

Power Diagram Enhanced Adaptive Isosurface Extraction from Signed Distance Fields

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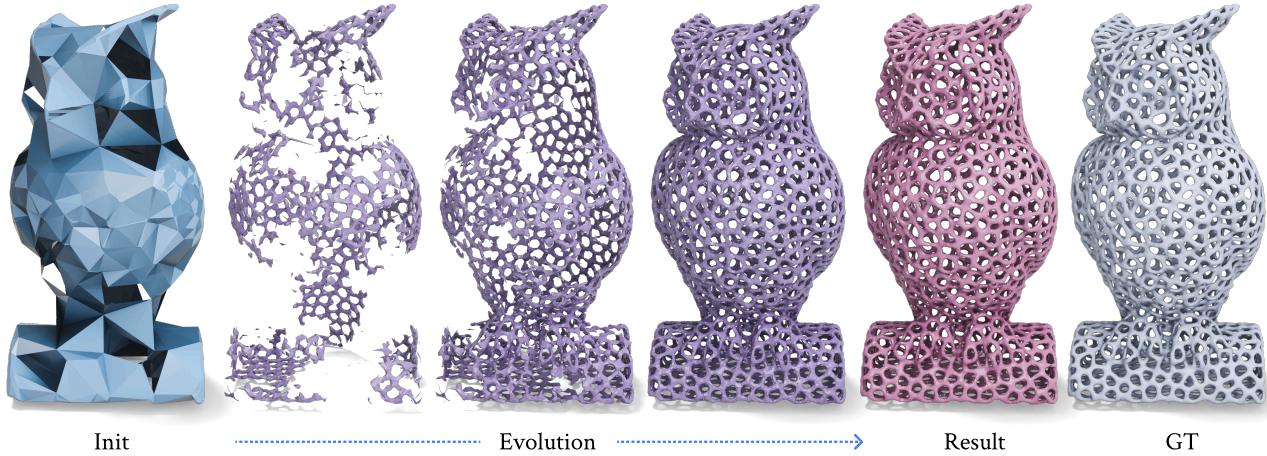


Figure 1. The workflow of our isosurface extraction. Starting from an initial coarse mesh surface, our method iteratively refines the geometry, progressively converging toward topologically faithful and geometrically accurate results.

Abstract

Extracting high-fidelity mesh surfaces from Signed Distance Fields (SDFs) has become a fundamental operation in geometry processing. Despite significant progress over the past decades, key challenges remain—namely, how to automatically capture the intricate geometric and topological structures encoded in the zero level set of SDFs. In this paper, we present a novel isosurface extraction algorithm that introduces two key innovations: 1) An incrementally constructed power diagram through the addition of sample points, which enables repeated updates to the extracted surface via its dual—regular Delaunay tetrahedralization; and 2) An adaptive point insertion strategy that identifies regions exhibiting the greatest discrepancy between the current mesh and the underlying continuous surface. As the teaser figure shows, our framework progressively refines the extracted mesh with minimal computational cost until it sufficiently approximates the underlying surface. Experimental results demonstrate that our approach outperforms state-of-the-art methods, particularly for models with intricate geometric variations and complex topologies.

1. Introduction

Signed Distance Fields (SDFs) serve as a fundamental implicit surface representation in geometry processing [7, 8, 10, 17, 18, 21, 22, 29]. With the development of geometric deep learning techniques, neural SDFs [30, 32] have become popular in many tasks, including surface reconstruction, shape completion, and shape generation. Consequently, extracting a topologically faithful and geometrically accurate mesh surface from SDFs has become increasingly important.

However, during mesh extraction, fully preserving geometric fidelity and capturing the true topological structure remains challenging due to the inherent implicit nature of SDFs. Traditional approaches, such as Marching Cubes [1, 6, 15] and Dual Contouring [11], as well as some learning-based variants [2, 14, 23], extract the mesh surface by discretizing space into uniform cubic units and analyzing

ing how the surface intersects the grid cells. They assume that the surface within each grid cell can be deemed sufficiently simple, thereby heavily relying on the pre-defined resolution of the grid. When the geometry is intricate—for instance, when the local feature size (LFS) is smaller than the cell size—geometric features (such as geometric details, feature lines, tip points, tiny tubes, and thin plates) may not be completely recovered or might even be lost in the extracted triangle mesh.

Recently, several geometry-aware mesh extraction algorithms [24, 26, 27] have been proposed for adaptive mesh extraction. For instance, Reach for the Spheres (RFTS) [27] operates under the assumption that each SDF sample corresponds to a spherical region that lies entirely either inside or outside the surface. Its extension, Reach for the Arc (RFTA) [26], leverages additional information from the SDF samples. Both methods perform well at low resolutions, when only a limited number of SDF samples are queried. However, RFTS struggles to capture high-frequency geometric variations, and RFTA is unable to modify the initial topology during the refinement process. In contrast, McGGrids [24] formulates adaptive grid construction as a probability sampling problem, solved via a Monte Carlo process. Despite its advantages, it still struggles to resolve tiny details and small topological features.



Figure 2. Compared with existing isosurface extraction approaches, our method excels at recovering intricate surface details.

In this paper, we improve adaptive mesh extraction approaches through two key innovations. First, we exploit the observation that a collection of samples with signed distance function (SDF) values defines a power diagram where squared SDF values act as weights. This power diagram contains a subset of facets that approximate the underlying surface. By incrementally constructing the power diagram with added samples, its dual tetrahedralization induces an adaptive and refined grid structure, enabling mesh extraction that closely aligns with the true surface. Second, we propose a novel point insertion strategy that prioritizes regions with the largest discrepancy between the current mesh and the underlying continuous surface. This simple yet effective strategy optimizes sample placement to maximize reconstruction accuracy. We further integrate these techniques into a progressive framework that requires only local updates per iteration, minimizing computational overhead. The algorithm terminates once a predefined accuracy toler-

ance is achieved. Extensive experiments demonstrate our method’s superior mesh extraction capabilities compared to state-of-the-art approaches, particularly for models with intricate details and complex topologies.

Our contributions include:

- A power diagram formulation that leverages SDF samples as weights, where incremental construction of the diagram and its dual tetrahedralization enable adaptive mesh extraction.
- A point insertion strategy that optimally targets regions with the largest reconstruction discrepancy between the mesh and the continuous surface.
- A progressive framework requiring only local updates per iteration, ensuring low computational cost. The algorithm terminates when the accuracy tolerance is met.

2. Related Work

Isosurface extraction from implicit fields is a fundamental problem in geometry processing with extensive applications. Existing approaches can be broadly categorized into two paradigms: operating on pre-defined grids and progressively evolving an initial surface estimate to match the target isosurface.

2.1. Extraction on Pre-defined Grids

The most widely used approach is the Marching Cubes [15], which uniformly partitions the space into cubic elements and determines surface geometry by examining field values at cube corners using a carefully designed lookup table. The Marching Tetrahedrons method [5] extends this idea to tetrahedral elements, offering better flexibility in handling complex geometries. Dual contouring techniques [11] further enhance feature preservation by incorporating gradient information.

To improve efficiency, hierarchical data structures like Octree [13, 20] can be used to increase resolution near surface regions and speed up computation. Although these approaches [12, 28] operate adaptively, they often use the signs at grid points to predict whether the surface intersects a grid cell, thus lacking a strong ability to detect unknown geometry.

The biggest advantage of these methods lies in their efficiency and parallelizability. However, they heavily rely on the resolution of pre-defined grid cells. Furthermore, locally refining the grid cells can easily cause mesh inconsistency between adjacent cells.

On this basis, recent data-driven methods, including Neural Marching Cubes [1] and Neural Dual Contouring [2], leverage learned features to recover surface details without explicit gradient information. Another differentiable approach [19] generates surfaces by optimizing point locations within Voronoi cells.

2.2. Progressive Evolution Methods

Progressive evolution methods offer an alternative paradigm. Early approaches in this category include surface tracking methods like Marching Triangles [9], which grow meshes from seed points, and Delaunay refinement approaches [4, 31] that incrementally improve mesh quality through careful vertex insertion.

This paradigm has gained significant attention recently due to its ability to iteratively correct surface topologies and refine geometric accuracy. Generally speaking, these methods utilize more information from the SDF to guide the refinement process. For instance, Reach for the Spheres [27] demonstrates that by exploiting sphere-surface tangency conditions, the surface can be better recovered even at low resolution. However, its fixed topology during optimization limits its applicability to complex models. Reach for the Arcs [26] addresses this topological limitation but requires repeatedly calling Poisson reconstruction, resulting in processing times orders of magnitude slower than conventional approaches, which makes it impractical for many applications.

McGrids [24] offers another recent innovation in progressive extraction through adaptive Voronoi diagrams for incremental sampling. It separates the phase of SDF-guided sampling from the phase of surface extraction. In this paper, our algorithm alternately adds sample points and locally updates the discrete surface. The two operations are highly interrelated: the position of the newly added sample is guided by the current surface, while the surface is updated within the region where the new point is inserted.

3. Overview

Our method extracts isosurfaces from Signed Distance Fields using both gradient and value information as input. The algorithm consists of two primary components that work in a tightly coupled manner:

First, a power diagram-based isosurface extraction framework. Given spatial sampling points and their gradient information, this framework extracts a surface that approximates the underlying implicit geometry. It supports incremental insertion of sampling points and enables localized updates to the extracted surface, avoiding costly global recompilation.

Second, a novel point insertion strategy that prioritizes regions exhibiting the largest discrepancy between the current mesh and the underlying continuous surface. This strategy identifies optimal locations for new sampling sites that, when incorporated into the power diagram, effectively improve problematic regions in the current reconstruction.

These two components alternate in a continuous cycle—extraction, evaluation, refinement, and re-extraction—progressively enhancing surface fidelity

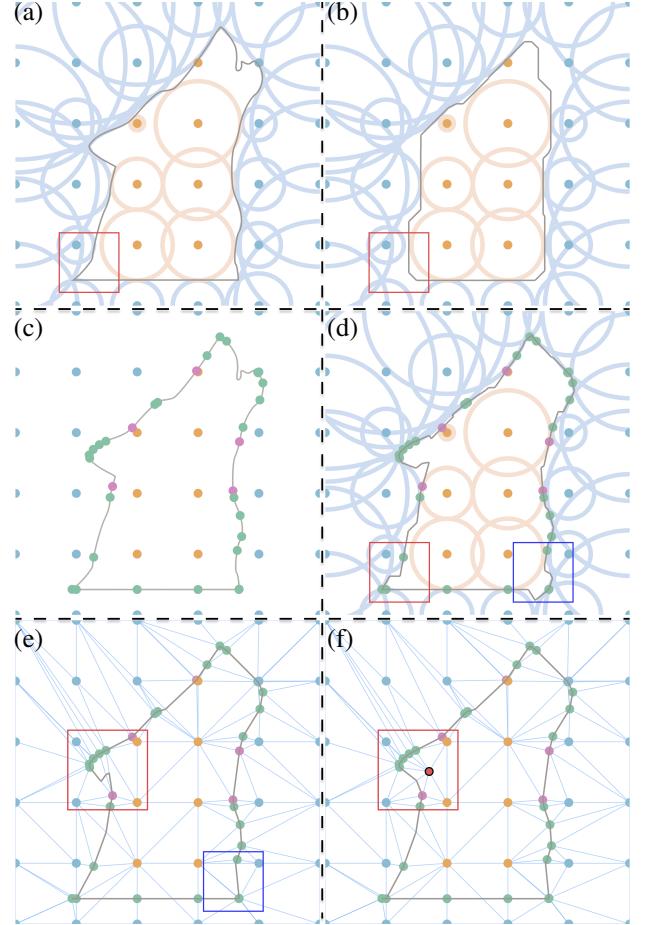


Figure 3. (a) Each sampling site generates a sphere with its center at the site and radius equal to its distance to the surface, creating tangential contact with the surface. (b) The power diagram computed from these sites produces boundaries that lie between inner and outer spheres without intersections, but fails to maintain tangency with the spheres (highlighted in red). (c) We incorporate projection points as additional power diagram sites, resulting in four distinct categories of sites. (d) The enhanced power diagram computation achieves tangency with spheres (red box) but introduces geometric instabilities manifesting as zigzag patterns (blue box). (e) By extracting the surface directly from the regular Delaunay, we eliminate the zigzag artifacts (blue box) while preserving the geometric properties. (f) Our power diagram-based representation naturally supports incremental construction. When inserting a new point (shown in red), all structural changes in both the regular Delaunay and the reconstructed surface are confined within the local region (red box).

until reaching either a predefined point budget or quality threshold. This framework implements a progressive approach that requires only local updates per iteration, thus minimizing computational overhead. The following sections provide detailed explanations of each component and their collaborative role in isosurface extraction algorithm.

4. Power Diagram-based Isosurface Extraction

We present our power diagram-based isosurface extraction approach in four progressive steps: First, we establish the theoretical foundation for isosurface extraction using power diagrams. Second, we demonstrate how incorporating projection points as additional sampling sites enhances extraction quality. Third, we employ the dual of the power diagram—regular Delaunay tetrahedralization—to overcome inherent instabilities in power diagram construction. Finally, we validate that this extraction framework supports localized incremental updates without requiring global recomputation.

4.1. Theoretical Foundation

RFTA [26] highlighted an essential geometric property: spheres constructed with centers at sampling points and radii equal to their unsigned surface distances maintain tangential contact with the surface. Figure 3(a) illustrates this property in the 2D case. In our framework, we classify sampling points as \mathbf{P}_- and \mathbf{P}_+ based on their negative and positive signed distance values, forming inner spheres \mathbf{S}_- and outer spheres \mathbf{S}_+ with radii equal to their unsigned distance values, as shown in Figure 3(a). By utilizing these points as power diagram sites and their squared surface distances as weights, we establish the following theorem:

Theorem 1. *For two sets of spheres \mathbf{S}^- and \mathbf{S}^+ , the boundary of the power diagram computed by using sphere centers as sites and squared distances as weights forms a valid surface between the two sets of spheres without intersecting any sphere.*

Proof. We prove by contradiction, as illustrated in the inset figure. Assume a power diagram boundary intersects with an inner sphere. This implies there exists a point x on the power boundary inside this inner sphere, with a negative power distance to some inner site p_i^- :

$$|x - p_i^-|^2 - \phi^2(p_i^-) < 0 \quad (1)$$

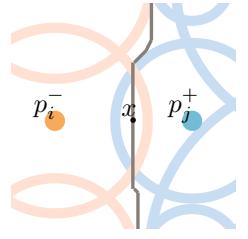
Here, p_i^- represents an inner point (the superscript “-” denotes an interior point), i is the index, and $\phi(p_i^-)$ is the distance from p_i^- to the surface.

Since x lies on the power diagram boundary, there must exist an outer site p_j^+ (where the superscript “+” denotes an exterior point) with equal power distance:

$$|x - p_j^+|^2 - \phi^2(p_j^+) = |x - p_i^-|^2 - \phi^2(p_i^-) < 0 \quad (2)$$

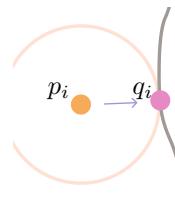
This indicates that x also lies inside the outer sphere centered at p_j^+ , creating an intersection between an inner and outer sphere at point x , as shown in the inset. This contradicts the fundamental property that inner and outer spheres cannot intersect, thereby proving the theorem. \square

This theorem establishes that the power diagram boundary provides a feasible surface approximation by maintaining the crucial non-intersection property with both sets of spheres, as illustrated in a 2D example in Figure 3(b).



4.2. Surface-Tangent Points Integration

Since SDF representation provides readily accessible gradient information, we can easily compute surface tangent points to enhance our reconstruction. For each sampling point p_i , we obtain its projection q_i onto the zero-level surface as shown in the inset figure.



We incorporate these projection points as additional power diagram sites alongside the original sampling points. Our power diagram sites are then classified into two categories, with \mathbf{P}_- and \mathbf{P}_+ representing exterior and interior sampling points, and \mathbf{P}_\perp and \mathbf{P}_\perp^\perp denoting their respective surface projections:

- Category 1: \mathbf{P}_- and \mathbf{P}_\perp^\perp
- Category 2: \mathbf{P}_+ and \mathbf{P}_\perp

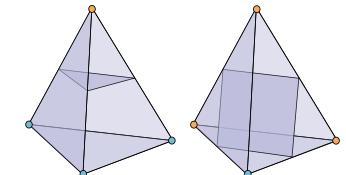
This classification creates power diagram boundaries that form planes tangent to the surface at the projection points. As a result, our reconstructed surface maintains proper geometric contact with the original surface at these projection points, significantly improving the fidelity of geometric features.

Figure 3(c) and (d) illustrate this point cloud classification and the corresponding power diagram extraction results in 2D scenarios.

4.3. Stable Surface Extraction

While power diagram boundaries ensure non-intersection with inner and outer spheres and maintain surface tangency, they inherently produce geometric instabilities in practice, as highlighted in the blue box in Figure 3(d). Leveraging the dual relationship between power diagrams and regular Delaunay tetrahedralization, we propose extracting the surface mesh \mathcal{M} directly from the regular Delaunay tetrahedralization \mathcal{D} by processing each tetrahedron containing different types of sites. This approach preserves the spatial guidance properties of power diagrams while eliminating their inherent geometric discontinuities.

For each tetrahedron containing different types of sites, there are



only two possible distributions: a 2-2 split or a 1-3 split, as shown in the inset. Consequently, the surface extracted within the tetrahedron is either triangular or quadrilateral.

For each edge connecting different types of sites, we extract a corresponding vertex using the following dualization rules, which depend on the properties of the two vertices forming the edge:

- $v_1 \in \mathbf{P}_-$ and $v_2 \in \mathbf{P}^{-\perp}$: v_2
- $v_1 \in \mathbf{P}_+$ and $v_2 \in \mathbf{P}^{+\perp}$: v_2
- $v_1 \in \mathbf{P}_-$ and $v_2 \in \mathbf{P}_+$: $\frac{v_1\phi(v_2)-v_2\phi(v_1)}{\phi(v_2)-\phi(v_1)}$
- $v_1 \in \mathbf{P}^{-\perp}$ and $v_2 \in \mathbf{P}^{+\perp}$: $\frac{v_1+v_2}{2}$

By connecting these dual vertices according to the cell's topology, we construct a mesh containing both triangular and quadrilateral elements that approximates the surface while avoiding the instabilities inherent in power diagram dualization, as demonstrated in Figure 3(e) for a 2D case.

4.4. Localized Incremental Update

A significant advantage of our approach is its natural adaptability to incremental construction. The regular Delaunay tetrahedralization inherently supports localized updates, allowing us to efficiently process the insertion of new sites without global recomputation. When inserting a new site, only a small neighborhood of existing elements requires modification, and correspondingly, only the associated portions of the dual mesh need to be updated.

This locality property ensures that the computational complexity of each update operation depends only on the size of the affected region, not on the total mesh complexity. As illustrated in Figure 3(f), when inserting a new point (marked in red) in a 2D scenario, all updates are confined within the highlighted red box. This localized behavior makes our approach particularly efficient for adaptive refinement and dynamic surface reconstruction tasks.

5. Algorithm

Starting from a surface extracted from uniformly distributed sampling points, we iteratively refine the surface by identifying regions with suboptimal reconstruction quality and strategically adding new points in those areas until convergence. Since our key algorithmic components are not executed sequentially but rather form a tightly coupled iterative process, we first present the overall pipeline before detailing each critical step.

5.1. Pipeline

Our algorithm proceeds through the following steps:

Step 1. Construct regular delaunay \mathcal{D} from uniform grid vertices and their projections, obtain initial surface \mathcal{M} (Section 4).

Step 2. Calculate the deviation metric δ between each triangle in \mathcal{M} and the underlying surface (Section 5.2) and organize them in max-heap \mathcal{Q} .

Step 3. Extract triangle with maximum deviation δ_c from \mathcal{Q} , add corresponding new site p_c and its projection q_c (Section 5.3). Update \mathcal{D} and \mathcal{M} (Section 4.4), compute quality metrics for affected and newly added faces (Section 5.2) and insert them into \mathcal{Q} .

Step 4. If the number of inserted points reaches k_{max} or $\delta_c < \epsilon$, proceed to Step 5. Else return to Step 3.

Step 5. Extract surface.

Each key step in our pipeline is detailed in the corresponding subsections referenced above. For better understanding, we also provide a 2D illustration of this pipeline in Figure 4.

5.2. Isosurface Approximation Quality Evaluation

A robust metric for assessing triangle fidelity to the zero-level set is crucial for our adaptive refinement strategy. Our approach leverages the geometric principle that for a triangle perfectly aligned with the isosurface, the SDF gradient $\nabla\phi(\mathbf{x})$ should parallel the triangle normal $\hat{\mathbf{n}}$ at any point \mathbf{x} on the triangle, as illustrated in Figure 5.

To quantify deviation from this ideal alignment, we measure the orthogonal component of the gradient for Δ :

$$\delta(\Delta) = \int_{\Delta} |\nabla\phi(\mathbf{x}) - (\nabla\phi(\mathbf{x}) \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}| dA \quad (3)$$

We select this gradient-based metric over simple SDF values for its enhanced sensitivity to topological variations through normal field analysis, as demonstrated in Figure 5.

For practical implementation, we approximate the integral by partitioning the triangle into three sub-triangles using angle bisectors and assuming constant gradient within each:

$$\delta(\Delta) \approx \sum_{i=1}^3 A_i |\nabla\phi(\mathbf{c}_i) - (\nabla\phi(\mathbf{c}_i) \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}| \quad (4)$$

where A_i is the area of the i -th sub-triangle and \mathbf{c}_i its centroid. This discretization provides an efficient and accurate quality metric for guiding point insertion.

5.3. Strategic Site Insertion for Local Refinement

After identifying regions with poor approximation quality through our deviation metric δ , we strategically place new sites to improve these problematic areas. For each triangle with high deviation, we select the dual vertex of its corresponding tetrahedron in the regular Delaunay tetrahedralization \mathcal{D} and its surface projection as new insertion sites.

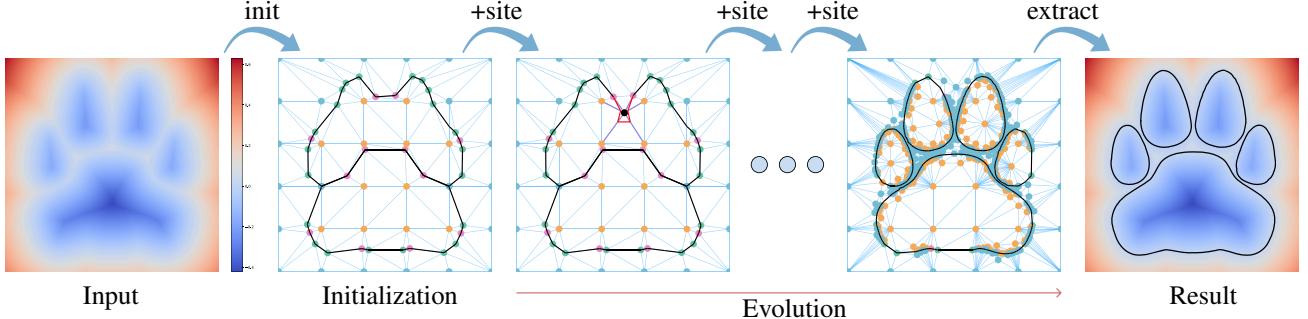


Figure 4. Given a signed distance field with associated gradients, we initialize the surface by constructing a regular Delaunay tetrahedralization from uniform samples and their surface projections. The algorithm then iteratively identifies suboptimal geometric elements (triangles in 3D, segments in 2D) and refines the reconstruction through strategic site insertion and incremental Delaunay updates until reaching either the point count limit or convergence threshold.

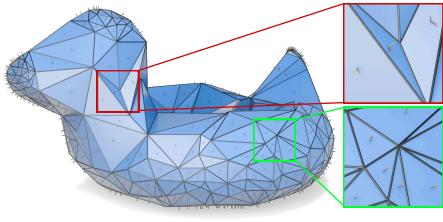
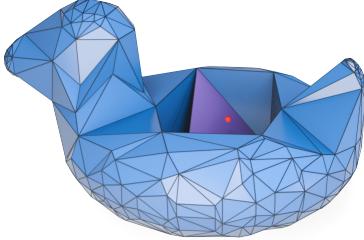


Figure 5. When a face is well-aligned with the zero-level set, the SDF gradient directions throughout its interior closely match the face normal (green box). Conversely, faces deviating significantly from the zero-level set exhibit interior points where gradient directions substantially differ from the face normal (red box).

As shown in the inset figure, the purple tetrahedron generates the problematic triangle, and the red point represents its dual vertex in the power diagram. We add both this point and its surface projection to the regular Delaunay construction to enhance the local reconstruction quality.

This approach is particularly effective because when significant inconsistency is detected between a triangle in \mathcal{M} and the zero-level set, the inserted sites must modify the local topology of precisely that region. The dual vertex serves as an ideal candidate for this purpose due to the one-to-one correspondence between triangles in \mathcal{M} and tetrahedra in \mathcal{D} .



GB RAM. The test dataset comprised 100 models curated from Thingi10k and models previously used in [26] and [27], supplemented with several custom-designed models. All models were normalized to fit within a unit cube of $[-1/2, 1/2]^3$, while the surface extraction was performed in the extended domain of $[-1, 1]^3$.

Comparison Methods This study presents a comparative analysis between our approach and five state-of-the-art methods: three recent techniques—RFTA (REACH for Arc) [26], RFTS (REACH for Sphere) [27], and MCGrids [24]—and two classical methods—Marching Cubes [16] and NDCx (Neural Dual Contouring) [3].

RFTA and RFTS utilize optimization-based approaches that exploit the tangential relationship between sample-centered spheres and surfaces for geometric reconstruction. Here, each sphere is centered at a sample point with its radius defined by the distance to the surface. For RFTS, we employed the default parameters provided in the authors’ GPyToolbox implementation [25], while RFTA was configured with 20 fine-tuning iterations and 30 maximum points per sphere. MCGrids employs an iterative isosurface extraction approach, whereas Marching Cubes and NDCx implement uniform spatial discretization strategies for surface extraction.

We categorize existing methods into two distinct classes: Direct Extraction Methods, which utilize fixed information for isosurface extraction without additional input during reconstruction (including Marching Cubes, NDCx, RFTS, and RFTA), and Iterative Refinement Methods, which progressively insert new points and adjust results during the extraction process (McGrids).

6.2. Comparison with Direct Extraction Methods

Figure 6 illustrates comparative analyses at uniform grid resolutions of 30^3 and 50^3 over the $[-1, 1]^3$ domain, where all algorithms receive identical grid point SDF values and gradient information for low-genus model reconstruction. Traditional methods show distinct limitations: Marching

6. Evaluation

6.1. Experiment Setting

Our experimental evaluation was conducted on a system equipped with an AMD Ryzen 9 5950X CPU and 72

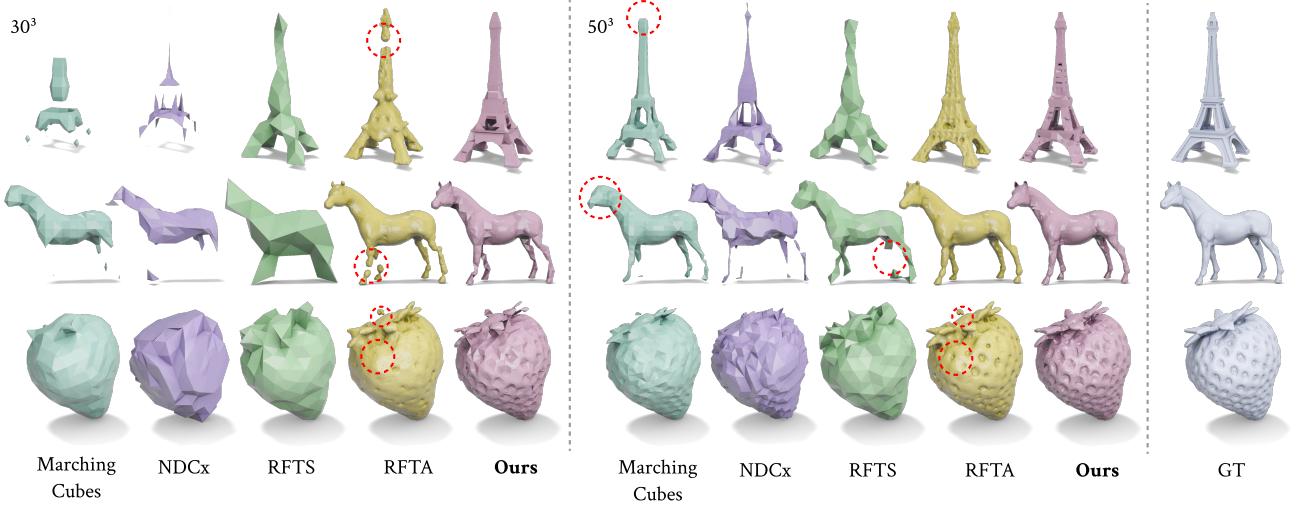


Figure 6. Comparison with state-of-the-art methods on three low-genus inputs. Results are generated using uniform 30^3 and 50^3 grid samples. Our method demonstrates superior expressiveness in capturing geometric details.

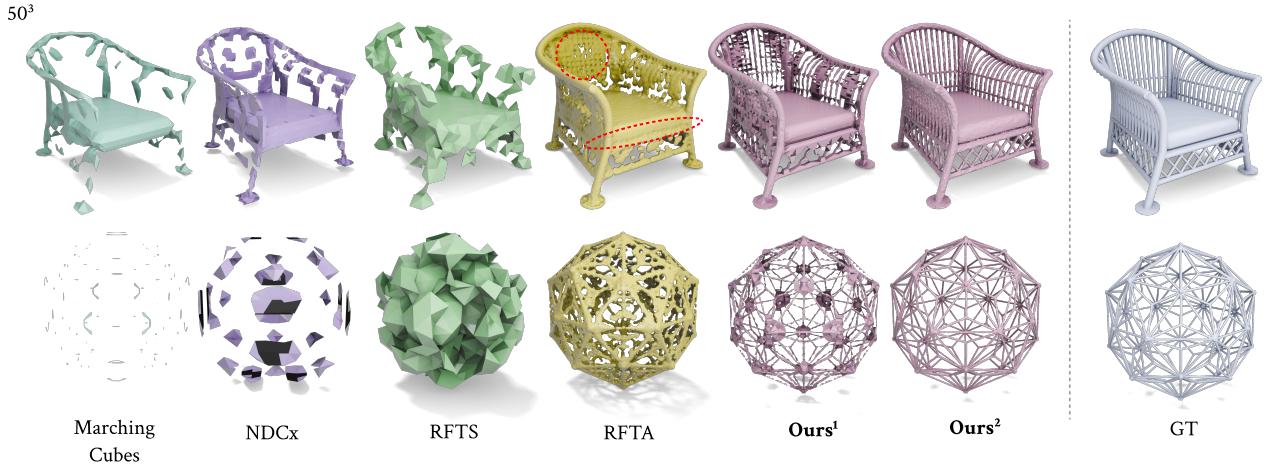


Figure 7. Comparison with state-of-the-art methods on three high-genus inputs. Previous methods and Ours¹ use uniform 50^3 grid samples, while Ours² starts from an 8^3 grid and applies our adaptive sampling strategy until reaching 62500 samples. Our method demonstrates better topological preservation, with further improvements achieved through our adaptive sampling strategy.

Cubes and Neural Dual Contouring utilize only SDF sign information, resulting in poor detail preservation; RFTS cannot handle topological variations, restricting its application to simple models; and RFTA, despite improving upon its predecessor, fails to recover intricate surface details. Our method leverages projection points derived from gradient information with regular Delaunay-based extraction to achieve superior reconstruction quality. This is demonstrated in Figure 6 with the strawberry model and in Figure 2 with the golf ball model, where our approach successfully reconstructs surface cavities that RFTA fails to capture. Additionally, at 50^3 resolution, RFTA’s tower reconstruction exhibits notable spherical artifacts.

For high-genus models (Figure 7), we present two implementations: Our¹, which uses standard 50^3 grid inputs, and Our², which employs an adaptive approach starting from an

8^3 grid and expanding to 50^3 total points (including both sample points and their projections) using the strategy detailed in Section 5. While conventional methods (Marching Cubes, NDCx, RFTS) fail to preserve intricate details such as chair backs, and RFTA shows only modest improvement, both of our implementations demonstrate better detail preservation. Our², through its adaptive point sampling strategy, achieves particularly significant improvements in detail preservation.

6.3. Comparison with Progressive Refinement Methods

McGrids introduces a Monte Carlo-based strategy that formulates adaptive grid construction as a probability sampling problem, employing Delaunay tetrahedralization followed by marching tetrahedra for isosurface extraction.



Figure 8. Comparison with McGrids [24]. Both methods employ an almost identical number of sampling points. Our method exhibits superior preservation of fine details and topological features.

The method implements a two-phase point addition strategy: initial Monte Carlo sampling followed by refinement. In comparative analysis (Figure 8) using approximately 20,000 sampling points for both methods, we achieve better results.

Our method demonstrates superior topology preservation for complex models, primarily due to our more comprehensive utilization of available information. While McGrids relies solely on SDF sign changes for extraction, our approach leverages both distance values and gradient information through regular Delaunay tetrahedralization, enabling more accurate surface inference and significantly enhancing extraction capabilities.

Furthermore, although MCGrids implements refinement capabilities in its second phase, it fails to detect extraction errors in extreme cases, such as models with radii smaller than its second-phase threshold. In contrast, our normal-based detection mechanism successfully identifies such cases.

Additionally, we evaluated the reconstruction accuracy by measuring the **L2 Chamfer** distance between the extracted isosurfaces and ground truth across our dataset of

100 models. We employed two strategies: Ours¹ used uniform 50^3 grid samples as input, while Ours² utilized our incremental construction approach until reaching 50,000 sampling points. Classical methods used uniform 50^3 grid information as input, while MCGrids progressively added points until reaching 50,000 samples. The resulting average L2 Chamfer distances were: **Marching Cubes:** 2.143×10^{-4} , **RFTA:** 0.421×10^{-4} , **Ours¹:** 0.209×10^{-4} , **McGrids:** 9.49×10^{-6} , and **Ours²:** 8.47×10^{-6} . These quantitative results further validate our method’s superior isosurface extraction capabilities, demonstrating improved accuracy in both uniform sampling and adaptive refinement scenarios.

6.4. Other Experiments

For all results presented in this paper, the running time consistently remains around **10 seconds**. Additional **timing data, ablation studies, and visualizations of intermediate stages in the isosurface extraction process** can be found in the **Supplementary Material**.

7. Conclusion, Limitations and Future Work

In this paper, we have presented a novel isosurface extraction method that advances capabilities in two key ways: first, by leveraging both SDF gradients and values with regular Delaunay tetrahedralization to utilize comprehensive information rather than merely sign changes; second, by introducing an iterative framework that couples extraction with adaptive refinement, guiding strategic point insertion based on quality assessment. This approach improves surface fidelity through localized updates without global recomputation. Experiments validate our method’s effectiveness across diverse geometric complexities, showing superior performance for models with intricate details, thin structures, and complex topologies.

Our method has two primary limitations. First, it requires SDF gradient information during execution, which may be unavailable in certain contexts. Second, while our approach achieves dynamic isosurface extraction by maximizing information utilization, our surface quality assessment process necessitates multiple SDF queries without specific optimizations to reduce query count. This may incur significant computational overhead in certain scenarios.

For future work, we aim to address these limitations in two directions. We plan to develop techniques for estimating tangent points based on power diagrams, eliminating the explicit requirement for SDF gradient information. Additionally, we intend to refine our isosurface evaluation criteria to reduce the number of SDF queries required, improving computational efficiency while maintaining extraction quality.

A. Iterative Evolution Analysis

Figure 13 demonstrates the iterative progression of our algorithm on two representative examples: a high-genus model and a model with intricate surface details. Starting from a uniform initialization with 15^3 sampling points, the algorithm iteratively incorporates new sites until reaching 50,000 sites. For each intermediate stage, the highlighted sites indicate the newly added positions during the transition to the subsequent stage. The visualization reveals two primary functions of our site addition strategy: (1) Topology correction - as evidenced in the first model, where new sites are strategically placed to either establish necessary connections or separate incorrectly merged regions; (2) Geometric refinement - as demonstrated in the second model, where the progressive addition of sites continuously enriches surface details. These results validate the effectiveness of our iterative strategy in progressively improving both topological accuracy and geometric fidelity.

B. Ablation Study

To evaluate the key components of our method, we conducted ablation studies on two critical aspects: 1) the effectiveness of incorporating projection points in the isosurface extraction process, and 2) the impact of our quality-driven iterative point addition strategy.

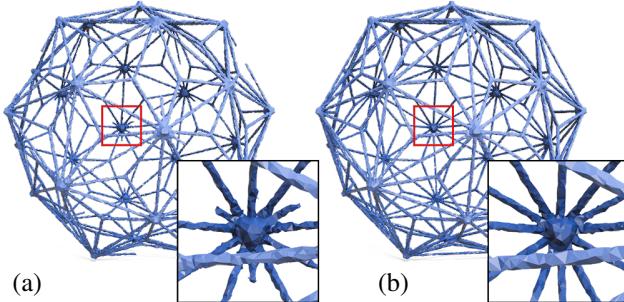


Figure 9. Comparison of results using (a) sampling points only versus (b) our complete method with both sampling points and their surface projections. Starting from a uniform 20^3 grid and refining to 40,000 points, the projection-enhanced approach demonstrates superior topological preservation.

B.1. Surface Projection Integration

The addition of projection points enhances the algorithm's ability to "sense" the surface more intuitively during isosurface extraction, which is crucial when handling complex topologies and rich geometric details. Figure 9 demonstrates the results using a uniform 15^3 grid as input, iteratively refined to 40,000 sampling points with: (a) our method using only sampling points without their projections, and (b) our complete method utilizing both sampling points and their surface projections.

The comparison clearly shows that the projection-enhanced approach preserves more complete topological structures, while the baseline method without projections exhibits numerous topological inaccuracies. This demonstrates how surface projections provide critical geometric guidance that significantly improves the algorithm's ability to reconstruct complex features, particularly in regions with intricate topology or thin structures that might otherwise be missed when relying solely on discrete sampling points.

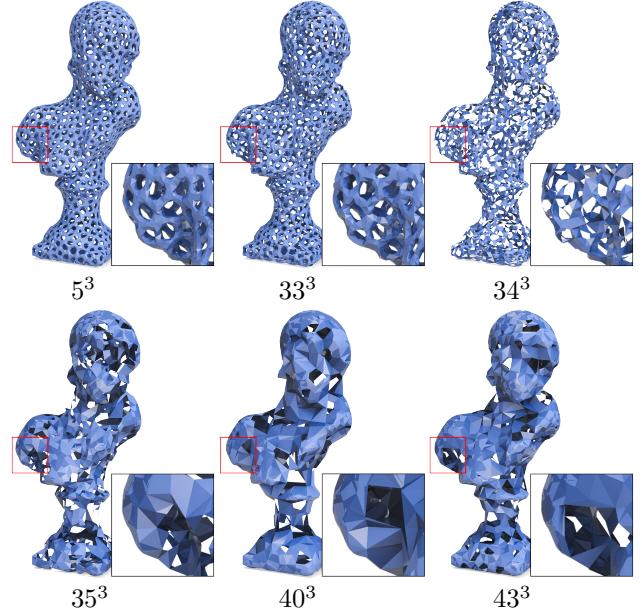


Figure 10. Extraction results using different initial uniform sampling densities while maintaining 40,000 total sampling points. When more points are determined by our quality-driven strategy rather than uniform initialization, topological preservation significantly improves, confirming the effectiveness of our adaptive refinement approach.

B.2. Quality-Driven Adaptive Refinement

To evaluate our iterative point addition strategy, we conducted experiments using different initial uniform grid resolutions while maintaining a fixed total of 40,000 sampling points. Figure 10 demonstrates that when a greater proportion of points are determined by our quality-driven strategy rather than uniform initialization, the reconstruction exhibits significantly better topological preservation. In contrast, models reconstructed primarily with uniformly distributed points show compromised topology, confirming that our adaptive approach more effectively allocates sampling points to topologically and geometrically critical regions.

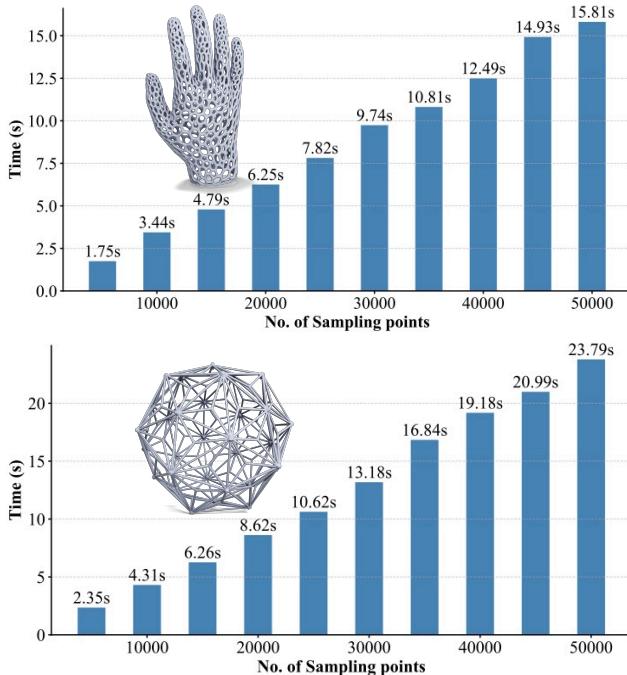


Figure 11. Performance scaling of our algorithm on two complex models with increasing sampling points. Benefiting from our localized update strategy, the computation time exhibits approximately linear growth with respect to the number of sampling points.

C. Computational Efficiency Analysis

Figure 11 illustrates the algorithm’s running time with varying numbers of iteratively added points using a complex model as an example. Due to our algorithm’s localized update approach, the time required to insert a new point remains relatively constant regardless of the total number of points already present in the regular Delaunay tetrahedralization. This locality property ensures that our method’s computational efficiency scales approximately linearly with increasing point count, rather than exhibiting higher complexity. The results validate the effectiveness of our localized update strategy, confirming the computational efficiency of our incremental framework.

D. Post-Processing Enhancements

In previous experiments, we validated our algorithm’s effectiveness in reconstructing complex topological structures. We further discovered that the surface quality can be significantly improved through a simple post-processing step. When we apply Laplacian smoothing to our reconstruction results (where topology is already largely correct) and then reproject all surface vertices back onto the zero-level set using gradient information, the overall quality of the reconstructed surface is further enhanced.

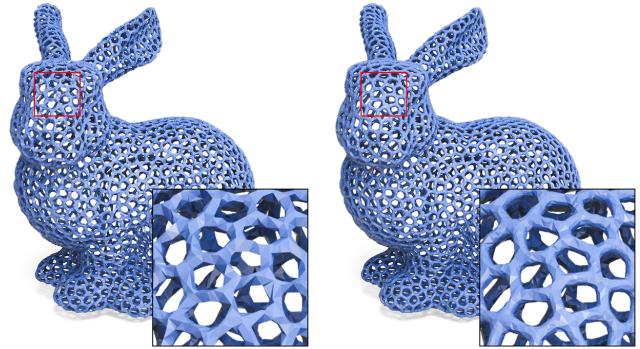


Figure 12. Comparison of isosurface extraction results before (left) and after (right) post-processing. The application of Laplacian smoothing followed by gradient-based reprojection significantly enhances surface smoothness and overall quality while preserving the captured topology.

As shown in Figure 12, the surface after smoothing and reprojection appears noticeably smoother than the original result while maintaining topological accuracy. It is important to note that for fair comparison with other methods, we did not apply this post-processing step to any results presented in the main body of the paper.

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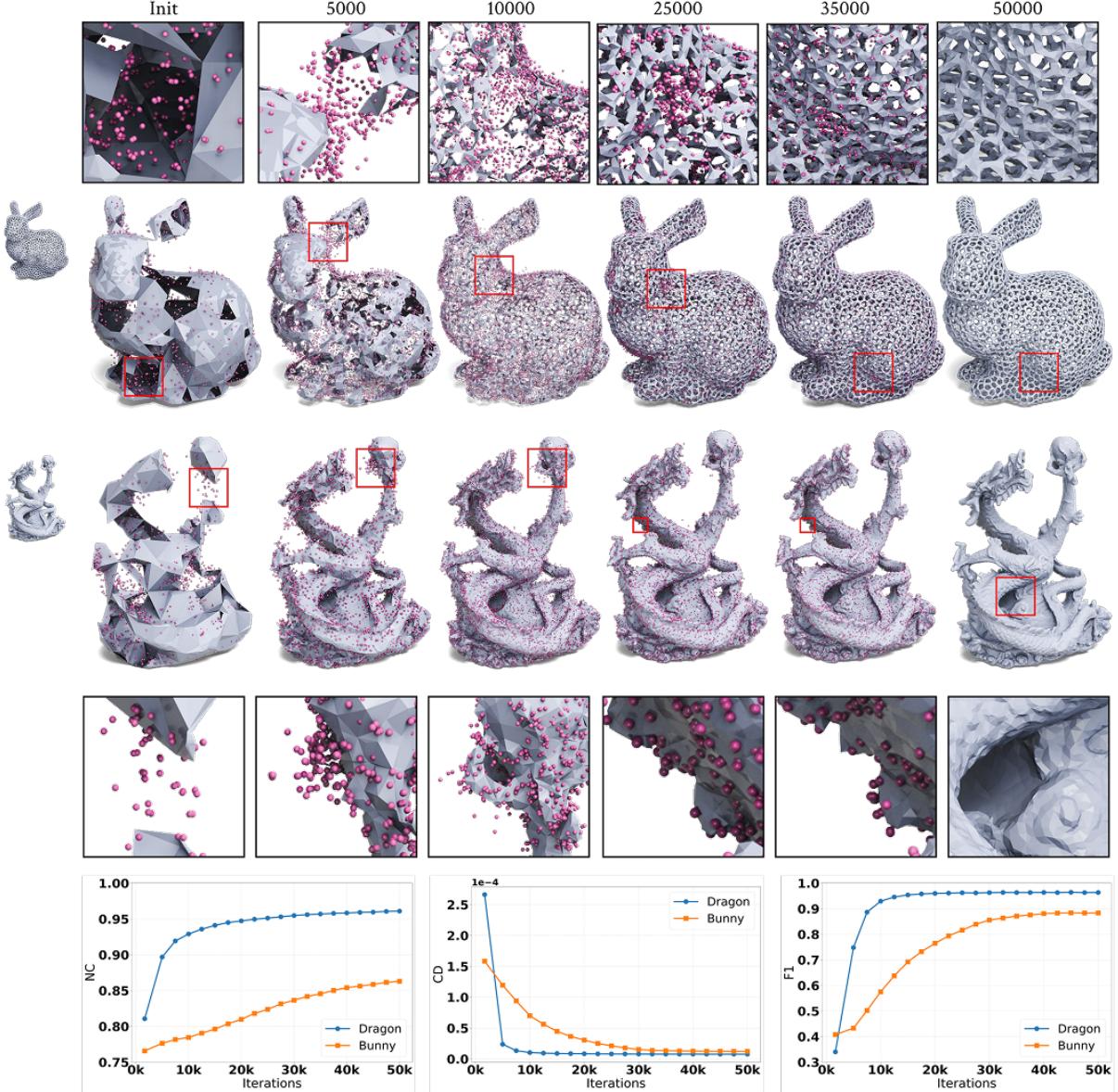


Figure 13. Intermediate stages of our algorithm demonstrated on two classic models with high genus and rich surface details. The process illustrates two key aspects of our site placement strategy: (1) Topology correction - shown in the first model, where new sites are strategically added to either establish necessary connections or separate incorrectly merged regions; (2) Geometric refinement - demonstrated in the second model, where progressive site addition continuously enriches surface details.

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