Food Model - Documentation

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1 Crop Producer

The crop producer makes decisions on the land allotted for various crops. He has an "expectation" for the price of crops and their yield.

1.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\ h\in H}} \mathsf{df}_y \left\{ \sum_{f\in C} \left(\pi^F_{yhnf}\mathbf{Q}^F_{yhnf} - \mathscr{C}^F_{yhnf}\mathbf{A}^F_{yhnf} - \frac{1}{2}\mathscr{C}^{\text{change}}_{yhn}\left(\mathbf{A}^F_{yhnf} - \mathbf{A}^F_{(y-1)snf}\right)^2\right) - \mathscr{C}^{\text{conv}}_{yhn}\sum_{f\in C}\left(\mathbf{A}^F_{yhnf} - \mathbf{A}^F_{(y-1)snf}\right)\right\} \tag{1.1}$$

such that for $f \in C$

$$\mathbf{Q}^F_{yhnf},\ \mathbf{A}^F_{yhnf} \quad \geq \quad 0 \tag{1.2a}$$

Total area used for crops is less than land available:

$$\mathbf{A}_n \quad \geq \quad \sum_{\substack{f\in C\\ h\in H}} \mathbf{A}^F_{yhnf} \qquad (\delta^1_{yhn}) \tag{1.2b}$$

Total Quantity is yield times area for that crop, both in real terms and expected terms:

$$\mathbf{Q}^F_{yhnf} \quad \leq \quad \mathcal{Y}_{yhnf}\mathbf{A}^F_{yhnf} \qquad (\delta^2_{yhnf}) \tag{1.2c}$$

Linking yield with climate yield factor:

$$\mathcal{Y}_{yhnf} \quad = \quad \mathsf{aCYF}\left(\pi^F_{yhnf}\right)^{\mathsf{e}} \qquad (\mathcal{Y}_{yhnf}) \tag{1.2d}$$

Fallow constraint where ${}_{\text{Fal}}\mathscr{O}$ is a number between 0 and 1 indicating the fraction of land that gets fallowed. ${}_{\text{Fal}}\mathscr{O}^{\text{Dur}}$ is the duration of fallow cycle:

$$\sum_{\substack{f\in C\\ h\in H}} \sum_{y'=y}^{y+{}_{\text{Fal}}\mathscr{O}^{\text{Dur}}} \mathbf{A}^F_{y'snf} \quad \leq \quad {}_{\text{Fal}}\mathscr{O}^{\text{Dur}}\mathbf{A}_n - {}_{\text{Fal}}\mathscr{O}\mathbf{A}_n \qquad (\delta^{{}_{\text{Fal}}\mathscr{O}}_{yn}) \tag{1.2e}$$

Crop rotation constraint where f is the primary crop rotated with f':

$${}_{\text{Rot}}\mathscr{O} \sum_{y'=y}^{y+{}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'}} \sum_{h\in H} \mathbf{A}^F_{y'snf} \quad \leq \quad {}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'} \sum_{y'=y}^{y+{}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'}} \sum_{h\in H} \mathbf{A}^F_{y'snf'} \qquad (\delta^{{}_{\text{Rot}}\mathscr{O}}_{ynff'}) \tag{1.2f}$$

1.2 KKT Conditions

These KKT conditions hold for $f \in C$

$$
\begin{aligned}
\delta^2_{yhnf} - \mathsf{df}_y \pi^F_{yhnf} &\geq 0 \quad (\mathbf{Q}^F_{yhnf}) && (1.3a)\\
\left.\begin{aligned}
\delta^1_{yhn} + \mathsf{df}_y \left(\mathscr{C}^F_{yhnf} + \mathscr{C}^{\text{conv}}_{yn} - \mathscr{C}^{\text{conv}}_{(y+1)n} + \mathscr{C}^{\text{change}}_{yhn} \mathbf{A}^F_{yhnf} + \mathscr{C}^{\text{change}}_{(y+1)n} \mathbf{A}^F_{yhnf} \right) \\
-\delta^2_{yhnf} \mathcal{Y}_{yhnf} - \mathsf{df}_y \left(\mathscr{C}^{\text{change}}_{yhn} \mathbf{A}^F_{(y-1)nf} + \mathscr{C}^{\text{change}}_{(y+1)n} \mathbf{A}^F_{(y+1)nf} \right) \\
+{}_{\text{Fal}}\mathscr{O} \sum_{y'=y-{}_{\text{Rot}}\mathscr{O}^{\text{Dur}}_{ff'}}^{y} \delta^{{}_{\text{Fal}}\mathscr{O}}_{yn} \\
\pm{}_{\text{Rot}}\mathscr{O} \sum_{y'=y-{}_{\text{Fal}}\mathscr{O}^{\text{Dur}}}^{y} \delta^{{}_{\text{Rot}}\mathscr{O}}_{ynff'}
\end{aligned}\right\} &\geq 0 \quad (\mathbf{A}^F_{yhnf}) && (1.3b)
\end{aligned}
$$

where the sign of the last term depends upon whether f is the main crop or the secondary crop used for rotation.

2 Livestock producer

2.1 Problem

$$
\text{Maximize} \quad : \quad \sum_{\substack{y \in Y \\ h \in H \\ f \notin C}} \mathsf{df}_y \left(\pi^F_{yhnf} \mathbf{Q}^F_{yhnf} + p^H_{yhn} \mathbf{Q}^H_{yhn} - \sum_{i \in N} (\mathscr{C}^{\text{cow,trans}}_{yhin} + \pi^{\text{cow}}_{yhi}) \mathcal{B}^{\text{buy}}_{yhin} - \mathcal{B}_{yhn} \mathscr{C}^{\text{cow}}_{yhn} \right) \tag{2.1}
$$

such that for $f \notin C$

$$
\begin{aligned}
\mathcal{B}^{\text{buy}}_{yhin}, \mathcal{B}_{yhn}, \mathcal{B}^{\text{slg}}_{yhn} &\geq 0 && && (2.2a)\\
\mathbf{Q}^F_{yhnf}, \mathbf{Q}^H_{yhn} &\geq 0 && && (2.2b)\\
\mathbf{Q}^F_{yhnf} &\leq \mathcal{Y}_{yhnf} \mathcal{B}_{yhn} && (f = \text{Milk}) \quad (\delta^2_{yhnf}) &&\\
\mathbf{Q}^F_{yhnf} &\leq \mathcal{Y}_{yhnf} \mathcal{B}^{\text{slg}}_{yhn} && (f = \text{Beef}) \quad (\delta^2_{yhnf}) &&\\
\mathbf{Q}^H_{yhn} &\leq \mathcal{Y}^H \mathcal{B}^{\text{slg}}_{yhn} && (\delta^3_{yhn}) && (2.2c)\\
\mathcal{B}^{\text{slg}}_{yhn} &\leq \mathcal{B}_{yhn} && (\delta^4_{yhn}) && (2.2d)\\
\mathcal{B}_{yhn} &\leq (1 + k - \kappa) \mathcal{B}_{(y-1)hn} - \mathcal{B}^{\text{slg}}_{yhn} + \sum_{i \in N} \left(\mathcal{B}^{\text{buy}}_{yhin} - \mathcal{B}^{\text{buy}}_{yhni} \right) && (\pi^{\text{cow}}_{yn}) && (2.2e)\\
\mathcal{B}^{\text{slg}}_{yhn} &\geq \kappa^{\text{death}}_{yhn} \mathcal{B}_{yhn} && (\delta^9_{yhn}) && (2.2f)\\
\mathcal{B}_{yhn} &\geq \mathcal{B}^{\text{herd}}_n && (\delta^{10}_{yhn}) && (2.2g)
\end{aligned}
$$

2.2 KKT Conditions

$$
\begin{aligned}
\left.\begin{aligned}
\mathsf{df}_y \mathscr{C}^{\text{cow}}_{yhn} - \delta^2_{yhnf} \mathcal{Y}_{yhnf} - \delta^4_{yhn} + \kappa^{\text{death}}_{yhn} \delta^9_{yhn} \\
-\delta^{10}_{yhn} + \pi^{\text{cow}}_{yhn} - (1 + k - \kappa) \pi^{\text{cow}}_{(y+1)hn}
\end{aligned}\right\} &\geq 0 && (f = \text{Milk}) \quad (\mathcal{B}_{yhn}) && (2.3a)\\
\mathsf{df}_y \left(\mathscr{C}^{\text{cow,trans}}_{yhin} + \pi^{\text{cow}}_{yhi} \right) + \left(\pi^{\text{cow}}_{yhi} - \pi^{\text{cow}}_{yhn} \right) &\geq 0 && (\mathcal{B}^{\text{buy}}_{yhin}) && (2.3b)\\
\delta^2_{yhnf} - \mathsf{df}_y \pi^F_{yhnf} &\geq 0 && (\mathbf{Q}^F_{yhnf}) &&\\
\delta^3_{yhn} - \mathsf{df}_y p^H_{yhn} &\geq 0 && (\mathbf{Q}^H_{yhn}) && (2.3c)\\
\delta^4_{yhn} - \delta^{10}_{yhn} - \delta^2_{yhnf} \mathcal{Y}_{yhnf} - \delta^3_{yhn} \mathcal{Y}^H + \pi^{\text{cow}}_{yhn} &\geq 0 && (f = \text{Beef}) \quad (\mathcal{B}^{\text{slg}}_{yhn}) && (2.3d)
\end{aligned}
$$

3 Distribution

3.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\h\in H\\f\in F}} \mathsf{df}_y \left\{ \sum_{n\in N} \left(\mathbf{Q}^{D_s}_{yhnf}\pi^W_{yhnf} - \mathbf{Q}^{D_b}_{yhnf}\pi^F_{yhnf} \right) - \sum_{r\in R} \mathscr{C}^R_{yhrf}\mathbf{Q}^D_{yhrf} \right\} \tag{3.1}$$

such that

$$\mathbf{Q}^{D_b}_{yhnf}, \mathbf{Q}^D_{yhrf}, \mathbf{Q}^{D_s}_{yhnf} \geq 0 \tag{3.2a}$$

$$\mathbf{Q}^{D_b}_{yhnf} + \sum_{r\in R_{\text{in}}} \mathbf{Q}^D_{yhrf} \geq \mathbf{Q}^{D_s}_{yhnf} + \sum_{r\in R_{\text{out}}} \mathbf{Q}^D_{yhrf} \qquad (\delta^6_{yhnf}) \tag{3.2b}$$

$$\mathbf{Q}^D_{yhrf} \leq \mathbf{Q}^{R,\text{CAP}}_{yrf} \qquad (\delta^7_{yhrf}) \tag{3.2c}$$

3.2 KKT Conditions

Representing s_r and d_r as the source and destination nodes of the transport system $r \in R$, we have the following KKT conditions.

$$\mathsf{df}_y\pi^F_{yhnf} - \delta^6_{yhnf} \geq 0 \qquad (\mathbf{Q}^{D_b}_{yhnf}) \tag{3.3a}$$

$$\delta^7_{yhrf} + \mathsf{df}_y\mathscr{C}^R_{yhrf} + \delta^6_{yhs_rf} - \delta^6_{yhd_rf} \geq 0 \qquad (\mathbf{Q}^D_{yhrf}) \tag{3.3b}$$

$$\delta^6_{yhnf} - \mathsf{df}_y\pi^W_{yhnf} \geq 0 \qquad (\mathbf{Q}^{D_s}_{yhnf}) \tag{3.3c}$$

4 Storage

4.1 Problem

$$\text{Maximize} \quad : \quad \sum_{\substack{y\in Y\\h\in H\\f\in F}} \pi^U_{yhnf}\mathbf{Q}^{W_s}_{yhnf} - \pi^W_{yhnf}\mathbf{Q}^{W_b}_{yhnf} - \left(\frac{1}{2}\mathscr{C}^{Wq}_{yhnf}\mathbf{Q}^W_{yhnf} + \mathscr{C}^{Wl}_{yhnf} \right) \mathbf{Q}^W_{yhnf} \tag{4.1}$$

such that

$$\mathbf{Q}^{W_b}_{yhnf}, \mathbf{Q}^{W_s}_{yhnf}, \mathbf{Q}^W_{yhnf} \geq 0 \tag{4.2a}$$

$$\mathbf{Q}^W_{yhnf} \leq \mathbf{Q}^{W,\text{CAP}}_{ynf} \qquad (\delta^8_{yhnf}) \tag{4.2b}$$

For the first season h, if h' is the last season

$$\mathbf{Q}^W_{yhnf} = \mathbf{Q}^W_{(y-1)h'nf} + \mathbf{Q}^{W_b}_{yhnf} - \mathbf{Q}^{W_s}_{yhnf} \qquad (\delta^{11}_{yhnf}) \tag{4.2c}$$

For other seasons

$$\mathbf{Q}^W_{yhnf} = \mathbf{Q}^W_{y(h-1)nf} + \mathbf{Q}^{W_b}_{yhnf} - \mathbf{Q}^{W_s}_{yhnf} \qquad (\delta^{11}_{yhnf}) \tag{4.2d}$$

4.2 KKT Conditions

$$\pi^W_{yhnf} - \delta^{11}_{yhnf} \geq 0 \qquad (\mathbf{Q}^{W_b}_{yhnf}) \tag{4.3a}$$

$$\delta^{11}_{yhnf} - \pi^U_{yhnf} \geq 0 \qquad (\mathbf{Q}^{W_s}_{yhnf}) \tag{4.3b}$$

For last season h, where h' is the first season

$$\mathscr{C}^{Wq}_{yhnf}\mathbf{Q}^W_{yhnf} + \mathscr{C}^{Wl}_{yhnf} + \delta^8_{yhnf} + \delta^{11}_{yhnf} - \delta^{11}_{(y+1)h'nf} \geq 0 \qquad (\mathbf{Q}^W_{yhnf}) \tag{4.3c}$$

For other seasons

$$\mathscr{C}^{Wq}_{yhnf}\mathbf{Q}^W_{yhnf} + \mathscr{C}^{Wl}_{yhnf} + \delta^8_{yhnf} + \delta^{11}_{yhnf} - \delta^{11}_{y(h+1)nf} \geq 0 \qquad (\mathbf{Q}^W_{yhnf}) \tag{4.3d}$$

5 Market Clearing

$$\mathbf{Q}^{F}_{yhnf} = \mathbf{Q}^{D_b}_{yhnf} \qquad (\pi^{F}_{yhnf}) \tag{5.1a}$$

$$\pi^{U}_{yhnf} = \alpha_{yhnf} - \beta_{yhnf}\mathbf{Q}^{W}_{yhnf} + \sum_{i\in F}\chi_{yhnfi}\pi^{U}_{yhni} \qquad (\pi^{U}_{yhnf}) \tag{5.1b}$$

$$\mathbf{Q}^{W_b}_{yhnf} = \mathbf{Q}^{D_s}_{yhnf} \qquad (\pi^{W}_{yhnf}) \tag{5.1c}$$

6 Electricity

6.1 Problem

We model electricity with a single operator. A subset of nodes can have production facilities. The production facilities have a minimum and maximum production capabilities. All nodes are connected by transmission lines, with a cost of transmission along each line. They have transmission capacities too. There is currently no possibility of expansion. The cost is minimized in the model.

$$\text{Minimize} \quad : \quad \sum_{\substack{y\in Y\\ n\in N}} \left(\mathscr{C}^{El}_{yhn}\mathbf{Q}^{E-gen}_{yhn} + \frac{1}{2}\mathscr{C}^{Eq}_{yhn}\mathbf{Q}^{E-gen\,2}_{yhn} \right) + \sum_{\substack{y\in Y\\ r\in R}} \mathscr{C}^{ER}_{yhr}\mathbf{Q}^{E}_{yhr} \tag{6.1}$$

such that

$$\mathbf{Q}^{E-gen}_{yhn} \geq 0 \tag{6.2a}$$

Generator limits

$$\mathbf{Q}^{E-gen}_{yhn} \geq \underline{\mathbf{Q}}^{E}_{yhn} \tag{6.2b}$$

$$\mathbf{Q}^{E-gen}_{yhn} \leq \overline{\mathbf{Q}}^{E}_{yhn} \tag{6.2c}$$

The Kirchoff Current Law in each node. This also ensures that the demand is met in each node. $\mathscr{L}^{E}_{yhr}$ is the transmission loss.

$$\mathbf{Q}^{E-gen}_{yhn} + \sum_{r\in R_{\text{in}}} \mathbf{Q}^{E}_{yhr}\left(1 - \mathscr{L}^{E}_{yhr}\right) \geq \mathbf{Q}^{E-dem}_{yhn} + \sum_{r\in R_{\text{out}}} \mathbf{Q}^{E}_{yhr} \tag{6.2d}$$

Transmission capacity

$$\mathbf{Q}^{E}_{yhr} \leq \overline{\mathbf{Q}}^{E}_{yhr} \tag{6.2e}$$