ECE 361 Probability for Engineers (Fall, 2016)
Homework 6

Assigned:	Thursday November 3, 2016	
Due:	Tuesday November 15, 2016	**(at the beginning of class 11am)**
Returned:	Monday November 21, 2016	**(in recitation)**

Your full name ____________________________

Your Drexel student ID ____________________________

ECE 361 homework policies:

1. There will be eight homework assignments, roughly one per week, but check the Course Calendar for exact dates.
2. Each homework will count equally towards your overall homework grade, although the lowest homework will be dropped.
3. Homework must be done INDIVIDUALLY. Although you are free to discuss course content in general with your classmates and peers, you are expected to NOT discuss particulars about homework problems with them or anyone else.
4. Similarly, although you are encouraged to read external sources (online or offline), it is expected that your homework handed in reflects YOUR work, and not work that you found elsewhere.
5. Homework will always be due at the BEGINNING of lecture. One of the Teaching Assistants will be in the lecture hall at the beginning of class to collect the assignments. At 11:10am (ten minutes into the start of lecture) the TA will leave the lecture with the assignments.
6. No submissions will be accepted after that time. No exceptions.
7. Homework will be returned to you during recitations.
8. Homework assignments should have your full NAME, your STUDENT ID, the ASSIGNMENT NUMBER, should be STAPLED, and should be written CLEARLY, with your final ANSWER clearly indicated, and all supporting WORK provided. Sloppy, unclear, or illegible work will not be graded.

Please answer the following questions:

1. (2 points) Let $Z \sim N(0,1)$ be a standard normal RV, with CDF $\Phi(z) = \mathbb{P}(Z \leq z)$.
   - For $z > 0$, give an expression for $\mathbb{P}(|Z| > z)$ in terms of z and $\Phi(\cdot)$.
   - Compute $\mathbb{P}(|Z| > k)$ for $k \in [5]$.
2. (2 points) Let $X \sim N(\mu, \sigma)$ be a normal RV for a given (μ, σ) pair. Let $Z \sim N(0,1)$ be a standard normal RV, with CDF $\Phi(z) = \mathbb{P}(Z \leq z)$. Fix $x > 0$ and find $\mathbb{P}(|X| > x)$. Your answer should be in terms of x, μ, σ and $\Phi(\cdot)$. *Hint: standardize X.*
3. (3 points) Let $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$ be independent normal RVs. Find the mean and the variance of W, where:
   - $W = aX + bY$
   - $W = aX - bY$
   - $W = aXY$
4. (4 points) Let $X \sim \mathrm{Uni}([0,1])$ and $Y \sim \mathrm{Uni}([0,1])$ be independent RVs, and consider the pair (X, Y) as a random point on the (x, y) plane, in fact in the unit box $[0,1]^2$. Find the PDF and CDF for each of the RVs given below. Plot the PDF and CDF for each RV.
   - $Z = \min(X, Y)$
   - $Z = \max(X, Y)$
   - $Z = X + Y$. *Hint: observe the support is $[0,2]$, and consider the two cases $z \in [0,1]$ and $z \in [1,2]$ separately. In each case you may use the total probability theorem, conditioning on the value of x, and then split the resulting integral over $x \in [0,1]$ into two sub-intervals, with the split determined by the value of z. Pay close attention to the boundaries, e.g., $\mathbb{P}(X \leq x) = 0$ for $x < 0$ and $\mathbb{P}(X \leq x) = 1$ for $x > 1$. Check that your final answer is a valid CDF: zero for $z < 0$, one for $z > 2$, and nondecreasing for $z \in [0,2]$.*
5. (2 points) Let $(X_1, \ldots, X_N)$ be independent and identically distributed RVs, with $X_n \sim N(0,1)$ for each $n \in [N]$.
   - Define $U = \min(X_1, \ldots, X_N)$. Find the PDF and CDF of U. Plot the PDF and CDF for $N = 1, 2, 5, 10$.
   - Define $V = \max(X_1, \ldots, X_N)$. Find the PDF and CDF of V. Plot the PDF and CDF for $N = 1, 2, 5, 10$.
6. (2 points) Let $(X_1, \ldots, X_N)$ be independent RVs, with $X_n \sim \exp(\lambda_n)$ for each $n \in [N]$, i.e., $F_{X_n}(x) = 1 - \mathrm{e}^{-\lambda_n x}$ for $x \geq 0$. Assume the parameters $(\lambda_1, \ldots, \lambda_N)$ obey $\lambda_n > 0$ for each $n \in [N]$. Define $Y = \min(X_1, \ldots, X_N)$. Find the PDF and CDF of Y.