Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

Consider a family of circles passing through two fixed points A(3,7) and B(6,5) show that the chords in which the circle $x^2+y^2-4x-6y-3=0$ cuts the members of the family are concurrent at a point.Find the coordinates of this point ?

II. SOLUTION

$$\mathbf{x}^2+\mathbf{y}^2-9\mathbf{x}-12\mathbf{y}+53=0 \tag{1}$$
$$\mathbf{x}^\top\mathbf{V}_1\mathbf{x}+2\mathbf{u}_1^\top\mathbf{x}+f_1 \tag{2}$$
$$\mathbf{x}^\top\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}+2\left(\frac{-9}{2}\right)-6\mathbf{x}+53=0 \tag{3}$$

Where

$$\mathbf{V}_1=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{4}$$
$$\mathbf{u}_1=\begin{pmatrix}\frac{-9}{2} & -6\end{pmatrix} \tag{5}$$
$$f_1=53 \tag{6}$$

Equation of circle with A and B as diameter
Equation of line passing through A and B

Direction vector

$$\mathbf{m}=\mathbf{A}-\mathbf{B} \tag{7}$$

Normal vector

$$\mathbf{n}=\mathbf{R}_{\frac{\pi}{2}}\mathbf{m} \tag{8}$$

where

$$\mathbf{R}_{\frac{\pi}{2}}=\begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix} \tag{9}$$

Equation of L_1

$$\mathbf{n}^\top\left(\mathbf{x}-\mathbf{A}\right)=0 \tag{10}$$
$$\mathbf{n}^\top\mathbf{x}-\mathbf{n}^\top\mathbf{A}=0 \tag{11}$$

Given circle

$$\mathbf{x}^2+\mathbf{y}^2-4\mathbf{x}-6\mathbf{y}-3=0 \tag{12}$$
$$\mathbf{x}^\top\mathbf{V}_2\mathbf{x}+2\mathbf{u}_2^\top\mathbf{x}+f_2 \tag{13}$$
$$\mathbf{x}^\top\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}+2\begin{pmatrix}-2 & -3\end{pmatrix}\mathbf{x}-3=0 \tag{14}$$

Where

$$\mathbf{V}_2=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{15}$$
$$\mathbf{u}_2=\begin{pmatrix}-2 & -3\end{pmatrix} \tag{16}$$
$$f_2=-3 \tag{17}$$

Common chord is given by

$$\mathbf{c}_1-\mathbf{c}_2+\lambda L_1 \tag{18}$$

Where
c_1 is circle having A and B as diameter
c_2 is given circle

$$\begin{pmatrix}-5 & -6\end{pmatrix}\mathbf{x}+56+\lambda L_1 \tag{19}$$
$$\begin{pmatrix}5 & 6\end{pmatrix}\mathbf{x}=56---\left(L_2\right) \tag{20}$$

Using python we get the L_1 and intersection point

$$\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x}=27---\left(L_1\right) \tag{21}$$
$$\begin{pmatrix}5 & 6\\2 & 3\end{pmatrix}\mathbf{x}=\begin{pmatrix}56\\27\end{pmatrix} \tag{22}$$
$$\mathbf{x}=\left(2,7.667\right) \tag{23}$$

III. CODE LINK

https://github.com/sssurajit/fwc/blob/main/matrix/circle/codes/circle.py

Execute the code by using the command
python3 circle.py

IV. FIGURE

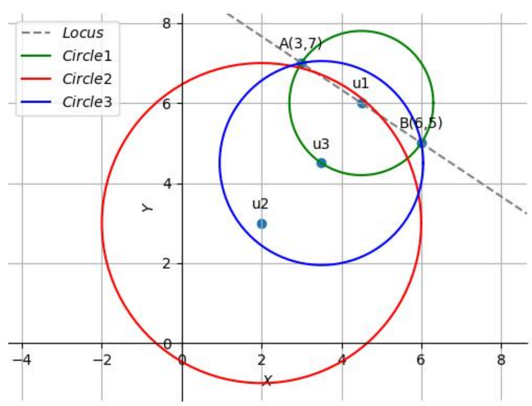
Locus
Circle1
Circle2
Circle3
A(3,7)
B(6,5)
u1
u2
u3
X
Y
-4
-2
0
2
4
6
8
0
2
4
6
8

Fig. 1.