<u>MECHATRONICS CONTROL & AUTOMATION LAB</u>

EXPERIMENT 1:
FIRST-ORDER SYSTEM

GROUP NUMBER: 2

PROGRAMME: MECHATRONICS

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DATE OF EXPERIMENT:

Wednesday, 11th March 2024

DATE OF SUBMISSION:

Wednesday, 18th March 2024

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Introduction

A first-order system is a system whose response to an input can be described by a first-order differential equation. This means that the system's output depends on the input and its state, and the rate of change of the output is directly proportional to the difference between the input and the current output.

Mathematically, a first-order system can be represented by the following differential equation:

$$a_0 \frac{dy(t)}{dt} + a_1 y(t) = f(t) \quad \text{with} \quad y(0) = y_0, \qquad (1.1)$$

An RC circuit consists of a resistor (R) and a capacitor (C) connected in series or parallel. When a voltage is applied to the circuit, the capacitor begins to charge or discharge, depending on the polarity of the voltage. This charging or discharging process follows a first-order dynamic behavior, meaning the voltage across the capacitor changes over time according to a first-order differential equation.

A DC servo system is a closed-loop control system used to control the position or velocity of a DC motor. It typically consists of a DC motor, a feedback device (such as an encoder or resolver), and a controller. In a first-order model of a DC servo system, the system's dynamic behavior can be described by a first-order differential equation.

Objectives

The objective of this laboratory experiment was to identify specific system parameters that influence system response. Additionally, the experiment aimed to model a first-order system and examine the impact of these parameters on its response to step or impulse input.

Methods

The first order of the system is defined as the first derivative with respect to time. First-order systems are those systems where the denominator of the transfer function is of the first order where the highest power of ‘s’ is 1. The standard form of the transfer function of a first-order system is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{(\tau s + 1)}$$

where Y(s) and U(s) are the Laplace transforms of the output and input variables while K represents the DC (steady-state) gain and τ is the time constant.

Whereas for the unit step input U(s)=1/s, the response of the system is:

$$Y(s) = G(s)U(s) = \frac{K}{(\tau s + 1)}\frac{1}{s} = \frac{K}{s(\tau s + 1)}$$

The inverse of the resulting Laplace transform is:

$$y(t) = L^{-1}\left[\frac{K}{s(\tau s + 1)}\right] = K\left\{L^{-1}\left[\frac{1}{s(\tau s + 1)}\right]\right\} = K(1 - e^{-t/\tau})$$

From the inverse equation, it is surmised that the value of y approaches K as t→ ∞. For a unit step input, the ultimate value of the output can be understood as the DC gain. The time constant is the amount of time that y(t) takes to get to 63.2% of its ultimate value. Thus, at t=τ, y(t)=0.632K for a unit step input. Ideally, y(t) = 0.632MK will be the result at t=τ for a step input of size M.

There are two cases that show the response of the first-order system to a unit step input. If the system’s output change is less than the input change applied, the system gain K<1. Meanwhile, if the system’s output change is more than the input change applied, the system gain is K>1.

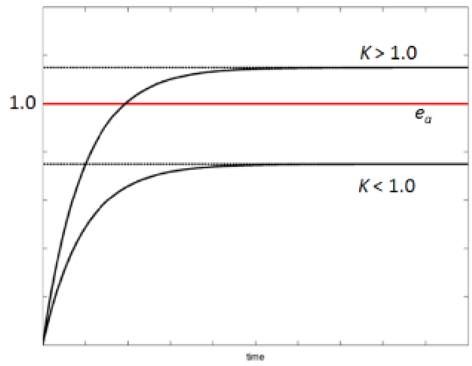
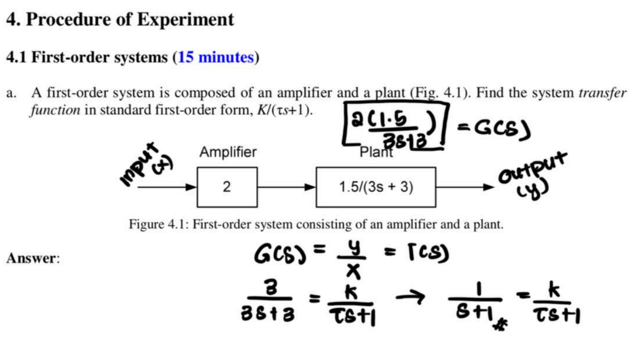


Figure 2.1: First-order system step response.

Discussion

Experiment 4.1



4. Procedure of Experiment

4.1 First-order systems (15 minutes)

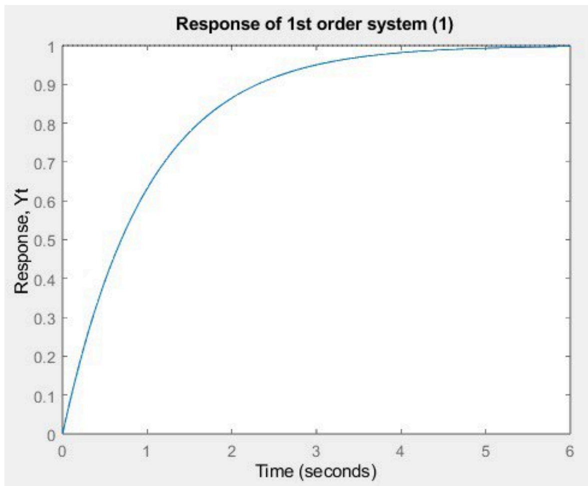
a. A first-order system is composed of an amplifier and a plant (Fig. 4.1). Find the system *transfer function* in standard first-order form, $K/(\tau s+1)$.

Figure 4.1: First-order system consisting of an amplifier and a plant.

Answer:

$G(s) = \frac{y}{x} = T(s)$

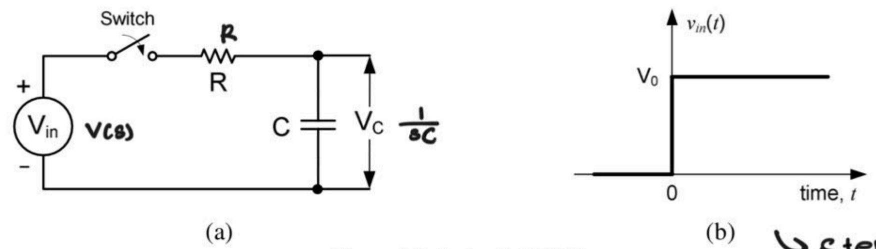
$\frac{3}{3s+3} = \frac{k}{\tau s+1} \rightarrow \frac{1}{s+1} = \frac{k}{\tau s+1}$



In the sketch of the output for the first-order system with a unit input, we observe two main phases which are transient and steady-state responses. Initially, the output undergoes a transient response, adjusting from its initial condition towards its steady-state value. The speed of this adjustment is determined by the system's time constant (τ), with smaller time constants leading to faster responses. Additionally, the amplification factor (K) affects the magnitude of the output response compared to the input. Once the transient response settles, the output reaches its steady-state value, representing the long-term behaviour of the system. The shape of the response, whether underdamped, critically damped, or overdamped, influences features such as oscillations, rise time, and settling time.

Experiment 4.2

4.2 RC transient analysis (25 minutes).



↳ step input

Figure 4.2: A simple RC filter.

a. Derive the differential equations for the transient state shown in Fig. 4.2.

Answer: KVL

$$-V_{in} + RI + \overbrace{\frac{1}{C}\int_0^t i\,dt}^{V_c} = 0$$

b. Find $v_C(t)$ immediately after the switch is closed (on).

Answer: $-V_{in} + V_R + V_c = 0$

$$V_c = V_{in} - V_R$$

step input

$$V_c(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_s(s)$$

$$V_c(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}\left(\frac{1}{s}\right) = \frac{\frac{1}{s}\left(\frac{1}{sC}\right)}{R + \frac{1}{sC}}$$

$$V_c(s) = \frac{\frac{1}{s^2 C}}{R + \frac{1}{sC}} \cdot \frac{(sC)}{(sC)} = \frac{\frac{1}{s}(s)}{sRC + 1\,(s)} = \frac{1}{s^2RC + s} = \frac{1}{s} - \frac{RC}{RCs+1}$$

$$V_c(t) = 1 - e^{-t/RC}$$

$-V_c$

c. If $V_0 = 5$V, $R = 10$ kΩ, and $C = 100$ μF, plot the transient curve of $v_C(t)$ and $v_R(t)$ in MATLAB for $t \le 8\tau$. Compute the time constant, τ, of the system. Show your work.

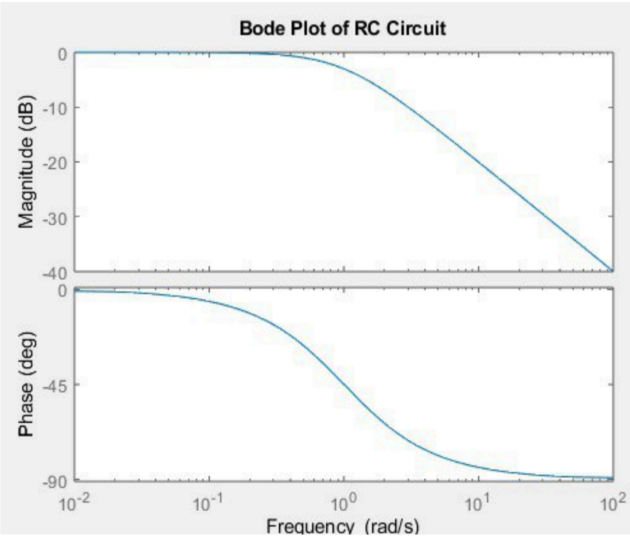
Answer:

$$\tau = RC = 10(10)^{-3} \times 10(10)^{-6} = 1$$

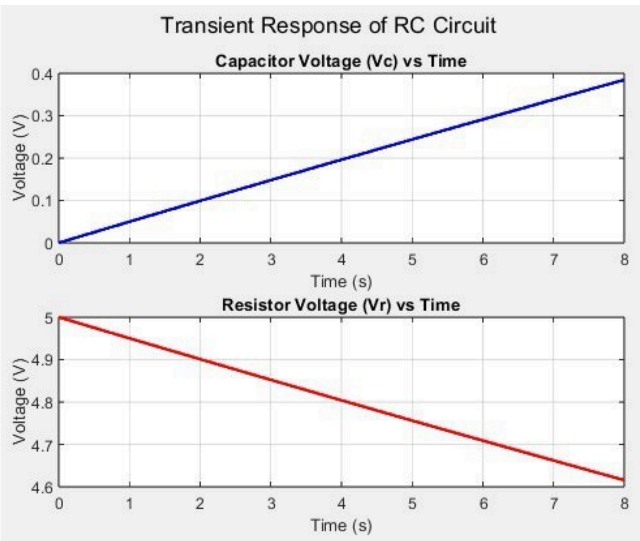
$$t \le 8\tau \rightarrow t \le 8 *$$

$$V_c = V_{in}\left[1 - e^{-t/RC}\right]$$

$$V_R = V_{in}\left[e^{-t/RC}\right]$$



The Bode plot of the RC circuit, acting as a low-pass filter, reveals how the output signal changes with frequency. At low frequencies, the signal passes through unattenuated (0 dB gain), but as frequency increases past the cutoff frequency, the output weakens at a rate of -20 dB per decade. The phase shift between input and output also increases with frequency, reaching -45° at cutoff frequency and -90° at high frequencies. This plot helps visualise the filtering effect, where low frequencies experience minimal attenuation while high frequencies are suppressed.



In conclusion, increasing the resistor to 1 MΩ will cause the capacitor to charge and discharge slower in the transient response, reflected by a slower rise time for Vc and a slower decay for Vr. The Bode plot will maintain its characteristic shape but with a shifted cutoff frequency to a lower value. This indicates that the low-pass filter behaviour persists, but the frequencies at which the attenuation starts and the attenuation rate at high frequencies might change depending on the specific capacitance value.

Experiment 4.3

Modeling DC servomotor

For the model of DC servomotor the following parameters are available:

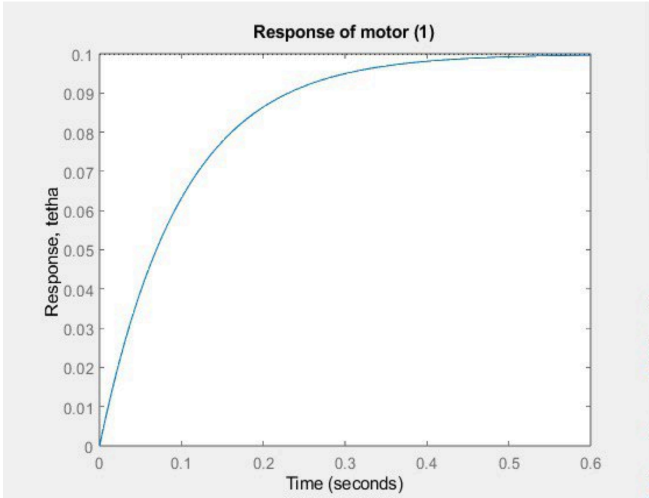
- Simulation interval, $0 \le t \le 3$ s
- Time step (for simulation), $\Delta t = 0.1$ ms
- Rotor's moment of inertia, $J = 0.01$ kg m^2
- Damping ratio, $b = 0.1$ Nms
- Back EMF constant, $K_e = 0.01$ Nm/A
- Torque constant, $K_T = 0.01$ Nm/A
- Amplifier gain, $K_a = 25$
- Armature electric resistance, $R_a = 1\ \Omega$
- Armature electric inductance, $L_a = 0.01$ H – 0.5 H (*varying*)

a. For the time being we assume that the armature electric inductance, $L_a \to 0$ (negligible small). Write a MATLAB routine for simulating the *step response* of the open-loop DC servomotor system with *amplifier driver*, e_a, as the input and *angular speed*, $\dot{\theta}$, as the output.

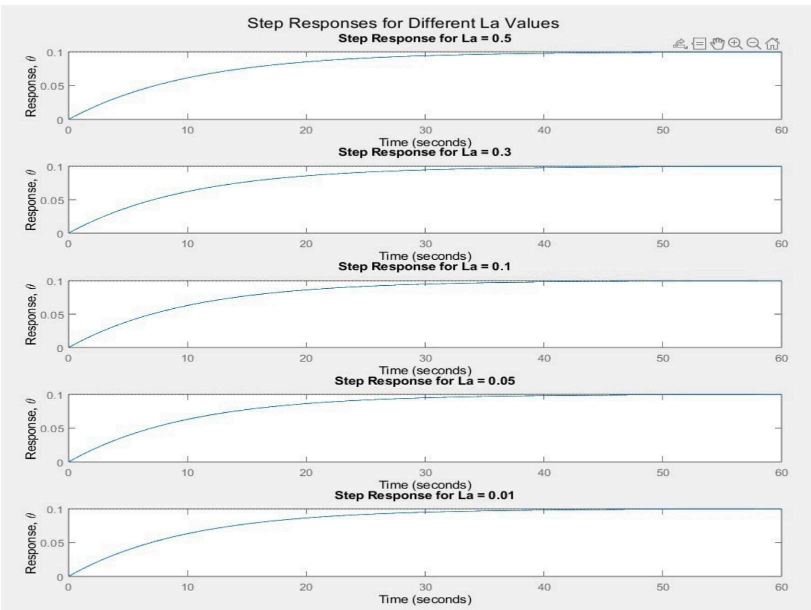
Answer (*use a new blank page if there is not enough space to answer*):

$$G(s) = \frac{s\Theta(s)}{e_a(s)} = \frac{K_T/(bR_a + K_T K_e)}{[JR_a/(bR_a + K_T K_e)]s+1}$$

$$G(s) = \frac{s\Theta(s)}{e_a(s)} = (0.01)/($$



Simulating the step response of the open-loop DC servomotor with an amplifier driver as the input and angular speed as the output using MATLAB provides a clear visual representation of how the motor reacts to sudden changes in the input signal. The plot shows the motor's angular speed over time, starting from rest and rapidly accelerating in response to the step input. The transient response phase depicts how quickly the motor reaches its desired speed, while the steady-state response phase shows the final, constant speed achieved by the motor. Features such as overshoot, settling time, and damping can be observed from the plot, offering valuable insights into the motor's performance characteristics. This analysis aids in understanding the motor's behavior and informs decisions regarding system design and control strategies.



From the sketch, variations in armature electric inductance influence the motor's transient response, steady-state behaviour, and overall performance. Changes in La impact the system's dynamics by altering the time constants and damping ratios, affecting how quickly the motor responds to input signals and how it settles into its final operating state. Higher values of La typically result in slower responses and smoother transitions, while lower values may lead to faster responses with potentially more pronounced oscillations.

In conclusion, the investigated DC motor model exhibits distinct behaviours depending on the armature electric inductance conditions. When the inductance is negligible, the motor's response is characterised by a rapid rise in angular speed, reflecting its high responsiveness to input signals. Conversely, varying La from 0.5 H down to 0.01 H introduces significant changes in the motor's transient and steady-state behaviours. Higher inductance values result in slower responses and smoother transitions, while lower values lead to faster responses with potential oscillations. This investigation highlights the importance of considering La in motor design and control, as it directly impacts the system's dynamic performance and overall efficiency.

Conclusion

In conclusion, this laboratory experiment focused specifically on first-order systems, aiming to identify key system parameters affecting their response and to model their behaviour under step or impulse inputs. Through meticulous experimentation and analysis, we explored how variations in parameters such as time constant and gain influence the dynamics of first-order systems. By observing the system's response to different inputs, we gained valuable insights into the fundamental characteristics and sensitivities of first-order systems. This investigation underscores the significance of understanding and manipulating system parameters in engineering applications, particularly in contexts where first-order systems play a crucial role. Overall, this experiment provided a comprehensive understanding of first-order system dynamics, laying a solid foundation for further exploration and application in real-world scenarios.