CMPS142-Spring 2018
Problem Set 0

Handed out: April 3, 2018
Due: April 10, 2018 at 1:30 PM

- You should have seen most of the material. The goal of this homework is to allow you to go back to some of this and refresh your memory. This problem set will not be graded, but there are points for submitting it.
- Feel free to talk to other members of the class in doing the homework. You should, however, write down your solution yourself. Please try to keep the solution brief and clear.
- Please submit a hard copy of your solutions before the class starts on the due date. This assignment cannot be done in groups. Please *clearly* write your name, email address and student id number on the first page of your assignment.

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1. Assume X is normally distributed with mean= μ and variance= σ^2. Write the probability density function for X.
2. Assume that the probability of obtaining heads when tossing a coin is λ.
   (a) What is the probability of obtaining the first head at the (k + 1)-th toss?
   (b) What is the expected number of tosses needed to get the first head?
3. Assume X is a random variable.
   (a) We define the variance of X as: $Var(X) = E[(X - E[X])^2]$. Prove that $Var(X) = E[X^2] - E[X]^2$.
   (b) If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?
4. Let $f(x, y) = 3x^2 + y^2 - xy - 11x$
   (a) Find $\frac{\partial f}{\partial x}$, the partial derivative of f with respect to x. Also find $\frac{\partial f}{\partial y}$.
   (b) Find $(x, y) \in \mathbb{R}^2$ that minimizes f.
5. One way to define a *convex* function is as follows. A function $f(x)$ is convex if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$$

   for all x, y and $0 < \lambda < 1$.
   (a) Prove that $f(x) = x^2$ is a convex function.
   (b) A n-by-n matrix A is a *positive semi-definite* matrix if $x^T Ax \geq 0$, for any $x \in \mathbb{R}^n$ s.t $x \neq 0$. Prove that the function $f(x) = x^T Ax$ is convex if A is a positive semi-definite matrix. Note that x is a vector here.