ECE 361 Probability for Engineers (Fall, 2016)
Homework 7

Assigned:	Thursday November 10, 2016	
Due:	Thursday December 1, 2016	**(at the beginning of class 11am)**
Returned:	Monday December 5, 2016	**(in recitation)**

Your full name ______________________________

Your Drexel student ID ______________________________

ECE 361 homework policies:

1. There will be eight homework assignments, roughly one per week, but check the Course Calendar for exact dates.
2. Each homework will count equally towards your overall homework grade, although the lowest homework will be dropped.
3. Homework must be done INDIVIDUALLY. Although you are free to discuss course content in general with your classmates and peers, you are expected to NOT discuss particulars about homework problems with them or anyone else.
4. Similarly, although you are encouraged to read external sources (online or offline), it is expected that your homework handed in reflects YOUR work, and not work that you found elsewhere.
5. Homework will always be due at the BEGINNING of lecture. One of the Teaching Assistants will be in the lecture hall at the beginning of class to collect the assignments. At 11:10am (ten minutes into the start of lecture) the TA will leave the lecture with the assignments.
6. No submissions will be accepted after that time. No exceptions.
7. Homework will be returned to you during recitations.
8. Homework assignments should have your full NAME, your STUDENT ID, the ASSIGNMENT NUMBER, should be STAPLED, and should be written CLEARLY, with your final ANSWER clearly indicated, and all supporting WORK provided. Sloppy, unclear, or illegible work will not be graded.

Please answer the following questions:

1. (3 points) Let (U,V) be independent continuous random variables, both uniformly distributed on $[0,1]$, and define the pair of RVs (X,Y) where $X = \min(U,V)$ and $Y = \max(U,V)$. Compute the PDF of $Z = X/Y$. *Hint: first find the joint CDF $F_{X,Y}(x,y)$ for $0 \leq x \leq y \leq 1$, then find the joint PDF $f_{X,Y}(x,y)$, then find $F_Z(z) = \mathbb{P}(X/Y \leq z)$, for $z \in [0,1]$, by conditioning on (X,Y), via $F_Z(z) = \int_0^1 \int_0^y \mathbb{P}(X/Y \leq z | X = x, Y = y) f_{X,Y}(x,y) \mathrm{d}x\mathrm{d}y$.*
2. (2 points) Let X_1, X_2 be independent and identically distributed RVs, both exponentially distributed with parameter $\lambda > 0$. Define $X = X_1 + X_2$. Please do the following:
   - Find the PDF for X.
   - Read about the gamma(k,θ) probability distribution, where k is the "shape" parameter and θ is the "scale" parameter. Show that the distribution of X is gamma, and find the appropriate values for (k,θ).
3. (2 points) Let X_1, X_2 be independent and identically distributed RVs, both distributed as standard normals. Define $X = X_1^2 + X_2^2$. Please do the following:
   - Find the PDF for X. *Hint: first find the CDF for X_1^2 and the PDF for X_1^2, then use the convolution formula to find the PDF for X. You may find it useful to use the fact that $\int_0^x \frac{1}{\sqrt{x_1(x-x_1)}} \mathrm{d}x_1 = \pi$.*
   - Read about the chi-squared $\chi^2(k)$ probability distribution, where $k \in \mathbb{N}$ is the "degrees of freedom" parameter. Show that the distribution of X is gamma, and find the appropriate values for k.
4. (2 points) Let X be a standard normal random variable and define $Y = 1/X$. Please do the following:
   - Find the PDF and CDF for Y. *Hint: consider $y < 0$ and $y \geq 0$ separately. Consider $y \geq 0$, and observe the equivalence of the events $\{Y \leq y\}$ and $\{X \leq 0$ or $X > 1/y\}$.*
   - Create two plots. The first plot should show the PDF of Y and the PDF of the standard normal distribution over the interval $[-5,+5]$. The second plot should show the CDF of Y and the CDF of the standard normal distribution over the interval $[-5,+5]$.
5. (3 points) Let (X,Y) be a pair of Bernoulli RVs with joint PMF
$$\mathbb{P}(X = Y = 1) = p,\ \mathbb{P}(X = 1, Y = 0) = (1-p)/2,\ \mathbb{P}(X = 0, Y = 1) = (1-p)/2,\ \mathbb{P}(X = Y = 0) = 0 \quad (1)$$
   for a parameter $p \in (0,1)$. Find the correlation of X, Y in terms of p.
6. (2 points) Let (X,Y) be a pair of discrete RVs, each with support $\{-1,0,+1\}$, with joint PMF:
$$\mathbb{P}(X = -1, Y = 0) = \mathbb{P}(X = 0, Y = -1) = \mathbb{P}(X = 0, Y = +1) = \mathbb{P}(X = +1, Y = 0) = \frac{1}{4}. \quad (2)$$
   Please do the following:
   - Find the correlation of (X,Y).
   - Determine whether (X,Y) are dependent or independent.
7. (3 points) Let (U,V) be independent continuous random variables, both uniformly distributed on $[0,1]$, and define the pair $(X,Y))$ where $X = \min(U,V)$ and $Y = \max(U,V)$. Compute the correlation of (X,Y). *Hints: recall the joint ($f_{X,Y}(x,y)$) and marginal ($f_X(x), f_Y(y)$) distributions from an earlier problem. Compute the means ($\mathbb{E}[X], \mathbb{E}[Y]$), compute the expected squared values ($\mathbb{E}[X^2], \mathbb{E}[Y^2]$), compute the variances ($\mathrm{Var}(X), \mathrm{Var}(Y)$), compute the covariance $\mathrm{Cov}(X,Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$ using the joint distribution $f_{X,Y}(x,y)$, and finally, compute the correlation.*
8. (2 points) Let (X,Y) be the coordinates of a point chosen uniformly at random in the unit circle $D = \{(x,y) : \sqrt{x^2+y^2} \leq 1\}$. Find the correlation of (X,Y).