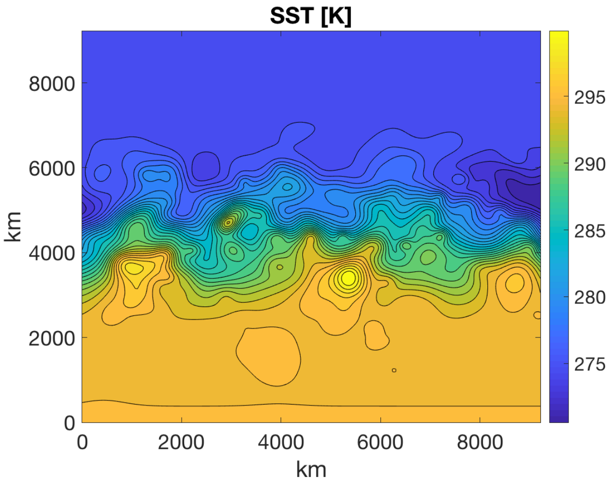
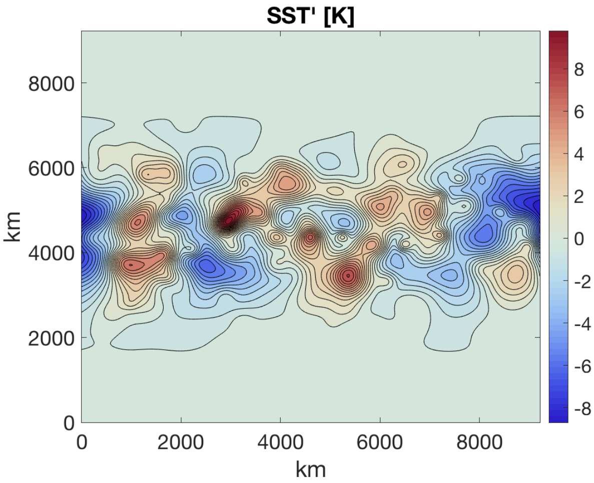
1 Idealized Front

Create the setup from Foussard et al. (2019) and consider



(a) Manufactured SST temperature



(b) Manufactured SST temperature anomaly from zonal mean

$$C_D = C_D^*(1 + \alpha T_o') \tag{1}$$
$$Q_s = \rho_a c_p C_D \|\mathbf{U}\|(T_o - T_a) \tag{2}$$
$$Q_L = \rho_a L_v C_D \|\mathbf{U}\|(q_o^* - q_a) \tag{3}$$

where the free parameters are, $T_o - T_a$ (ΔT), α, and relative humidity (RH).

A wind field is applied (see three cases below) and the wind stress is calculated as $\tau_{xy} = \rho_a C_D \|\vec{u}\|\vec{u}$. Let $\hat{u}$ be the unit vector in the direction of the wind at each point. The along-wind (aw) and cross-wind (cw) components of the SST gradients are

$$\nabla_{aw} T_o = (\nabla T_o \cdot \hat{u}) \cdot \hat{u} \tag{4}$$
$$\nabla_{cw} T_o = \nabla T_o - ((\nabla T_o \cdot \hat{u}) \cdot \hat{u}) \tag{5}$$

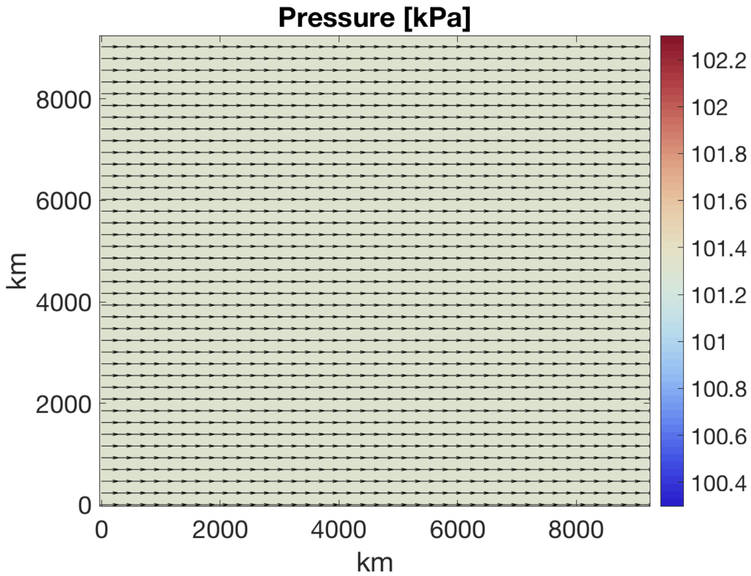
1.1 Constant Pressure and Velocity ($\vec{u} = (\bar{u}, 0)$, $p = p_0$)

With a constant pressure and velocity field the curl of the stress is directly proportional to the cross-wind SST gradient, and the divergence of the stress is directly proportional to the down-wind SST gradient (see Section A.2) at each point. For a particular setup with $C_D^* = 1E-3$, $\rho_a = 1.2$ kg/m^{-3}, $\alpha = 1E-3$, $\bar{u} = 4$ m/s, the slope of the curl of the stress (y-axis) to the cross-wind SST gradient (x-axis) is 1.92E-5 as seen in Figure 2.

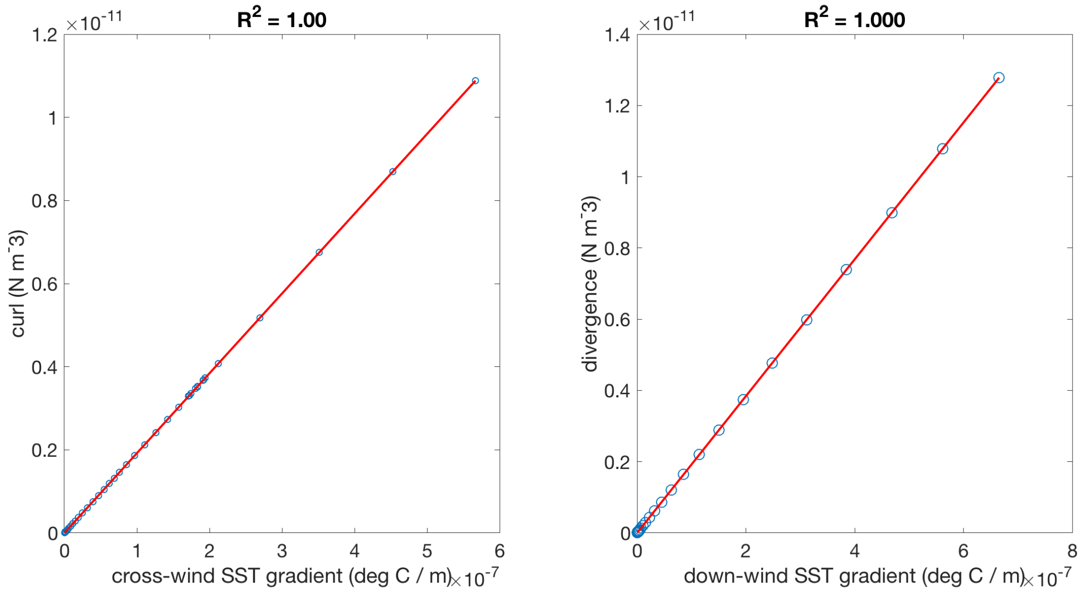
1.2 Chelton Coupling Coefficient ($u = \overline{u} + \gamma T'$, $p = \overline{p}$)

From (Chelton et al., 2004), the coupling coefficient is between 0.2 and 0.44. A detailed derivation is in Section A.3, but the conclusion that if $\gamma T'$ is small with respect to $\bar{u}$, then the slope of the divergence plotted against the "down-wind" SST gradient will be $C_D^* \rho_a 2\gamma\bar{u}$, which for this setup would be $C_D^* \rho_a 2\gamma\bar{u} = 0.0019$ and the slope of the best fit line for this setup is 0.002 as shown in Figure 4.

1.3 Baroclinic Wave ($u = u_{\textbf{geostrophic}}$)

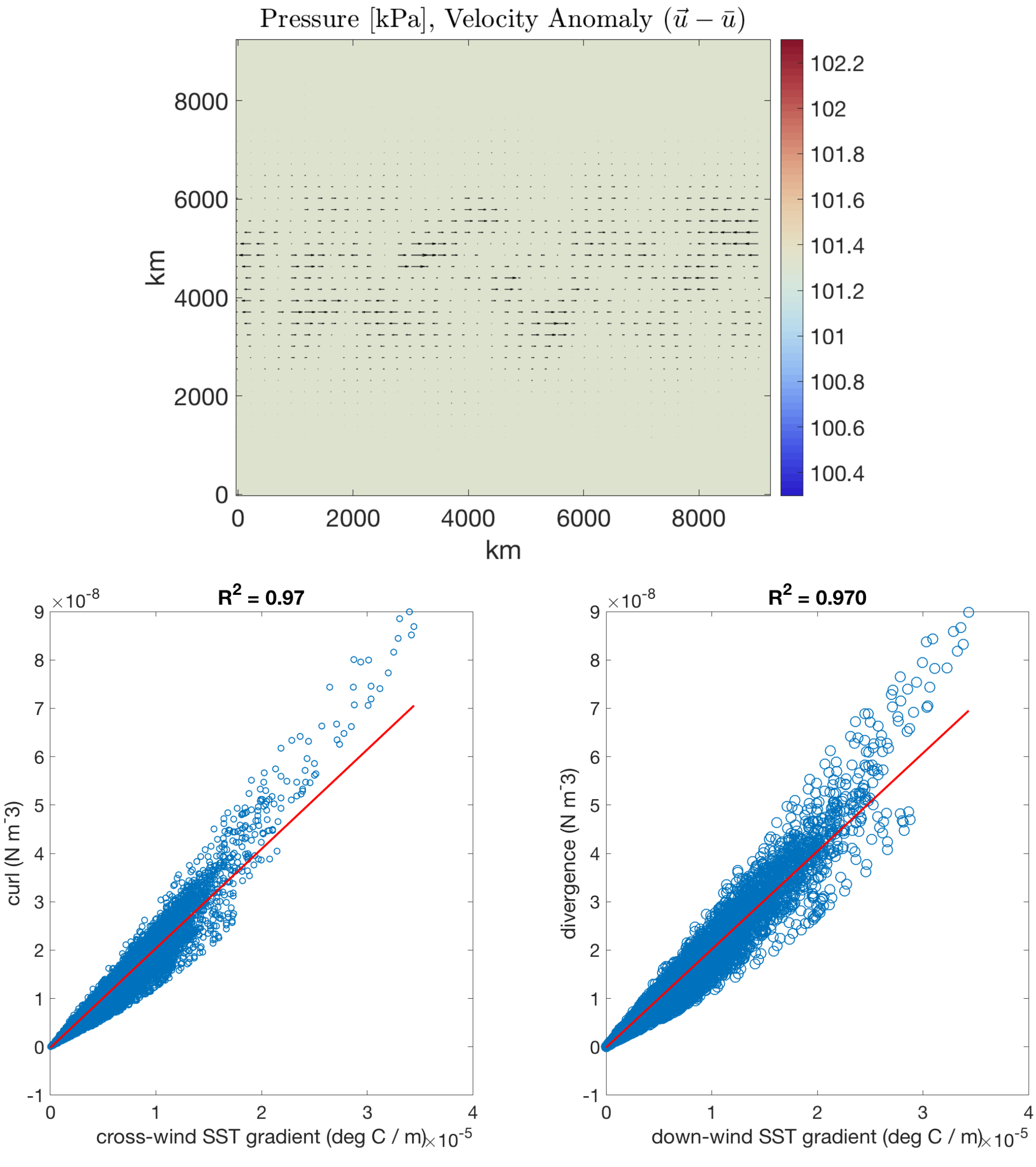


(a) Manufactured SST temperature



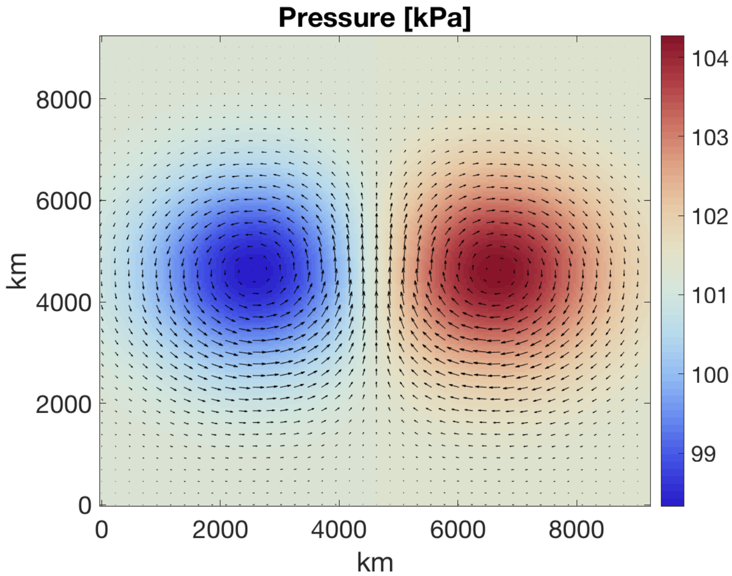
(b) Sample of 10 points in blue circles (however all points in the domain fall on red line which has a slope of 1.92E-5)

Figure 2

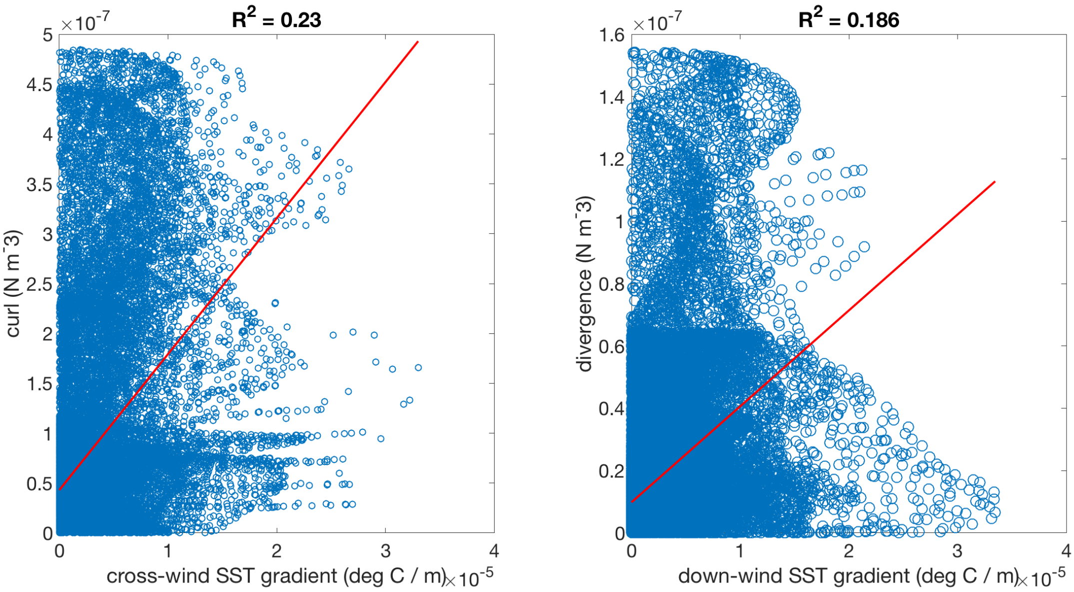


(a) All points in the domain are represented by blue circles, the slope of the divergence-down-wind gradient fitted line is 0.002.

Figure 3



(a) The pressure field is two Gaussians creating a wavelength of $\lambda = 4000\text{km}$.



(b) All points in the domain are represented by blue circles

Figure 4

2 ERA5

2.1 Which years to pick?

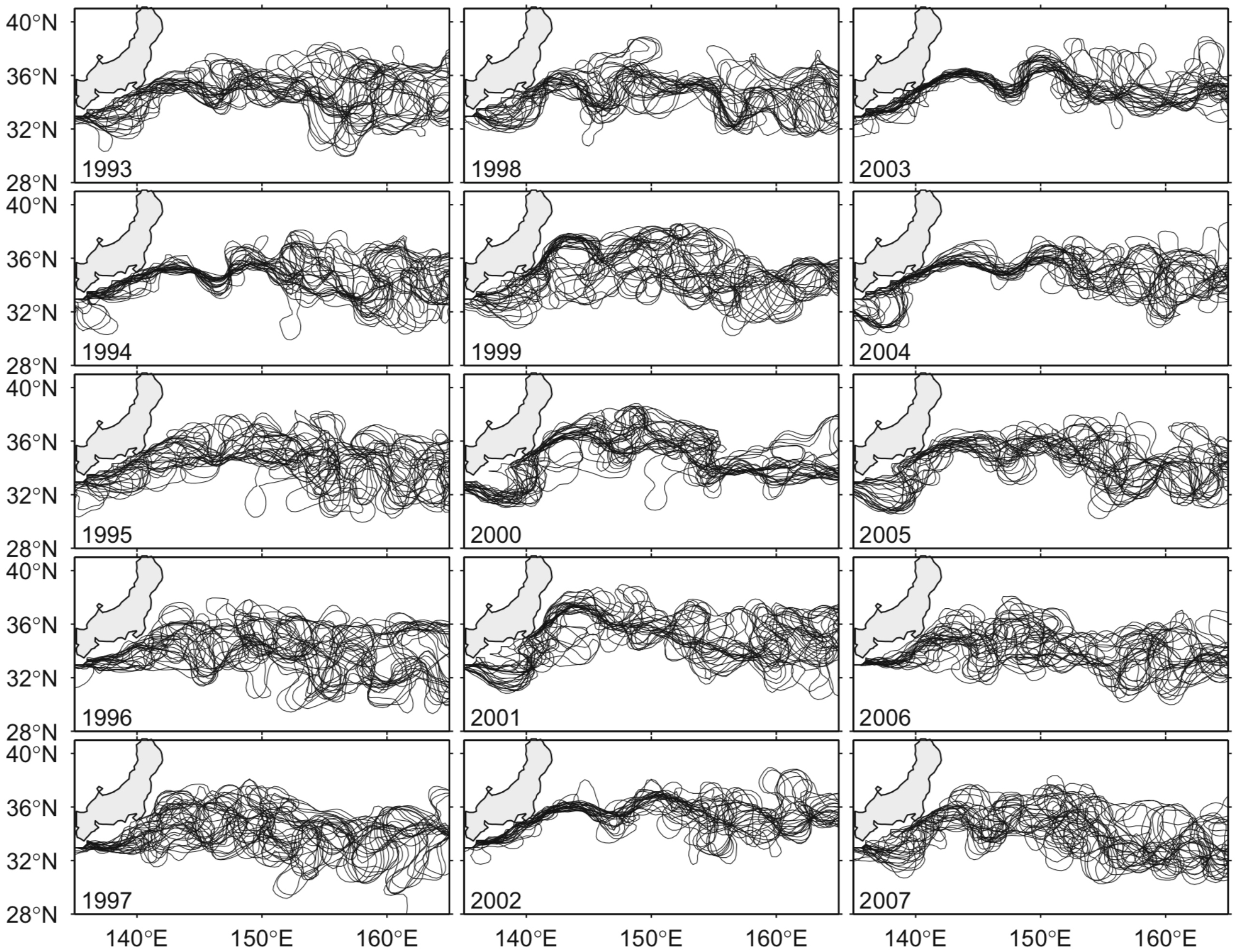


Figure 5: **?**

$\bar{\bullet}$	zonal average	$\bullet'$	zonal anomaly
$\langle\bullet\rangle$	temporal average	C_D^*	1E-3
T_o	Sea Surface Temperature	T_a	2m Temperature
q_o	saturated specific humidity at the local SST	a_a	specific humidity at the 2m temperature
RH	relative humidity at 2m	T_d	dew point temperature at 2m
c_p	specific heat of air	L_v	latent heat of vaporization

ERA5 has 0.25° resolution.

$$C_D^{s,L} = C_D^*(1 + \alpha_{s,L} T') \tag{6}$$

$$Q_s = \rho_a c_p C_D^s \|\mathbf{U}\|(T_o - T_a) \tag{7}$$

$$Q_L = \rho_a L_v C_D^L \|\mathbf{U}\|(q_o^* - q_a) \tag{8}$$

2.2 Full Atmosphere Full Ocean

Use Q_s and Q_L from ERA5 (along with the SST, 2m dewpoint temperature, 2m temperature, surface pressure, and 10m horizontal wind speed) to calculate α at every point in space and time.

$$T_a = \overline{T_a} + T_a' \tag{9}$$
$$T_d = \overline{T_d} + T_d' \tag{10}$$
$$p_0 = \overline{p_0} + p_0' \tag{11}$$
$$u_{10} = \overline{u_{10}} + u_{10}' \tag{12}$$
$$v_{10} = \overline{v_{10}} + v_{10}' \tag{13}$$
$$\|\mathbf{U}\| = \sqrt{u_{10}^2 + v_{10}^2} \tag{14}$$
$$q_o = q_o(\overline{T_o} + T_o', \overline{p_0} + p_0') \tag{15}$$
$$q_a = q_a(\overline{T_a} + T_a', \overline{T_d} + T_d', \overline{p_0} + p_0') \tag{16}$$

$$\alpha_s = \frac{1}{T'}\left(1 - \frac{Q_S}{\rho_a c_p C_D^* \|\mathbf{U}\|(T_o - T_a)}\right) \tag{17}$$
$$\alpha_L = \frac{1}{T'}\left(1 - \frac{Q_S}{\rho_a L_v C_D^* \|\mathbf{U}\|(q_o - q_a)}\right) \tag{18}$$

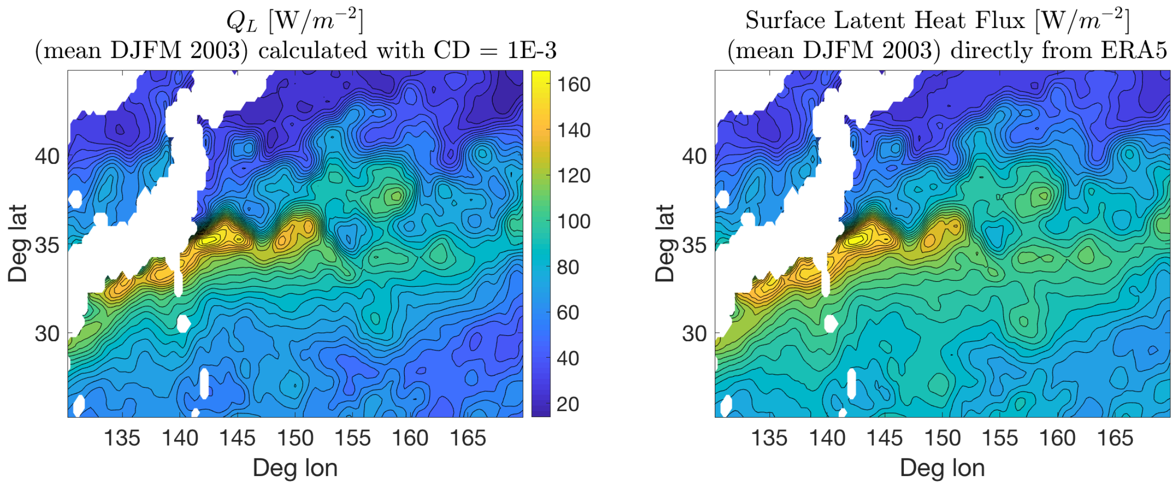


Figure 6: The mean SLHF from Equation 29 and directly from the reanalysis data

2.3 Full Atmosphere, Smooth Ocean

Compute the flux from a smooth sea surface where $T_o' = 0$.

$$T_a = \overline{T_a} + T_a' \tag{19}$$
$$T_o = \overline{T_o} \tag{20}$$
$$T_d = \overline{T_d} + T_d' \tag{21}$$
$$p_0 = \overline{p_0} + p_0' \tag{22}$$
$$u_{10} = \overline{u_{10}} + u_{10}' \tag{23}$$
$$v_{10} = \overline{v_{10}} + v_{10}' \tag{24}$$
$$\|\mathbf{U}\| = \sqrt{u_{10}^2 + v_{10}^2} \tag{25}$$
$$q_o = q_o(\overline{T_o}, \overline{p_0} + p_0') \tag{26}$$
$$q_a = q_a(\overline{T_a} + T_a', \overline{T_d} + T_d', \overline{p_0} + p_0') \tag{27}$$

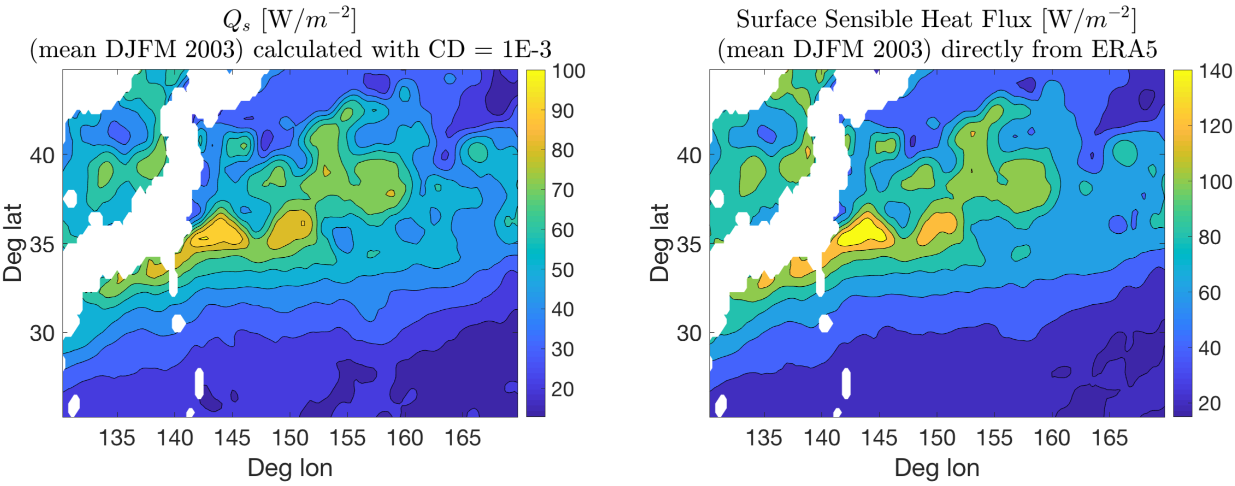


Figure 7: The mean SSHF from Equation 28 and directly from the reanalysis data

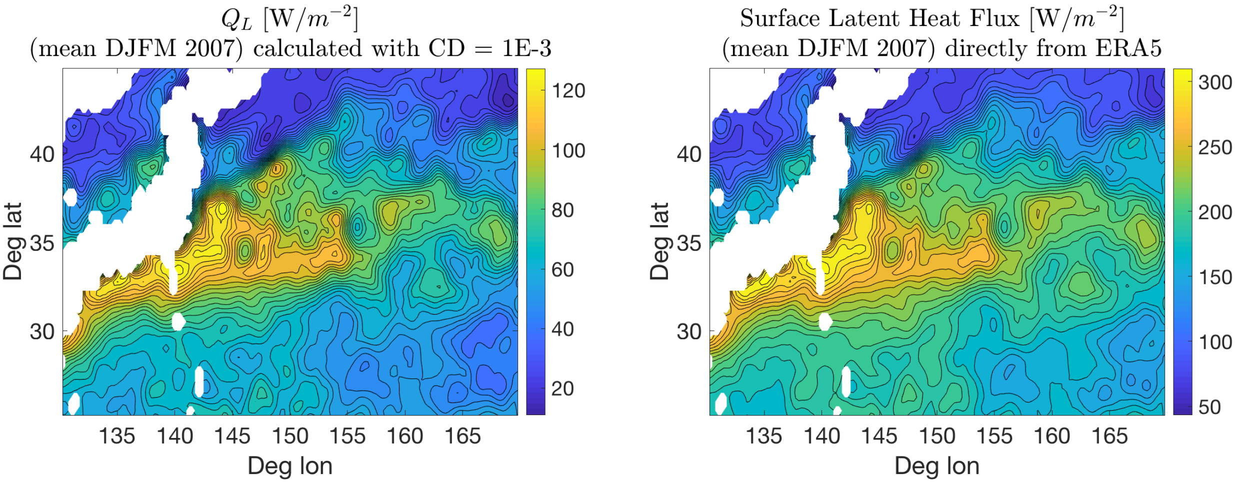


Figure 8: The mean SLHF from Equation 29 and directly from the reanalysis data

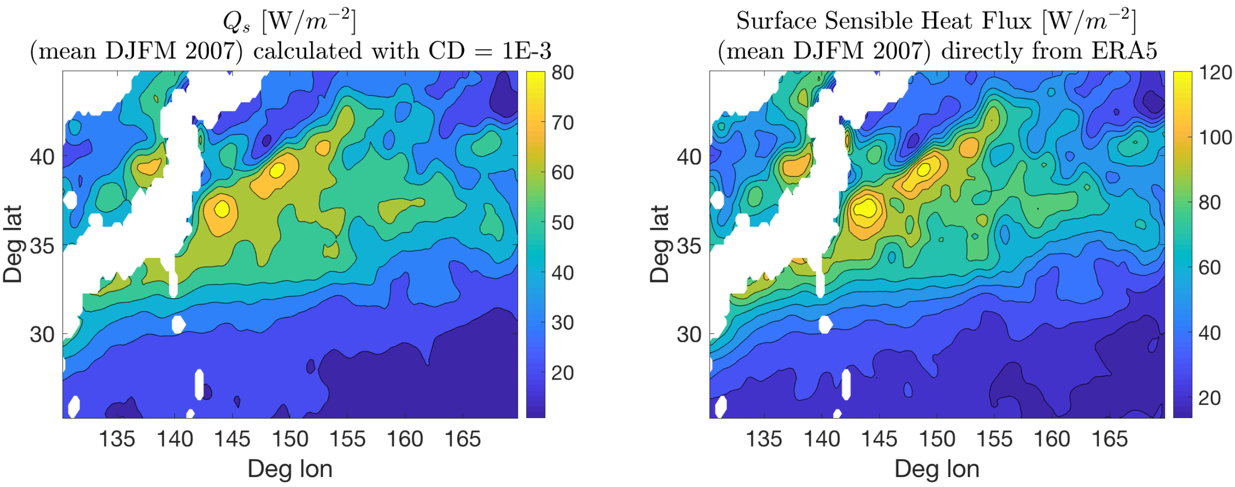


Figure 9: The mean SSHF from Equation 28 and directly from the reanalysis data

$$Q_s = \rho_a c_p C_D^* \|\mathbf{U}\|(T_o - T_a) \tag{28}$$

$$Q_L = \rho_a L_v C_D^* \|\mathbf{U}\|(q_o^* - q_a) \tag{29}$$

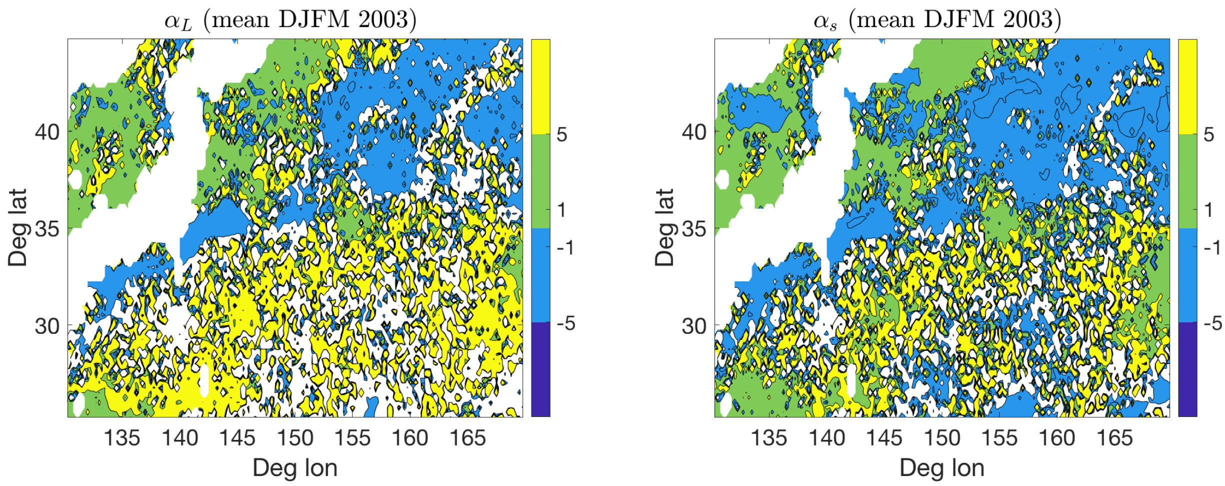


Figure 10: The mean α values (i.e. α is calculated every 6 hours then point-wise averaged in time). The color contours are limited to ± 5 since at some points the value of α contains "Inf" values.

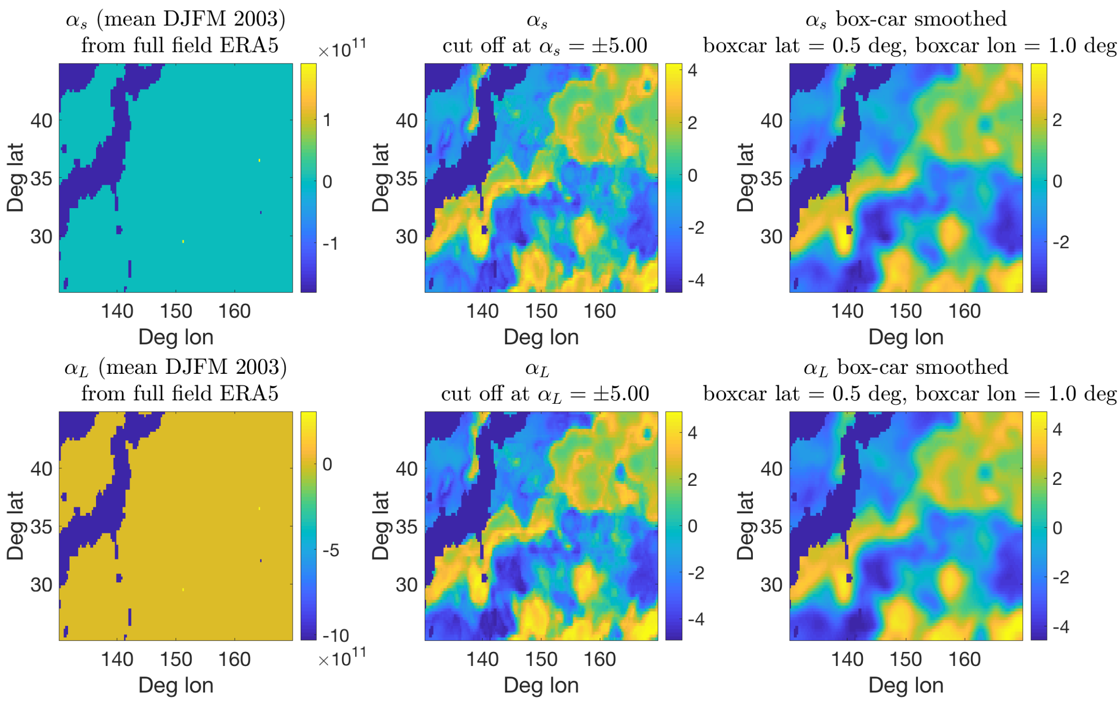


Figure 11: using a cut-off and box-car smoothing the α fields.

2.4 Vanished Anomaly

Compute the flux from a smooth sea surface where $T' = 0$.

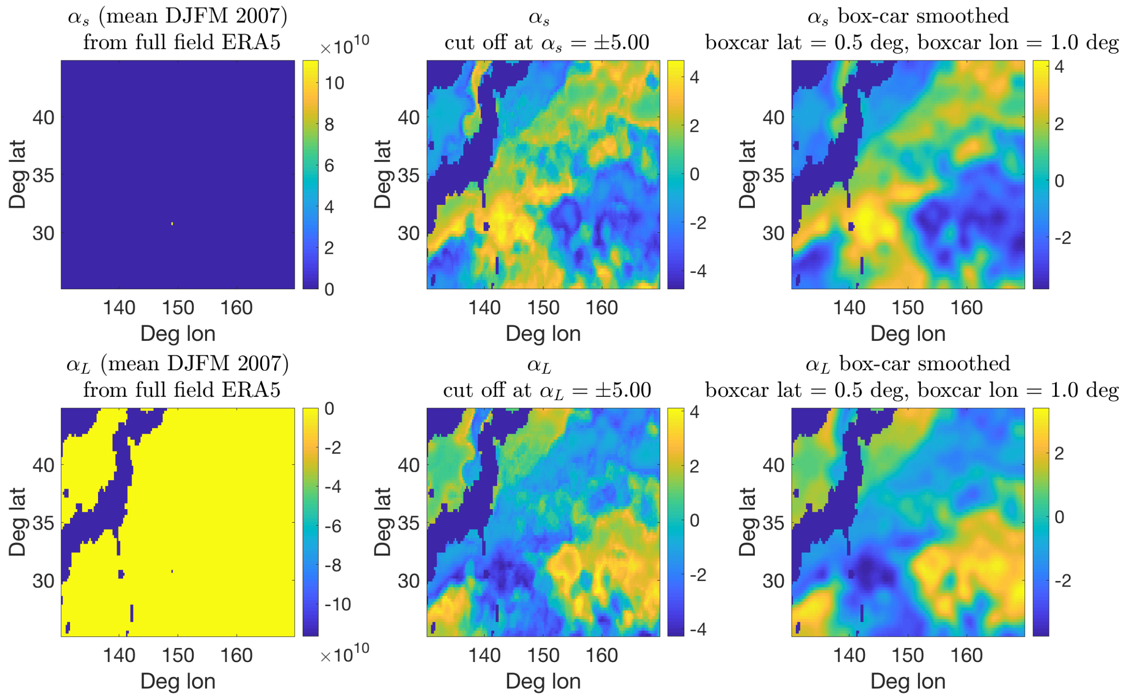


Figure 12: using a cut-off and box-car smoothing the α fields.

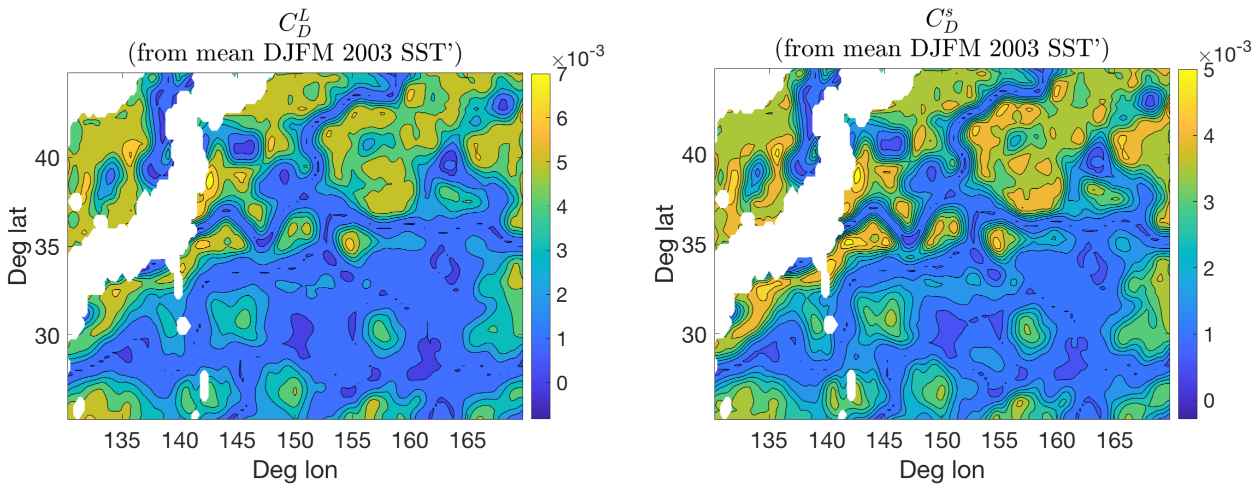
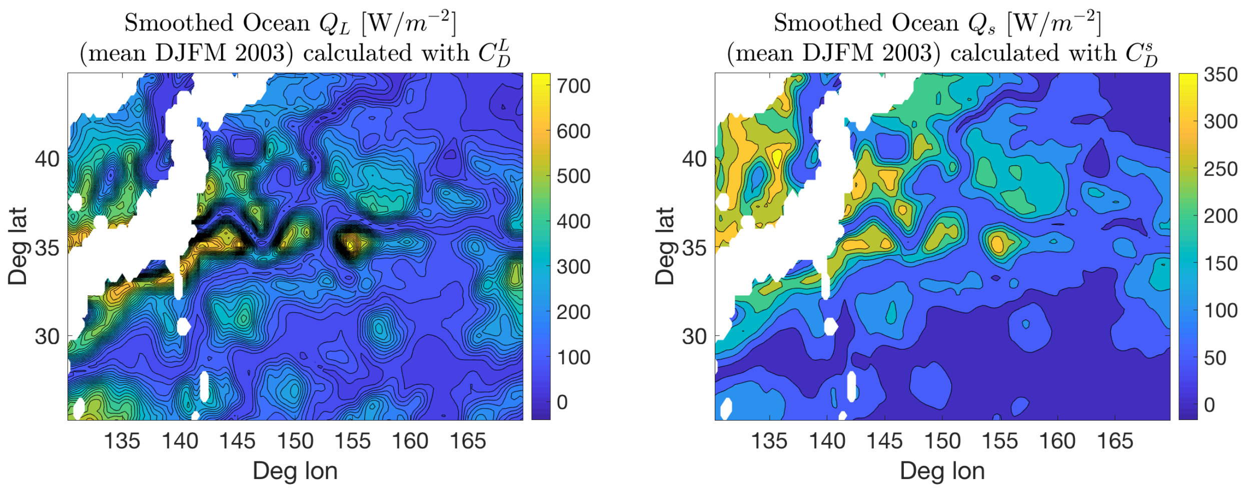
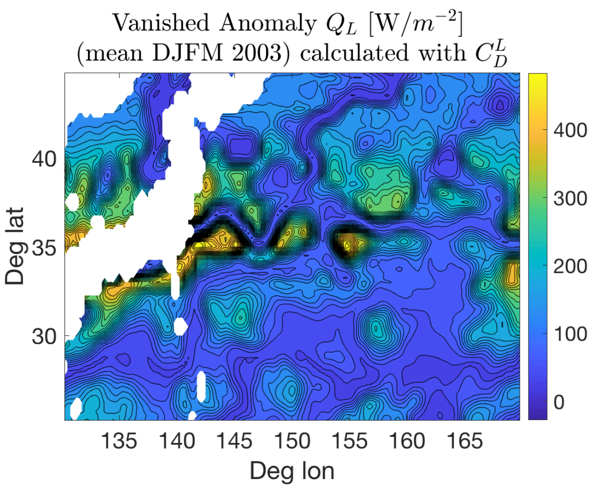


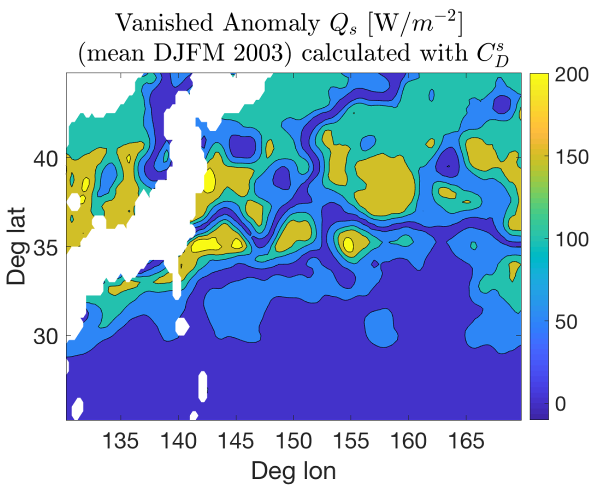
Figure 13: The drag coefficients using ERA5 SST' and smoothed α fields.

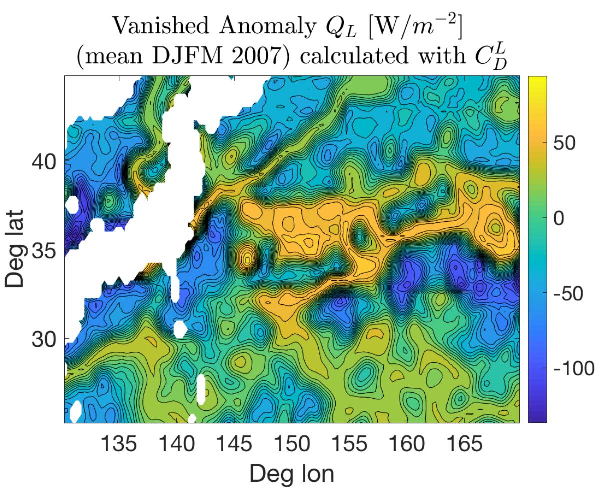


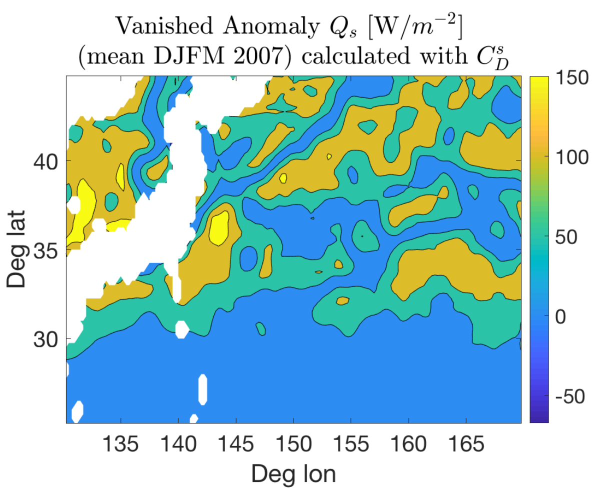
$$
\begin{aligned}
T_a &= \overline{T_a} &(30)\\
T_o &= \overline{T_o} &(31)\\
T_d &= \overline{T_d} &(32)\\
p_0 &= \overline{p_0} &(33)\\
u_{10} &= \overline{u_{10}} &(34)\\
v_{10} &= \overline{v_{10}} &(35)\\
\|\mathbf{U}\| &= \sqrt{u_{10}^2 + v_{10}^2} &(36)\\
q_o &= q_o(\overline{T_o}, \overline{p_0}) &(37)\\
q_a &= q_a(\overline{T_a}, \overline{T_d}, \overline{p_0}) &(38)
\end{aligned}
$$

$$
\begin{aligned}
Q_s &= \rho_a c_p C_D^* \|\mathbf{U}\| (T_o - T_a) &(39)\\
Q_L &= \rho_a L_v C_D^* \|\mathbf{U}\| (q_o^* - q_a) &(40)
\end{aligned}
$$









3 Idealized Front vs ERA 5

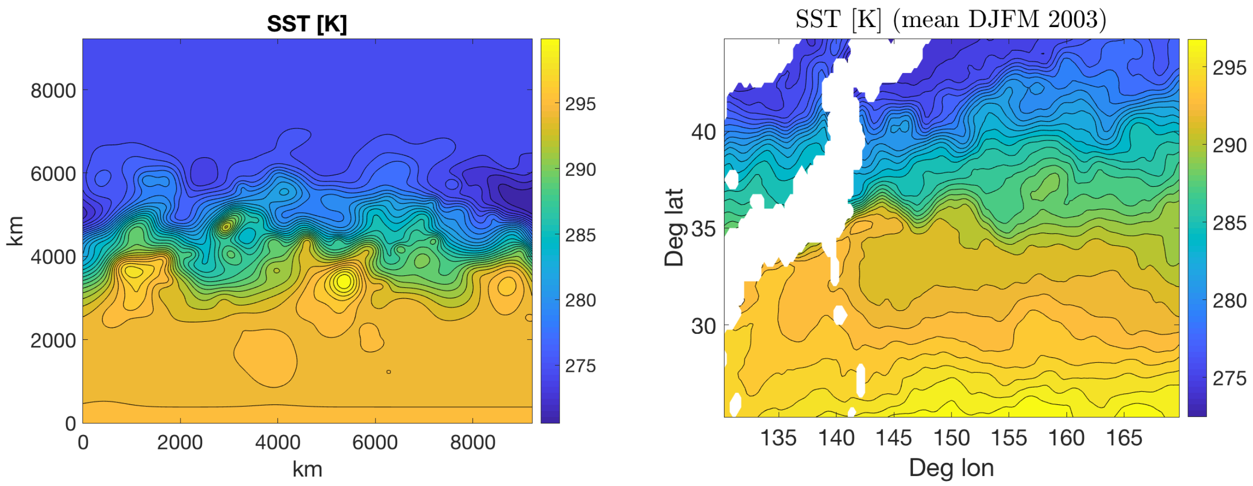


Figure 17: The mean SST from reanalysis is qualitatively similar to that from the idealized from in terms of the range of temperatures,

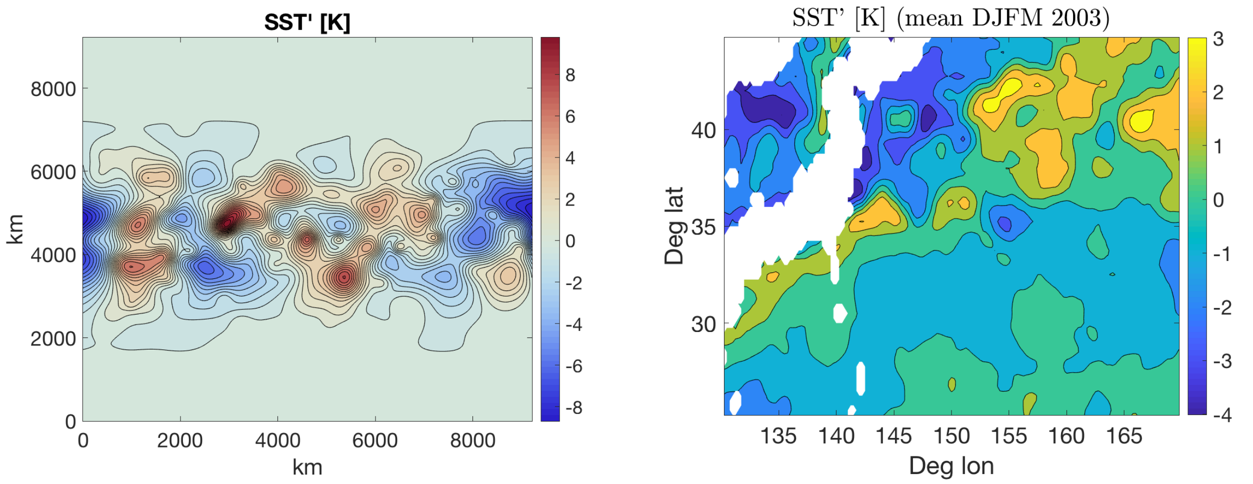
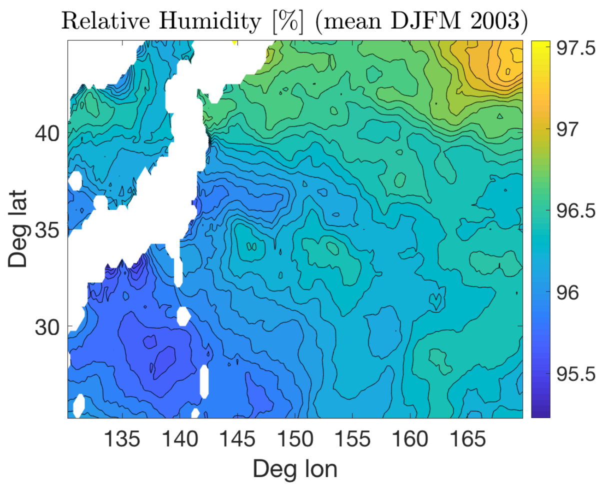
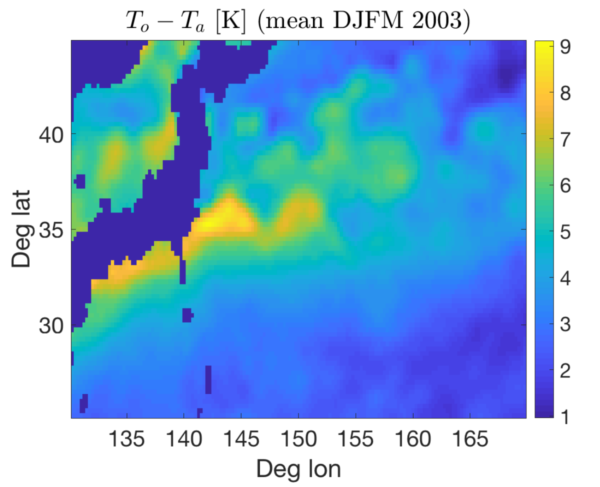


Figure 18: The mean anomaly (i.e. calculating the anomaly from the zonal mean every 6 hours then averaging all the anomalies together) from the ERA5 reanalysis data has smaller amplitudes than that from the idealized front.



(a) The RH from ERA5 is much larger in general than the 80% assumed by the idealized front experiment.



(b) The DT from ERA5 is much larger in general than the 0.5 assumed by the idealized front experiment.

A Proving the curl/divergence and cross-wind/down-wind gradient relationships

Variables:

C_D^*	Reference drag coefficient (1E-3)
ρ_a	air density [kg m^{-3}]
T_C	temperature field in (y) without eddies in [K][1]
T'	temperature perturbation (eddies only) in (x, y) in [K][2]
T	total temperature field $(= T_C + T')$ in (x, y) in [K]
$\vec{u}$	velocity vector with components (u, v) in [m/s]
x, y	spatial coordinates [m]

A.1 stress calculations

For tuning parameter α, the surface stress is

$$\tau = \rho_a C_D^* (1 + \alpha T')\vec{u}\|u\|$$

The divergence of the stress is

$$\nabla \cdot \tau = C_D^* \rho_a \left(\frac{(1 + \alpha T')(4\frac{\partial u}{\partial x}u^3 + 2\frac{\partial v}{\partial x}u^2 v + 2\frac{\partial u}{\partial x}uv^2)}{2\sqrt{u^4 + u^2v^2}} + \frac{(1 + \alpha T')(4\frac{\partial v}{\partial y}v^3 + 2\frac{\partial u}{\partial y}uv^2 + 2\frac{\partial v}{\partial y}u^2)}{2\sqrt{v^4 + u^2v^2}} + \ldots \right.$$
$$\left. \alpha\frac{\partial T'}{\partial x}\sqrt{u^4 + u^2v^2} - \alpha\frac{\partial T'}{\partial y}\sqrt{v^4 + u^2v^2} \right)$$

The curl of the stress is

$$\nabla \times \tau = C_D^* \rho_a \left(\frac{(1 + \alpha T')(4\frac{\partial u}{\partial y}u^3 + 2\frac{\partial v}{\partial y}u^2 v + 2\frac{\partial u}{\partial y}uv^2)}{2\sqrt{u^4 + u^2v^2}} - \frac{(1 + \alpha T')(4\frac{\partial v}{\partial x}v^3 + 2\frac{\partial u}{\partial x}uv^2 + 2\frac{\partial v}{\partial x}u^2)}{2\sqrt{v^4 + u^2v^2}} + \ldots \right.$$
$$\left. \alpha\frac{\partial T'}{\partial y}\sqrt{u^4 + u^2v^2} - \alpha\frac{\partial T'}{\partial x}\sqrt{v^4 + u^2v^2} \right)$$

The gradient of the sea surface temperature in the "down-wind" or the "downwind" direction is

$$\nabla SST_{\|} = \left(\frac{u\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2 + v^2}}, \frac{v\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2 + v^2}} \right)$$

The gradient of the sea surface temperature in the "cross-wind" direction is

$$\nabla SST_{\perp} = \left(\frac{\partial T}{\partial x} - \frac{u\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2 + v^2}}, \frac{\partial T}{\partial y} - \frac{v\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2 + v^2}} \right)$$

A.2 In the limit that $\vec{u} = (\bar{u}, 0)$ everywhere

This means that $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = v = 0$ and $u = \bar{u}$

So the above quantities become:

Divergence:

$$\nabla \cdot \tau = C_D^* \rho_a \alpha \bar{u}^2 \frac{\partial T'}{\partial x}$$

Down-wind gradient:

$$\begin{aligned} \nabla SST_{\parallel} &= \left(\frac{\partial T}{\partial x}, 0 \right) \\ &= \left(\frac{\partial}{\partial x}(T_C + T'), 0) \right) \\ &= \left(\frac{\partial T'}{\partial x}, 0 \right) \end{aligned}$$

Which, when plotted against eachother, will have a slope of $C_D^* \rho_a \alpha \bar{u}^2$.

Curl:

$$\nabla \times \tau = -C_D^* \rho_a \alpha \bar{u}^2 \frac{\partial T'}{\partial y}$$

Cross-wind gradient:

$$\begin{aligned} \nabla SST_{\perp} &= \left(\frac{\partial T}{\partial x} - \frac{\partial T'}{\partial x}, \frac{\partial T}{\partial y} - 0 \right) \\ \nabla SST_{\perp} &= \left(0, \frac{\partial}{\partial y}(T_C + T') \right) \\ \nabla SST_{\perp} &= \left(0, \frac{\partial}{\partial y} T_C + \frac{\partial}{\partial y} T' \right) \end{aligned}$$

Which, when plotted against eachother, will have a slope of approximately $-C_D^* \rho_a \alpha \bar{u}^2$ if $\frac{\partial T_C}{\partial y}$ is small relative to $\frac{\partial T'}{\partial y}$.

A.3 In the limit that the velocity field is determined by a coupling coefficient

$\vec{u} = (\bar{u} + T'\gamma, 0)$ for a coupling coefficient γ and the drag coefficient is nolonger a function of temperature (i.e. $\alpha = 0$)

The divergence of the stress is now

$$\nabla \cdot \tau = C_D^* \rho_a \left(\frac{(4 \frac{\partial u}{\partial x} u^3)}{2u^2} \right)$$
$$\nabla \cdot \tau = C_D^* \rho_a \left(2u \frac{\partial u}{\partial x} \right)$$
$$\nabla \cdot \tau = C_D^* \rho_a \left(2u\gamma \frac{\partial T'}{\partial x} \right)$$
$$\nabla \cdot \tau = C_D^* \rho_a \left(2\gamma \frac{\partial T'}{\partial x} (\bar{u} + \gamma T') \right)$$

and the gradient of the sea surface temperature in the ”downwind” direction is

$$\nabla SST_{\parallel} = \left(\frac{\partial T'}{\partial x}, 0 \right)$$

If $\gamma T'$ is small with respect to $\bar{u}$, then the slope of the divergence plotted against the ”downwind” SST gradient will be $C_D^* \rho_a 2\gamma\bar{u}$.

The curl of the stress is now

$$\nabla \times \tau = C_D^* \rho_a \left(\frac{(4 \frac{\partial u}{\partial y} u^3)}{2u^2} \right)$$
$$\nabla \times \tau = C_D^* \rho_a \left(2u \frac{\partial u}{\partial y} \right)$$
$$\nabla \times \tau = C_D^* \rho_a \left(2u\gamma \frac{\partial T'}{\partial y} \right)$$
$$\nabla \times \tau = C_D^* \rho_a \left(2\gamma \frac{\partial T'}{\partial y} (\bar{u} + \gamma T') \right)$$

and the gradient of the sea surface temperature in the ”cross-wind” direction is

$$\nabla SST_{\perp} = \left(0, \frac{\partial}{\partial y} T_C + \frac{\partial}{\partial y} T' \right)$$

so in the limit that $\gamma T'$ is small with respect to $\bar{u}$ and $\frac{\partial}{\partial y} T_C$ is small with respect to $\frac{\partial}{\partial y} T'$, the resulting slope of these two quantities plotted against eachother would also be $C_D^* \rho_a 2\gamma\bar{u}$.

References

Chelton, D. B., Schlax, M. G., Freilich, M. H., and Milliff, R. F. (2004). Satellite Radar Measurements Reveal Short-scale Features in the Wind Stress Field over the World Ocean. *Science*, 303(February):978–983.

Foussard, A., Lapeyre, G., and Plougonven, R. (2019). Storm track response to oceanic eddies in idealized atmospheric simulations. *Journal of Climate*, 32(2):445–463.