



Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is

II. CONSTRUCTION

Symbol	Value	Description
U_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	center of given circle
r_1	2	radius of given circle
$\begin{pmatrix} a & b \end{pmatrix}$	$\begin{pmatrix} 1 & 2 \end{pmatrix}$	point on circle

III. SOLUTION

$$U_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

Radius of Circle-1,

$$r_1 = p \quad (2)$$

The general form of conic is

$$X^T V X + 2U^T X + f = 0 \quad (3)$$

For circle 1

$$X^T V_1 X + 2U_1^T X + f_1 = 0 \quad (4)$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f_1 = 0 \quad (5)$$

as circles are orthogonal

$$r_1^2 + r_2^2 = U_1 - U_2^2 \quad (6)$$

$$U_1^2 + U_2^2 - 2U_1^T U_2 = p^2 + r_2^2 \quad (7)$$

$$r_2^2 = U_2^2 - f_2 \quad (8)$$

By solving the equations (6) and (7)

$$f_2 = p^2 \quad (9)$$

For circle 2

$$X^T V_2 X + 2U_2^T X + f_2 = 0 \quad (10)$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} -g & -t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + p^2 = 0 \quad (11)$$

$$x^2 + y^2 - 2gx - 2ty + p^2 = 0 \quad (12)$$

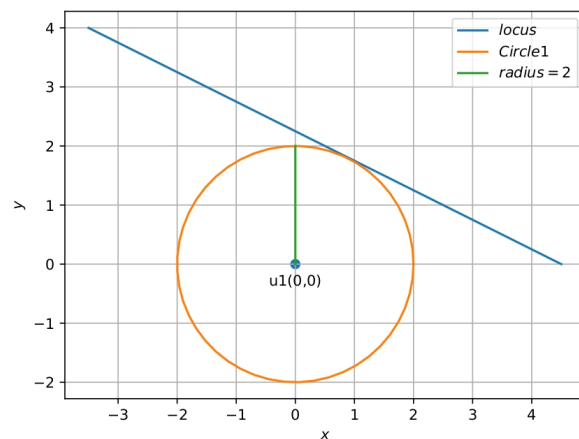
By substituting (a,b) in equation (13)

$$a^2 + b^2 - 2ga - 2tb + p^2 = 0 \quad (13)$$

The locus is

$$2ga + 2tb - (a^2 + b^2 + p^2) = 0 \quad (14)$$

IV. FIGURE



V. CODE LINK

<https://github.com/sssurajit/fwc/blob/main/line/codes/line.py>

Execute the code by using the command
python3 circle.py