ECE 361 Probability for Engineers (Fall, 2016)
Homework 1

Assigned:	Tuesday September 20, 2016	
Due:	Tuesday September 27, 2016	**(at the beginning of class 11am)**
Returned:	Monday October 3, 2016	**(in recitation)**

Your full name ______________________________

Your Drexel student ID ______________________________

ECE 361 homework policies:

1. There will be eight homework assignments, roughly one per week, but check the Course Calendar for exact dates.
2. Each homework will count equally towards your overall homework grade, although the lowest homework will be dropped.
3. Homework must be done INDIVIDUALLY. Although you are free to discuss course content in general with your classmates and peers, you are expected to NOT discuss particulars about homework problems with them or anyone else.
4. Similarly, although you are encouraged to read external sources (online or offline), it is expected that your homework handed in reflects YOUR work, and not work that you found elsewhere.
5. Homework will always be due at the BEGINNING of lecture. One of the Teaching Assistants will be in the lecture hall at the beginning of class to collect the assignments. At 11:10am (ten minutes into the start of lecture) the TA will leave the lecture with the assignments.
6. No submissions will be accepted after that time. No exceptions.
7. Homework will be returned to you during recitations.
8. Homework assignments should have your full NAME, your STUDENT ID, the ASSIGNMENT NUMBER, should be STAPLED, and should be written CLEARLY, with your final ANSWER clearly indicated, and all supporting WORK provided. Sloppy, unclear, or illegible work will not be graded.

For each of the following you are to:

1. Write the sample space Ω.
2. Give the probability of each outcome in the sample space.
3. Show the probabilities sum to one.

Please answer the following questions:

1. (3 points) Consider flipping a *fair* coin. The coin is repeatedly flipped until a head occurs, and the outcome of the experiment is the number of times the coin is flipped. *Hint: recall the geometric series* $a^0 + a^1 + a^2 + \cdots$ *has a sum* $1/(1-a)$ *for* $0 \leq a < 1$.

   Solution.

   (a) The sample space is $\Omega = \mathbb{N}$, where $\mathbb{N} = \{1, 2, 3, \ldots\}$ is the natural numbers.

   (b) The outcome n occurs when the coin comes up tails for the first $n-1$ tosses, and then comes up heads on toss n. Let p_n denote the probability that the n tosses are required. Then
   $$p_1 = 1/2, \; p_2 = (1/2)(1/2), \; p_3 = (1/2)(1/2)(1/2), \tag{1}$$
   and so $p_n = (1/2)^n$ for each $n \in \mathbb{N}$.

   (c) Observe
   $$p_1 + p_2 + \cdots = \sum_{n=1}^{\infty} (1/2)^n = -1 + \sum_{n=0}^{\infty} (1/2)^n = -1 + \frac{1}{1 - 1/2} = 1. \tag{2}$$

2. (3 points) Consider flipping a *fair* coin. The coin is repeatedly flipped until either of the following occurs: *two* successive heads occur (i.e., two heads in a row) or *two* successive tails occur (i.e., two tails in a row). The outcome of the experiment is the number of times the coin is flipped. *Hint: there are many ways to solve this problem, but here is one way. Define the event* A_n *for those outcomes where strictly more than* n *flips are required, and* B_n *for those outcomes where exactly* n *flips are required. Use logic to characterize* A_n *and thereby compute* $\mathbb{P}(A_n)$. *Now* A_n^c *(where* c *denotes complement) is the event that* n *or fewer flips are required. Express* A_n^c *in terms of* B_n *and* A_{n-1}^c, *and use this relation to obtain* $p_n = \mathbb{P}(B_n)$.

3. (3 points) Two *indistinguishable* coins, each *fair*, are each simultaneously flipped. This process is repeated until both coins show a head, and the outcome of the experiment is the number of times the pair of coins is flipped (not the total number of flipped coins).

4. (3 points) Two *distinguishable* coins, each *fair*, are each simultaneously flipped. This process is repeated until both coins show a head, and the outcome of the experiment is the number of times the pair of coins is flipped (not the total number of flipped coins).

5. (3 points) Consider flipping a *biased* coin, where $p = \mathbb{P}(\text{heads})$, for some $p \in (0, 1)$ (the open interval of real numbers between 0 and 1). The coin is repeatedly flipped until a head occurs, and the outcome of the experiment is the number of times the coin is flipped.

6. (3 points) Consider flipping a *biased* coin, where $p = \mathbb{P}(\text{heads})$, for some $p \in (0, 1)$ (the open interval of real numbers between 0 and 1). The coin is repeatedly flipped until either of the following occurs: *two* successive heads occur (i.e., two heads in a row) or *two* successive tails occur (i.e., two tails in a row). The outcome of the experiment is the number of times the coin is flipped. *Hint: the same approach of events* A_n, A_n^c, B_n *from the hint for problem 2 applies here, but the probabilities of* A_n *now depend upon* p. *Carefully consider* n *even vs.* n *odd.*

7. (3 points) Consider three *biased coins*. Coin 1 has bias p, coin 2 has bias q, and coin 3 has bias r, for $0 < p < q < r < 1$. You are to pick two of the three coins. Once picked, you then flip the two coins simultaneously. If the coins show the same face, you win. If the coins show different faces, you lose. The outcome of the experiment is win or lose. Give the probability of winning for each of the three choices (p, q), (p, r), and (q, r).
8. (3 points) Consider three *biased coins*. Coin 1 has bias p, coin 2 has bias q, and coin 3 has bias r, for $0 < p < q < r < 1$. There are three players and each player takes one of the three coins. The three players simultaneously toss their coins. If all three coins show the same face then there is no winner. If the coin faces disagree, however, the player with the unique face wins and the two players with the shared face lose. The outcome of the experiment is the winner of the game.
9. **Extra credit (3 points).** In question 7. above, which selection of coins gives the best probability of winning? That is, given (p, q, r) with $0 < p < q < r < 1$, give a *very simple* expression to pick the pair of coins that maximizes the chance of winning. Credit only for the correct expression in its most simple form.
10. **Extra credit (3 points).** In question 8. above, suppose the biases of the three coins (p, q, r) are known to you and your two competitors. Suppose the coins are picked as follows: player 1 picks one of the three coins, player 2 picks one of the two remaining coins, and player 3 picks the remaining coin. As a function of (p, q, r), give a *very simple* expression for the coin player 1 should pick to maximize her chance of winning. Credit only for the correct expression in its most simple form.