CH03_12

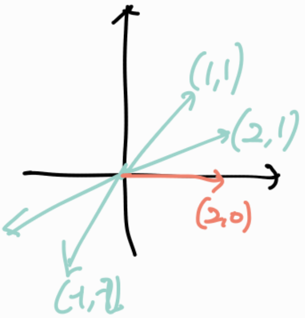
## <EigenValues and EigenVectors>

$A\vec{x} = \vec{b}$

$\underbrace{A\vec{x}}_{\vec{b}} = \underbrace{\lambda\vec{x}}_{\vec{x}\text{의 상수배}}$ ; 방향은 바뀌지 않고, 크기만 바뀌는 벡터

$\lambda$ : EigenValue , $\lambda\vec{x}$ : EigenVector

ex)




$$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \end{pmatrix}$$

Projection Vector의 경우, 정사영이기 때문에 방향이 바뀜.

$A\vec{x} = \lambda\vec{x}$

$A\vec{x} - \lambda\vec{x} = \vec{0}$

$A\vec{x} - \lambda \cdot I\vec{x} = \vec{0}$

$(A - \lambda I)\vec{x} = 0$

$\det(A-\lambda I)=0$ 이 되어야 함.

$$A\begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow A - \lambda I = \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}$$

$\det(A) = ad - bc$

★ $\det(A-\lambda I) = (a-\lambda)(d-\lambda) - bc = 0$

$\lambda^2 = (a+d)\lambda + ad - bc = 0$

$\lambda_1$ , $\lambda_2$ 값을 원래 식에 다시 대입.

$$P_v = \frac{\vec{v} \cdot \vec{v}^T}{\|\vec{v}\|^2}$$

ex) $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longrightarrow \|\vec{v}\| = \sqrt{5}$

$$P_v = \frac{\vec{v} \cdot \vec{v}^T}{\|\vec{v}\|^2} = \frac{1}{5} \times \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

$$P_v - \lambda I = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \frac{1}{5}\left[\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} - \begin{pmatrix} 5\lambda & 0 \\ 0 & 5\lambda \end{pmatrix}\right]$$

$$\det(P_v - \lambda I) = (1-5\lambda)(4-5\lambda) - 4$$

$$= 25\lambda^2 - 25\lambda = 0$$

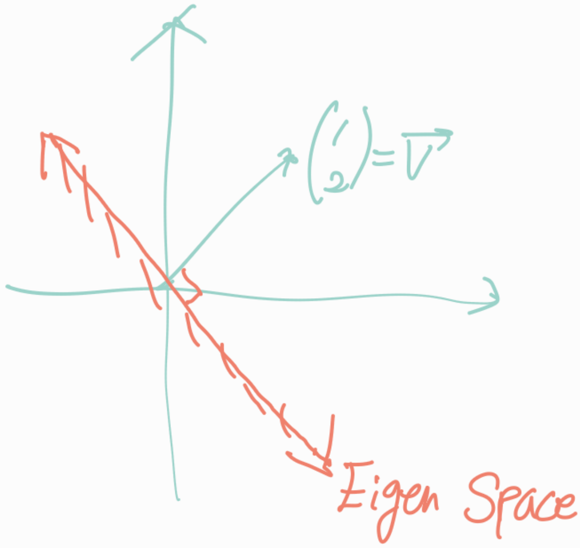
$$= \lambda(1-\lambda) = 0 \qquad \therefore \lambda = 0 \text{ or } 1$$

$\lambda = 0, 1$ 대입하면,

① $\lambda = 0$,

$$\cancel{\frac{1}{5}}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x+2y \\ 2x+4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightsquigarrow \begin{matrix} x+2y=0 \\ y = -\frac{1}{2}x \end{matrix}$$




② $\lambda = 1$,

$$\frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \times 5 \\ y \times 5 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4x-2y \\ -2x+y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad 2x = y.$$

변환을 했을 때 방향이 바뀌지 않기 때문에 중심을 잡아주는 벡터들이다. Eigen Vectors