ECE 361 Probability for Engineers (Fall, 2016)
Midterm Exam

Name **Student ID** **Signature**

Question	Maximum	Score
1. (a)	2	
1. (b)	2	
2. (a)	2	
2. (b)	2	
3. (a)	2	
3. (b)	2	
4. (a)	2	
4. (b)	2	
4. (c)	2	
4. (d)	2	
4. (e)	2	
4. (f)	2	
5. (a)	2	
5. (b)	2	
TOTAL	28	
6. (a)	2	
6. (b)	2	
TOTAL	32	

1. **4 points.** Presents. Fix integers k, n with $0 < k < n$. A mischevious parent places k toys in n (closed) boxes and wraps up each box as a present. Thus k of the boxes hold toys and $n-k$ of the boxes are empty. A child is then allowed to open these presents one by one, stopping as soon as *any one* of the toys is found. Let the RV X denote the number of presents opened. *Hint: the event that x presents are opened means the first $(x-1)$ presents opened did not contain a toy and the xth present did contain a toy. There are $\binom{n}{x}$ ways of selecting x of the n presents and $x!$ ways of ordering them, for a total of $\binom{n}{x}x!$ ways of selecting x ordered presents. Compute how many of these ways result in first finding a toy on the xth present.*

   (a) **2 points.** Give the support $\mathcal{X}$ of the RV X in terms of k, n.

(b) **2 points.** Give the probability mass function $p_X = (p_X(x), x \in \mathcal{X})$.

2. **4 points.** Rock, paper, scissors. Alejandro and Brunhilda play the game "rock paper scissors". That is, the two players each select one of these three items simultaneously and the winner of the game is determined by the following rule: rock beats scissors, scissors beats paper, paper beats rock, and if the two players select the same item then the outcome is a tie. Alejandro selects his item $X \in \{r, p, s\}$ (independently of Brunhilda) with probabilities $p_X = (p_X(r), p_X(p), p_X(s))$, where p_X is nonnegative and sums to one. Brunhilda selects her item $Y \in \{r, p, s\}$ (independently of Alejandro) with probabilities $p_Y = (p_Y(r), p_Y(p), p_Y(s))$, where p_Y is nonnegative and sums to one.

   (a) **2 points.** What is the probability that Alejandro wins the game?

(b) **2 points.** What is the probability of a tie?

3. **4 points.** Mini World Series. Two teams, the Aardwolves (a South African fox-like animal) and the Babirusas (a large wild pig of Indonesia) play a series of games, where in each game the Aardwolves win with probability p or the Babirusas win with probability $1-p$, for some $p \in [0,1]$. The outcomes of the games are independent of one another. The two teams play a "best of three" series, meaning the first team to win two games is the winner of the series.

   (a) **2 points.** What is the probability that the Aardwolves win the series?

(b) **2 points.** What is the probability mass function on the number of games played in the series?

4. **12 points.** Colors. Each of three players (Adebayo, Baldemar, and Cuthbert), denoted A, B, C, simultaneously and independently picks one of two colors, coquelicot (a shade of orange) or smaragdine (a shade of green), denoted c, s. Let p be the probability that a player selects coquelicot, and $1-p$ be the probability a player selects smaragdine. Define the following RVs, where a majority means two or three players:

   - Define the RV X to equal: *i*) -1 if the majority select coquelicot, *ii*) $+1$ if the majority select smaragdine, or *iii*) 0 if there is no majority.
   - Define the RV Y to equal: *i*) -1 if all three players select coquelicot, *ii*) $+1$ if all three players select smaragdine, or *iii*) 0 if the players select a mixture of colors between them.
   - Define the RV $Z = XY$.

   (a) **2 points.** Give the joint PMF for (X, Y) in terms of p. *Hint: first fill in the remaining seven entries in the following table, and use it to construct the table $p_{X,Y}$ with three rows ($X \in \{-1, 0, +1\}$) and three columns ($Y \in \{-1, 0, +1\}$). Note, some entries in $p_{X,Y}$ may equal zero. Clearly label the rows and columns of the $p_{X,Y}$ table to indicate the corresponding values of X, Y.*

A	B	C	$p(A,B,C)$	X	Y
c	c	c	p^3	-1	-1

(1)

(b) **2 points.** Give the PMF for X in terms of p

(c) **2 points.** Give the PMF for Z in terms of p

(d) **2 points.** Give $\mathbb{E}[X]$ in terms of p

(e) **2 points.** Give $\mathbb{E}[Z]$ in terms of p

(f) **2 points.** Give $\mathrm{Var}(Z)$ in terms of p

5. **4 points.** Coins. Agathangelos and Benedykta hold separate biased coins and perform the following experiment. Agathangelos flips his coin n_A times (for $n_A \in \mathbb{N}$); his coin comes up heads with probability $p_A \in [0,1]$ or tails with probability $1 - p_A$. Benedykta flips her coin n_B times (for $n_B \in \mathbb{N}$); her coin comes up heads with probability $p_B \in [0,1]$ or tails with probability $1 - p_B$. Let the RV X be the number of times that Agathangelos's coin shows a head and let the RV Y be the number of times that Benedykta's coin shows a head. Let the RV $Z = aX + bY$, for (a,b) real numbers.

   (a) **2 points.** Give an expression for $\mathbb{E}[Z]$ in terms of $(a, b, p_A, p_B, n_A, n_B)$.

(b) **2 points.** Give an expression for $\mathrm{Var}(Z)$ in terms of $(a, b, p_A, p_B, n_A, n_B)$.

6. **Extra Credit: 4 points.** Voting. Two political candidates, Akulina and Beulah, are competing in an election with 3 voters named Chinatsu, Damayanti, and Edgard. Each of the voters votes randomly and independently, casting a vote for Akulina with probability p or for Beulah with probability $1-p$, for $p \in [0,1]$. The winner is the candidate with the majority vote.

   (a) **2 points.** What is the probability that Chinatsu voted for Akulina given that Akulina wins the election?

(b) **2 points.** What is the probability that all three voters voted for Akulina given that Akulina wins the election?