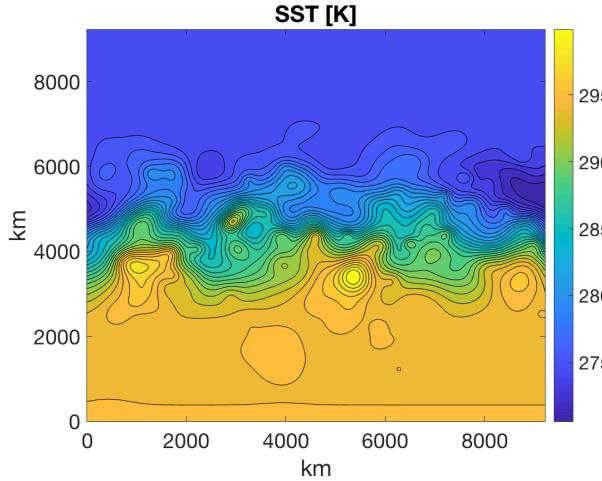
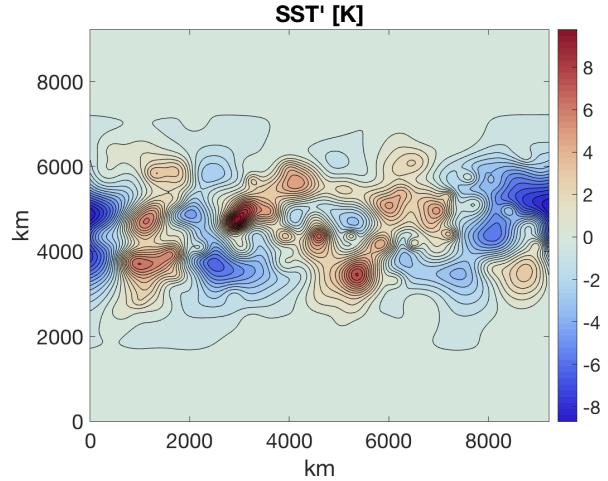


1 Idealized Front

Create the setup from Foussard et al. (2019) and consider



(a) Manufactured SST temperature



(b) Manufactured SST temperature anomaly from zonal mean

$$C_D = C_D^*(1 + \alpha T'_o) \quad (1)$$

$$Q_s = \rho_a c_p C_D \|\mathbf{U}\| (T_o - T_a) \quad (2)$$

$$Q_L = \rho_a L_v C_D \|\mathbf{U}\| (q_o^* - q_a) \quad (3)$$

where the free parameters are, $T_o - T_a$ (ΔT), α , and relative humidity (RH).

A wind field is applied (see three cases below) and the wind stress is calculated as $\tau_{xy} = \rho_a C_D \|\vec{u}\| \vec{u}$. Let \hat{u} be the unit vector in the direction of the wind at each point. The along-wind (aw) and cross-wind (cw) components of the SST gradients are

$$\nabla_{aw} T_o = (\nabla T_o \cdot \hat{u}) \cdot \hat{u} \quad (4)$$

$$\nabla_{cw} T_o = \nabla T_o - ((\nabla T_o \cdot \hat{u}) \cdot \hat{u}) \quad (5)$$

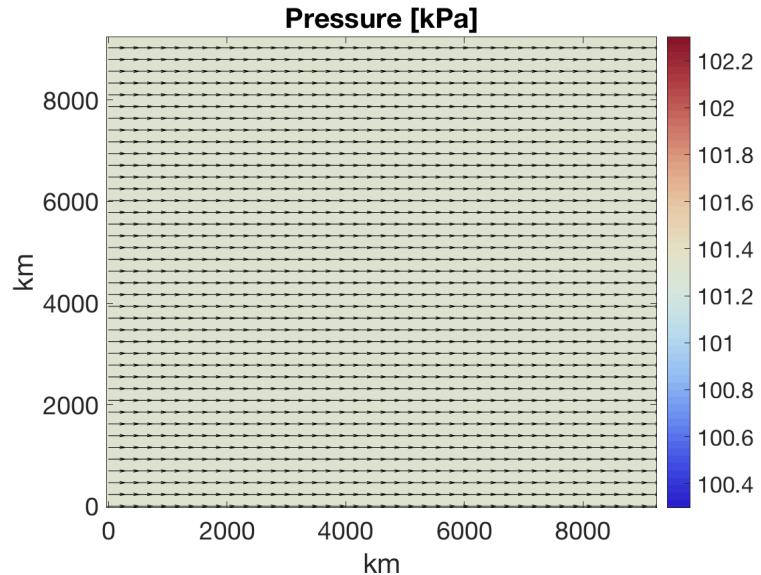
1.1 Constant Pressure and Velocity ($\vec{u} = (\bar{u}, 0)$, $p = p_0$)

With a constant pressure and velocity field the curl of the stress is directly proportional to the cross-wind SST gradient, and the divergence of the stress is directly proportional to the down-wind SST gradient (see Section A.2) at each point. For a particular setup with $C_D^* = 1E - 3$, $\rho_a = 1.2 \text{ kg/m}^{-3}$, $\alpha = 1E - 3$, $\bar{u} = 4 \text{ m/s}$, the slope of the curl of the stress (y-axis) to the cross-wind SST gradient (x-axis) is $1.92E-5$ as seen in Figure 2.

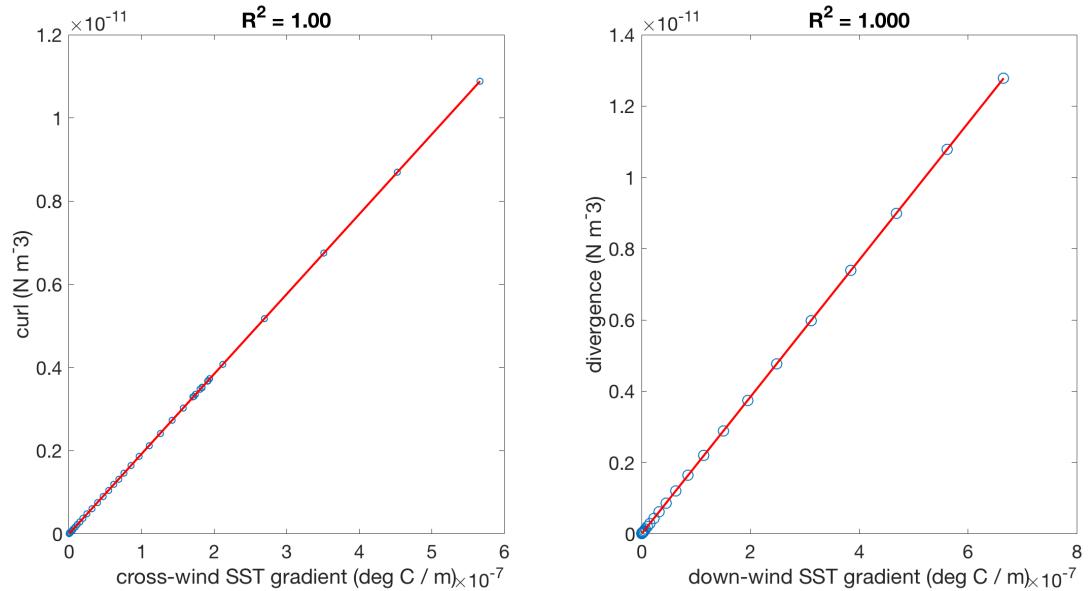
1.2 Chelton Coupling Coefficient ($u = \bar{u} + \gamma T'$, $p = \bar{p}$)

From (Chelton et al., 2004), the coupling coefficient is between 0.2 and 0.44. A detailed derivation is in Section A.3, but the conclusion that if $\gamma T'$ is small with respect to \bar{u} , then the slope of the divergence plotted against the "down-wind" SST gradient will be $C_D^* \rho_a 2\gamma \bar{u}$, which for this setup would be $C_D^* \rho_a 2\gamma \bar{u} = 0.0019$ and the slope of the best fit line for this setup is 0.002 as shown in Figure 4.

1.3 Baroclinic Wave ($u = u_{\text{geostrophic}}$)

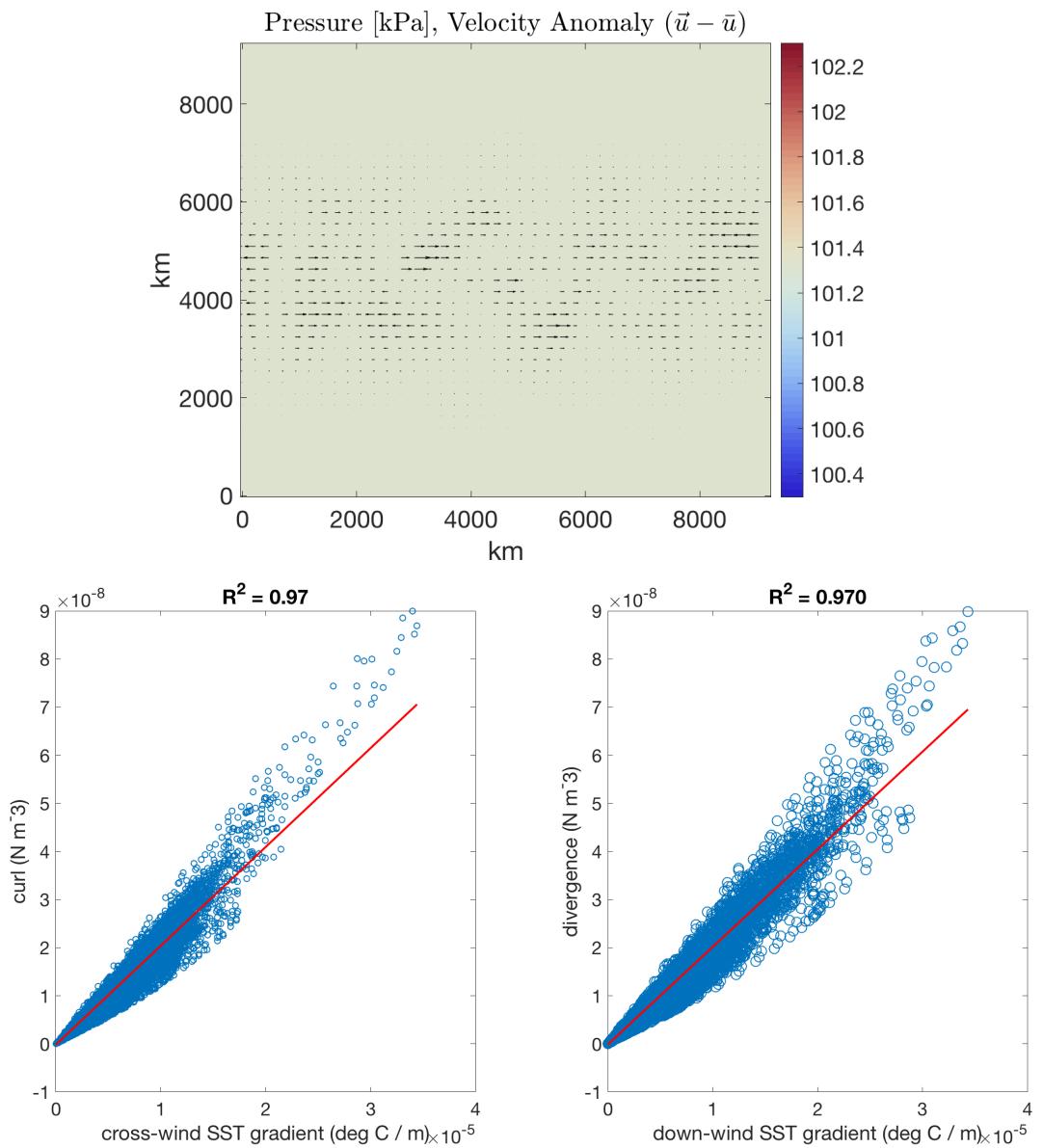


(a) Manufactured SST temperature



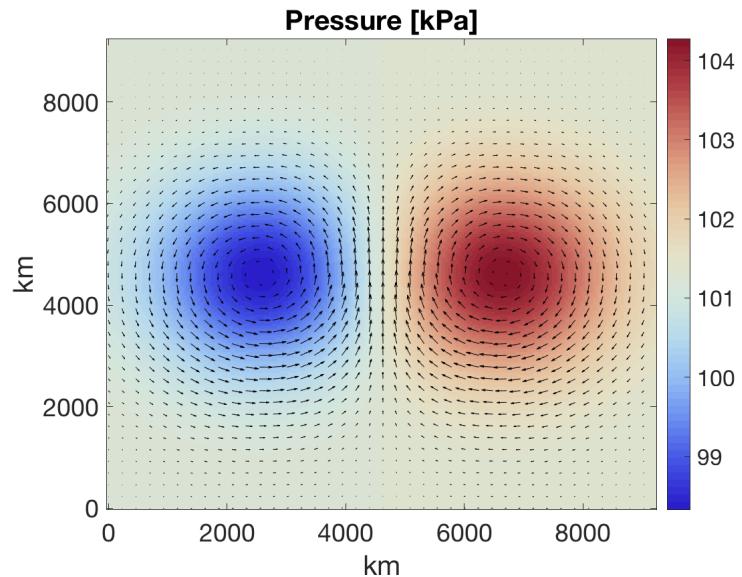
(b) Sample of 10 points in blue circles (however all points in the domain fall on red line which has a slope of 1.92E-5)

Figure 2

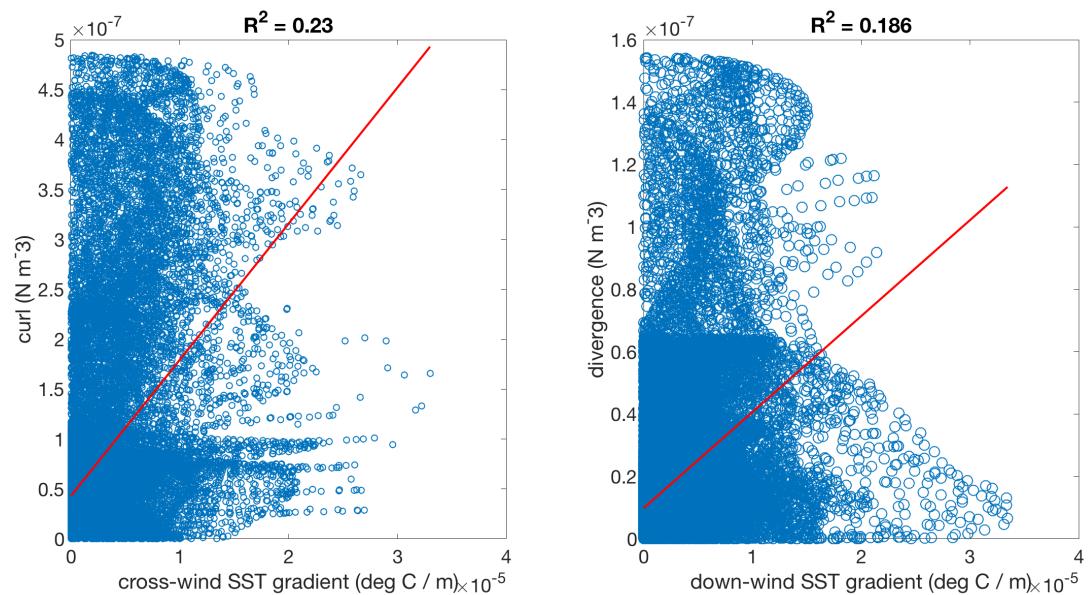


(a) All points in the domain are represented by blue circles, the slope of the divergence-down-wind gradient fitted line is 0.002.

Figure 3



(a) The pressure field is two Gaussians creating a wavelength of $\lambda = 4000\text{km}$.



(b) All points in the domain are represented by blue circles

Figure 4

2 ERA5

2.1 Which years to pick?

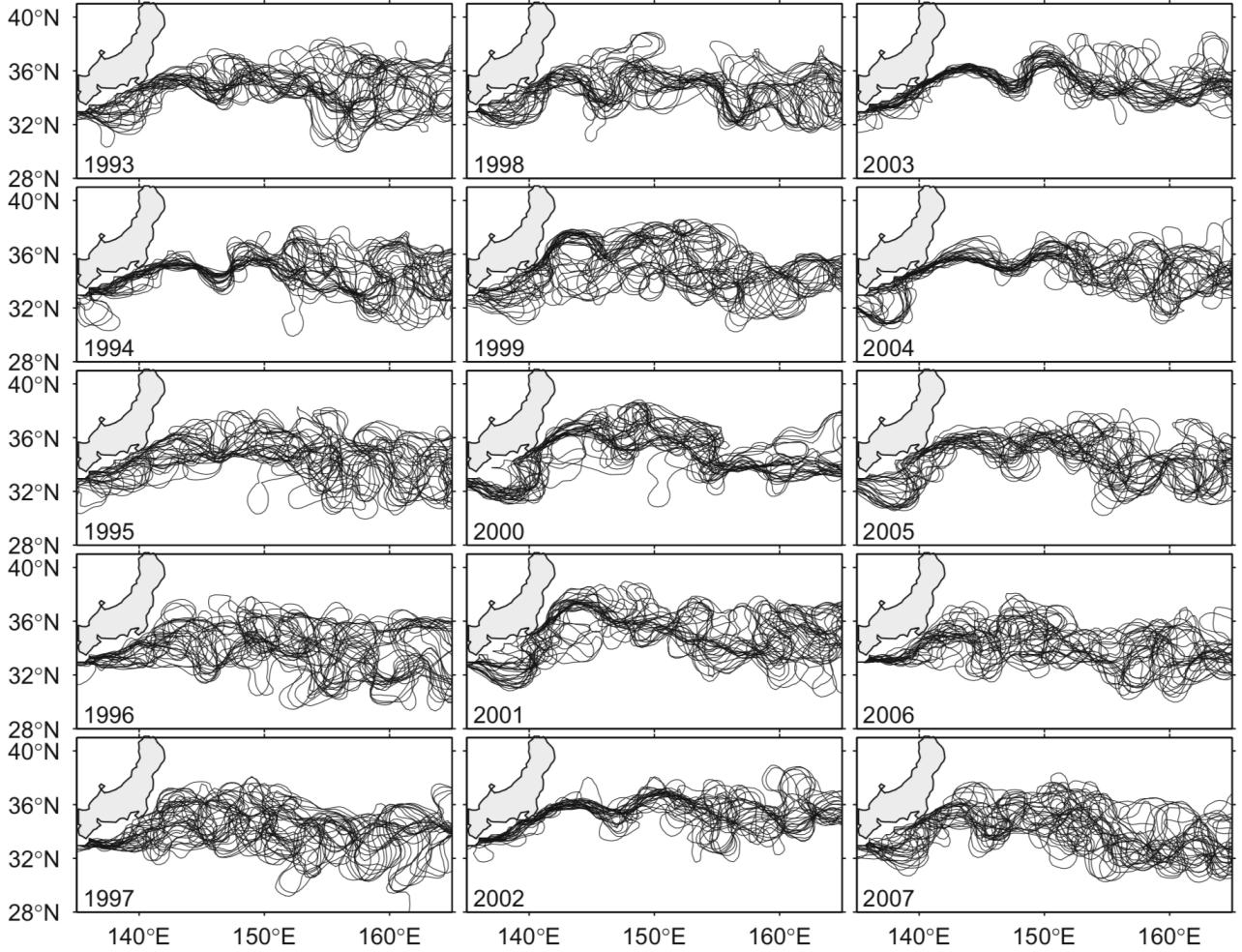


Figure 5: ?

•	zonal average	•'	zonal anomaly
$\langle \bullet \rangle$	temporal average	C_D^*	1E-3
T_o	Sea Surface Temperature	T_a	2m Temperature
q_o	saturated specific humidity at the local SST	a_a	specific humidity at the 2m temperature
RH	relative humidity at 2m	T_d	dew point temperature at 2m
c_p	specific heat of air	L_v	latent heat of vaporization

ERA5 has 0.25° resolution.

$$C_D^{s,L} = C_D^*(1 + \alpha_{s,L} T') \quad (6)$$

$$Q_s = \rho_a c_p C_D^s \| \mathbf{U} \| (T_o - T_a) \quad (7)$$

$$Q_L = \rho_a L_v C_D^L \| \mathbf{U} \| (q_o^* - q_a) \quad (8)$$

2.2 Full Atmosphere Full Ocean

Use Q_s and Q_L from ERA5 (along with the SST, 2m dewpoint temperature, 2m temperature, surface pressure, and 10m horizontal wind speed) to calculate α at every point in space and time.

$$T_a = \bar{T}_a + T'_a \quad (9)$$

$$T_d = \bar{T}_d + T'_d \quad (10)$$

$$p_0 = \bar{p}_0 + p'_0 \quad (11)$$

$$u_{10} = \bar{u}_{10} + u'_{10} \quad (12)$$

$$v_{10} = \bar{v}_{10} + v'_{10} \quad (13)$$

$$\|\mathbf{U}\| = \sqrt{u_{10}^2 + v_{10}^2} \quad (14)$$

$$q_o = q_o(\bar{T}_o + T'_o, \bar{p}_0 + p'_0) \quad (15)$$

$$q_a = q_a(\bar{T}_a + T'_a, \bar{T}_d + T'_d, \bar{p}_0 + p'_0) \quad (16)$$

$$\alpha_s = \frac{1}{T'} \left(1 - \frac{Q_S}{\rho_a c_p C_D^* \|\mathbf{U}\| (T_o - T_a)} \right) \quad (17)$$

$$\alpha_L = \frac{1}{T'} \left(1 - \frac{Q_S}{\rho_a L_v C_D^* \|\mathbf{U}\| (q_o - q_a)} \right) \quad (18)$$

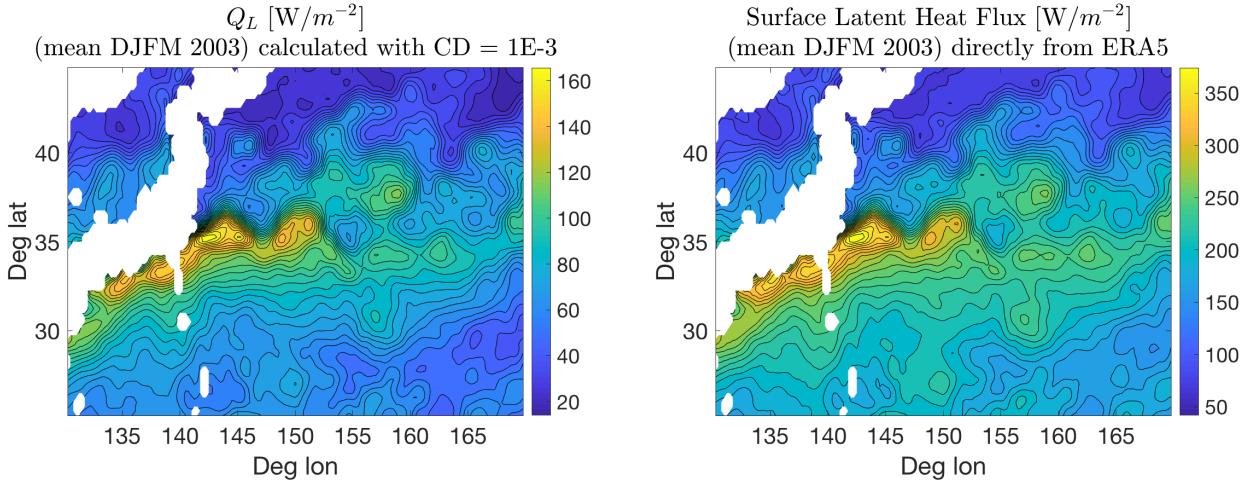


Figure 6: The mean SLHF from Equation 29 and directly from the reanalysis data

2.3 Full Atmosphere, Smooth Ocean

Compute the flux from a smooth sea surface where $T'_o = 0$.

$$T_a = \bar{T}_a + T'_a \quad (19)$$

$$T_o = \bar{T}_o \quad (20)$$

$$T_d = \bar{T}_d + T'_d \quad (21)$$

$$p_0 = \bar{p}_0 + p'_0 \quad (22)$$

$$u_{10} = \bar{u}_{10} + u'_{10} \quad (23)$$

$$v_{10} = \bar{v}_{10} + v'_{10} \quad (24)$$

$$\|\mathbf{U}\| = \sqrt{u_{10}^2 + v_{10}^2} \quad (25)$$

$$q_o = q_o(\bar{T}_o, \bar{p}_0 + p'_0) \quad (26)$$

$$q_a = q_a(\bar{T}_a + T'_a, \bar{T}_d + T'_d, \bar{p}_0 + p'_0) \quad (27)$$

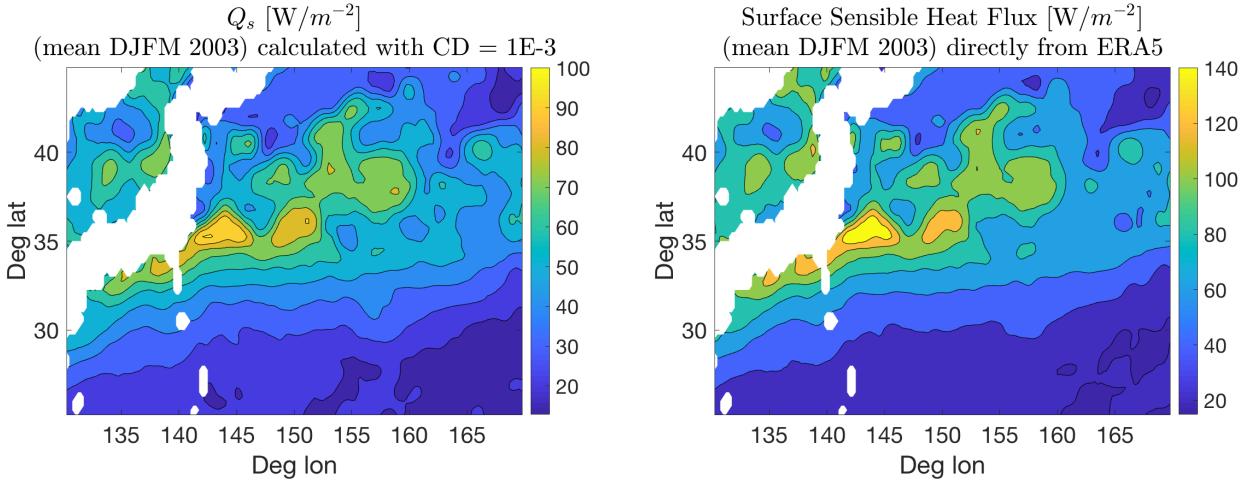


Figure 7: The mean SSHF from Equation 28 and directly from the reanalysis data

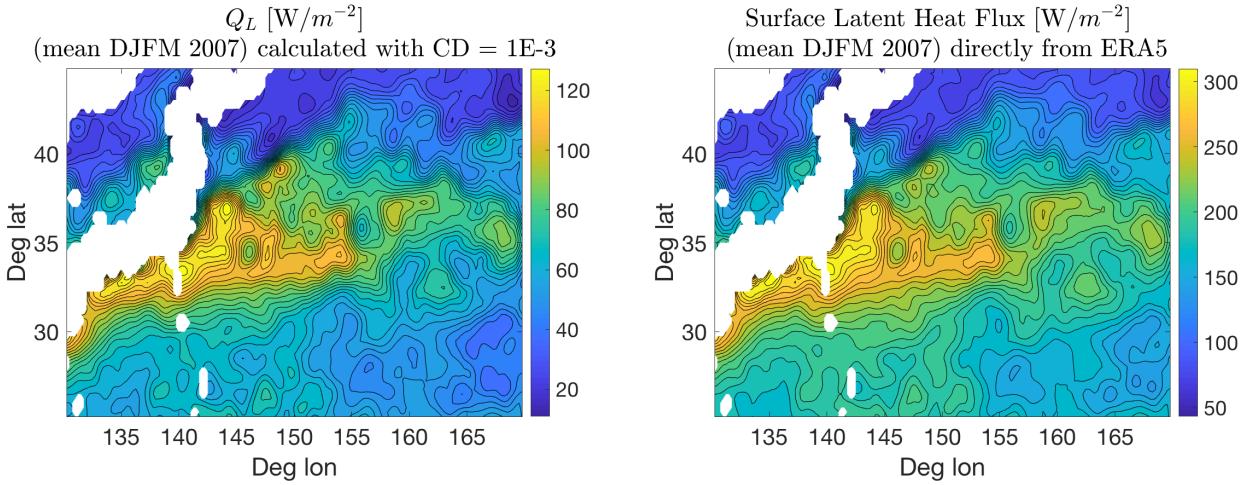


Figure 8: The mean SLHF from Equation 29 and directly from the reanalysis data

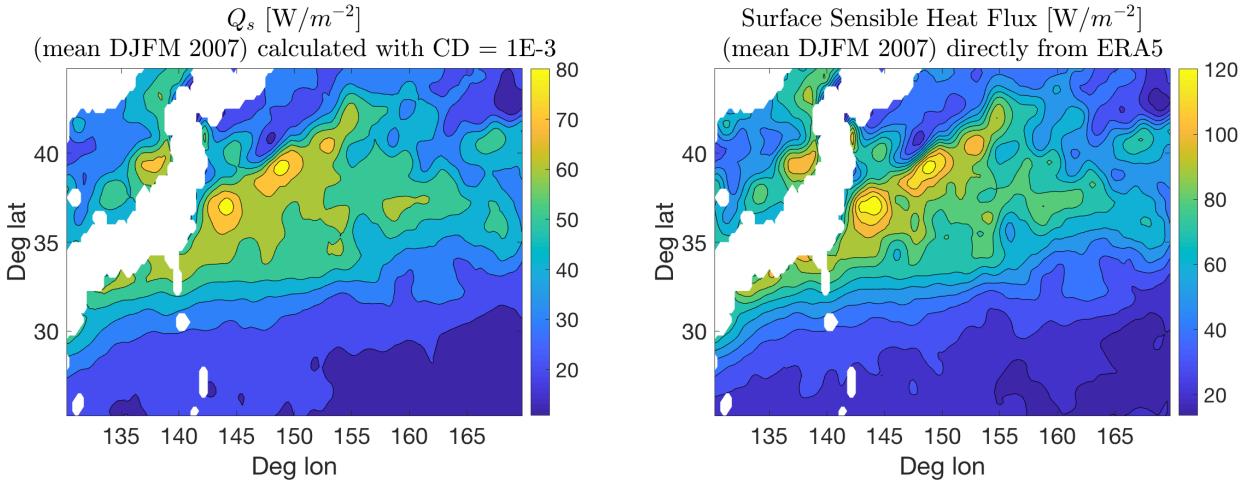


Figure 9: The mean SSHF from Equation 28 and directly from the reanalysis data

$$Q_s = \rho_a c_p C_D^* \|\mathbf{U}\| (T_o - T_a) \quad (28)$$

$$Q_L = \rho_a L_v C_D^* \|\mathbf{U}\| (q_o^* - q_a) \quad (29)$$

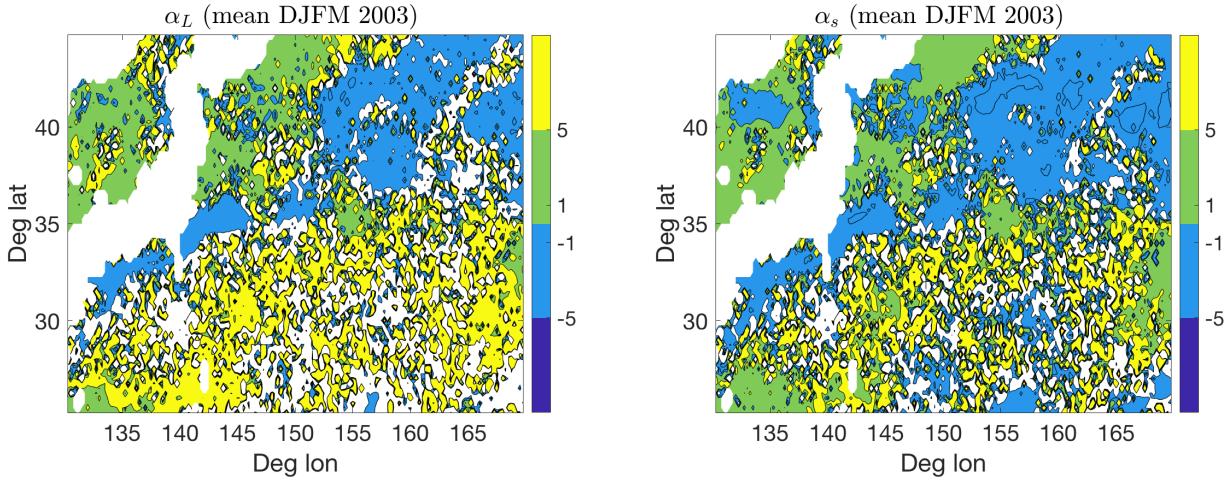


Figure 10: The mean α values (i.e. α is calculated every 6 hours then point-wise averaged in time). The color contours are limited to ± 5 since at some points the value of α contains "Inf" values.

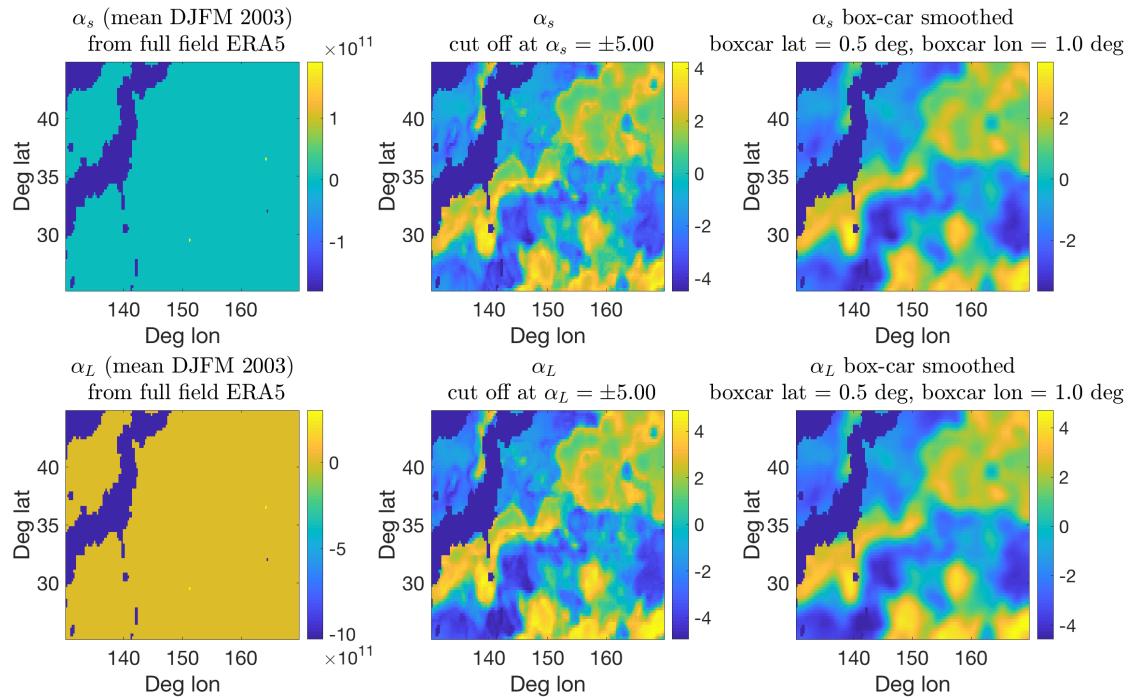


Figure 11: using a cut-off and box-car smoothing the α fields.

2.4 Vanished Anomaly

Compute the flux from a smooth sea surface where $T' = 0$.

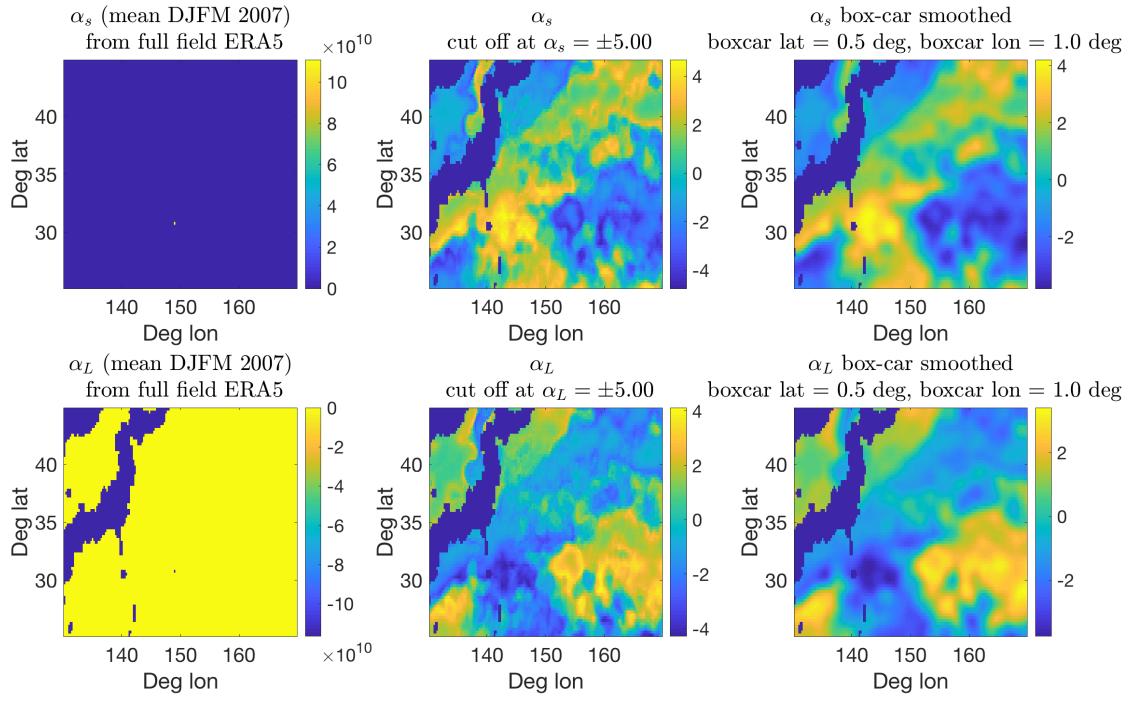


Figure 12: using a cut-off and box-car smoothing the α fields.

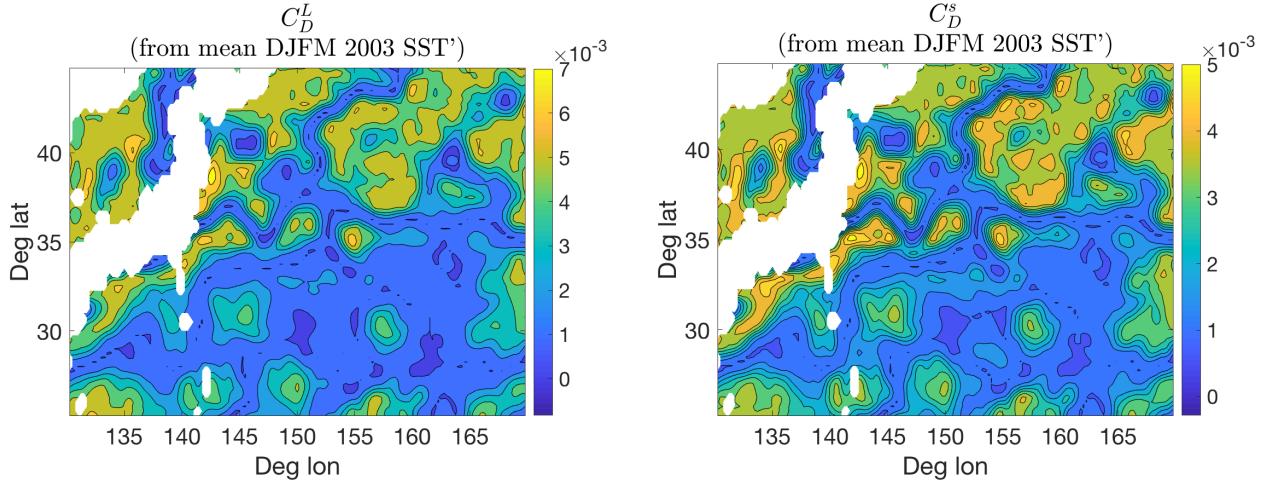
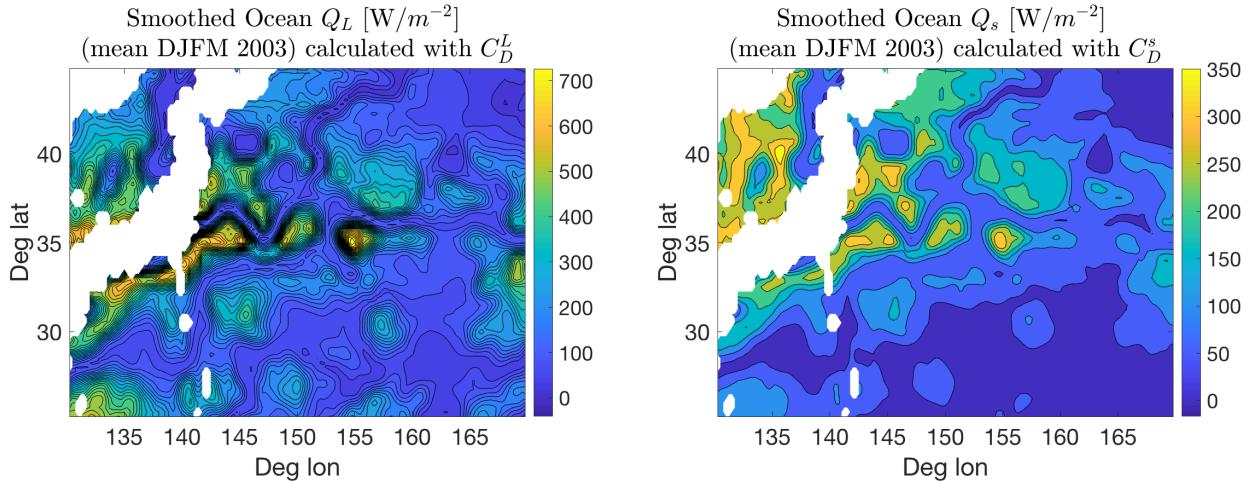


Figure 13: The drag coefficients using ERA5 SST' and smoothed α fields.



$$T_a = \overline{T}_a \quad (30)$$

$$T_o = \overline{T}_o \quad (31)$$

$$T_d = \overline{T}_d \quad (32)$$

$$p_0 = \overline{p}_0 \quad (33)$$

$$u_{10} = \overline{u}_{10} \quad (34)$$

$$v_{10} = \overline{v}_{10} \quad (35)$$

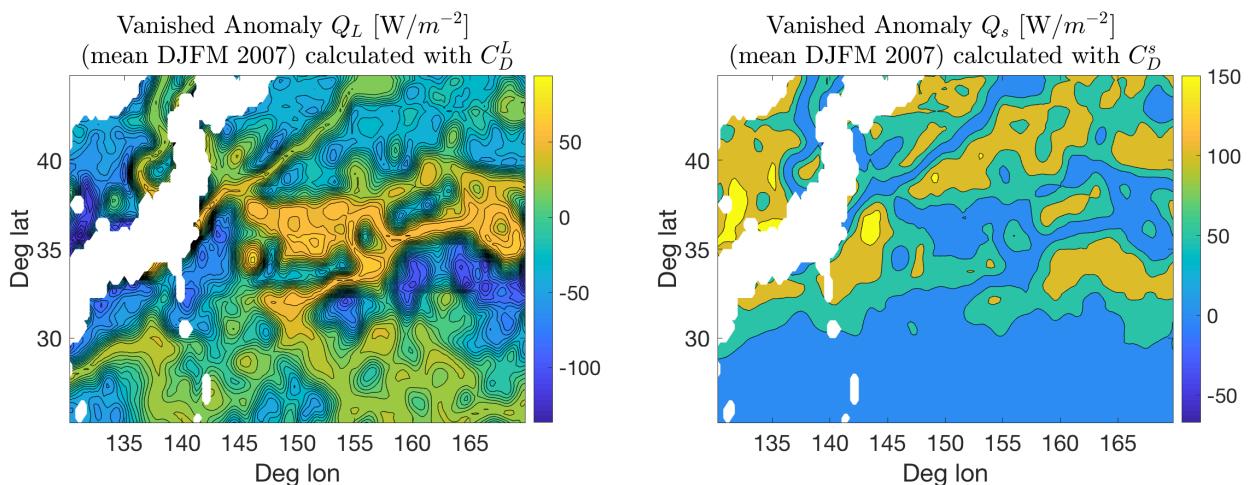
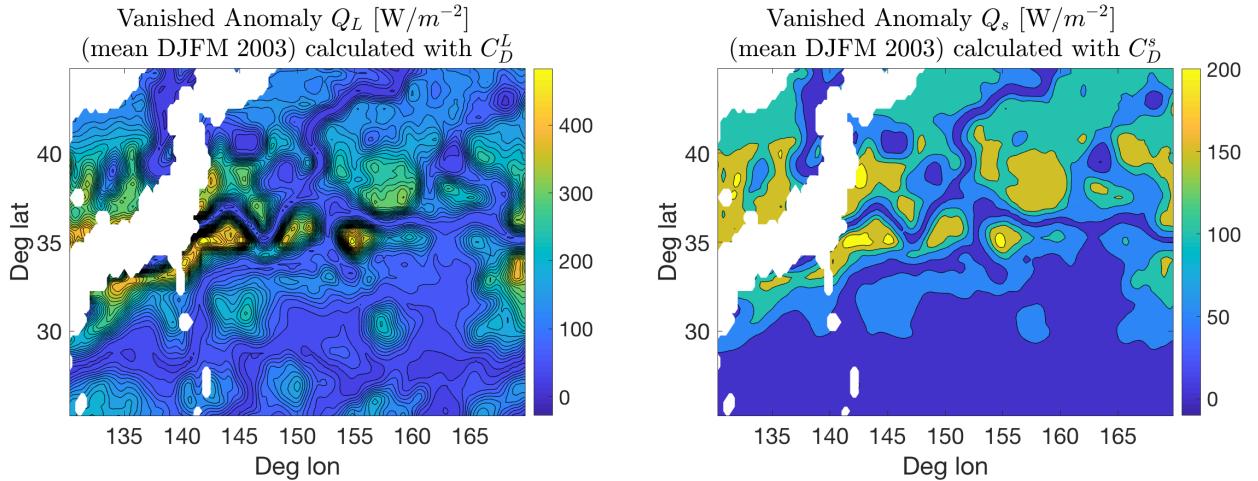
$$\|\mathbf{U}\| = \sqrt{u_{10}^2 + v_{10}^2} \quad (36)$$

$$q_o = q_o(\overline{T}_o, \overline{p}_0) \quad (37)$$

$$q_a = q_a(\overline{T}_a, \overline{T}_d, \overline{p}_0) \quad (38)$$

$$Q_s = \rho_a c_p C_D^* \|\mathbf{U}\| (T_o - T_a) \quad (39)$$

$$Q_L = \rho_a L_v C_D^* \|\mathbf{U}\| (q_o^* - q_a) \quad (40)$$



3 Idealized Front vs ERA 5

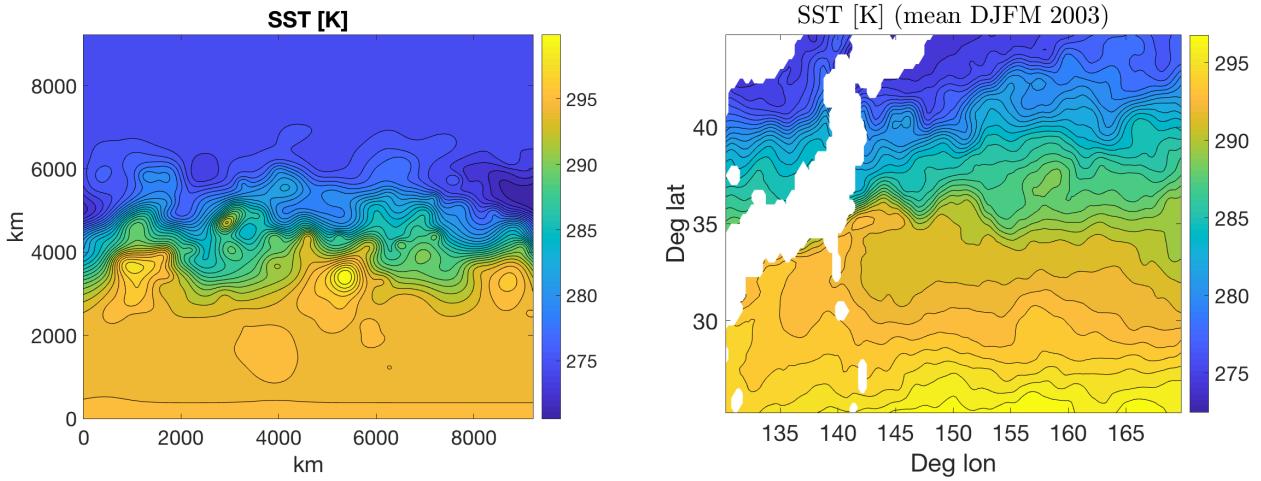


Figure 17: The mean SST from reanalysis is qualitatively similar to that from the idealized from in terms of the range of temperatures,

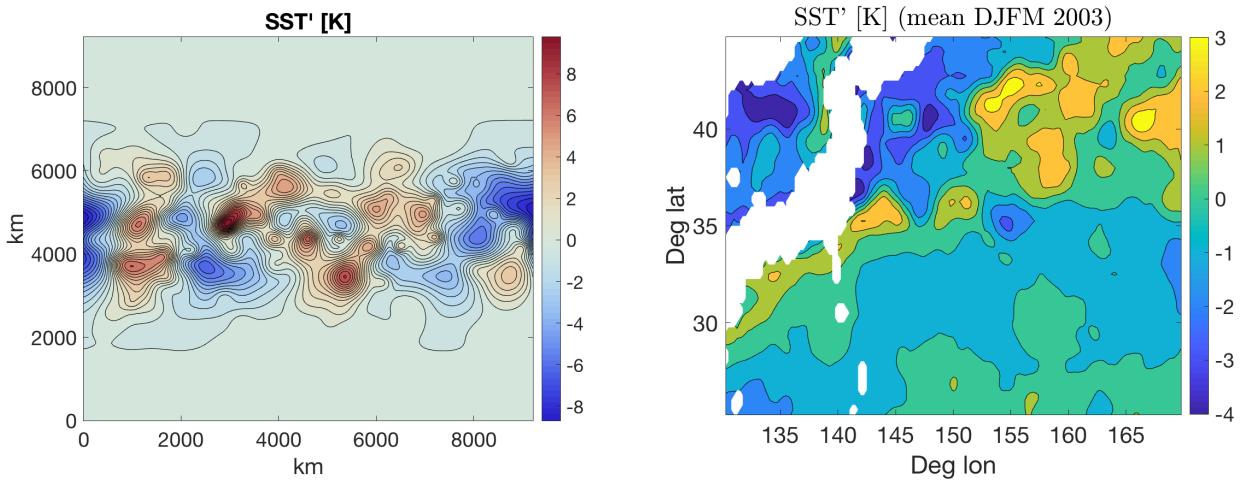
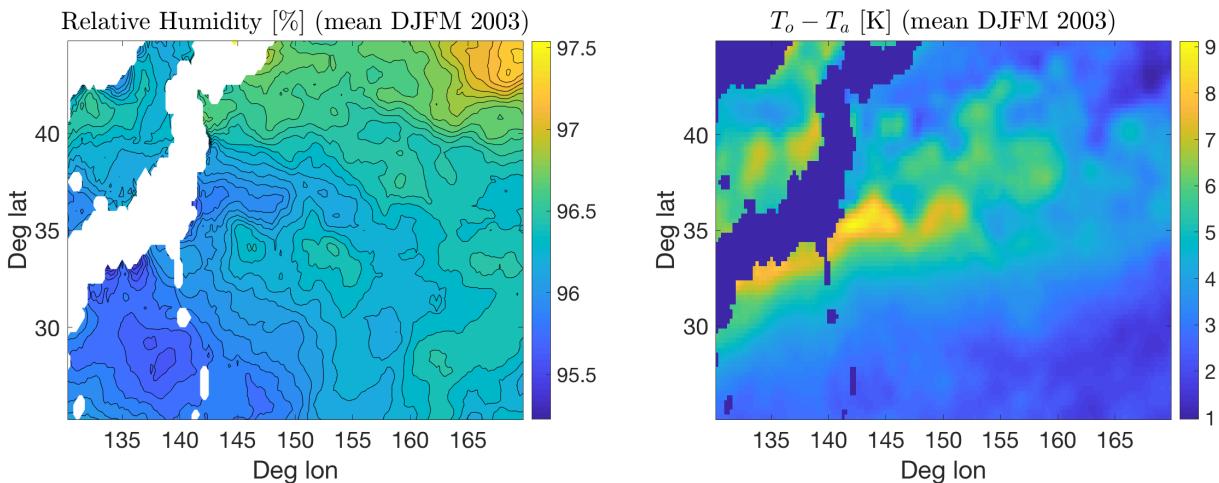


Figure 18: The mean anomaly (i.e. calculating the anomaly from the zonal mean every 6 hours then averaging all the anomalies together) from the ERA5 reanalysis data has smaller amplitudes than that from the idealized front.



- (a) The RH from ERA5 is much larger in general than the 80% assumed by the idealized front experiment.
- (b) The DT from ERA5 is much larger in general than the 0.5 assumed by the idealized front experiment.

A Proving the curl/divergence and cross-wind/down-wind gradient relationships

Variables:

C_D^*	Reference drag coefficient (1E-3)
ρ_a	air density [kg m^{-3}]
T_C	temperature field in (y) without eddies in [K] ¹
T'	temperature perturbation (eddies only) in (x, y) in [K] ²
T	total temperature field ($= T_C + T'$) in (x, y) in [K]
\vec{u}	velocity vector with components (u, v) in [m/s]
x, y	spatial coordinates [m]

A.1 stress calculations

For tuning parameter α , the surface stress is

$$\tau = \rho_a C_D^* (1 + \alpha T') \vec{u} \|u\|$$

The divergence of the stress is

$$\nabla \cdot \tau = C_D^* \rho_a \left(\frac{(1 + \alpha T') (4 \frac{\partial u}{\partial x} u^3 + 2 \frac{\partial v}{\partial x} u^2 v + 2 \frac{\partial u}{\partial x} u v^2)}{2 \sqrt{u^4 + u^2 v^2}} + \frac{(1 + \alpha T') (4 \frac{\partial v}{\partial y} v^3 + 2 \frac{\partial u}{\partial y} u v^2 + 2 \frac{\partial v}{\partial y} u^2)}{2 \sqrt{v^4 + u^2 v^2}} + \dots \right. \\ \left. \alpha \frac{\partial T'}{\partial x} \sqrt{u^4 + u^2 v^2} - \alpha \frac{\partial T'}{\partial y} \sqrt{v^4 + u^2 v^2} \right)$$

The curl of the stress is

$$\nabla \times \tau = C_D^* \rho_a \left(\frac{(1 + \alpha T') (4 \frac{\partial u}{\partial y} u^3 + 2 \frac{\partial v}{\partial y} u^2 v + 2 \frac{\partial u}{\partial y} u v^2)}{2 \sqrt{u^4 + u^2 v^2}} - \frac{(1 + \alpha T') (4 \frac{\partial v}{\partial x} v^3 + 2 \frac{\partial u}{\partial x} u v^2 + 2 \frac{\partial v}{\partial x} u^2)}{2 \sqrt{v^4 + u^2 v^2}} + \dots \right. \\ \left. \alpha \frac{\partial T'}{\partial y} \sqrt{u^4 + u^2 v^2} - \alpha \frac{\partial T'}{\partial x} \sqrt{v^4 + u^2 v^2} \right)$$

The gradient of the sea surface temperature in the "down-wind" or the "downwind" direction is

$$\nabla SST_{\parallel} = \left(\frac{u \left(\frac{\frac{\partial T}{\partial x} u}{\sqrt{u^2 + v^2}} + \frac{\frac{\partial T}{\partial y} v}{\sqrt{u^2 + v^2}} \right)}{\sqrt{u^2 + v^2}}, \frac{v \left(\frac{\frac{\partial T}{\partial x} u}{\sqrt{u^2 + v^2}} + \frac{\frac{\partial T}{\partial y} v}{\sqrt{u^2 + v^2}} \right)}{\sqrt{u^2 + v^2}} \right)$$

The gradient of the sea surface temperature in the "cross-wind" direction is

$$\nabla SST_{\perp} = \left(\frac{\partial T}{\partial x} - \frac{u \left(\frac{\frac{\partial T}{\partial x} u}{\sqrt{u^2 + v^2}} + \frac{\frac{\partial T}{\partial y} v}{\sqrt{u^2 + v^2}} \right)}{\sqrt{u^2 + v^2}}, \frac{\partial T}{\partial y} - \frac{v \left(\frac{\frac{\partial T}{\partial x} u}{\sqrt{u^2 + v^2}} + \frac{\frac{\partial T}{\partial y} v}{\sqrt{u^2 + v^2}} \right)}{\sqrt{u^2 + v^2}} \right)$$

A.2 In the limit that $\vec{u} = (\bar{u}, 0)$ everywhere

This means that $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = v = 0$ and $u = \bar{u}$

So the above quantities become:

Divergence:

$$\nabla \cdot \tau = C_D^* \rho_a \alpha \bar{u}^2 \frac{\partial T'}{\partial x}$$

Down-wind gradient:

$$\begin{aligned} \nabla SST_{\parallel} &= \left(\frac{\partial T}{\partial x}, 0 \right) \\ &= \left(\frac{\partial}{\partial x} (T_C + T'), 0 \right) \\ &= \left(\frac{\partial T'}{\partial x}, 0 \right) \end{aligned}$$

Which, when plotted against each other, will have a slope of $C_D^* \rho_a \alpha \bar{u}^2$.

Curl:

$$\nabla \times \tau = -C_D^* \rho_a \alpha \bar{u}^2 \frac{\partial T'}{\partial y}$$

Cross-wind gradient:

$$\begin{aligned} \nabla SST_{\perp} &= \left(\frac{\partial T}{\partial x} - \frac{\partial T'}{\partial x}, \frac{\partial T}{\partial y} - 0 \right) \\ \nabla SST_{\perp} &= \left(0, \frac{\partial}{\partial y} (T_C + T') \right) \\ \nabla SST_{\perp} &= \left(0, \frac{\partial}{\partial y} T_C + \frac{\partial}{\partial y} T' \right) \end{aligned}$$

Which, when plotted against each other, will have a slope of approximately $-C_D^* \rho_a \alpha \bar{u}^2$ if $\frac{\partial T_C}{\partial y}$ is small relative to $\frac{\partial T'}{\partial y}$.

A.3 In the limit that the velocity field is determined by a coupling coefficient

$\vec{u} = (\bar{u} + T' \gamma, 0)$ for a coupling coefficient γ and the drag coefficient is no longer a function of temperature (i.e. $\alpha = 0$)

The divergence of the stress is now

$$\begin{aligned}
\nabla \cdot \tau &= C_D^* \rho_a \left(\frac{(4 \frac{\partial u}{\partial x} u^3)}{2u^2} \right) \\
\nabla \cdot \tau &= C_D^* \rho_a \left(2u \frac{\partial u}{\partial x} \right) \\
\nabla \cdot \tau &= C_D^* \rho_a \left(2u\gamma \frac{\partial T'}{\partial x} \right) \\
\nabla \cdot \tau &= C_D^* \rho_a \left(2\gamma \frac{\partial T'}{\partial x} (\bar{u} + \gamma T') \right)
\end{aligned}$$

and the gradient of the sea surface temperature in the "downwind" direction is

$$\nabla SST_{\parallel} = \left(\frac{\partial T'}{\partial x}, 0 \right)$$

If $\gamma T'$ is small with respect to \bar{u} , then the slope of the divergence plotted against the "downwind" SST gradient will be $C_D^* \rho_a 2\gamma \bar{u}$.

The curl of the stress is now

$$\begin{aligned}
\nabla \times \tau &= C_D^* \rho_a \left(\frac{(4 \frac{\partial u}{\partial y} u^3)}{2u^2} \right) \\
\nabla \times \tau &= C_D^* \rho_a \left(2u \frac{\partial u}{\partial y} \right) \\
\nabla \times \tau &= C_D^* \rho_a \left(2u\gamma \frac{\partial T'}{\partial y} \right) \\
\nabla \times \tau &= C_D^* \rho_a \left(2\gamma \frac{\partial T'}{\partial y} (\bar{u} + \gamma T') \right)
\end{aligned}$$

and the gradient of the sea surface temperature in the "cross-wind" direction is

$$\nabla SST_{\perp} = \left(0, \frac{\partial}{\partial y} T_C + \frac{\partial}{\partial y} T' \right)$$

so in the limit that $\gamma T'$ is small with respect to \bar{u} and $\frac{\partial}{\partial y} T_C$ is small with respect to $\frac{\partial}{\partial y} T'$, the resulting slope of these two quantities plotted against each other would also be $C_D^* \rho_a 2\gamma \bar{u}$.

References

- Chelton, D. B., Schlax, M. G., Freilich, M. H., and Milliff, R. F. (2004). Satellite Radar Measurements Reveal Short-scale Features in the Wind Stress Field over the World Ocean. *Science*, 303(February):978–983.
- Foussard, A., Lapeyre, G., and Plougouven, R. (2019). Storm track response to oceanic eddies in idealized atmospheric simulations. *Journal of Climate*, 32(2):445–463.