

Supplemental Materials: Position-free Multiple-bounce Computations for Smith Microfacet BSDFs

ANONYMOUS AUTHOR(S)
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1 FULL SPHERICAL SHADOWING-MASKING FUNCTION

In this section, we provide proofs and analysis for our height-uncorrelated and height-correlated shadowing-masking functions.

1.1 Height-uncorrelated shadowing-masking function

As illustrated in Fig. 1, in the upper hemisphere, the projected area of microfacets towards direction ω can be written in the integral form as

$$\text{projected area} = \omega \cdot \omega_g = \int_{\Omega^+} G_1(\omega, \omega_m) \langle \omega, \omega_m \rangle D(\omega_m) d\omega_m, \quad (1)$$

where G_1 determines whether a microfacet will contribute to the projected area. And two possible non-contributing cases are (1) locally, a microfacet has a backward orientation w.r.t. the given direction ω , and (2) distantly, a microfacet will be occluded by other microfacets regardless of its own orientation ω_m .

Using the local / distant separated form of G_1 and taking advantage of the distant term G_1^{dist} 's independence of any microfacet's normal, Eqn. 1 can be solved [Heitz et al. 2016] in the following form:

$$G_1^{\text{dist}}(\omega) = \frac{1}{1 + \Lambda(\omega)}, \quad (2)$$

where Λ is computed from the normal distribution function by integrating, as described by Smith [1967].

However, when the direction of interest ω is from below the surface, Eqn. 1 does not hold anymore, since $\omega \cdot \omega_g$ will be negative. Our first goal is to extend Eqn. 1 to handle this case. This is done by replacing the projected area with $|\omega \cdot \omega_g|$, so the projected area is always positive and is physically correct no matter which direction of interest ω is provided. This leads to our spherical distant shadowing / masking term:

$$G_1^{\text{dist}}(\omega) = \left| \frac{1}{1 + \Lambda(\omega)} \right| = \begin{cases} 1/(1 + \Lambda(\omega)), & \text{if } \omega \cdot \omega_g > 0, \\ -1/(1 + \Lambda(\omega)), & \text{if } \omega \cdot \omega_g \leq 0, \end{cases} \quad (3)$$

In Fig. 2, we validate the correctness of Eqn. 3, by comparing with the numerical solution of the integral form of $G_1^{\text{dist}}(\omega)$ derived from our extended spherical form of Eqn. 1:

$$G_1^{\text{dist}}(\omega) = \frac{|\omega \cdot \omega_g|}{\int_{\Omega^\pm} G_1^{\text{local}}(\omega, \omega_m) \langle \omega, \omega_m \rangle D(\omega_m) d\omega_m}. \quad (4)$$

As expected, the two solutions match very well. The curves for other configurations (other NDFs, like Beckmann or other roughness) also match well.

One important and interesting observation, is that when ω is from below the macrosurface, the value of G_1^{dist} could be greater than 1. This observation can be strictly validated by re-looking at Fig. 1. As illustrated, the entire shadowing / masking term G_1 gives the

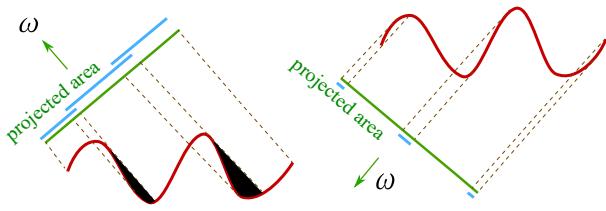


Fig. 1. For both ω from above the macrosurface (left) or from below the macrosurface (right), the blue area represents the projected area of microfacets which are not back-facing, without considering shadowing, and the green area represents the projected area considering shadowing, thus G_1^{dist} is the ratio between the areas of blue and green.

projected area, marked as green, while if we ignore the shadowing between microfacets, i.e., project a microfacet as long as it's not back-facing, we will end up with the blue area, predicted only by G_{local} . By its mathematical definition, G_1^{dist} is the ratio between the areas of blue and green.

When projecting upwards, since there will never be holes on a surface, the blue area is always guaranteed to be not smaller than the green area. Therefore, the distant shadowing / masking term only performs pruning in this case. However, when projected downwards, very few microfacets are front-facing w.r.t. ω , and therefore cannot pass the local test. This results in much smaller blue area. In this case, the G_1^{dist} term should be greater than 1.

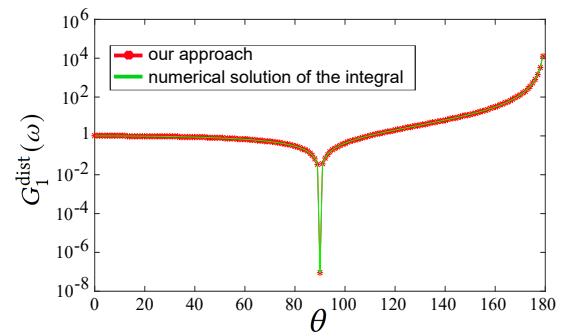


Fig. 2. We validate our spherical distant shadowing / masking term $G_1^{\text{dist}}(\omega)$ computed with our Eqn. 3 against the numerical solution of Eqn. 6. $G_1^{\text{dist}}(\omega)$ is visualized logarithmically as a function of the angle θ between ω and ω_g . NDF: GGX model with $\alpha = 1.0$.

115 1.2 Height-correlated shadowing-masking function

116 As shown in Ross et al. [2005], the height-correlated shadowing-
 117 masking function for reflection is:

$$118 \quad G_2^{\text{dist}}(\omega_i, \omega_o) = \frac{1}{1 + \Lambda(\omega_i) + \Lambda(\omega_o)}, \quad (5)$$

120 where both ω_i and ω_o are above the macrosurface.

122 To extend Eqn. 5 to the full-spherical domain, we first derive the
 123 integral form of $G_2^{\text{dist}}(\omega_i, \omega_o)$ by :

$$\begin{aligned} 124 \quad & 1 + \Lambda(\omega_i) + \Lambda(\omega_o) \\ 125 \quad &= \frac{\int_{\Omega^+} \langle \omega_i, \omega_m \rangle D(\omega_m) d\omega_m}{\omega_i \cdot \omega_g} + \frac{\int_{\Omega^+} \langle -\omega_o, \omega_m \rangle D(\omega_m) d\omega_m}{\omega_o \cdot \omega_g} \\ 126 \quad &= \frac{\int_{\Omega^+} D(\omega_m) [\langle \omega_i, \omega_m \rangle (\omega_o \cdot \omega_g) + \langle -\omega_o, \omega_m \rangle (\omega_i \cdot \omega_g)] d\omega_m}{(\omega_i \cdot \omega_g)(\omega_o \cdot \omega_g)}, \end{aligned} \quad (6)$$

$$\begin{aligned} 131 \quad & \frac{1}{1 + \Lambda(\omega_i) + \Lambda(\omega_o)} \\ 132 \quad &= \frac{(\omega_i \cdot \omega_g)(\omega_o \cdot \omega_g)}{\int_{\Omega^+} D(\omega_m) [\langle \omega_i, \omega_m \rangle (\omega_o \cdot \omega_g) + \langle -\omega_o, \omega_m \rangle (\omega_i \cdot \omega_g)] d\omega_m} \end{aligned} \quad (7)$$

137 where ω_i and ω_o are still above the macrosurface.

139 Since only one of the directions could be below the macrosurface,
 140 we assume it's the incoming direction for simplicity. Starting from
 141 Eqn. 7, we have:

$$\begin{aligned} 142 \quad & \int_{\Omega^+} D(\omega_m) [\langle \omega_i, \omega_m \rangle (\omega_o \cdot \omega_g) + \langle -\omega_o, \omega_m \rangle (\omega_i \cdot \omega_g)] d\omega_m \\ 143 \quad &= (1 + \Lambda(\omega_i) + \Lambda(\omega_o))(\omega_i \cdot \omega_g)(\omega_o \cdot \omega_g). \end{aligned} \quad (8)$$

145 If the incoming direction is below the macrosurface, and we denote
 146 it as $-\omega_i$, then we have:

$$\begin{aligned} 148 \quad & \int_{\Omega^+} D(\omega_m) [\langle -\omega_i, \omega_m \rangle (\omega_o \cdot \omega_g) + \langle -\omega_o, \omega_m \rangle (\omega_i \cdot \omega_g)] d\omega_m \\ 149 \quad &= (\Lambda(\omega_i)(\omega_i \cdot \omega_g)(\omega_o \cdot \omega_g) + \Lambda(\omega_o)(\omega_i \cdot \omega_g)(\omega_o \cdot \omega_g), \\ 150 \quad &= (\Lambda(\omega_i) + \Lambda(\omega_o))(\omega_i \cdot \omega_g)(\omega_o \cdot \omega_g), \end{aligned} \quad (9)$$

152 thus, we get:

$$154 \quad G_2^{\text{dist}}(-\omega_i, \omega_o) = \frac{1}{\Lambda(\omega_i) + \Lambda(\omega_o)}, \text{ if } \omega_i \cdot \omega_g > 0 \quad (10)$$

156 If we use ω_i to denote the direction below the macrosurface, we
 157 have the equivalence formulation:

$$159 \quad G_2^{\text{dist}}(\omega_i, \omega_o) = \frac{1}{\Lambda(-\omega_i) + \Lambda(\omega_o)}, \text{ if } \omega_i \cdot \omega_g < 0. \quad (11)$$

162 Now we have the complete definition of the height-correlated
 163 shadowing-masking function:

$$164 \quad G_2^{\text{dist}}(\omega_i, \omega_o) = \begin{cases} \frac{1}{-\Lambda(-\omega_i) + \Lambda(\omega_o)}, & \text{if } \omega_i \cdot \omega_g > 0, \\ \frac{1}{\Lambda(-\omega_i) + \Lambda(\omega_o)}, & \text{if } \omega_i \cdot \omega_g \leq 0. \end{cases} \quad (12)$$

167 The height-correlated shadowing-masking function for the last
 168 bounce is the same as the height-correlated shadowing-masking
 169 function:

$$170 \quad G_2^{i=k}(\omega_i, \omega_o) = G_2^{\text{dist}}(\omega_i, \omega_o). \quad (13)$$

172 1.3 Height-correlated shadowing-masking function for the

173 middle bounce

174 For the middle bounce, the shadowing-masking function has differ-
 175 ent meaning from the last bounce. For bounce i ($i < k$), shadowing-
 176 masking function $G_2^{i < k}(\omega_i, \omega_o)$ means the probability that the light
 177 ray with incident direction ω_i arrives at the surface and the outgoing
 178 ray with direction ω_o is blocked.

179 If ω_o is below the macrosurface, the ray will always intersect
 180 with the microgeometry, resulting in

$$184 \quad G_2^{i < k}(\omega_i, \omega_o) = \frac{1}{1 + \Lambda(\omega_i)}, \text{ if } \omega_o \cdot \omega_g < 0. \quad (14)$$

187 If ω_o is above the macrosurface, we derive a novel formula-
 188 tion. Starting from the original definition of the height-correlated
 189 shadowing-masking function [Heitz et al. 2016]:

$$191 \quad G_2^{\text{dist}}(\omega_i, \omega_o) = \int_{-\infty}^{+\infty} G_1^{\text{dist}}(\omega_i, h) G_1^{\text{dist}}(\omega_o, h) P^1(h) dh, \quad (15)$$

193 where $P^1(h)$ is the height distribution function.

194 We propose a novel formulation for the middle bounce from
 195 Eqn. 15 :

$$\begin{aligned} 197 \quad & G_2^{i < k}(\omega_i, \omega_o) \\ 198 \quad &= \int_{-\infty}^{+\infty} G_1^{\text{dist}}(\omega_i, h) (1 - G_1^{\text{dist}}(\omega_o, h)) P^1(h) dh, \\ 199 \quad &= \int_{-\infty}^{+\infty} G_1^{\text{dist}}(\omega_i, h) P^1(h) dh - \int_{-\infty}^{+\infty} G_1^{\text{dist}}(\omega_i, h) G_1^{\text{dist}}(\omega_o, h) P^1(h) dh, \\ 200 \quad &= G_1^{\text{dist}}(\omega_i) - G_2^{\text{dist}}(\omega_i, \omega_o), \\ 201 \quad &= \frac{1}{|\Lambda(\omega_i) + 1|} - \frac{1}{|\Lambda(-\omega_i)| + \Lambda(\omega_o)}. \end{aligned} \quad (16)$$

204 Thus, the complete formulation of the middle bounce is:

$$\begin{aligned} 205 \quad & G_2^{i < k}(\omega_i, \omega_o) = \begin{cases} \frac{1}{|\Lambda(\omega_i) + 1|} - \frac{1}{|\Lambda(-\omega_i)| + \Lambda(\omega_o)}, & \text{if } \omega_o \cdot \omega_g > 0, \\ \frac{1}{|\Lambda(\omega_i) + 1|}, & \text{if } \omega_o \cdot \omega_g \leq 0. \end{cases} \end{aligned} \quad (17)$$

2 RECIPROCITY OF OUR MODELS

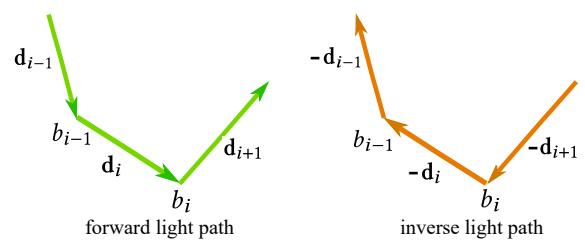


Fig. 3. A subpath (d_{i-1} to d_i) from the forward light path (green) and its inverse (orange).

In this section, we analyze the reciprocity of our models (height-uncorrelated and height-correlated). We first prove that height-uncorrelated model has reciprocity, and then provide a counter example to show that our height-correlated model doesn't have reciprocity.

2.1 Reciprocity of our height-uncorrelated model

As shown in Section 3.4, it's obvious that the vertex terms have reciprocity. Here, we prove the reciprocity of the segment term. Since we are using a separable model for the shadowing-masking function, the product of the shadowing terms of incident direction ω_i and exit direction ω_o have reciprocity, given any light paths. Thus, without loss of generality, we only proof the reciprocity of the internal direction.

Given a direction \mathbf{d}_i in the forward path, as shown in Fig. 3 (left), its segment term is computed by:

$$s_i^{\text{forward}} = e_i p_i, \quad (18)$$

where $e_i = 1$, since the direction is below the macrosurface, and p_i is defined as:

$$p_i = G_1(-\mathbf{d}_i, \frac{-\mathbf{d}_i + \mathbf{d}_{i+1}}{\|\mathbf{d}_i + \mathbf{d}_{i+1}\|}) = G_1^{\text{dist}}(-\mathbf{d}_i). \quad (19)$$

Thus, we have

$$s_i^{\text{forward}} = G_1^{\text{dist}}(-\mathbf{d}_i) = \frac{1}{1 + \Lambda(-\mathbf{d}_i)}. \quad (20)$$

Given a direction in the inverse path, as shown in Fig. 3 (right), its segment term is computed by:

$$s_i^{\text{inverse}} = e_i p_i, \quad (21)$$

$$\begin{aligned} e_i &= 1 - G_1(-\mathbf{d}_i, \frac{-\mathbf{d}_i + \mathbf{d}_{i+1}}{\|\mathbf{d}_i + \mathbf{d}_{i+1}\|}) = 1 - G_1^{\text{dist}}(-\mathbf{d}_i) \\ &= 1 - \frac{1}{1 + \Lambda(-\mathbf{d}_i)} = \frac{\Lambda(-\mathbf{d}_i)}{1 + \Lambda(-\mathbf{d}_i)} \\ &= \frac{\Lambda(-\mathbf{d}_i)}{1 + \Lambda(-\mathbf{d}_i)}, \end{aligned} \quad (22)$$

$$\begin{aligned} p_i &= G_1(\mathbf{d}_i, \frac{-\mathbf{d}_{i-1} + \mathbf{d}_i}{\|\mathbf{d}_{i-1} + \mathbf{d}_i\|}) \\ &= G_1^{\text{dist}}(\mathbf{d}_i) \\ &= \frac{-1}{1 + \Lambda(\mathbf{d}_i)} \\ &= \frac{1}{\Lambda(-\mathbf{d}_i)}. \end{aligned} \quad (23)$$

Thus, we have

$$\begin{aligned} s_i^{\text{inverse}} &= \frac{\Lambda(-\mathbf{d}_i)}{1 + \Lambda(-\mathbf{d}_i)} \frac{1}{\Lambda(-\mathbf{d}_i)} \\ &= \frac{1}{1 + \Lambda(-\mathbf{d}_i)} \\ &= s_i^{\text{forward}} \end{aligned} \quad (24)$$

Finally, we prove that the segment term has the reciprocity. Although the direction of \mathbf{d}_i is pointing downwards in our proof, the proof still holds, when it points upwards.

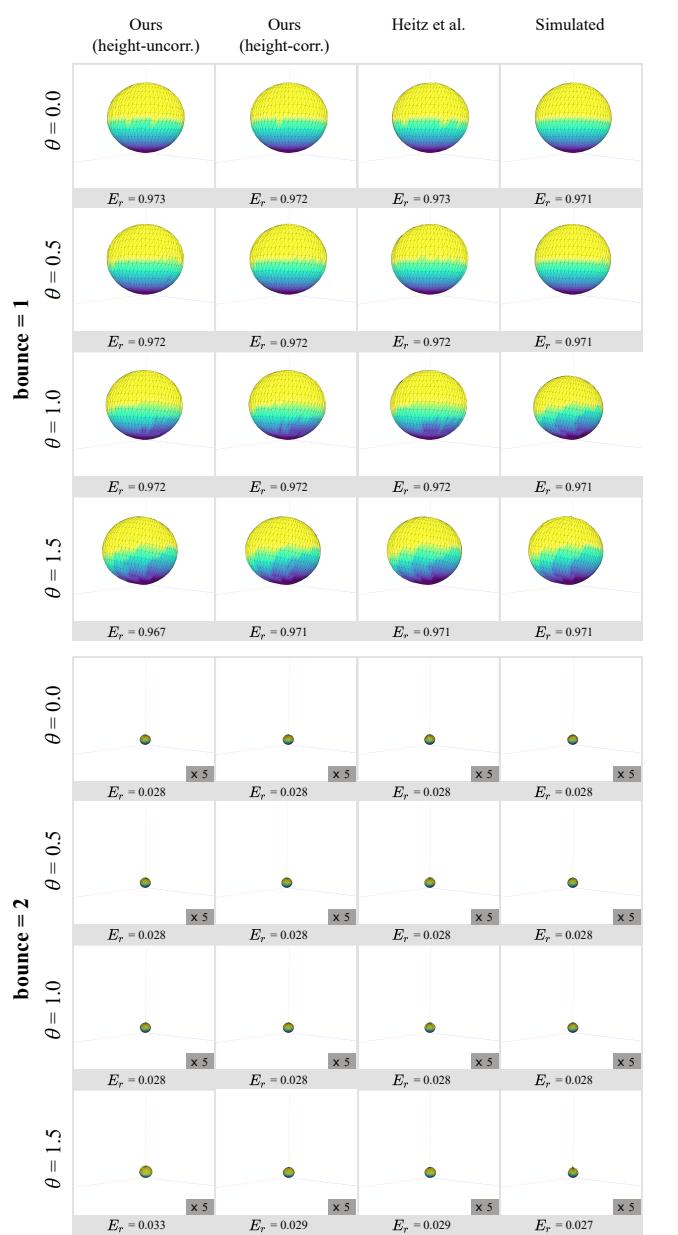


Fig. 4. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough diffuse material with roughness 0.25. θ is the angle between the incident direction and the normal to the macrosurface.

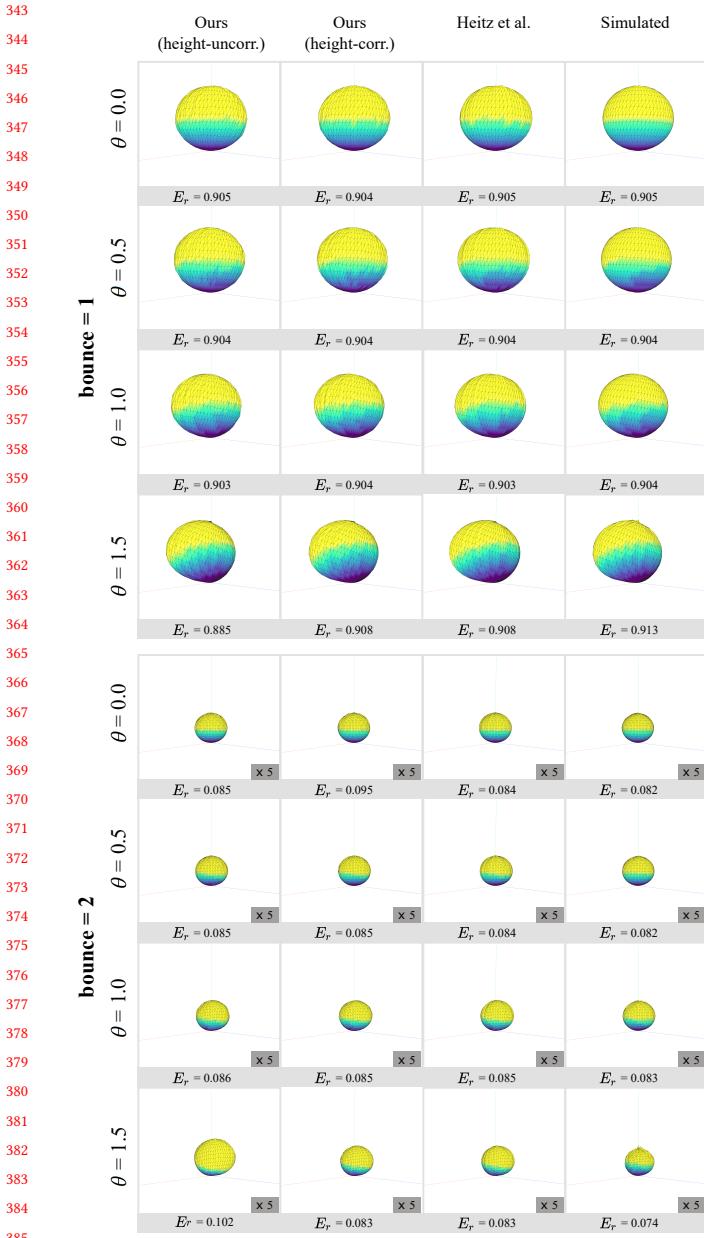


Fig. 5. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough diffuse material with roughness 0.5. θ is the angle between the incident direction and the normal to the macrosurface.

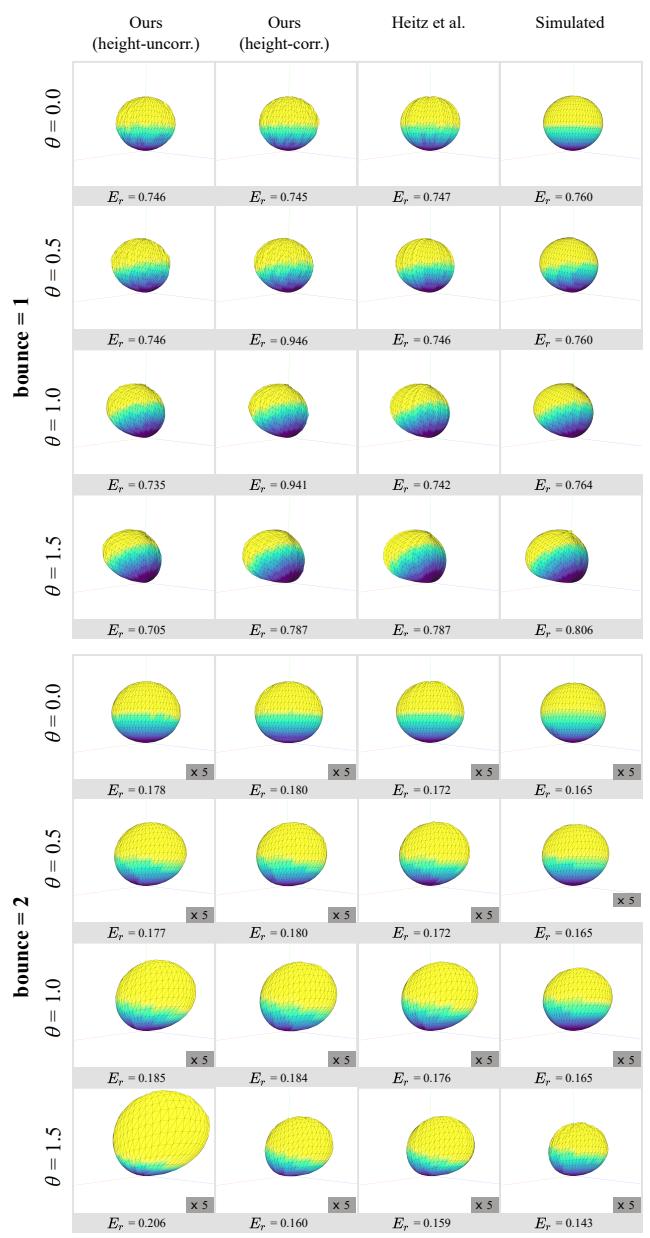


Fig. 6. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough diffuse material with roughness 1.0. θ is the angle between the incident direction and the normal to the macrosurface.

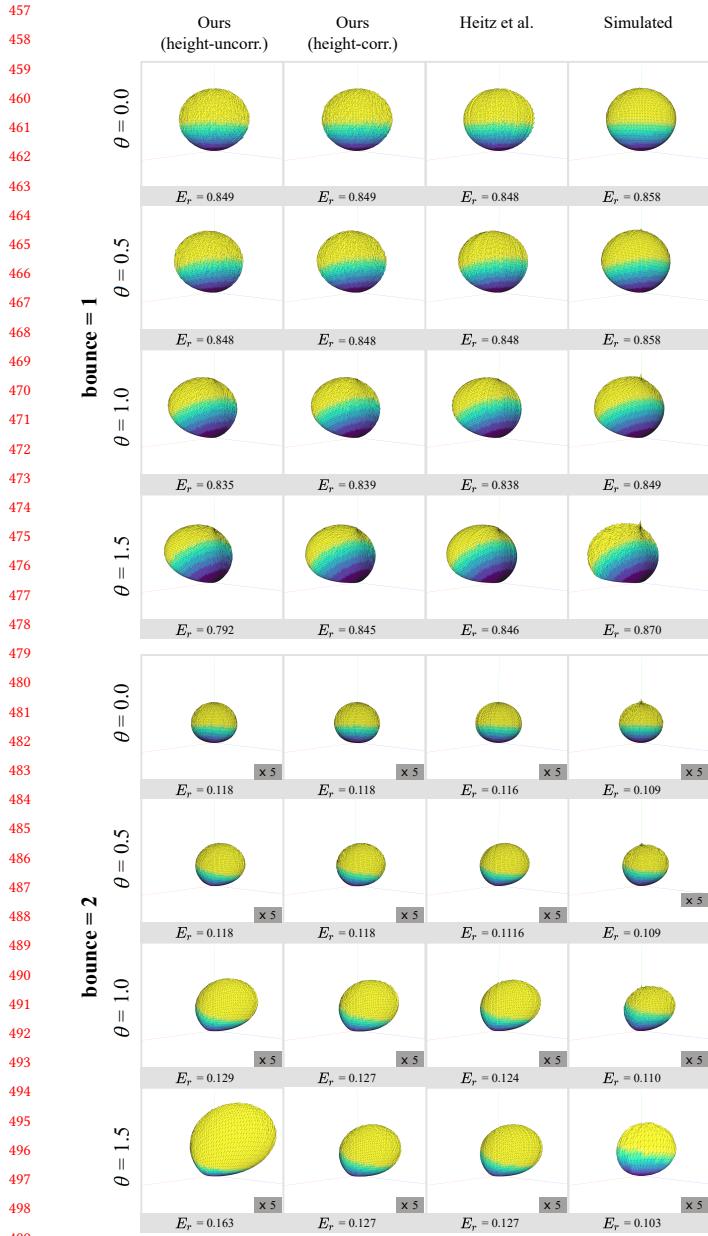


Fig. 7. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for anisotropic rough diffuse material with roughness 1.0 and 0.1. θ is the angle between the incident direction and the normal to the macrosurface.

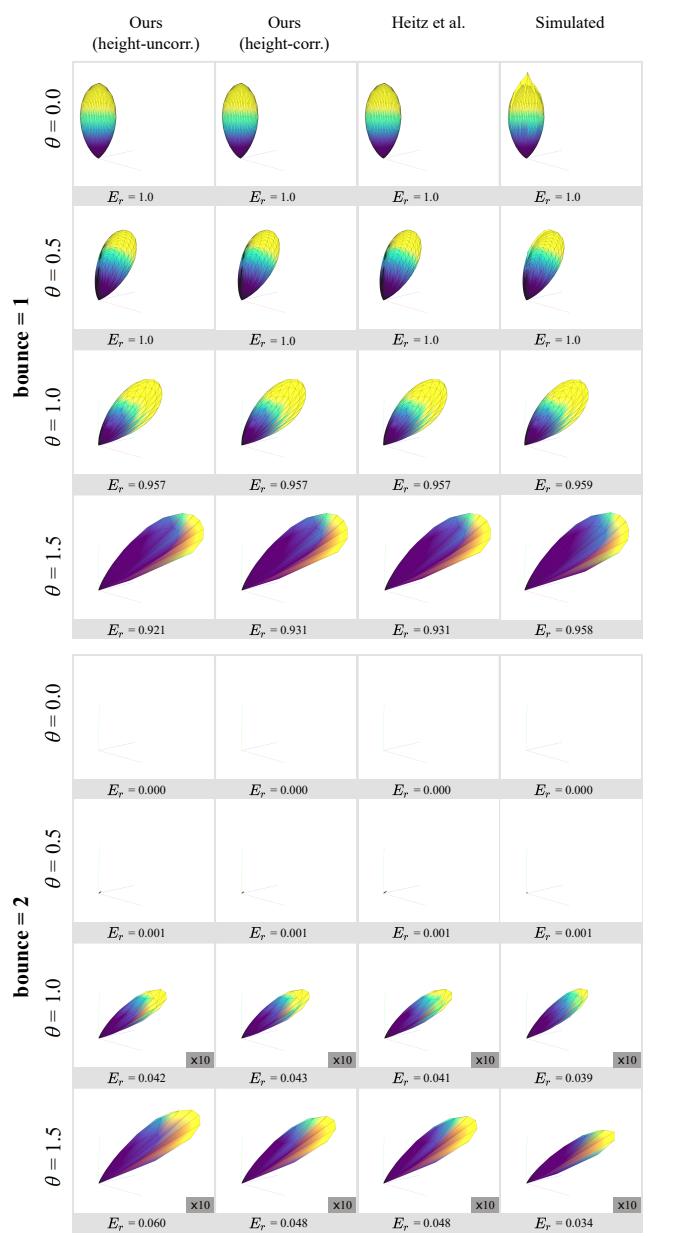


Fig. 8. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough conductor material with roughness 0.25. θ is the angle between the incident direction and the normal to the macrosurface.

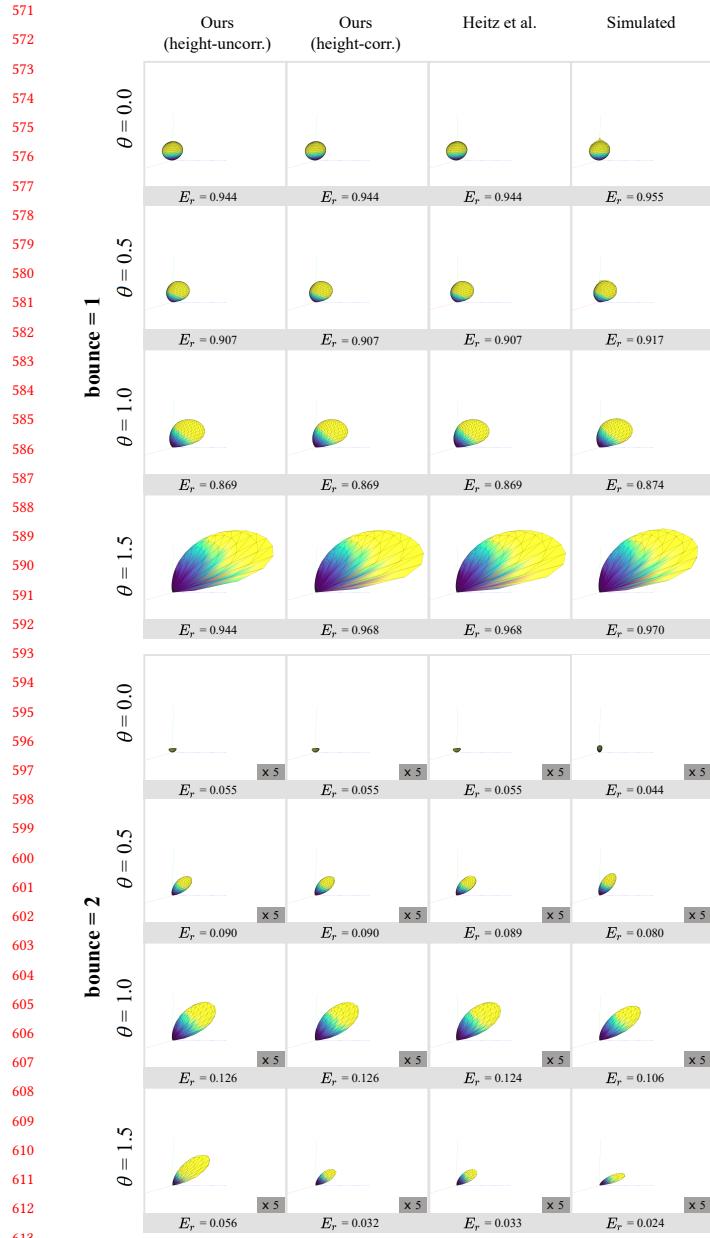


Fig. 9. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough conductor material with roughness 0.5. θ is the angle between the incident direction and the normal to the macrosurface.

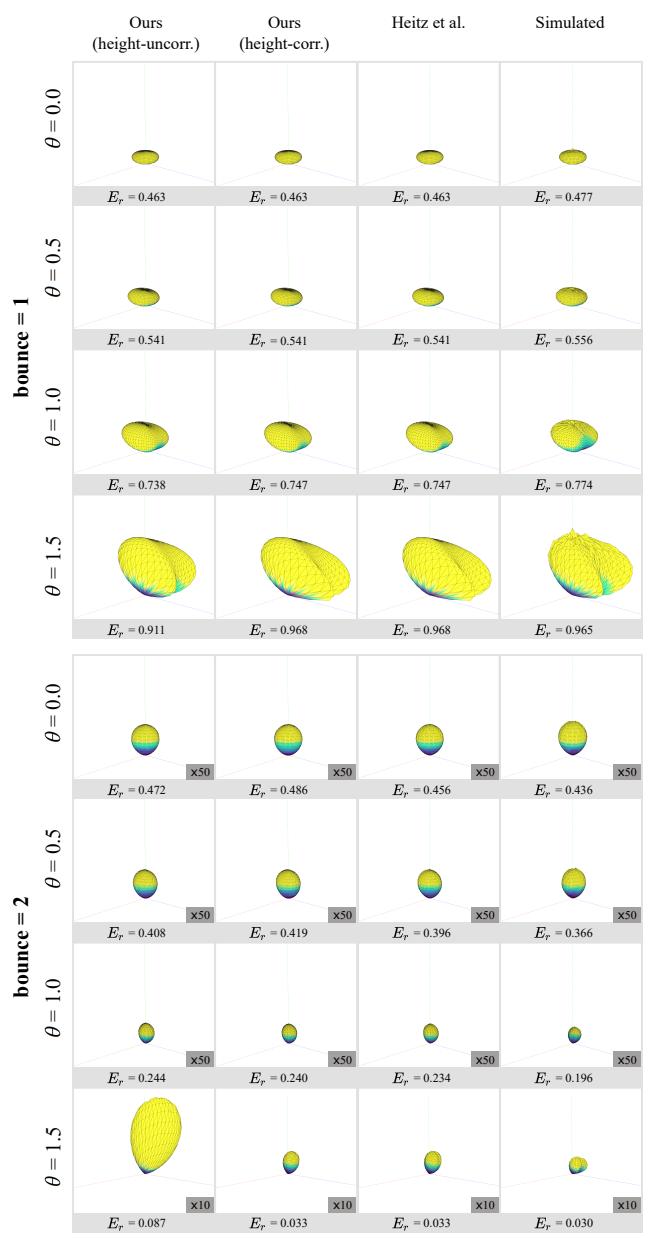


Fig. 10. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough conductor material with roughness 1.0. θ is the angle between the incident direction and the normal to the macrosurface.

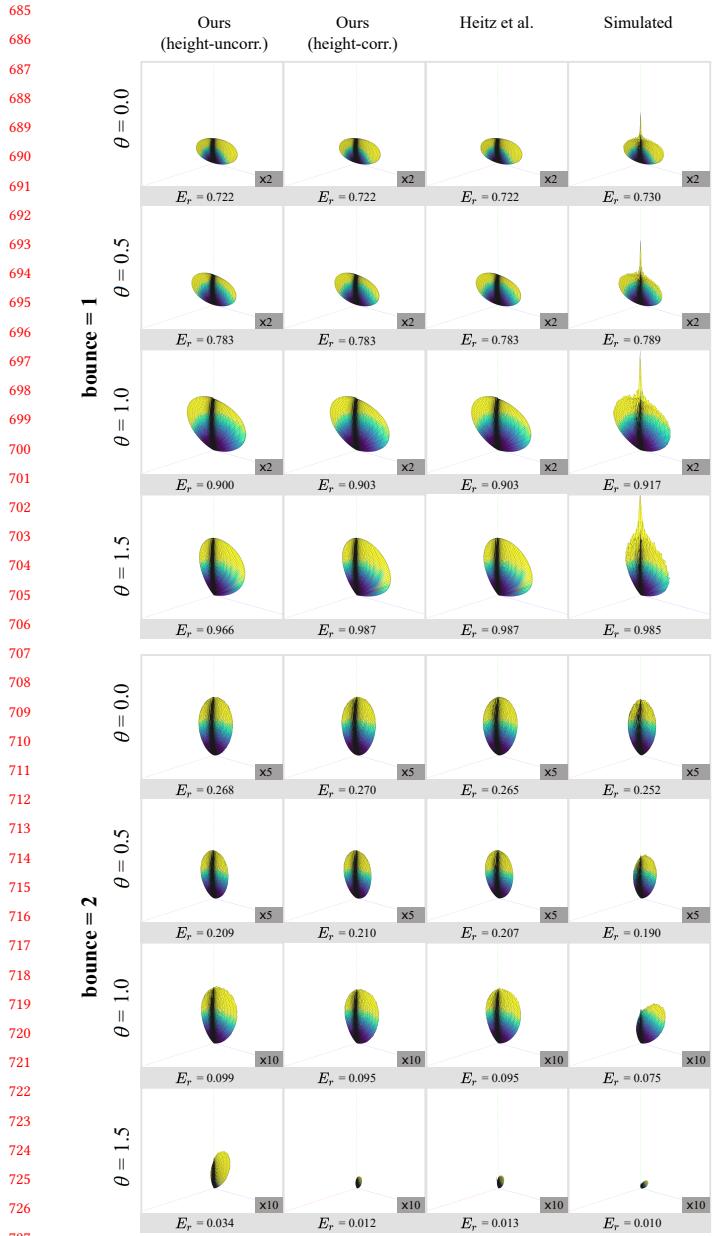


Fig. 11. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for anisotropic rough conductor material with roughness 1.0 and 0.1. θ is the angle between the incident direction and the normal to the macrosurface.

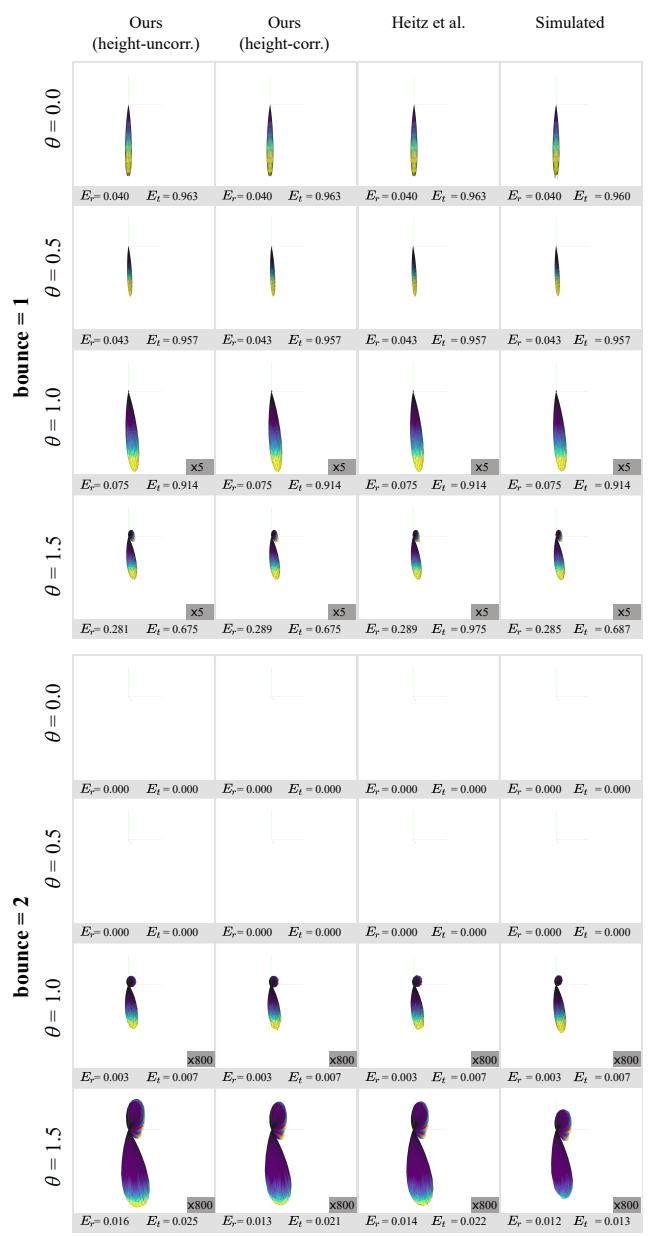


Fig. 12. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough dielectric material with roughness 0.25. θ is the angle between the incident direction and the normal to the macrosurface.

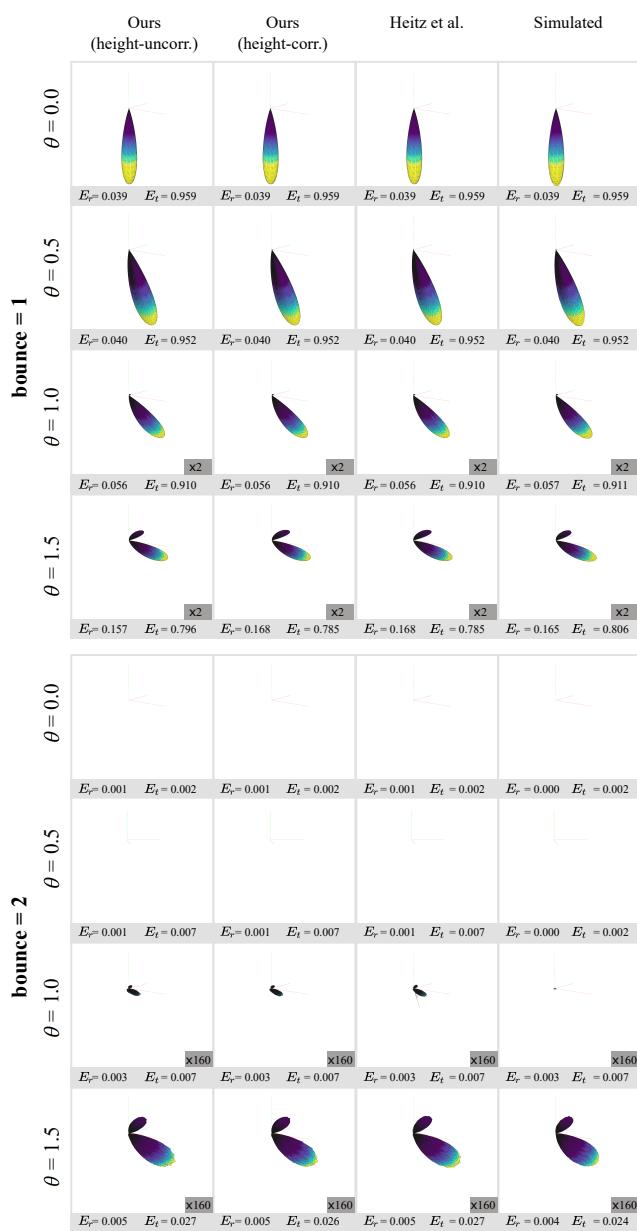


Fig. 13. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough dielectric material with roughness 0.5. θ is the angle between the incident direction and the normal to the macrosurface.

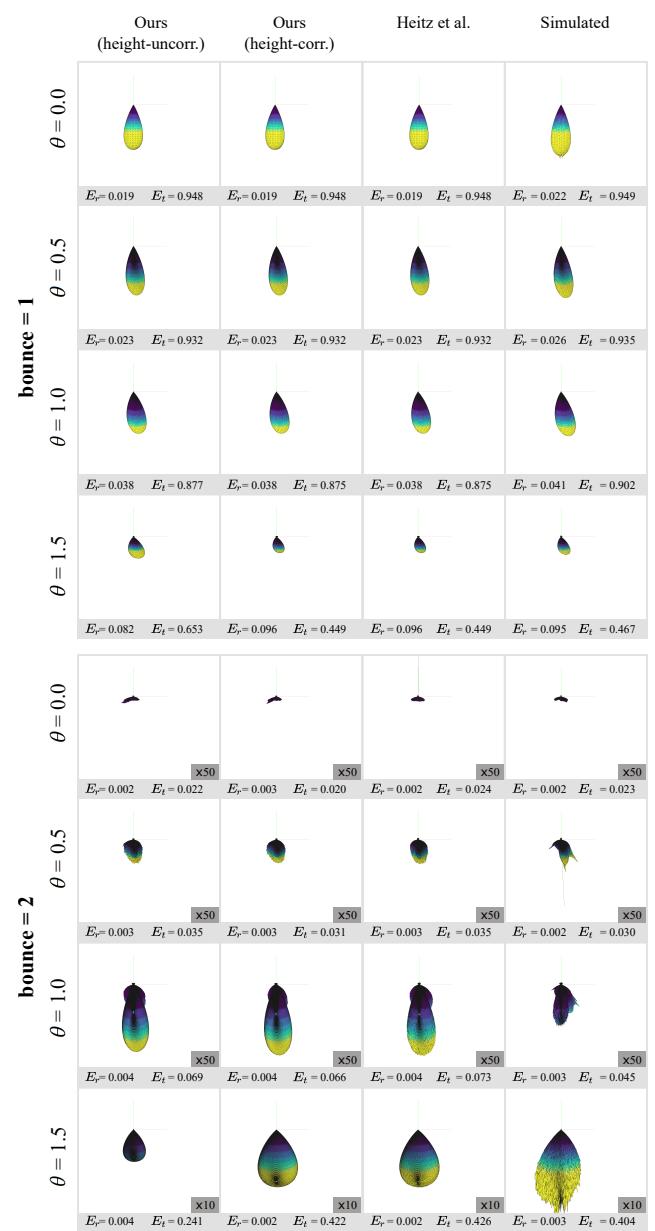


Fig. 14. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for rough dielectric material with roughness 1.0. θ is the angle between the incident direction and the normal to the macrosurface.

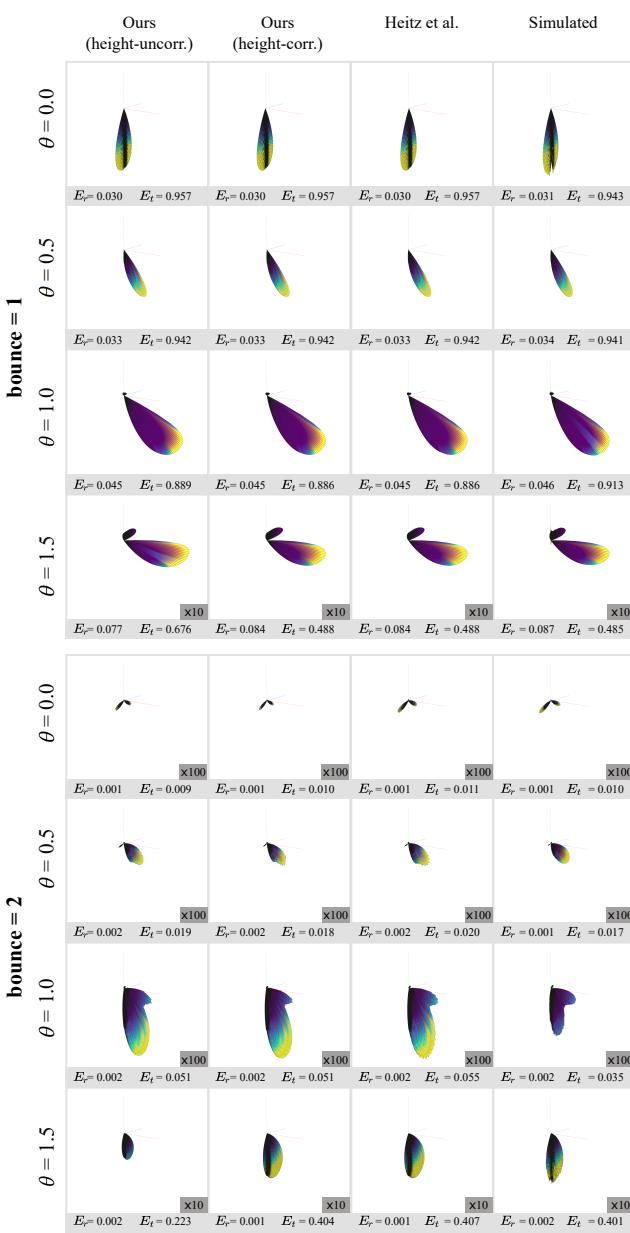


Fig. 15. Comparison between our multiple-bounce BSDF models (both height-uncorrelated and height-correlated), Heitz et al. [2016] model and simulated data, for anisotropic rough dielectric material with roughness 1.0 and 0.1. θ is the angle between the incident direction and the normal to the macrosurface.

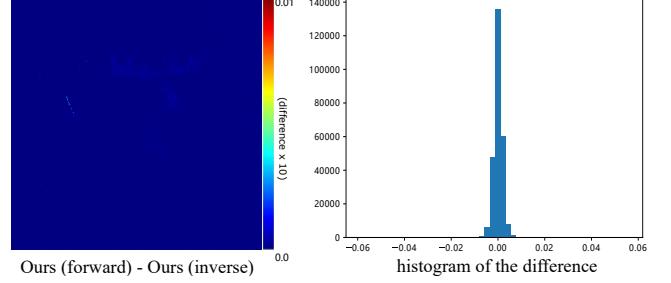


Fig. 16. The difference between renderings using $\rho(\omega_i, \omega_o)$ and $\rho(\omega_o, \omega_i)$. The difference is because of the noise, rather than the model. We show the histogram of the difference. From its distribution, we can conclude that the expectation of the difference image is about zero.

2.2 Reciprocity of our height-correlated model

We use the same example as shown in Fig. 3, to demonstrate that our height-correlated model does not have reciprocity. Since the vertex terms are symmetric, the main reason for non-reciprocity is the segment term.

Given the forward path shown in Fig. 3 (left), the segment term is:

$$\begin{aligned} s_i^{\text{forward}} &= G_2^{(i < k)}(-\mathbf{d}_{i-1}, \mathbf{d}_i) G_2^{(i=k)}(-\mathbf{d}_i, \mathbf{d}_{i+1}) \\ &= G_1^{\text{dist}}(-\mathbf{d}_{i-1}) G_2^{\text{dist}}(-\mathbf{d}_i, \mathbf{d}_{i+1}) \\ &= \frac{1}{1 + \Lambda(-\mathbf{d}_{i-1})} \frac{1}{1 + \Lambda(-\mathbf{d}_i) + \Lambda(\mathbf{d}_{i+1})}. \end{aligned} \quad (25)$$

The segment term for the inverse path shown in Fig. 3 (right) is:

$$\begin{aligned} s_i^{\text{inverse}} &= G_2^{(i < k)}(-\mathbf{d}_i, \mathbf{d}_{i+1}) G_2^{(i=k)}(\mathbf{d}_i, -\mathbf{d}_{i-1}) \\ &= \left[G_1^{\text{dist}}(\mathbf{d}_{i+1}) - G_2^{\text{dist}}(-\mathbf{d}_i, \mathbf{d}_{i+1}) \right] G_2^{\text{dist}}(\mathbf{d}_i, -\mathbf{d}_{i-1}) \\ &= \left[\frac{1}{1 + \Lambda(\mathbf{d}_{i+1})} - \frac{1}{1 + \Lambda(\mathbf{d}_{i+1}) + \Lambda(-\mathbf{d}_i)} \right] \frac{1}{\Lambda(-\mathbf{d}_i) + \Lambda(-\mathbf{d}_{i-1})} \\ &= \frac{\Lambda(-\mathbf{d}_i)}{(1 + \Lambda(\mathbf{d}_{i+1}))(1 + \Lambda(\mathbf{d}_{i+1}) + \Lambda(-\mathbf{d}_i))} \frac{1}{\Lambda(-\mathbf{d}_i) + \Lambda(-\mathbf{d}_{i-1})}. \end{aligned} \quad (26)$$

Since $s_i^{\text{forward}} \neq s_i^{\text{inverse}}$, our height-correlated model does not have reciprocity.

3 LOBE VISUALIZATION

In Figs. 4 and the following eleven figures, we compare the visualized lobes for individual bounce between our methods (height-uncorrelated and height-correlated), Heitz et al. [2016] and simulated data which is obtained by ray tracing on a generated surface with Beckmann distribution [Heitz and Dupuy 2015]. We perform the comparison on rough diffuse (albedo set as 1), rough conductor (Fresnel set as 1) and rough dielectric BSDFs, considering both isotropic ($\alpha = 0.25, 0.5, 1$) and anisotropic ($\alpha = (1.0, 0.1)$) cases. We visualize the lobes with ω_i elevation angles of 0.0, 0.5, 1.0 and 1.5 radians. E_r and E_t denote the total amount of reflected and transmitted energies, respectively.

For all the bounces with all the incident angles, our height-correlated model produces very similar results as Heitz et al. [2016], while our height-uncorrelated model has larger difference from Heitz et al. [2016] mostly at grazing angles.

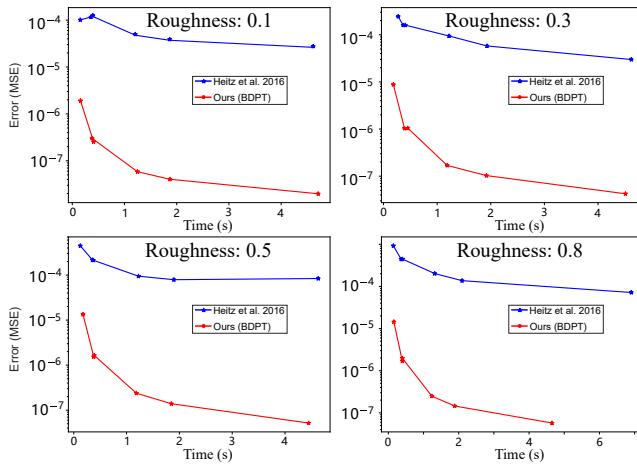


Fig. 17. The error (MSE) with logarithm scale of our method (height-correlated) and Heitz et al. [2016] over varying rendering time on the Single Slab scene with varying roughness.

4 CONVERGENCE VALIDATION

In Figure 17, we show the Mean Square Error (MSE) as a function of varying rendering time for our method (BDPT, height-correlated) and Heitz et al. [2016] in the Single Slab scene with varying roughness, considering directional lighting only. With only two samples per pixel, our method is able to produce very close result to the ground truth, while Heitz et al. [2016] produces result with a lot of noise. Increasing the number of samples (rendering time) improves the quality for both methods, but our method remains consistently better.

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