{ynf} \qquad (\delta^1_{yn}) \tag{1.2b}$$

$$\mathcal{Q}^F_{ynf} \leq \mathcal{Y}_{ynf} A^F_{ynf} \qquad (\delta^2_{ynf}) \tag{1.2c}$$

1.2 KKT Conditions

These KKT conditions hold for $f \in C$

$$\delta^2_{ynf} - \mathsf{df}_y \pi^F_{ynf} \geq 0 \qquad (\mathcal{Q}^F_{ynf}) \tag{1.3a}$$

$$\left.\begin{aligned} \delta^1_{yn} + \mathsf{df}_y \left(\mathscr{C}^F_{ynf} + \mathscr{C}^{\text{conv}}_{yn} - \mathscr{C}^{\text{conv}}_{(y+1)n} + \mathscr{C}^{\text{change}}_{yn} A^F_{ynf} + \mathscr{C}^{\text{change}}_{(y+1)n} A^F_{ynf} \right) \\ - \delta^2_{ynf} \mathcal{Y}_{ynf} - \mathsf{df}_y \left(\mathscr{C}^{\text{change}}_{yn} A^F_{(y-1)nf} + \mathscr{C}^{\text{change}}_{(y+1)n} A^F_{(y+1)nf} \right) \end{aligned}\right\} \geq 0 \qquad (A^F_{ynf}) \tag{1.3b}$$

2 Livestock producer

2.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\ f\notin C}} \mathsf{df}_y \left(\pi^F_{ynf}\mathcal{Q}^F_{ynf} + p^H_{yn}\mathcal{Q}^H_{yn} - \sum_{i\in N}(\mathscr{C}^{\text{cow,trans}}_{yin} + \pi^{\text{cow}}_{yi})\mathcal{B}^{\text{buy}}_{yin} - \mathcal{B}_{yn}\mathscr{C}^{\text{cow}}_{yn} \right) \tag{2.1}$$

such that for $f \notin C$

$$\mathcal{B}_{yn},\ \mathcal{B}^{\text{buy}}_{yin},\ \mathcal{Q}^F_{ynf},\ \mathcal{Q}^H_{yn},\ \mathcal{B}^{\text{slg}}_{yn} \geq 0 \tag{2.2a}$$

$$\mathcal{Q}^F_{ynf} \leq \mathcal{Y}_{ynf}\mathcal{B}_{yn} \quad (f = \text{Milk}) \quad (\delta^2_{ynf})$$

$$\mathcal{Q}^F_{ynf} \leq \mathcal{Y}_{ynf}\mathcal{B}^{\text{slg}}_{yn} \quad (f = \text{Beef}) \quad (\delta^2_{ynf})$$

$$\mathcal{Q}^H_{yn} \leq \mathcal{Y}^H_{yn}\mathcal{B}^{\text{slg}}_{yn} \quad (\delta^3_{yn}) \tag{2.2b}$$

$$\mathcal{B}^{\text{slg}}_{yn} \leq \mathcal{B}_{yn} \quad (\delta^4_{yn}) \tag{2.2c}$$

$$\mathcal{B}_{(y+1)n} \leq (1+k-\kappa)\mathcal{B}_{yn} - \mathcal{B}^{\text{slg}}_{yn} + \sum_{i\in N}\left(\mathcal{B}^{\text{buy}}_{yin} - \mathcal{B}^{\text{buy}}_{yni}\right) \quad (\pi^{\text{cow}}_{yn}) \tag{2.2d}$$

2.2 KKT Conditions

$$\left.\begin{aligned} &\mathsf{df}_y\mathscr{C}^{\text{cow}}_{yn} - \delta^2_{ynf}\mathcal{Y}_{ynf} - \delta^4_{yn} \\ &+\pi^{\text{cow}}_{(y-1)n} - (1+k-\kappa)\pi^{\text{cow}}_{yn} \end{aligned}\right\} \geq 0 \quad (f = \text{Milk}) \quad (\mathcal{B}_{yn}) \tag{2.3a}$$

$$\mathsf{df}_y\left(\mathscr{C}^{\text{cow,trans}}_{yin} + \pi^{\text{cow}}_{yi}\right) + \left(\pi^{\text{cow}}_{yi} - \pi^{\text{cow}}_{yn}\right) \geq 0 \quad (\mathcal{B}^{\text{buy}}_{yin}) \tag{2.3b}$$

$$\delta^2_{ynf} - \mathsf{df}_y\pi^F_{ynf} \geq 0 \quad (\mathcal{Q}^F_{ynf})$$

$$\delta^3_{yn} - \mathsf{df}_y p^H_{yn} \geq 0 \quad (\mathcal{Q}^H_{yn}) \tag{2.3c}$$

$$\delta^4_{yn} - \delta^2_{ynf}\mathcal{Y}_{ynf} - \delta^3_{yn}\mathcal{Y}^H_{yn} + \pi^{\text{cow}}_{yn} \geq 0 \quad (f = \text{Beef}) \quad (\mathcal{B}^{\text{slg}}_{yn}) \tag{2.3d}$$

3 Distribution

3.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\ f\in F}} \mathsf{df}_y \left\{ \sum_{n\in N} \left(\pi^F_{ynf} \left(\mathcal{Q}^{D_s}_{ynf} - \mathcal{Q}^{D_b}_{ynf} \right) \right) - \sum_{r\in R} \mathscr{C}^R_{yrf} \mathcal{Q}^D_{yrf} \right\} \tag{3.1}$$

such that

$$\mathcal{Q}^{D_b}_{ynf},\ \mathcal{Q}^D_{yrf},\ \mathcal{Q}^{D_s}_{ynf} \quad \geq \quad 0 \tag{3.2a}$$

$$\mathcal{Q}^{D_b}_{ynf} + \sum_{r\in R_{\text{in}}} \mathcal{Q}^D_{yrf} \quad \geq \quad \mathcal{Q}^{D_s}_{ynf} + \sum_{r\in R_{\text{out}}} \mathcal{Q}^D_{yrf} \qquad (\delta^6_{ynf}) \tag{3.2b}$$

$$\mathcal{Q}^D_{yrf} \quad \leq \quad \mathcal{Q}^{R,\text{CAP}}_{yrf} \qquad (\delta^7_{yrf}) \tag{3.2c}$$

3.2 KKT Conditions

Representing s_r and d_r as the source and destination nodes of the transport system $r \in R$, we have the following KKT conditions.

$$\mathsf{df}_y \pi^F_{ynf} - \delta^6_{ynf} \quad \geq \quad 0 \qquad (\mathcal{Q}^{D_b}_{ynf}) \tag{3.3a}$$

$$\delta^7_{yrf} + \mathsf{df}_y \mathscr{C}^R_{yrf} + \delta^6_{ys_rf} - \delta^6_{yd_rf} \quad \geq \quad 0 \qquad (\mathcal{Q}^D_{yrf}) \tag{3.3b}$$

$$\delta^6_{ynf} - \mathsf{df}_y \pi^F_{ynf} \quad \geq \quad 0 \qquad (\mathcal{Q}^{D_s}_{ynf}) \tag{3.3c}$$

4 Storage

4.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y \in Y \\ f \in F}} \left(\pi^{U}_{ynf} - \pi^{F}_{ynf} - \frac{1}{2} \mathscr{C}^{Sq}_{ynf} \mathcal{Q}^{S}_{ynf} - \mathscr{C}^{Sl}_{ynf} \right) \mathcal{Q}^{S}_{ynf} \tag{4.1}$$

such that

$$\mathcal{Q}^{S}_{ynf} \quad \geq \quad 0 \tag{4.2a}$$

$$\mathcal{Q}^{S}_{ynf} \quad \leq \quad \mathcal{Q}^{S,\text{CAP}}_{ynf} \qquad (\delta^{8}_{ynf}) \tag{4.2b}$$

4.2 KKT Conditions

$$\pi^{F}_{ynf} - \pi^{U}_{ynf} + \mathscr{C}^{Sq}_{ynf} \mathcal{Q}^{S}_{ynf} + \mathscr{C}^{Sl}_{ynf} + \delta^{8}_{ynf} \quad \geq \quad 0 \qquad (\mathcal{Q}^{S}_{ynf}) \tag{4.3a}$$

5 Market Clearing

$$\mathcal{Q}^{F}_{ynf} = \mathcal{Q}^{D_b}_{ynf} \qquad (\pi^{F}_{ynf}) \qquad (5.1a)$$

$$\pi^{U}_{ynf} = \alpha_{ynf} - \beta_{ynf}\mathcal{Q}^{S}_{ynf} + \sum_{i \in F} \chi_{ynfi}\pi^{U}_{yni} \qquad (\pi^{U}_{ynf}) \qquad (5.1b)$$