$$
\begin{aligned}
Q_s &= \rho_a c_p C_D^*(1+\alpha_s T_o')\|\mathbf{U}\|(T_o - T_a) && (1)\\
Q_L &= \rho_a L_v C_D^*(1+\alpha_L T_o')\|\mathbf{U}\|(q_o^* - q_a) && (2)
\end{aligned}
$$

and the total flux is

$$F = Q_s + Q_L \tag{3}$$

At large scales, $T_a^{LP} \propto T_o^{LP}$ such that the low-pass-filtered fields

sensible

$$
\begin{aligned}
Q_s \propto\, & C_D^s(1+\alpha_s T_o')(U + U')(T_o + T_o' - T_a - T_a')\\
Q_s \propto\, & C_D^s T_o U - C_D^s T_a' U - C_D^s T_a U + C_D^s T_o' U - C_D^s T_a U' - C_D^s T_a' U' + C_D^s T_o U' + \ldots\\
& C_D^s T_o' U' + C_D^s T_o'^2 U \alpha_s + C_D^s T_o'^2 U' \alpha_s - C_D^s T_a T_o' U \alpha_s - C_D^s T_a' T_o' U \alpha_s + \ldots\\
& C_D^s T_o T_o' U \alpha_s - C_D^s T_a T_o' U' \alpha_s - C_D^s T_a' T_o' U' \alpha_s + C_D^s T_o T_o' U' \alpha_s\\
\overline{Q_s} \propto\, & C_D^s T_o U - \cancelto{0}{C_D^s T_a' U} - C_D^s T_a U + \cancelto{0}{C_D^s T_o' U} - \cancelto{0}{C_D^s T_a U'} - C_D^s \overline{T_a' U'} + \cancelto{0}{C_D^s T_o U'} + \ldots\\
& C_D^s \overline{T_o' U'} + C_D^s \overline{T_o'^2} U \alpha_s + C_D^s \overline{T_o'^2 U'} \alpha_s - \cancelto{0}{C_D^s T_a T_o' U} \alpha_s - C_D^s \overline{T_a' T_o'} U \alpha_s + \ldots\\
& \cancelto{0}{C_D^s T_o T_o' U} \alpha_s - C_D^s T_a \overline{T_o' U'} \alpha_s - C_D^s \overline{T_a' T_o' U'} \alpha_s + C_D^s T_o \overline{T_o' U'} \alpha_s\\
\overline{Q_s} \propto\, & C_D^s T_o U - C_D^s T_a U - C_D^s \overline{T_a' U'} + C_D^s \overline{T_o' U'} + C_D^s \overline{T_o'^2} U \alpha_s + C_D^s \overline{T_o'^2 U'} \alpha_s + \ldots\\
& - C_D^s \overline{T_a' T_o'} U \alpha_s - C_D^s T_a \overline{T_o' U'} \alpha_s - C_D^s \overline{T_a' T_o' U'} \alpha_s + C_D^s T_o \overline{T_o' U'} \alpha_s
\end{aligned}
$$