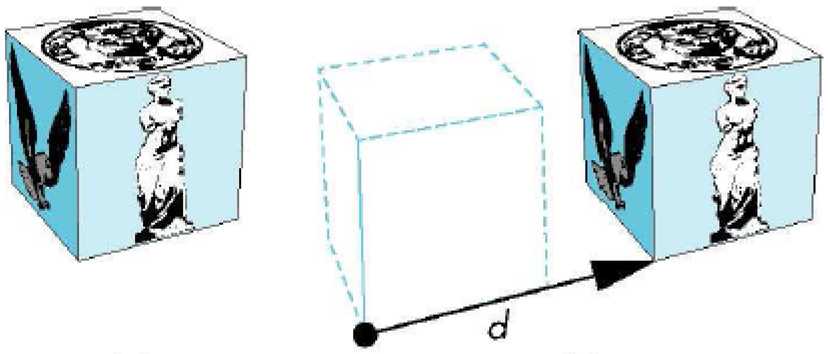
Računarska grafika
(2OER7O02)

Geometrijske transformacije

Predavanja

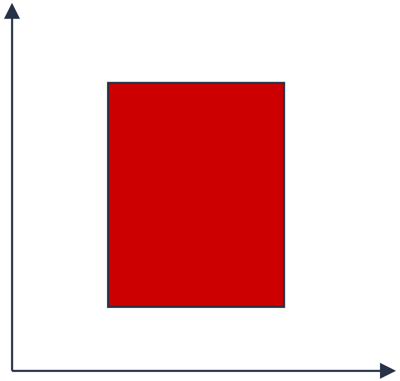
Geometrijske transformacije

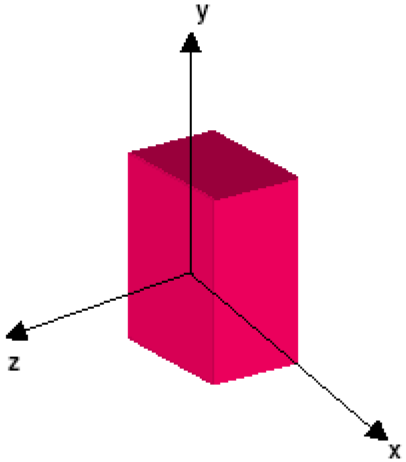
- Geometrijske transformacije preslikavaju originalnu tačku u njenu sliku.



Podela geometrijskih transformacija

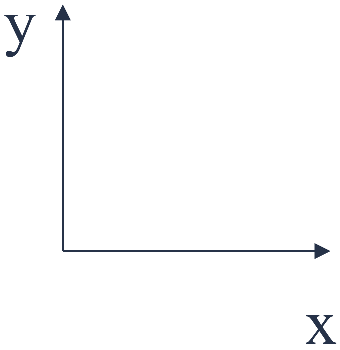
- Dvodimenzionalne transformacije (2D)
- Trodimenzionalne transformacije (3D)





2D koordinatni sistem

- 2D desni pravougli koordinatni sistem je definisan na sledeći način:
  - koordinatna osa prve koordinate (X) se prevodi u osu drugu koordinate (Y) rotacijom oko koordinatnog početka za 90° u smeru suprotnom od kretanja kazaljke na časovniku.



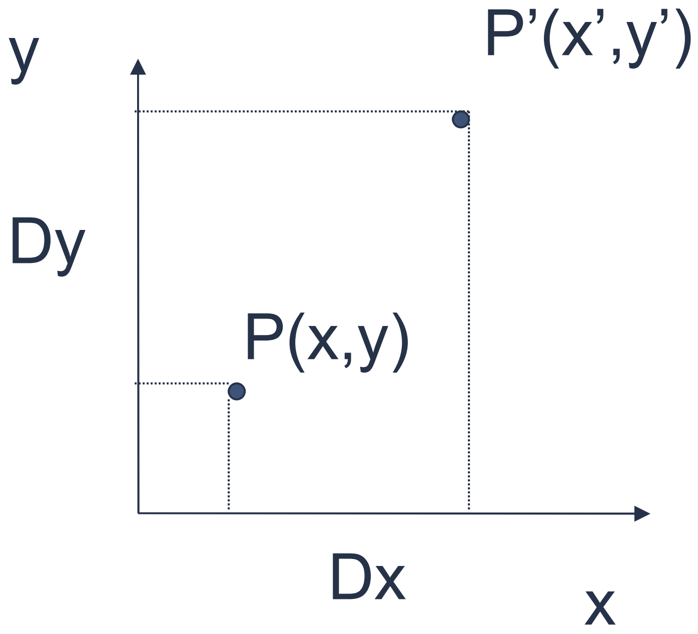
Elementarne 2D transformacije

- Translacija
- Skaliranje
- Rotacija
- Refleksija
- Smicanje

Elementarne 2D transformacije

- Uz pomoć elementarnih transformacija mogu se konstruisati skoro sve transformacije.
- Dakle, bilo koja transformacija može da se dobije kao proizvod sukcesivnog izvršavanja nekih elementarnih transformacija.

Translacija



$$x' = x + Dx$$
$$y' = y + Dy$$

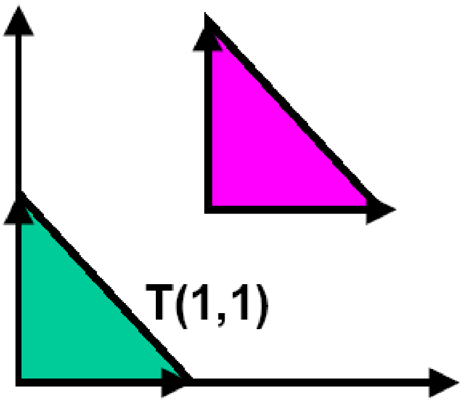
$$P = [x \quad y]$$
$$P' = [x' \quad y']$$
$$T = [Dx \quad Dy]$$

Translacija

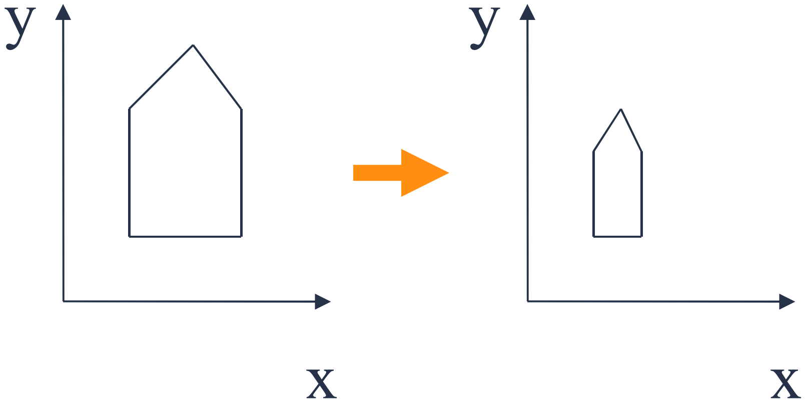
$$[x' \quad y'] = [x \quad y] + [Dx \quad Dy]$$

$$x' = x + Dx$$
$$y' = y + Dy$$

$$\boxed{P' = P + T}$$



Skaliranje



$$P = [x \quad y]$$
$$P' = [x' \quad y']$$

$$x' = x \cdot Sx$$
$$y' = y \cdot Sy$$

$$S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

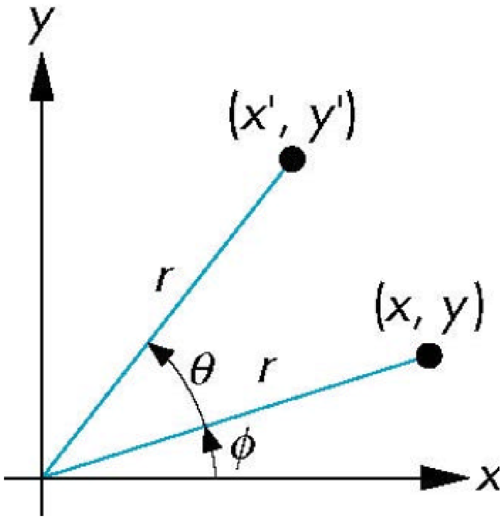
$$[x' \quad y'] = [x \quad y] \cdot \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$\boxed{P' = P \cdot S}$$

Skaliranje

- Za $S_x = S_y$ radi se o **uniformnom** skaliranju.
- Ako je $S > 1$ radi se o **uvećanju**.
- Ako je $S < 1$ radi se o **smanjenju**.

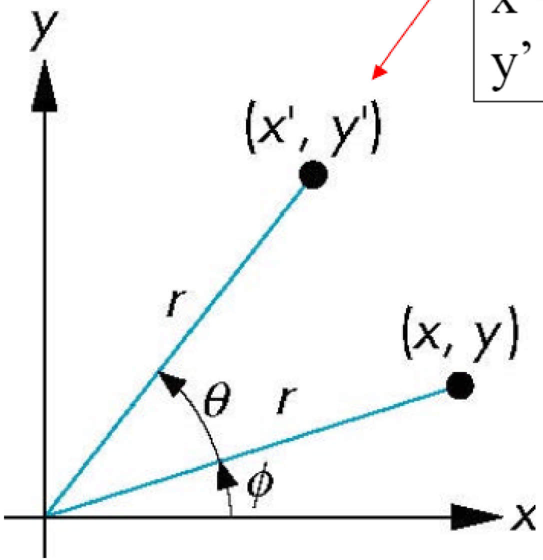
Rotacija



$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$x' = \rho\cos(\phi+\theta)$$
$$y' = \rho\sin(\phi+\theta)$$

$$x' = \rho\cos\phi\cos\theta - \rho\sin\phi\sin\theta = x\cos\theta - y\sin\theta$$
$$y' = \rho\cos\phi\sin\theta + \rho\sin\phi\cos\theta = x\sin\theta + y\cos\theta$$

Rotacija



$$x' = x \cos\theta - y \sin\theta$$
$$y' = x \sin\theta + y \cos\theta$$

$$P = [x \quad y]$$
$$P' = [x' \quad y']$$

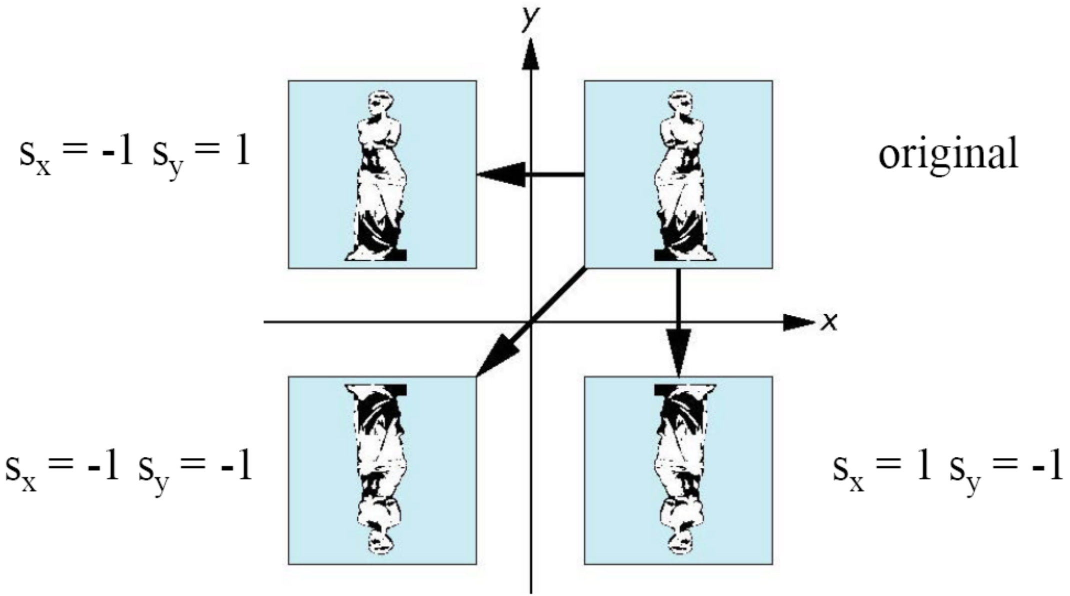
$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$[x' \quad y'] = [x \quad y] \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\boxed{P' = P \cdot R}$$

Refleksija

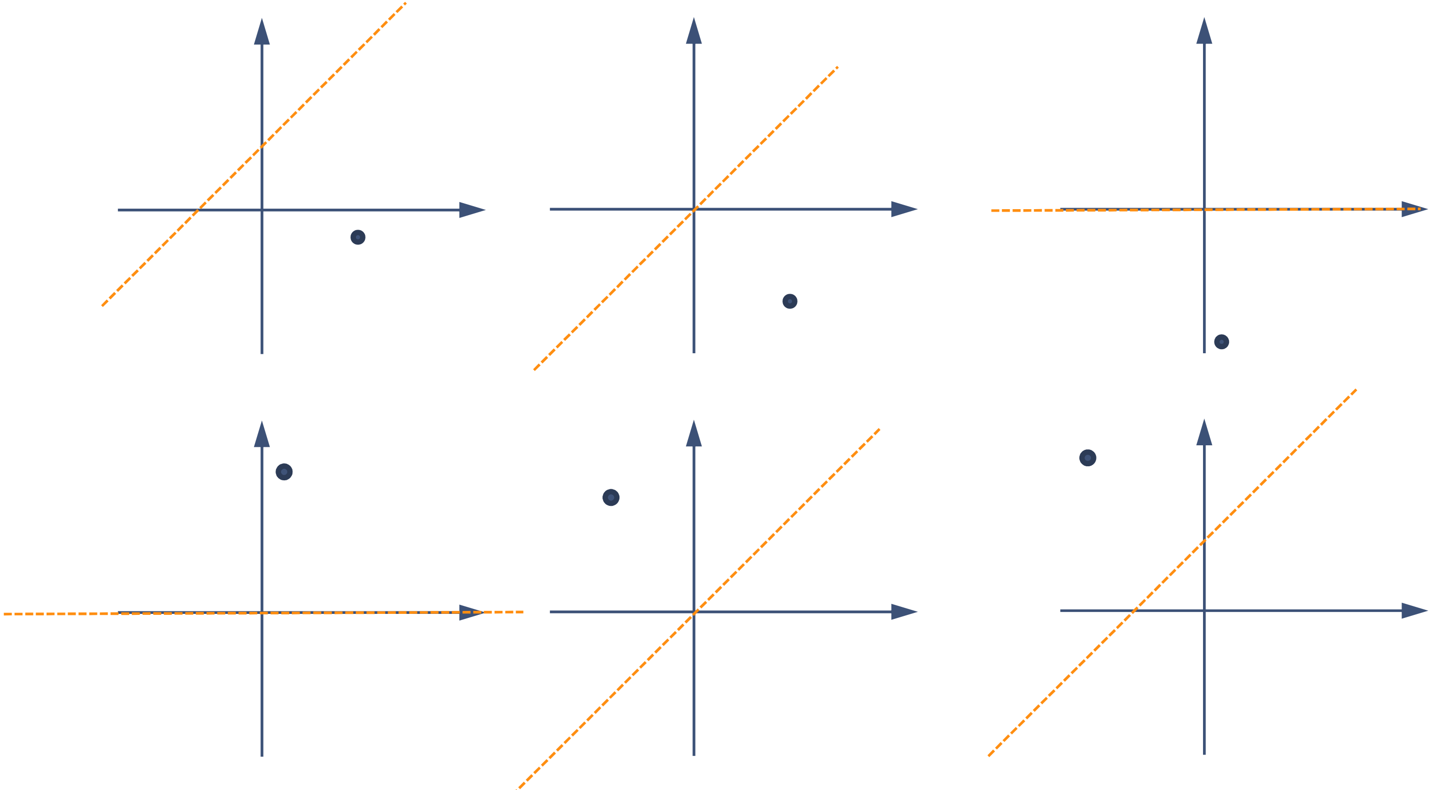
- **Refleksija** je skaliranje sa negativnim faktorom skaliranja.



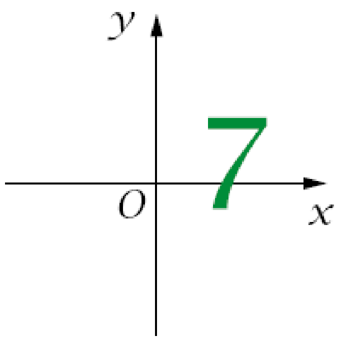
Refleksija u odnosu na proizvoljnu osu

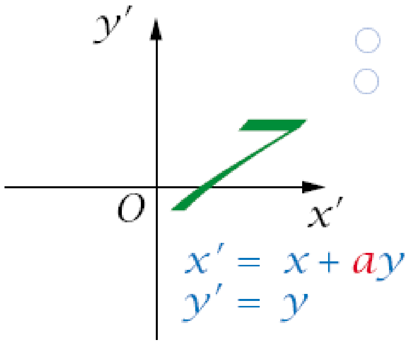
- Translacijom se dovede koordinatni početak na datu osu
- Rotacijom se X-osa poklopi sa datom osom
- Primeni se refleksija prema X-osi
- Inverzna rotacija
- Inverzna translacija

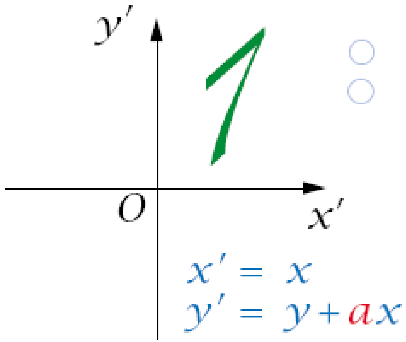
Refleksija u odnosu na proizvoljnu osu



Smicanje (*Shear*)







Smicanje

$$H = \begin{bmatrix} 1 & H_y \\ H_x & 1 \end{bmatrix}$$

P = [x   y]
P’= [x’   y’]

$$[x' \quad y'] = [x \quad y] \cdot \begin{bmatrix} 1 & H_y \\ H_x & 1 \end{bmatrix}$$

$$\boxed{P' = P \cdot H}$$

Smicanje samo u pravcu X-ose: H_Y=0;
Smicanje samo u pravcu Y-ose: H_X=0

Homogene koordinate

- $P' = P + T$
- $P' = P \cdot S$
- $P' = P \cdot R$
- $P' = P \cdot H$

NEUNIFORMNOST!!!

Homogene koordinate

- Kako bi se postigla uniformnost transformacija, uvode se **homogene** koordinate:

$$(x,y) \longrightarrow (x,y,w)$$

Homogene koordinate

- Jedna tačka može da ima više homogenih koordinata. Na primer, (2,3,6) i (4,6,12) predstavljaju jednu istu 2D tačku.
- (0,0,0) nije dozvoljena. To je tačka u beskonačnosti.
- (x,y,w) i (x',y',w') predstavljaju jednu istu tačku ukoliko su jedne koordinate umnožak druge.
- U računarskoj grafici se koristi sledeći oblik homogenih koordinata:

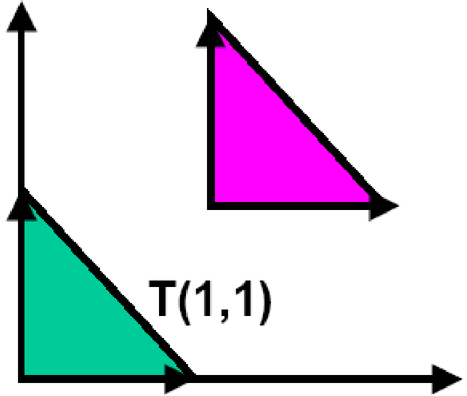
$$(x,y,1)$$

Translacija – homogene koordinate

$$[x' \quad y' \quad 1] = [x \quad y \quad 1] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Dx & Dy & 1 \end{bmatrix}$$

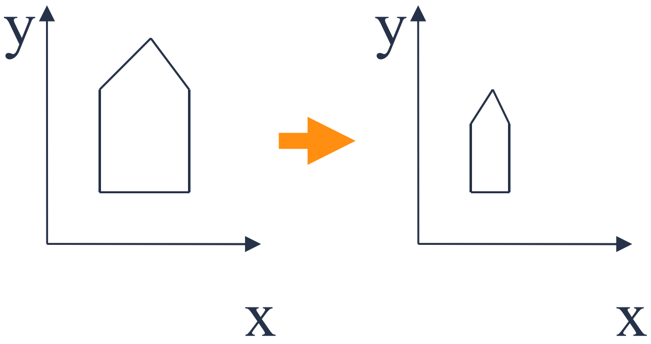
$$x' = x + Dx$$
$$y' = y + Dy$$

$$\boxed{P' = P \cdot T}$$



$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Dx & Dy & 1 \end{bmatrix}$$

Skaliranje – homogene koordinate



x' = x · Sx
y' = y · Sy

P = [x y 1]
P'= [x' y' 1]

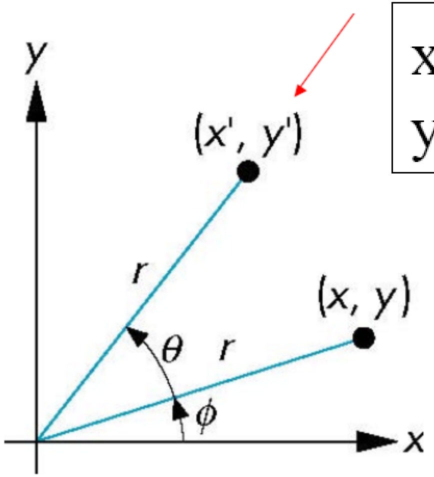
$$[x' \quad y' \quad 1] = [x \quad y \quad 1] \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{P' = P \cdot S}$$

Rotacija – homogene koordinate

$$P = [x \quad y \quad 1]$$
$$P' = [x' \quad y' \quad 1]$$



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

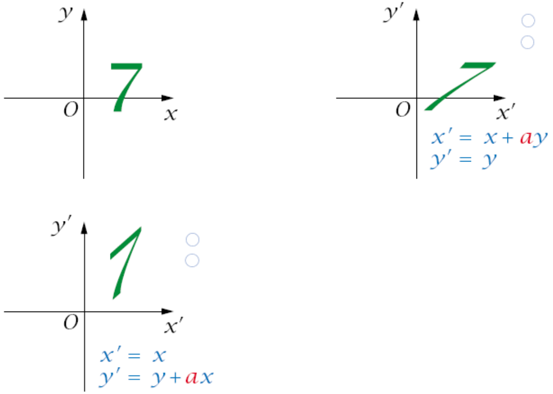
$$[x' \quad y' \quad 1] = [x \quad y \quad 1] \cdot \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{P' = P \cdot R}$$

Smicanje – homogene koordinate

$$H = \begin{bmatrix} 1 & H_y & 0 \\ H_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

P = [x y 1]
P’= [x’ y’ 1]



$$[x' \quad y' \quad 1] = [x \quad y \quad 1] \cdot \begin{bmatrix} 1 & H_y & 0 \\ H_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

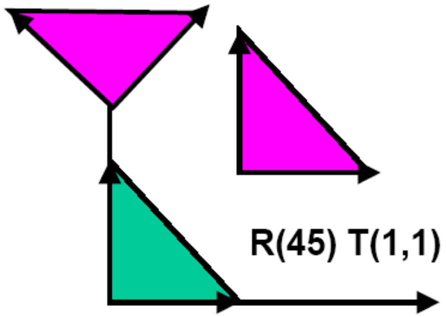
$$\boxed{P' = P \cdot H}$$

Inverzne transformacije

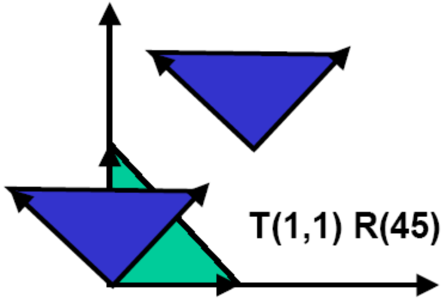
- $T(x,y) \longrightarrow T(-x,-y)$
- $S(Sx,Sy) \longrightarrow S(1/Sx,1/Sy)$
- $R(\theta) \longrightarrow R(-\theta)$
- $H(Hx,Hy) \longrightarrow H(-Hx,-Hy)$

Komponovanje transformacija

- $T = T_1 \cdot T_2$
- $S = S_1 \cdot S_2$
- $R = R_1 \cdot R_2$



T(1,1) ·R(45)



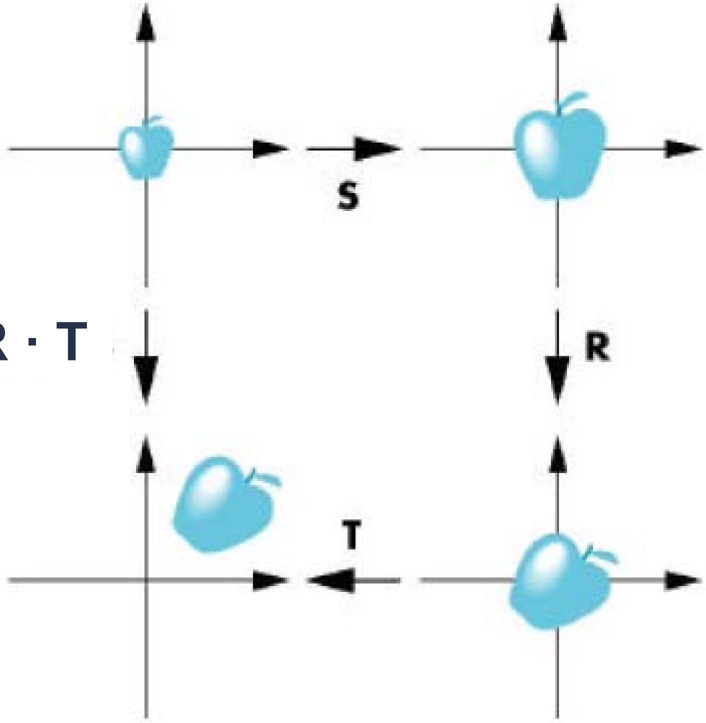
R(45) · T(1,1)

- $F = T \cdot S$ (translacija + skaliranje)

- $T \cdot R \neq R \cdot T$

Komponovanje transformacija

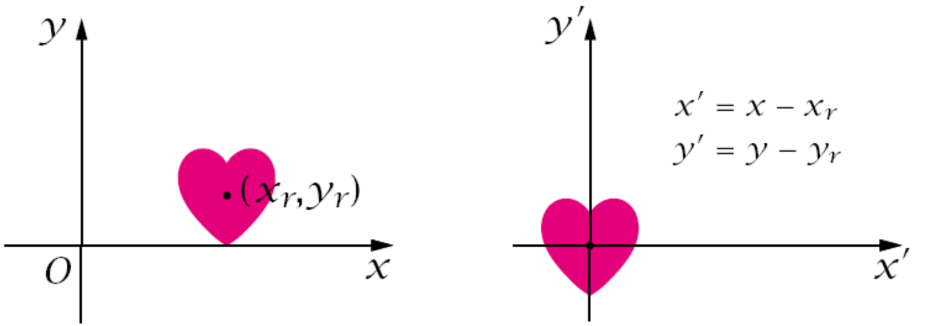
$M = S \cdot R \cdot T$



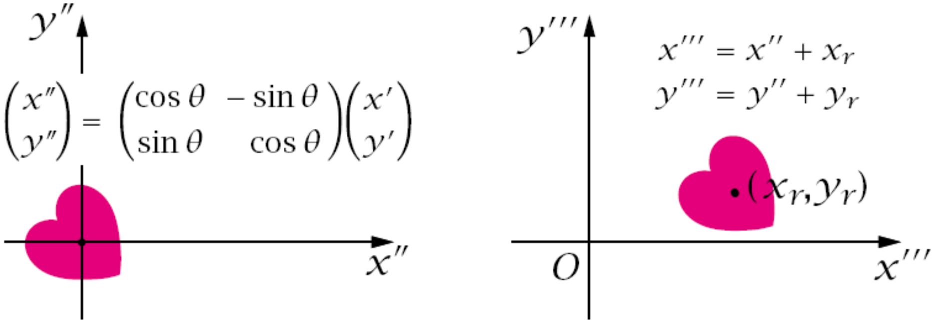
Važeće relacije

- $T(\alpha_1, \beta_1)T(\alpha_2, \beta_2) = T(\alpha_1 + \alpha_2, \beta_1 + \beta_2)$
- $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$
- $S(\alpha_1, \beta_1)S(\alpha_2, \beta_2) = S(\alpha_1\alpha_2, \beta_1\beta_2)$

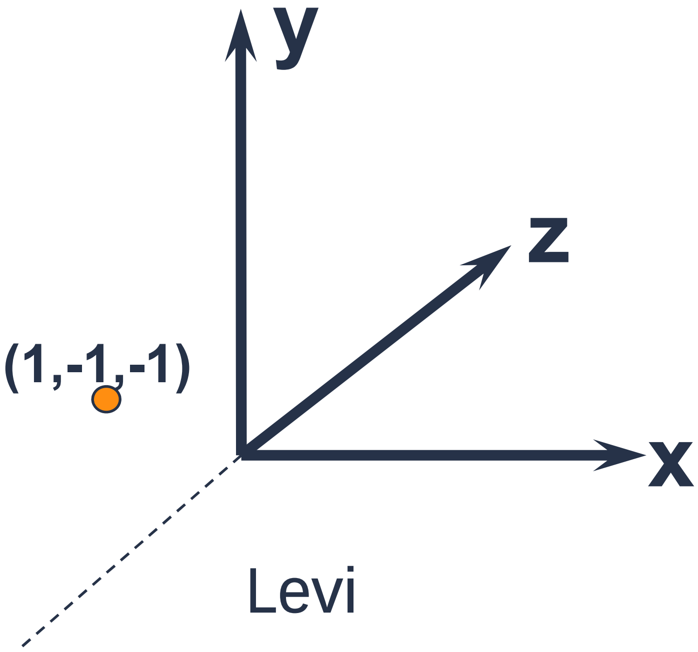
Rotacija oko proizvoljne tačke

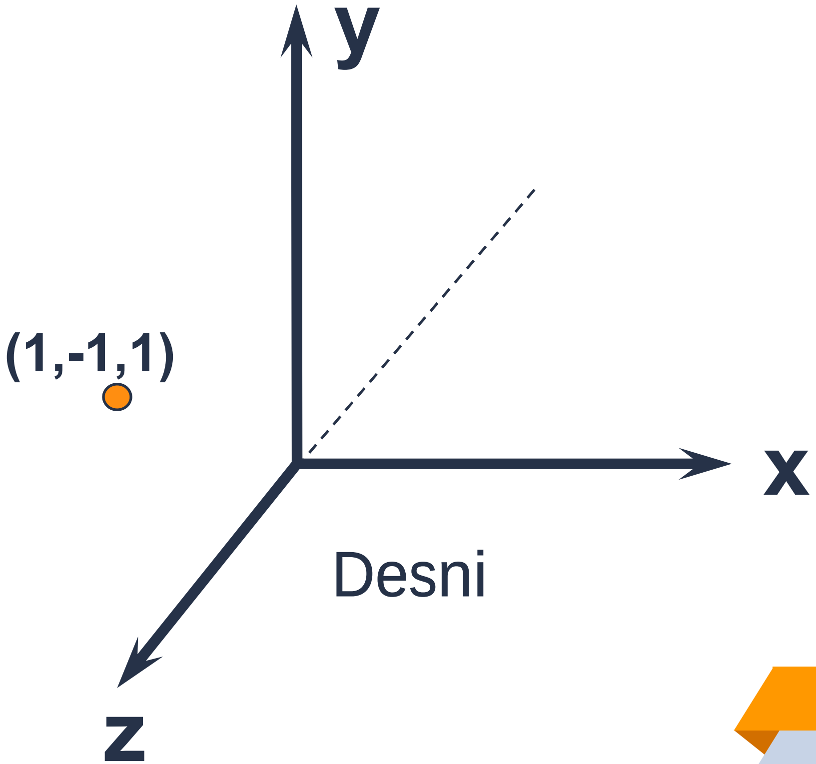


M = T(-x,-y) · R(θ) · T (x,y)



3D koordinatni sistemi



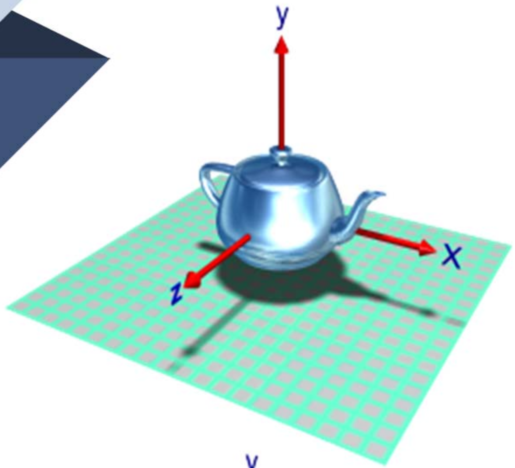


3D transformacije

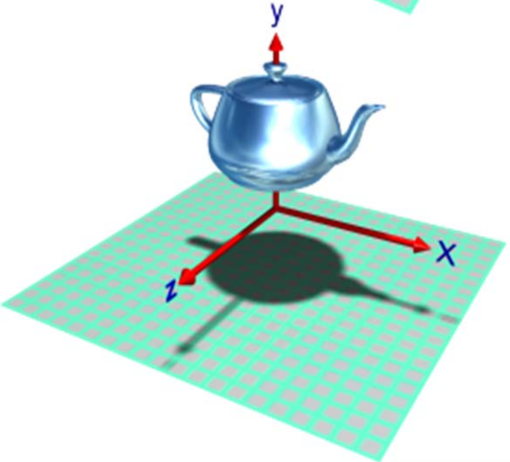
- Postoji formalna sličnost sa transformacijama u 2D grafici:
  - dodaje se jedan član jednačina (za koordinatu z),
  - dodaje se jedna jednačina (za z')
  - posledica je da matrica transformacije postaje 4x4

$$[x' \quad y' \quad z' \quad 1] = [x \quad y \quad z \quad 1] \cdot \begin{bmatrix} A1 & A2 & A3 & 0 \\ B1 & B2 & B3 & 0 \\ C1 & C2 & C3 & 0 \\ D1 & D2 & D3 & 1 \end{bmatrix}$$

Translacija – homogene koordinate



$$[x' \quad y' \quad z' \quad 1] = [x \quad y \quad z \quad 1] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Dx & Dy & Dz & 1 \end{bmatrix}$$



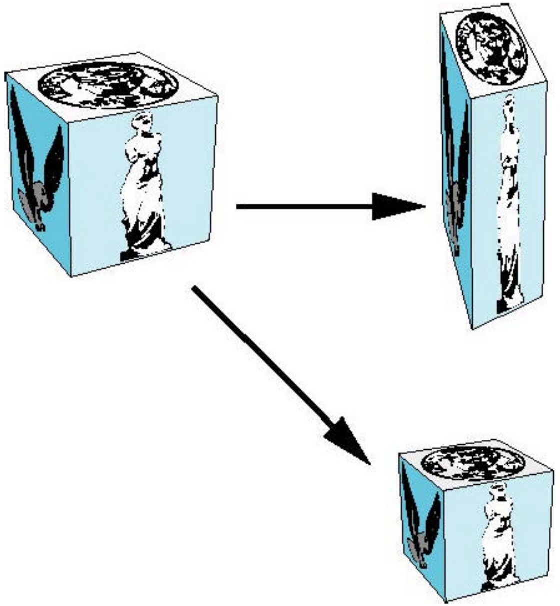
$$x' = x + Dx$$
$$y' = y + Dy$$
$$z' = z + Dz$$

$$\boxed{P' = P \cdot T}$$

Skaliranje – homogene koordinate

$$P = [x \quad y \quad z \quad 1]$$
$$P' = [x' \quad y' \quad z' \quad 1]$$

$$[x' \quad y' \quad 1] = [x \quad y \quad 1] \cdot \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$x' = x \cdot Sx$$
$$y' = y \cdot Sy$$
$$z' = z \cdot Sz$$

$$\boxed{P' = P \cdot S}$$

Rotacija – homogene koordinate

- Za razliku od 2D transformacija, gde je postojala jedna elementarna rotacija (oko koordinatnog početka) u 3D grafici postoje 3 elementarne rotacije (oko svake ose koordinatnog sistema).
- Pozitivan smer rotacije oko ose koordinatnog sistema određen je pravilom desne zavojnice.

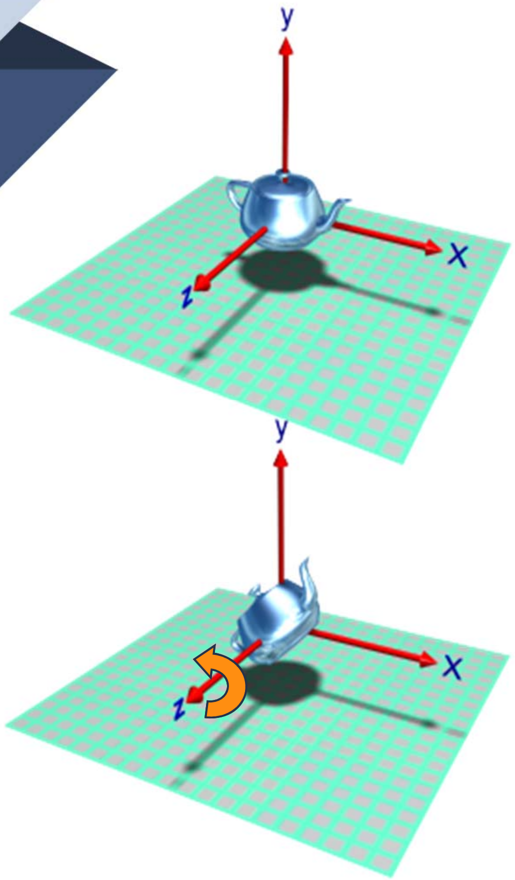
Rotacija – homogene koordinate

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{P' = P \cdot R}$$



Rotacija od 45^{o} oko Z ose

Smicanje – homogene koordinate

Smicanje duž X ose:

$x' = x$
$y' = H_Y \cdot x + y$
$z' = H_Z \cdot x + z$

$$[x\ y\ z\ 1] = [x'\ y'\ z'\ 1] \cdot \begin{bmatrix} 1 & Hy & Hz & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Smicanje duž Y ose:

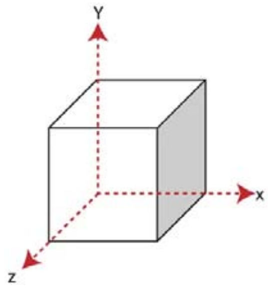
$x' = x + H_X \cdot y$
$y' = y$
$z' = H_Z \cdot y + z$

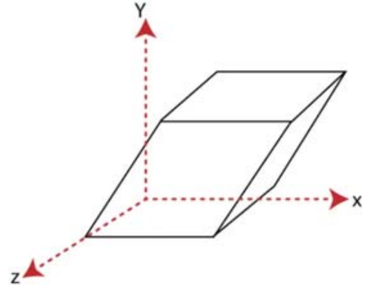
$$[x\ y\ z\ 1] = [x'\ y'\ z'\ 1] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ Hx & 1 & Hz & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Smicanje duž Z ose:

$x' = x + H_X \cdot z$
$y' = y + H_Y \cdot z$
$z' = z$

$$[x\ y\ z\ 1] = [x'\ y'\ z'\ 1] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ Hx & Hy & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





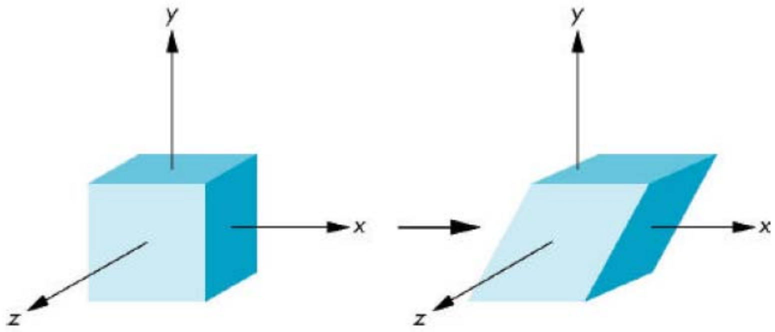
$$\boxed{P' = P \cdot H}$$

Smicanje – homogene koordinate

$$H_x = \begin{bmatrix} 1 & Hy & Hz & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Hx & 1 & Hz & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{P' = P \cdot H}$$

$$H_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ Hx & Hy & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



PITANJA