Variables:

C_D^*	Reference drag coefficient (1E-3)
ρ_a	air density [kg m^{-3}]
T_C	temperature field in (y) without eddies in [K][1]
T'	temperature perturbation (eddies only) in (x, y) in [K][2]
T	total temperature field ($= T_C + T'$) in (x, y) in [K]
$\vec{u}$	velocity vector with components (u, v) in [m/s]
x, y	spatial coordinates [m]

stress calculations

For tuning parameter α, the surface stress is

$$\tau = \rho_a C_D^*(1 + \alpha T')\vec{u}\|u\|$$

The divergence of the stress is

$$\nabla \cdot \tau = C_D^* \rho_a \left(\frac{(1 + \alpha T')(4\frac{\partial u}{\partial x}u^3 + 2\frac{\partial v}{\partial x}u^2 v + 2\frac{\partial u}{\partial x}uv^2)}{2\sqrt{u^4 + u^2v^2}} + \frac{(1 + \alpha T')(4\frac{\partial v}{\partial y}v^3 + 2\frac{\partial u}{\partial y}uv^2 + 2\frac{\partial v}{\partial y}u^2)}{2\sqrt{v^4 + u^2v^2}} + \dots \right.$$
$$\left. \alpha\frac{\partial T'}{\partial x}\sqrt{u^4 + u^2v^2} - \alpha\frac{\partial T'}{\partial y}\sqrt{v^4 + u^2v^2} \right)$$

The curl of the stress is

$$\nabla \times \tau = C_D^* \rho_a \left(\frac{(1 + \alpha T')(4\frac{\partial u}{\partial y}u^3 + 2\frac{\partial v}{\partial y}u^2 v + 2\frac{\partial u}{\partial y}uv^2)}{2\sqrt{u^4 + u^2v^2}} - \frac{(1 + \alpha T')(4\frac{\partial v}{\partial x}v^3 + 2\frac{\partial u}{\partial x}uv^2 + 2\frac{\partial v}{\partial x}u^2)}{2\sqrt{v^4 + u^2v^2}} + \dots \right.$$
$$\left. \alpha\frac{\partial T'}{\partial y}\sqrt{u^4 + u^2v^2} - \alpha\frac{\partial T'}{\partial x}\sqrt{v^4 + u^2v^2} \right)$$

The gradient of the sea surface temperature in the "along-wind" or the "downwind" direction is

$$\nabla SST_{\|} = \left(\frac{u\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2+v^2}}, \frac{v\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2+v^2}} \right)$$

The gradient of the sea surface temperature in the "cross-wind" direction is

$$\nabla SST_{\perp} = \left(\frac{\partial T}{\partial x} - \frac{u\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2+v^2}}, \frac{\partial T}{\partial y} - \frac{v\left(\frac{\frac{\partial T}{\partial x}u}{\sqrt{u^2+v^2}} + \frac{\frac{\partial T}{\partial y}v}{\sqrt{u^2+v^2}}\right)}{\sqrt{u^2+v^2}} \right)$$

Taking Limits

In the limit that $\vec{u} = (\bar{u}, 0)$ everywhere

This means that $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = v = 0$ and $u = \bar{u}$

So the above quantities become:

Divergence:

$$\nabla \cdot \tau = C_D^* \rho_a \alpha \bar{u}^2 \frac{\partial T'}{\partial x}$$

Down-wind gradient:

$$\begin{aligned}\nabla SST_{\parallel} &= \left(\frac{\partial T}{\partial x}, 0 \right) \\ &= \left(\frac{\partial}{\partial x}(T_C + T'), 0) \right) \\ &= \left(\frac{\partial T'}{\partial x}, 0 \right)\end{aligned}$$

Which, when plotted against eachother, will have a slope of $C_D^* \rho_a \alpha \bar{u}^2$.

Curl:

$$\nabla \times \tau = -C_D^* \rho_a \alpha \bar{u}^2 \frac{\partial T'}{\partial y}$$

Cross-wind gradient:

$$\begin{aligned}\nabla SST_{\perp} &= \left(\frac{\partial T}{\partial x} - \frac{\partial T'}{\partial x}, \frac{\partial T}{\partial y} - 0 \right) \\ \nabla SST_{\perp} &= \left(0, \frac{\partial}{\partial y}(T_C + T') \right) \\ \nabla SST_{\perp} &= \left(0, \frac{\partial}{\partial y} T_C + \frac{\partial}{\partial y} T' \right)\end{aligned}$$

Which, when plotted against eachother, will have a slope of approximately $-C_D^* \rho_a \alpha \bar{u}^2$ if $\frac{\partial T_C}{\partial y}$ is small relative to $\frac{\partial T'}{\partial y}$.

In the limit that the velocity field is determined by a coupling coefficient

$\vec{u} = (\bar{u} + T'\gamma, 0)$ for a coupling coefficient γ and the drag coefficient is nolonger a function of temperature (i.e. $\alpha = 0$)

The divergence of the stress is now

$$\begin{aligned}\nabla \cdot \tau =& C_D^* \rho_a \left(\frac{(4 \frac{\partial u}{\partial x} u^3)}{2u^2} \right) \\ \nabla \cdot \tau =& C_D^* \rho_a \left(2u \frac{\partial u}{\partial x} \right) \\ \nabla \cdot \tau =& C_D^* \rho_a \left(2u\gamma \frac{\partial T'}{\partial x} \right) \\ \nabla \cdot \tau =& C_D^* \rho_a \left(2\gamma \frac{\partial T'}{\partial x} (\bar{u} + \gamma T') \right)\end{aligned}$$

and the gradient of the sea surface temperature in the "downwind" direction is

$$\nabla SST_{\parallel} = \left(\frac{\partial T'}{\partial x}, 0\right)$$

If $\gamma T'$ is small with respect to $\bar{u}$, then the slope of the divergence plotted against the "down-wind" SST gradient will be $C_D^* \rho_a 2\gamma\bar{u}$.

The curl of the stress is now

$$\begin{aligned}
\nabla \times \tau =& C_D^* \rho_a \left(\frac{(4\frac{\partial u}{\partial y}u^3)}{2u^2}\right) \\
\nabla \times \tau =& C_D^* \rho_a \left(2u\frac{\partial u}{\partial y}\right) \\
\nabla \times \tau =& C_D^* \rho_a \left(2u\gamma\frac{\partial T'}{\partial y}\right) \\
\nabla \times \tau =& C_D^* \rho_a \left(2\gamma\frac{\partial T'}{\partial y}(\bar{u} + \gamma T')\right)
\end{aligned}$$

and the gradient of the sea surface temperature in the "cross-wind" direction is

$$\nabla SST_{\perp} = \left(0, \frac{\partial}{\partial y}T_C + \frac{\partial}{\partial y}T'\right)$$

so in the limit that $\gamma T'$ is small with respect to $\bar{u}$ and $\frac{\partial}{\partial y}T_C$ is small with respect to $\frac{\partial}{\partial y}T'$, the resulting slope of these two quantities plotted against eachother would also be $C_D^* \rho_a 2\gamma\bar{u}$.