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Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

If a circle passes through the point (a,b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is

II. CONSTRUCTION

Symbol	Value	Description
$\mathbf{U_1}$	$\begin{pmatrix}0\\0\end{pmatrix}$	center of given circle
$\mathbf{r_1}$	2	radius of given circle
$\begin{pmatrix}a & b\end{pmatrix}$	$\begin{pmatrix}1 & 2\end{pmatrix}$	point on circle

III. SOLUTION

$$\mathbf{U_1}=\begin{pmatrix}0\\0\end{pmatrix} \tag{1}$$

Radius of Circle-1,

$$\mathbf{r_1}=\mathbf{p} \tag{2}$$

The general form of conic is

$$\mathbf{X^\top V X}+2\mathbf{U^\top X}+\mathbf{f}=0 \tag{3}$$

For circle 1

$$\mathbf{X^\top V_1 X}+2\mathbf{U_1}^\top\mathbf{X}+\mathbf{f_1}=0 \tag{4}$$

$$\begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+2\begin{pmatrix}0 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+f1=0 \tag{5}$$

as circles are orthogonal

$$\mathbf{r_1}^2+\mathbf{r_2}^2=\mathbf{U_1}-\mathbf{U_2}^2 \tag{6}$$

$$\mathbf{U_1}^2+\mathbf{U_2}^2-2\mathbf{U_1}^\top\mathbf{U_2}=\mathbf{p}^2+\mathbf{r_2}^2 \tag{7}$$

$$\mathbf{r_2}^2=\mathbf{U_2}^2-\mathbf{f_2} \tag{8}$$

By solving the equations (6) and (7)

$$\mathbf{f_2}=\mathbf{p}^2 \tag{9}$$

For circle 2

$$\mathbf{X^\top V_2 X}+2\mathbf{U_2}^\top\mathbf{X}+\mathbf{f_2}=0 \tag{10}$$

$$\begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+2\begin{pmatrix}-g & -t\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+p^2=0 \tag{11}$$

$$\mathbf{x}^2+\mathbf{y}^2-\mathbf{2gx}-\mathbf{2ty}+\mathbf{p}^2=0 \tag{12}$$

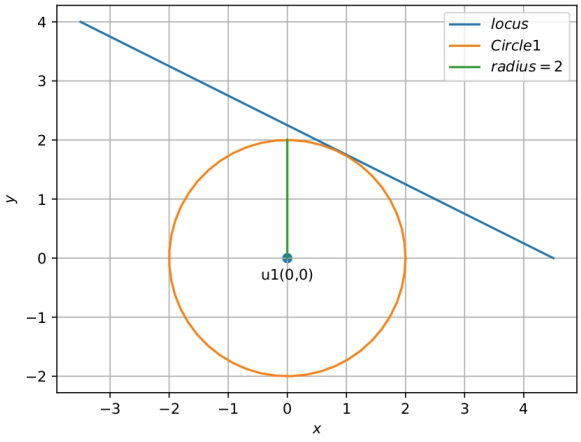
By substituting (a,b) in equation (13)

$$\mathbf{a}^2+\mathbf{b}^2-\mathbf{2ga}-\mathbf{2tb}+\mathbf{p}^2=0 \tag{13}$$

The locus is

$$\mathbf{2ga}+\mathbf{2tb}-(\mathbf{a}^2+\mathbf{b}^2+\mathbf{p}^2)=0 \tag{14}$$

IV. FIGURE



V. CODE LINK

https://github.com/sssurajit/fwc/blob/main/circle/codes/circle.py

Execute the code by using the command
python3 circle.py