Carrier Concentration:

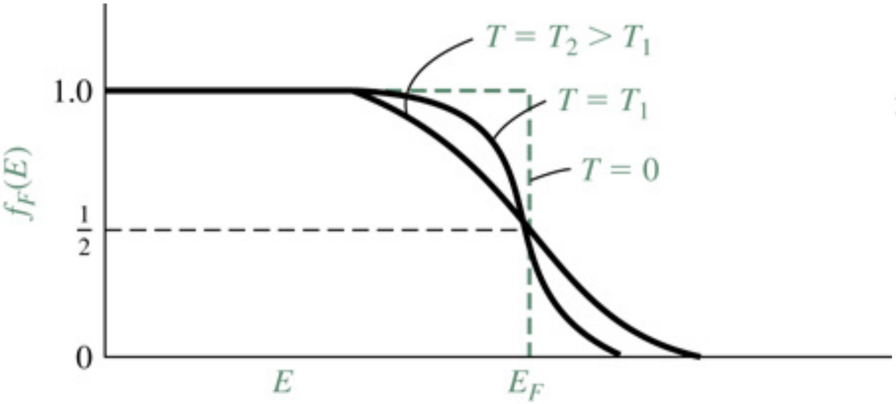
Fermi-Dirac Statistics

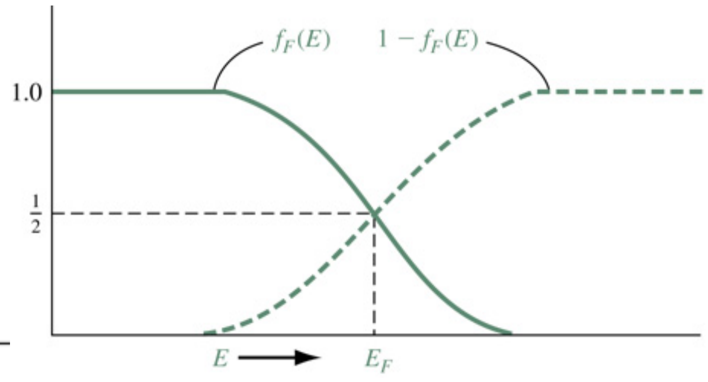
Fermi Function

- Probability of finding an electron at energy level E is

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

- where E_F is the Fermi level (or chemical potential, or electrochemical potential)





Temperature dependence of Fermi-Dirac distribution

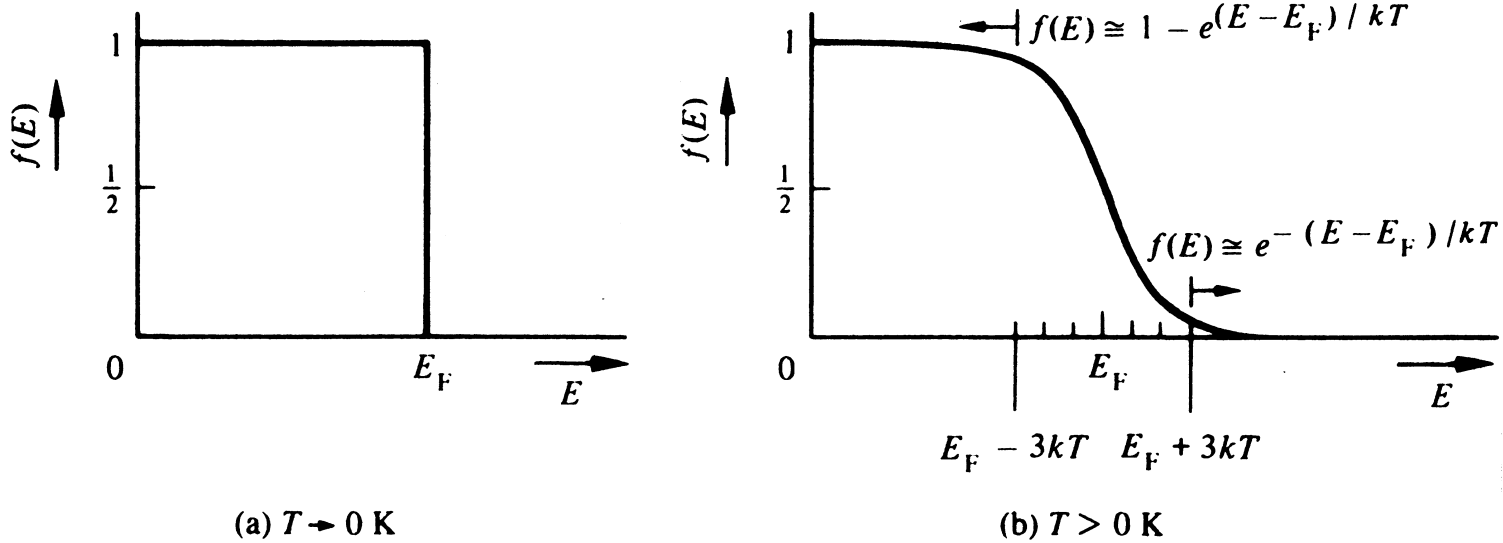
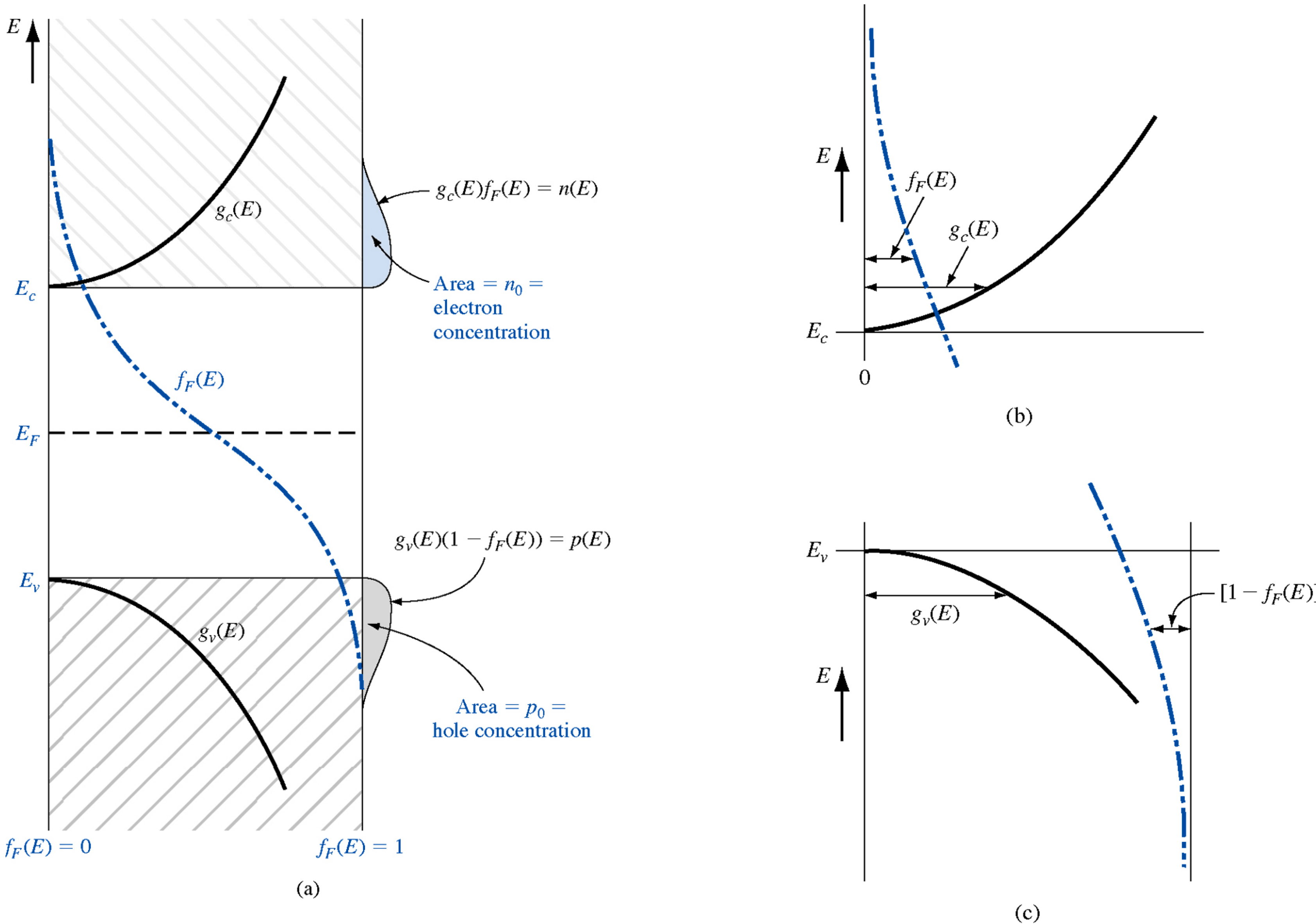


Figure 2.15

Product of the Fermi Function f(E) and the density of states g(E) gives carrier concentration n(E)



Fermi-level positioning and carrier distributions

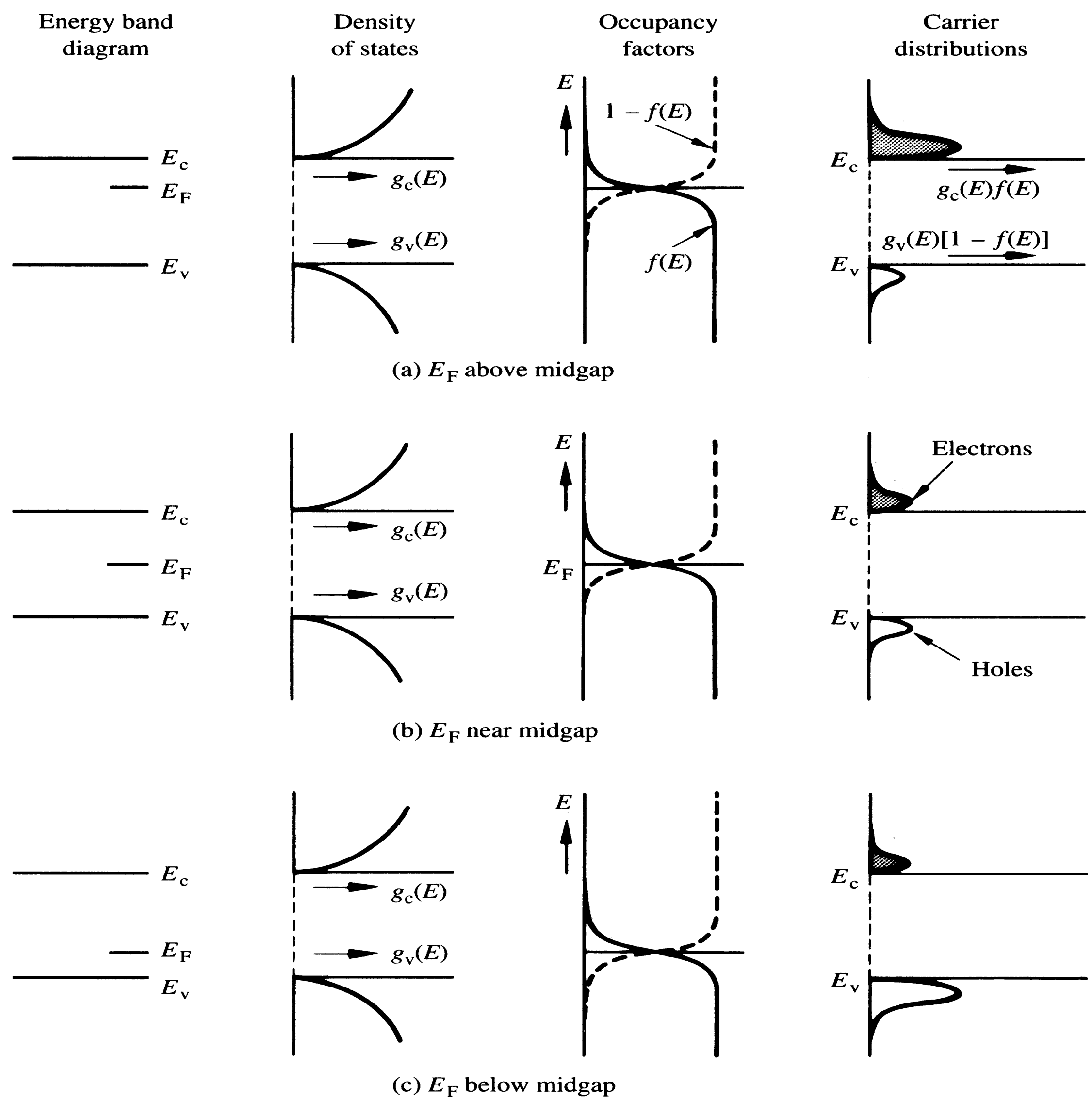


Figure 2.16

Equilibrium distribution of carriers

Distribution of carriers = DOS × probability of occupancy

$n(E) = g(E) \times f(E)$

(where DOS = Density of states)

Total number of electrons in conduction band =

$$n_0 = \int_{E_C}^{E_{top}} g_C(E)\, f(E)\, \mathrm{d}E$$

Total number of holes in valence band=

$$p_0 = \int_{E_{Bottom}}^{E_V} g_V(E)\left(1 - f(E)\right)\mathrm{d}E$$

Electron and Hole Concentration

$$n_o = \int_{E_c}^{\infty} \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_c} \frac{1}{1 + e^{\frac{E-E_F}{kT}}} dE$$

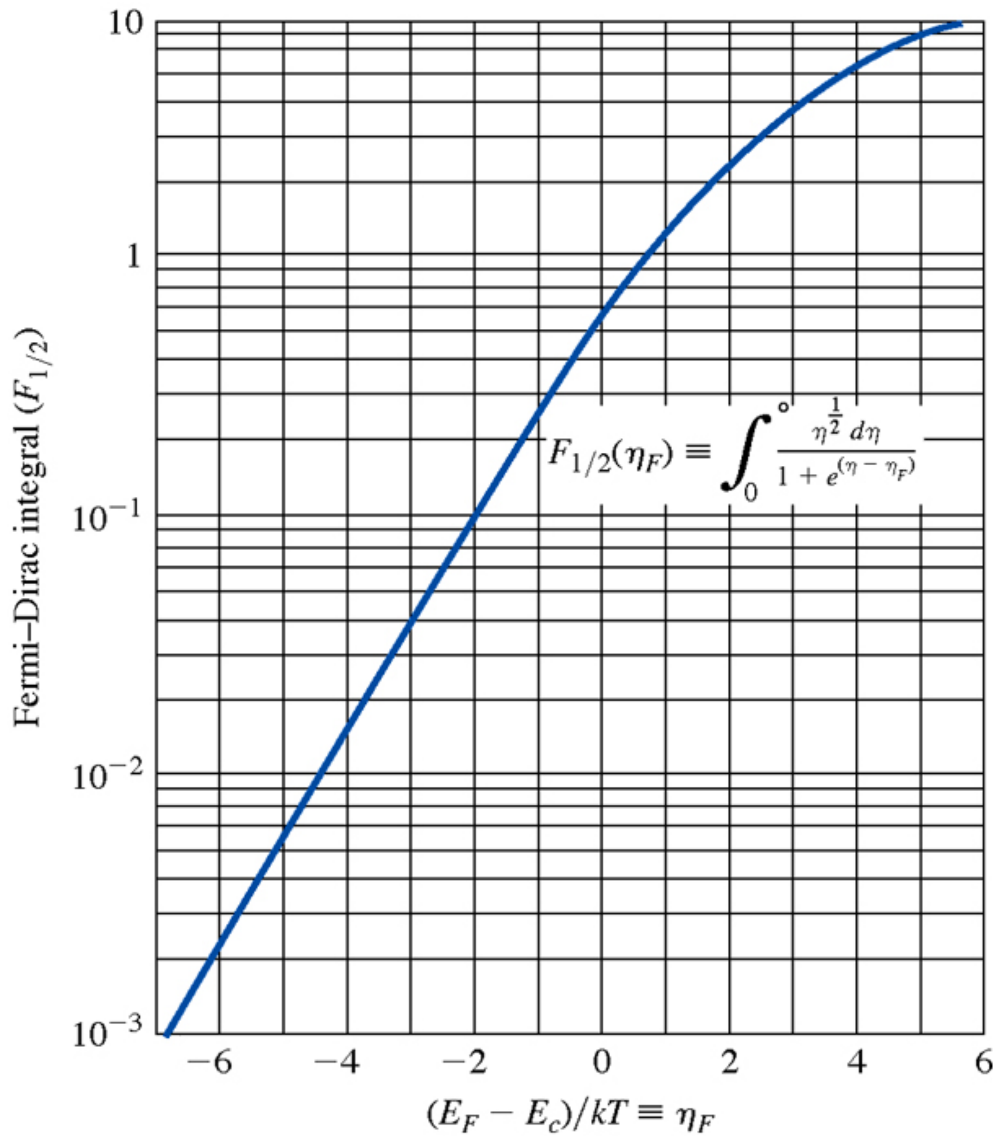
$$p_o = \int_{-\infty}^{E_v} \frac{8\pi\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_v - E} \frac{1}{1 + e^{\frac{E_F-E}{kT}}} dE$$

Which does not have a closed form integral. At T = 0:

$$n_0 = \int_{E_C}^{E_F} g_C(E)\, dE \qquad at \quad T = 0K$$

$$n_0 = \frac{2}{3}\frac{\sqrt{2}}{\pi^2}\frac{m_e^{*3/2}}{\hbar^3}\left(E_F - E_C\right)^{3/2} \qquad \text{for } E_F \geq E_C$$

Fermi-Dirac Integral of order 1/2



For Non-Degenerate Semiconductors

$$n_0 = N_c e^{-(E_c - E_F)/KT}$$

$$p_0 = N_v e^{(E_v - E_F)/KT}$$

or

$$n_0 = n_i e^{(E_F - E_i)/KT}$$

$$p_0 = n_i e^{(E_i - E_F)/KT}$$

Note :

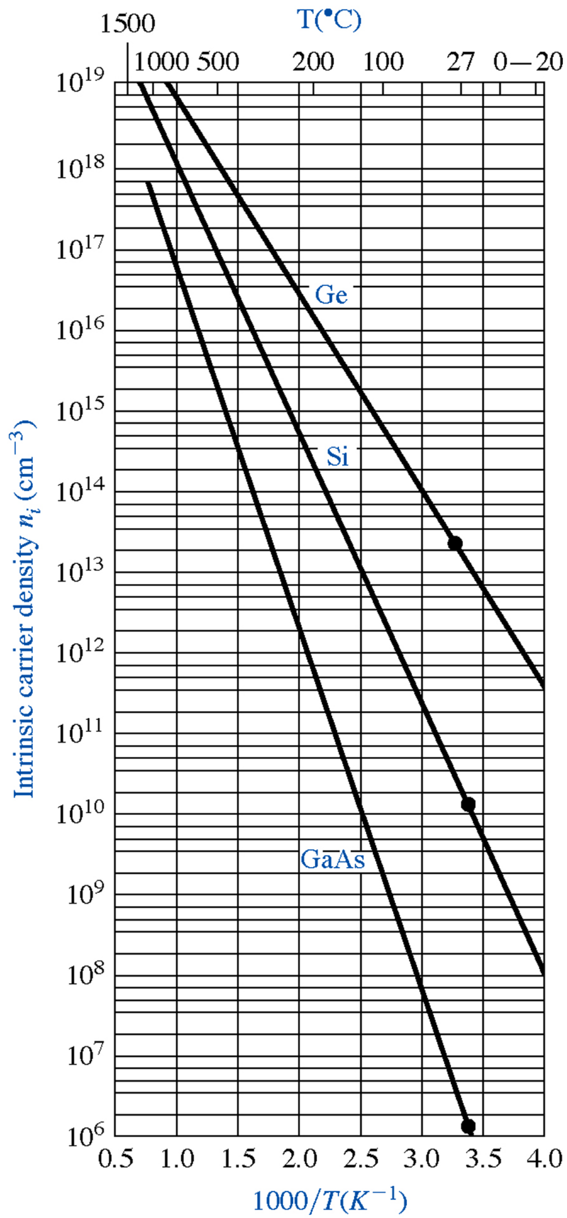
$$n_0 \times p_0 = n_i^2 \quad \text{mass} - \text{action law}$$

$$n_i = \sqrt{N_v N_c}\, e^{-E_g/2KT}$$

Charge Neutrality:
Total positive charge = total negative charge

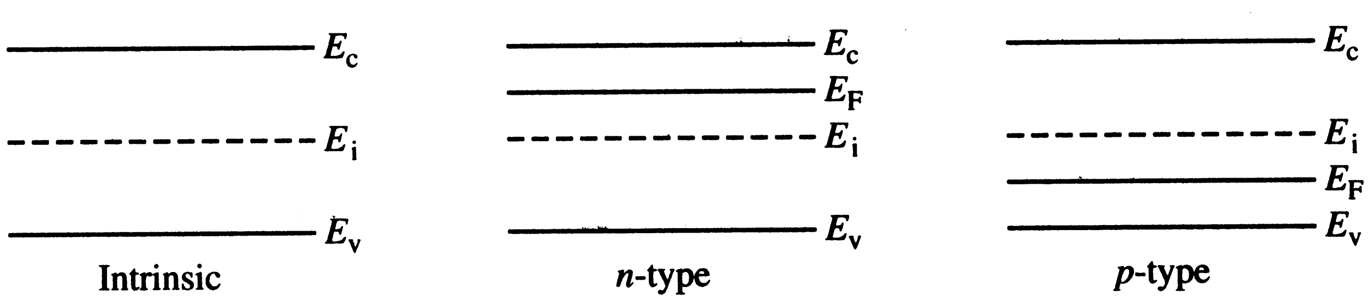
$$n_0 + N_a^- = p_0 + N_d^+$$

Intrinsic Carrier Concentration and its temperature dependence

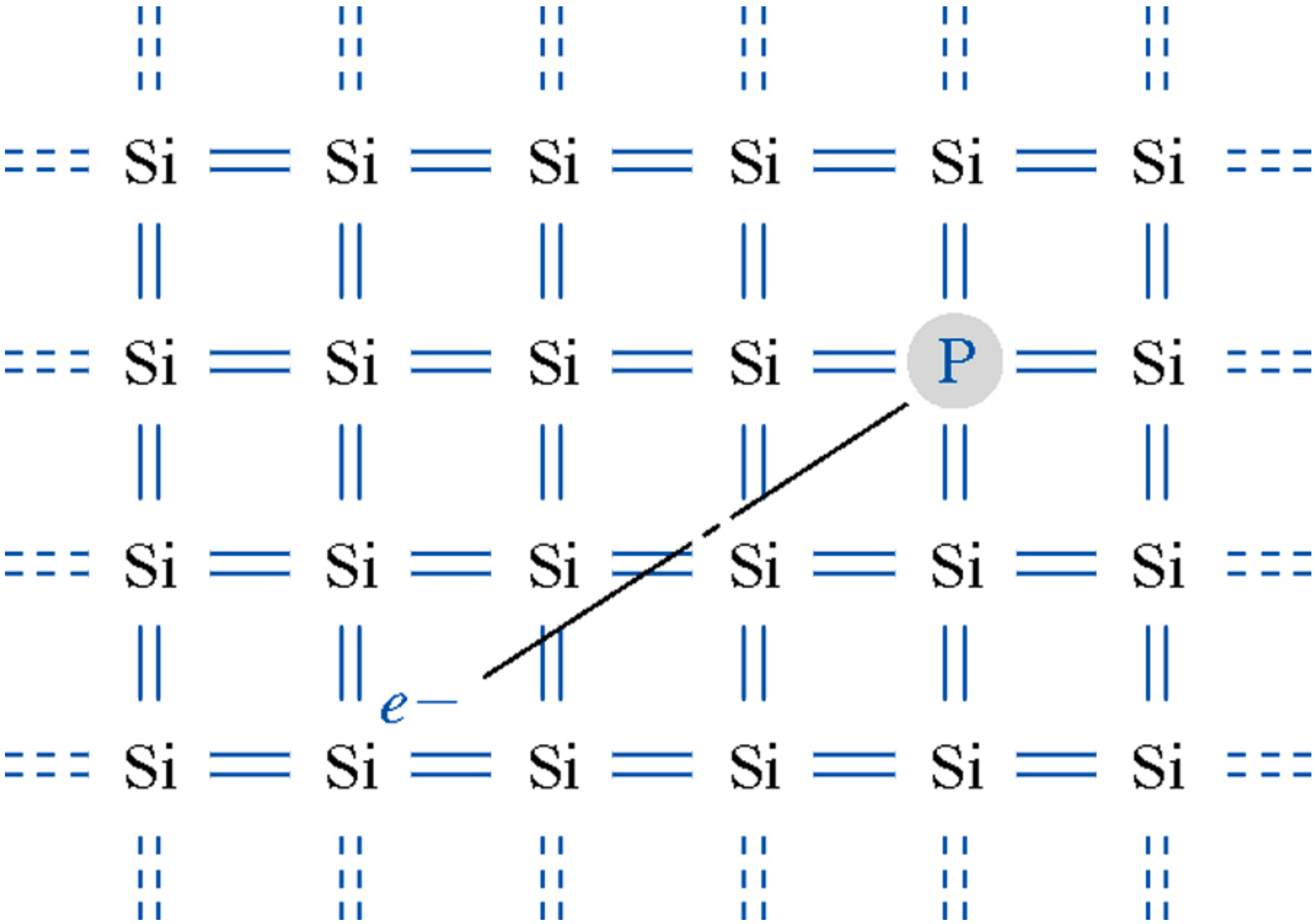


Visualization of carrier distribution (continued)

Another more useful way to convey the carrier distribution is to draw the following band diagrams. The position of E_F with respect to E_i is used to indicate whether is n-type, p-type or intrinsic.

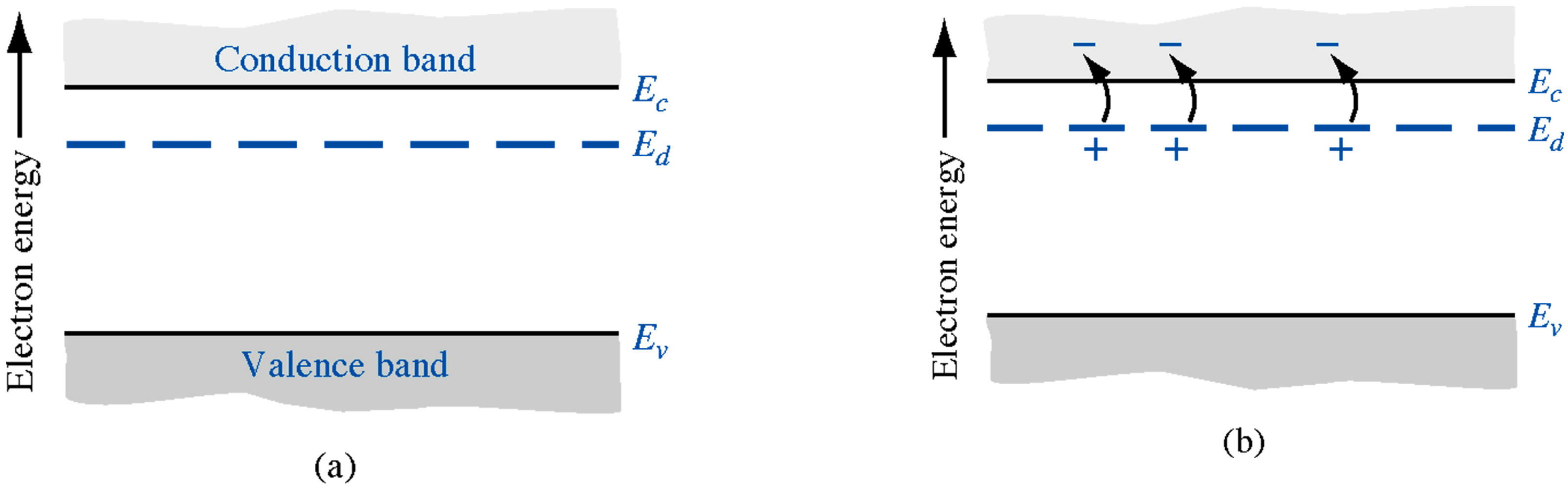


N-type dopant



Donor atoms produce electrons in conduction band

Notice that electrons are mobile while the positively charged donor ions are not. That is, these mobile electrons participate in conduction, but ionized donor atoms are immobile. The affect conduction indirectly by changing the mobility

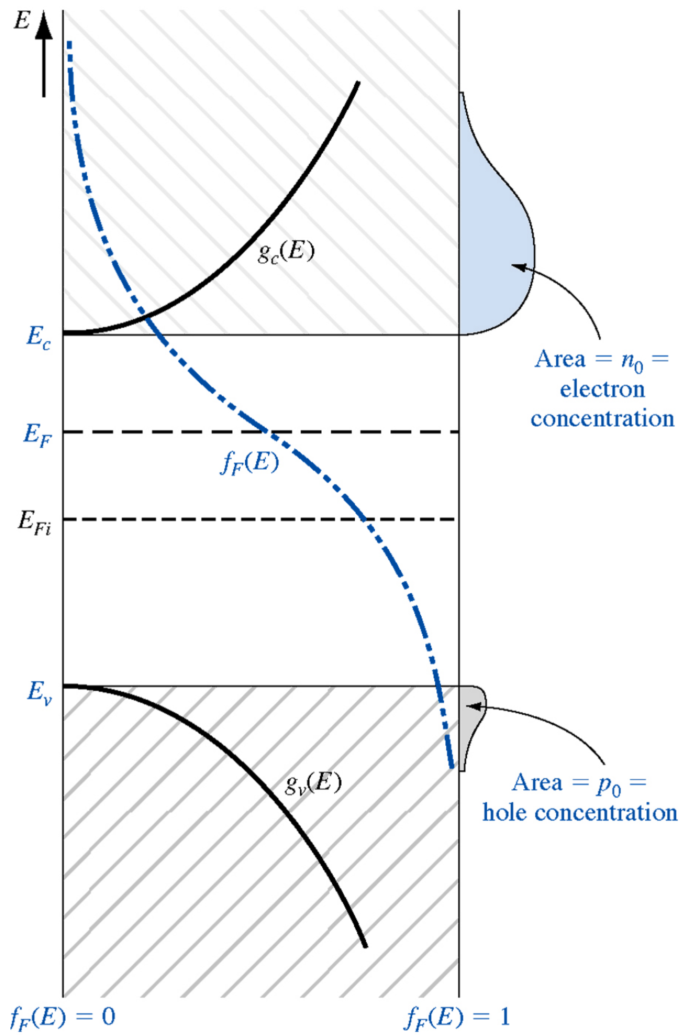


Number of carriers at energy level E = Density of States x probability of occupancy

$$n(E) = g_C(E) \times f(E)$$

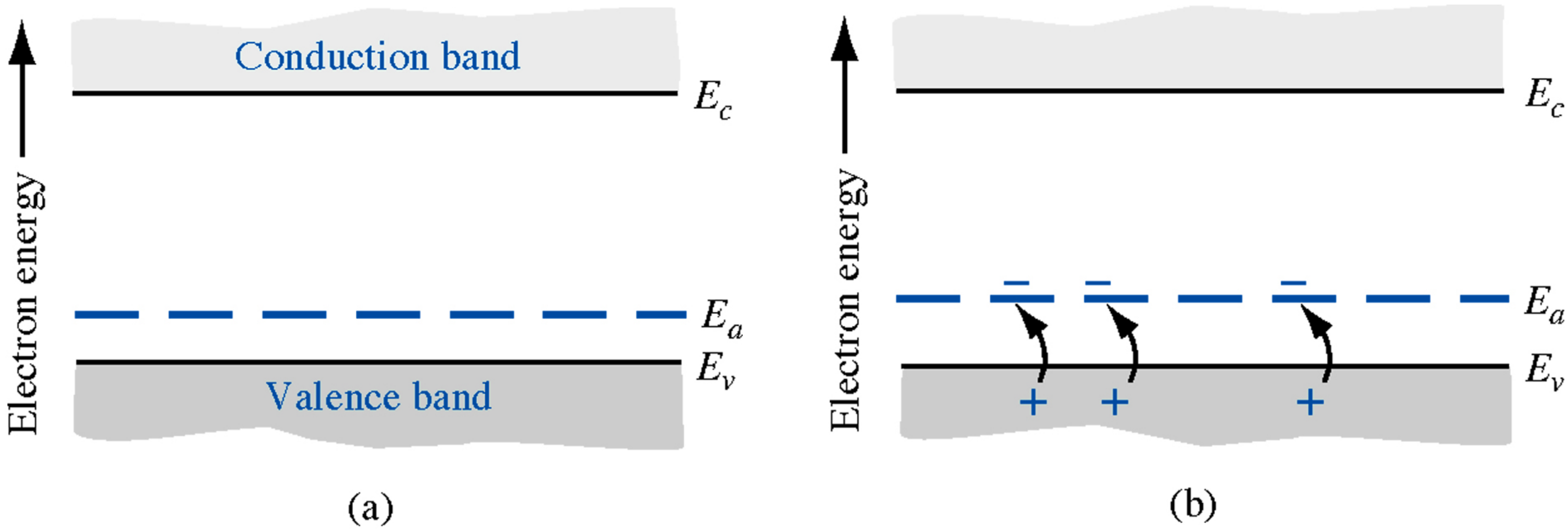
Total Carriers in conduction band = total # of mobile electrons:

$$n_0 = \int_{E_C}^{E_{top}} g_C(E)\, f(E)\, \mathrm{d}E$$

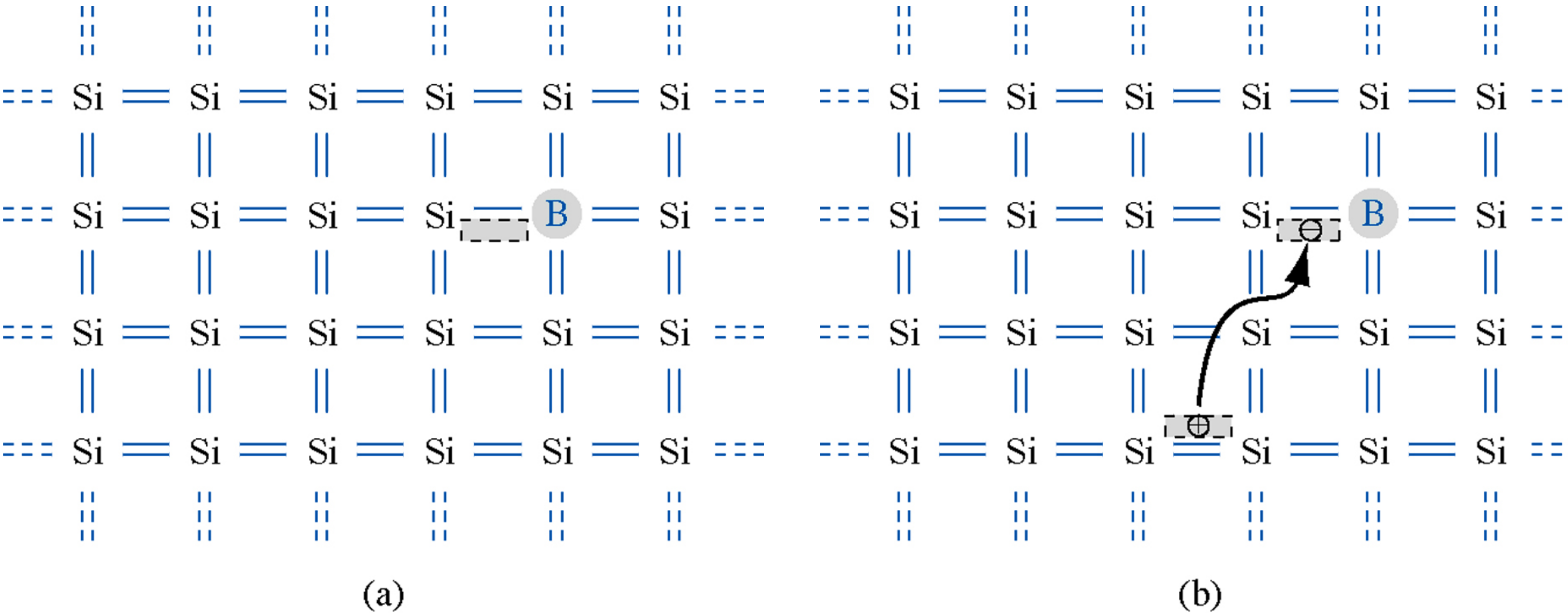


Holes. Acceptor energy levels.

Notice that holes are mobile while negatively charged acceptor ions are not. That is, holes participate in conduction, but acceptors do so indirectly by changing the mobility



Empty states are 'holes'.



P-type material

Number of carriers at energy level E =

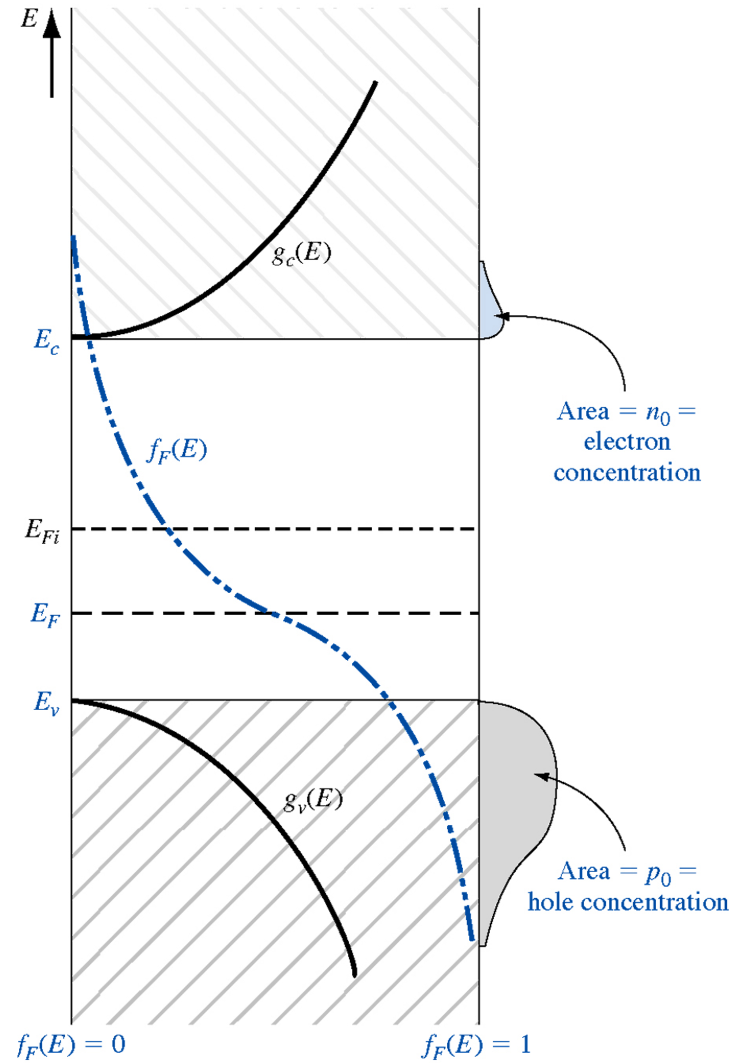
Density of States x probability of occupancy

$$n(E) = g_V(E) \times \left(1 - f(E)\right)$$

Total Carriers in valence band =

total # of (mobile) holes:

$$p_0 = \int_{E\text{bottom}}^{E_V} g_V(E) \times \left(1 - f(E)\right) \mathrm{d}E$$



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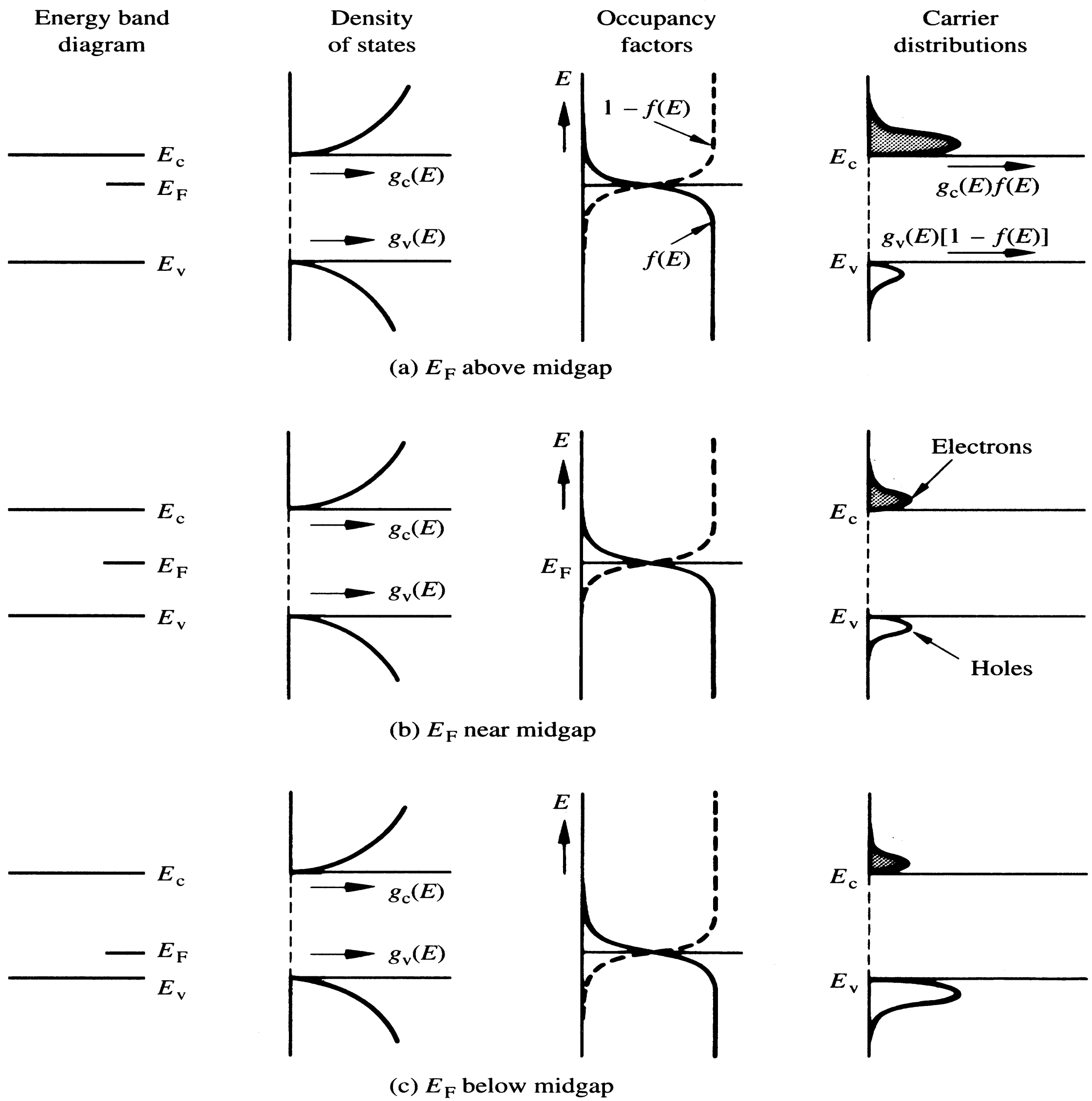
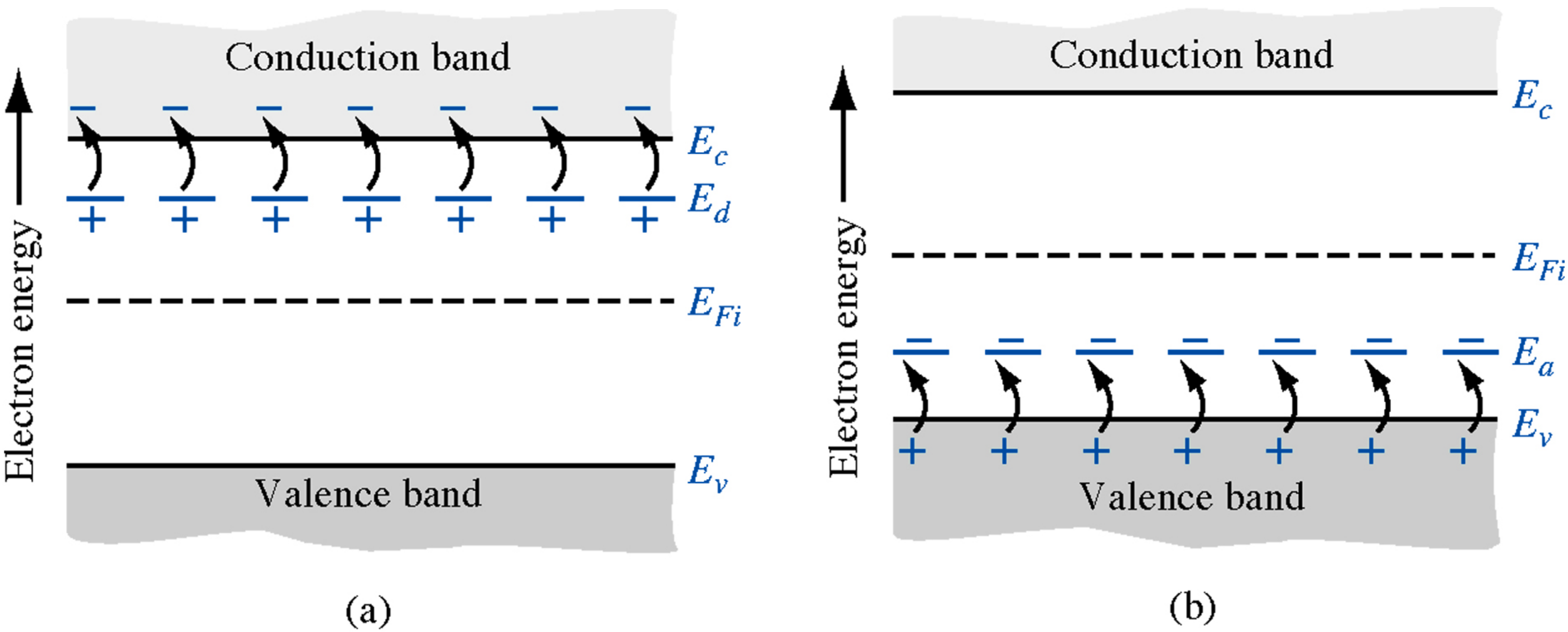


Figure 2.16

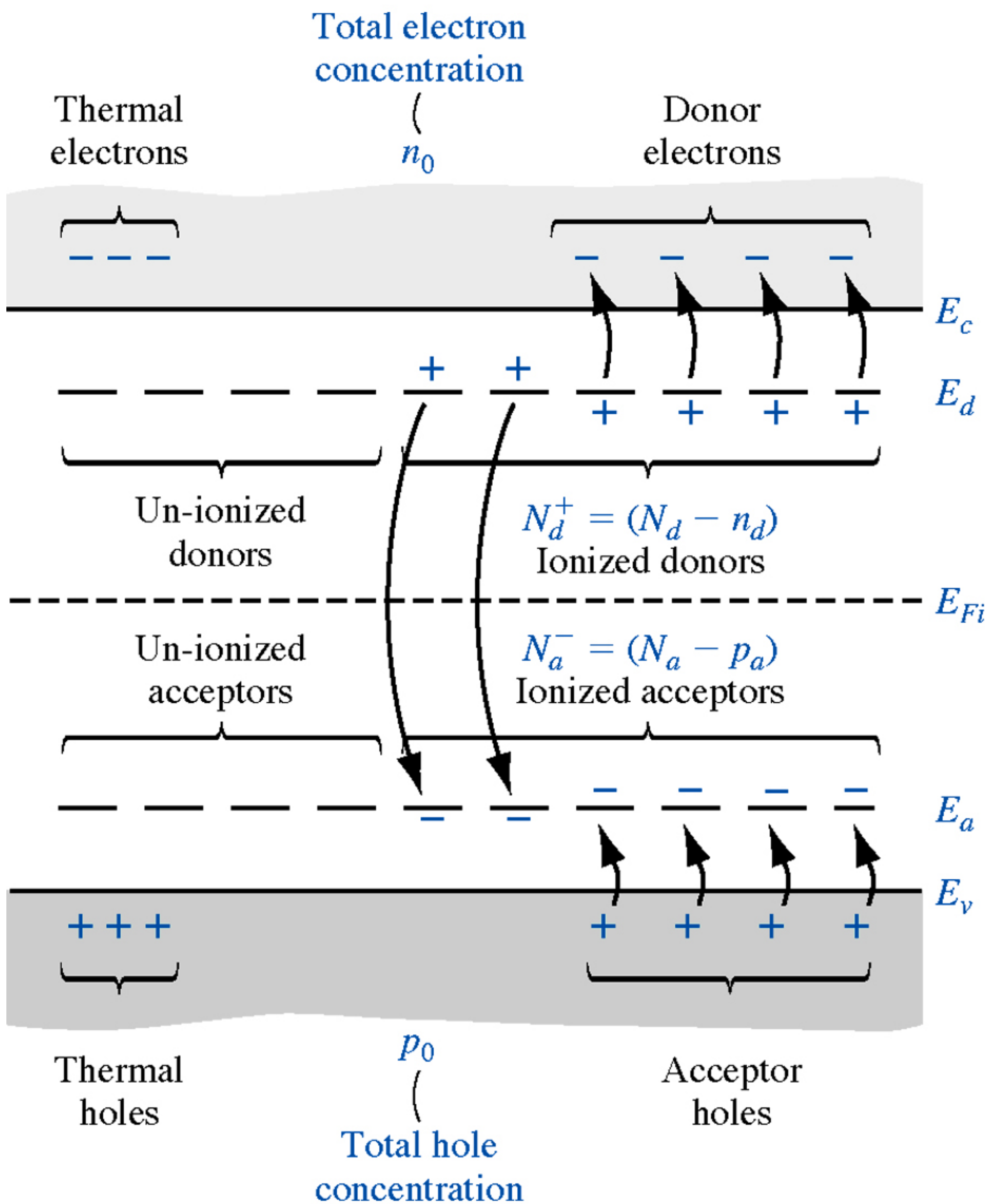
Charge Neutrality:

$$n_0 + N_a^- = p_0 + N_d^+$$

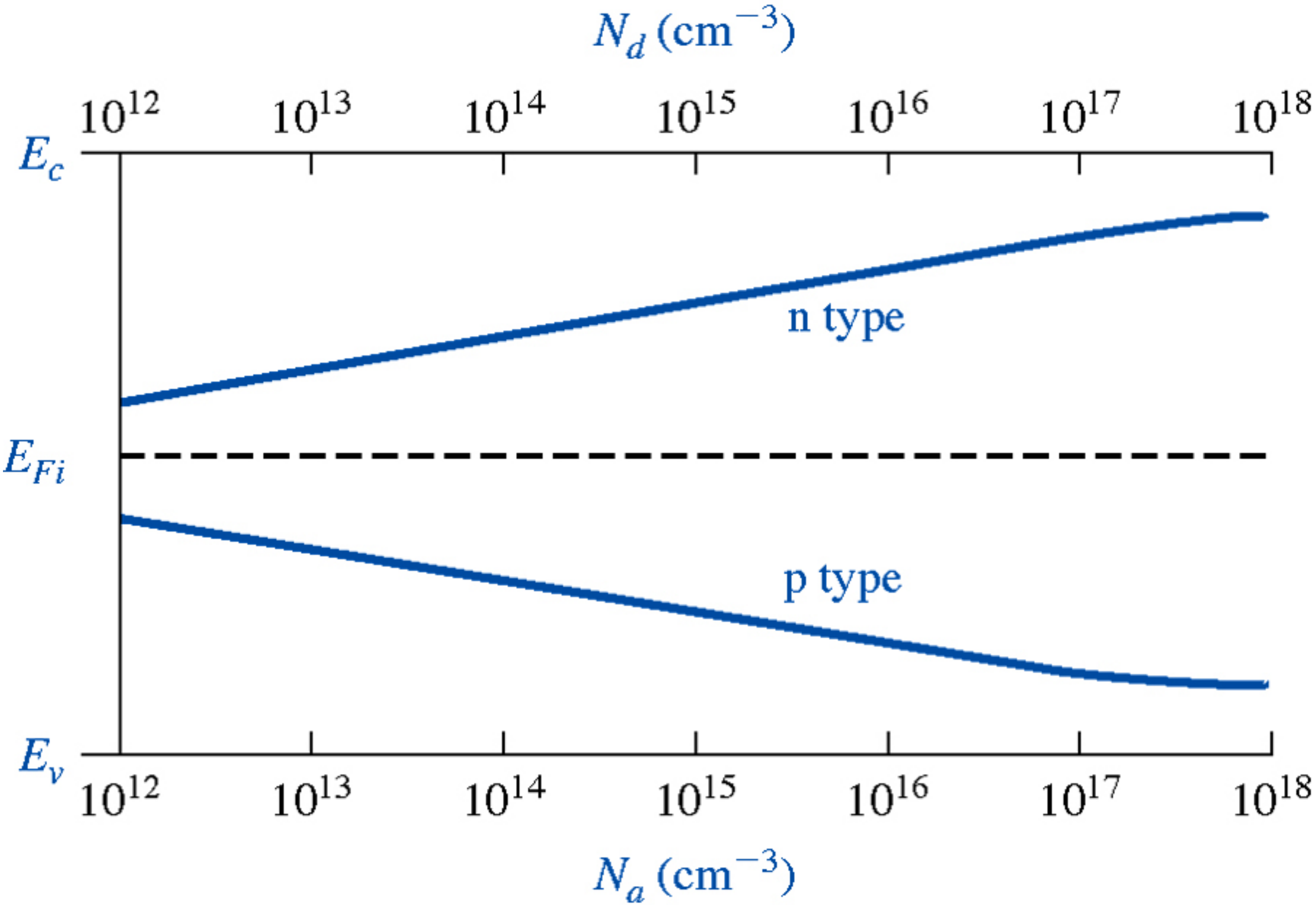
Donor and Acceptor ions



Compensated Material



Fermi level variation with carrier concentration



Fermi Level variation with Temperature at various doping concentrations