ECE 361 Probability for Engineers (Fall, 2016)
Homework 3

Assigned:	Thursday October 6, 2016	
Due:	Thursday October 13, 2016	**(at the beginning of class 11am)**
Returned:	Monday October 17, 2016	**(in recitation)**

Your full name ______________________________

Your Drexel student ID ______________________________

ECE 361 homework policies:

1. There will be eight homework assignments, roughly one per week, but check the Course Calendar for exact dates.
2. Each homework will count equally towards your overall homework grade, although the lowest homework will be dropped.
3. Homework must be done INDIVIDUALLY. Although you are free to discuss course content in general with your classmates and peers, you are expected to NOT discuss particulars about homework problems with them or anyone else.
4. Similarly, although you are encouraged to read external sources (online or offline), it is expected that your homework handed in reflects YOUR work, and not work that you found elsewhere.
5. Homework will always be due at the BEGINNING of lecture. One of the Teaching Assistants will be in the lecture hall at the beginning of class to collect the assignments. At 11:10am (ten minutes into the start of lecture) the TA will leave the lecture with the assignments.
6. No submissions will be accepted after that time. No exceptions.
7. Homework will be returned to you during recitations.
8. Homework assignments should have your full NAME, your STUDENT ID, the ASSIGNMENT NUMBER, should be STAPLED, and should be written CLEARLY, with your final ANSWER clearly indicated, and all supporting WORK provided. Sloppy, unclear, or illegible work will not be graded.

Please answer the following questions:

1. (3 points) Let X, Y be a pair of independent and identically distributed random variables, each one uniformly distributed over $\{0, \ldots, N\}$, for $N = 10$. That is, $X \sim \text{Uni}(0, N)$ and $Y \sim \text{Uni}(0, N)$. Let $Z = X - Y$. Find the probability mass function for Z. *Hint: first find the support for Z, say $\mathcal{Z}$. Next, for any $z \in \mathcal{Z}$, find the set of all pairs (x, y) such that $x - y = z$. Finally, the probability that $(X, Y) = (x, y) = p_X(x)p_Y(y) = \frac{1}{N+1} \times \frac{1}{N+1}$, as (X, Y) are independent.*
2. (3 points) Let X, Y be as in the previous problem. Let $Z = X + Y$. Find the probability mass function for Z.
3. (2 points) Let X, Ybe as in the previous problem. Find $\mathbb{E}[X - Y]$ and $\mathbb{E}[X + Y]$.
4. (4 points) Fix $N = 10$. Let X be a random variable with support $[N]$ where

$$p_X(x) = \frac{2}{N}\left(1 - \frac{x}{N+1}\right), \ x \in [N].$$

   Please do the following:

   - Plot the PMF $p_X = (p_X(x), x \in \mathcal{X})$.
   - Establish whether or not p_X is a valid PMF.
   - Find $\mathbb{E}[X]$.
   - Find $\text{Var}(X)$.

5. (3 points) A wireless channel flips each bit sent over the channel independently with probability $1/1000$. A packet of length 1000 bits is sent over the channel. Error correction allows errors in a packet to be corrected provided there are five or fewer bit errors in a packet, otherwise the packet is in error. Find the probability that a packet is in error. *Hint: use the binomial distribution.*
6. (3 points) Recall the setup of the previous problem. To reduce the packet error rate the transmitter sends each packet twice. The information in the packet is lost precisely when *both* packets holding that information have a packet error. Find the probability that the information in a pair of packets is lost.
7. (3 points) A fair coin is tossed $N = 10$ times. Let X be the number of heads and $N - X$ be the number of tails. Let $Z = |X - (N - X)| = |2X - N|$ be the discrepancy between the number of heads and tails.

   - Write the PMF p_Z
   - Plot the PMF p_Z.
   - Establish whether or not p_Z is a valid PMF.

8. (3 points) Harry and Mary take turns flipping a biased coin with bias $p \in (0, 1)$, with Harry flipping the first coin. The first person to flip a heads wins the coin. Find the probability that Harry wins the coin. *Hint: Let $X \sim \text{Geo}(p)$ be the toss on which the first head occurs, and let $E = \{X \text{ odd}\}$ be the event that Harry wins. Then $\mathbb{P}(E) = \mathbb{P}(X = 1) + \mathbb{P}(X = 3) + \cdots$. Recall the geometric series formula from the first homework.*
9. (3 points) Recall the setup of the previous problem. For which values of p is it beneficial to be the first person to flip the coin?
10. (3 points) Recall the setup of the previous problem. Now, Harry flips the coin first, then Mary flips twice, then Harry flips once, then Mary flips twice, etc. Find the probability that Harry wins the coin.
11. (3 points) Recall the setup of the previous problem. For which values of p is it beneficial to be the first person to flip the coin?