Assignment - 12.10.3.5

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I. PROBLEM

Show that each of the given three vectors is a unit vector:$\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right), \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right), \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$ Also, Show that they are mutually perpendicular to eatch other.

II. SOLUTION

Given

$$\mathbf{A}=\begin{pmatrix}\frac{2}{7}\\ \frac{3}{7}\\ \frac{6}{7}\end{pmatrix}, \mathbf{B}=\begin{pmatrix}\frac{3}{7}\\ -\frac{6}{7}\\ \frac{2}{7}\end{pmatrix}, \mathbf{C}=\begin{pmatrix}\frac{6}{7}\\ \frac{2}{7}\\ -\frac{3}{7}\end{pmatrix} \tag{1}$$

$$\|\mathbf{A}\| = \mathbf{A}^{\top}\mathbf{A} \tag{2}$$

$$= \sqrt{\begin{pmatrix}\frac{2}{7} & \frac{3}{7} & \frac{6}{7}\end{pmatrix}\begin{pmatrix}\frac{2}{7}\\ \frac{3}{7}\\ \frac{6}{7}\end{pmatrix}} \tag{3}$$

$$= \sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}} \tag{4}$$

$$= \sqrt{\frac{49}{49}} \tag{5}$$

$$= 1 \tag{6}$$

$$\|\mathbf{B}\| = \mathbf{B}^{\top}\mathbf{B} \tag{7}$$

$$= \sqrt{\begin{pmatrix}\frac{3}{7} & -\frac{6}{7} & \frac{2}{7}\end{pmatrix}\begin{pmatrix}\frac{3}{7}\\ -\frac{6}{7}\\ \frac{2}{7}\end{pmatrix}} \tag{8}$$

$$= \sqrt{\frac{9}{49}+\frac{36}{49}+\frac{4}{49}} \tag{9}$$

$$= \sqrt{\frac{49}{49}} \tag{10}$$

$$= 1 \tag{11}$$

$$\|\mathbf{C}\| = \mathbf{C}^{\top}\mathbf{C} \tag{12}$$

$$= \sqrt{\begin{pmatrix}\frac{6}{7} & \frac{2}{7} & -\frac{3}{7}\end{pmatrix}\begin{pmatrix}\frac{6}{7}\\ \frac{2}{7}\\ -\frac{3}{7}\end{pmatrix}} \tag{13}$$

$$= \sqrt{\frac{36}{49}+\frac{4}{49}+\frac{9}{49}} \tag{14}$$

$$= \sqrt{\frac{49}{49}} \tag{15}$$

$$= 1 \tag{16}$$

Now, we need to show that they are mutually perprndicular to eatch other.

$$\mathbf{A}^{\top}\mathbf{B} = \begin{pmatrix}\frac{2}{7} & \frac{3}{7} & \frac{6}{7}\end{pmatrix}\begin{pmatrix}\frac{3}{7}\\ -\frac{6}{7}\\ \frac{2}{7}\end{pmatrix} \tag{17}$$

$$= \frac{6}{49}-\frac{18}{49}+\frac{12}{49} \tag{18}$$

$$= 0 \tag{19}$$

$$\mathbf{B}^{\top}\mathbf{C} = \begin{pmatrix}\frac{3}{7} & -\frac{6}{7} & \frac{2}{7}\end{pmatrix}\begin{pmatrix}\frac{6}{7}\\ \frac{2}{7}\\ -\frac{3}{7}\end{pmatrix} \tag{20}$$

$$= \frac{18}{49}-\frac{12}{49}-\frac{6}{49} \tag{21}$$

$$= 0 \tag{22}$$

$$\mathbf{C}^{\top}\mathbf{A} = \begin{pmatrix}\frac{6}{7} & \frac{2}{7} & -\frac{3}{7}\end{pmatrix}\begin{pmatrix}\frac{2}{7}\\ \frac{3}{7}\\ \frac{6}{7}\end{pmatrix} \tag{23}$$

$$= \frac{12}{49}+\frac{6}{49}-\frac{18}{49} \tag{24}$$

$$= 0 \tag{25}$$

So,

$$\mathbf{A}^{\top}\mathbf{B} = \mathbf{B}^{\top}\mathbf{C} = \mathbf{C}^{\top}\mathbf{A} = 0$$

Thus, they are mutully perpendiculars to eatch other