



Assignment - 12.10.3.5

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I. PROBLEM

Show that each of the given three vectors is a unit vector: $\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$, $\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$, $\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$
Also, Show that they are mutually perpendicular to each other.

II. SOLUTION

Given

$$\mathbf{A} = \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{3}{7} \\ -\frac{6}{7} \\ \frac{2}{7} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{6}{7} \\ \frac{2}{7} \\ -\frac{3}{7} \end{pmatrix} \quad (1)$$

$$\|\mathbf{A}\| = \mathbf{A}^T \mathbf{A} \quad (2)$$

$$= \sqrt{\begin{pmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \end{pmatrix} \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix}} \quad (3)$$

$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} \quad (4)$$

$$= \sqrt{\frac{49}{49}} \quad (5)$$

$$= 1 \quad (6)$$

$$\|\mathbf{B}\| = \mathbf{B}^T \mathbf{B} \quad (7)$$

$$= \sqrt{\begin{pmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \end{pmatrix} \begin{pmatrix} \frac{3}{7} \\ -\frac{6}{7} \\ \frac{2}{7} \end{pmatrix}} \quad (8)$$

$$= \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} \quad (9)$$

$$= \sqrt{\frac{49}{49}} \quad (10)$$

$$= 1 \quad (11)$$

$$\|\mathbf{C}\| = \mathbf{C}^T \mathbf{C} \quad (12)$$

$$= \sqrt{\begin{pmatrix} \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix} \begin{pmatrix} \frac{6}{7} \\ \frac{2}{7} \\ -\frac{3}{7} \end{pmatrix}} \quad (13)$$

$$= \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} \quad (14)$$

$$= \sqrt{\frac{49}{49}} \quad (15)$$

$$= 1 \quad (16)$$

Now, we need to show that they are mutually perpendicular to each other.

$$\mathbf{A}^T \mathbf{B} = \begin{pmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \end{pmatrix} \begin{pmatrix} \frac{3}{7} \\ -\frac{6}{7} \\ \frac{2}{7} \end{pmatrix} \quad (17)$$

$$= \frac{6}{49} - \frac{18}{49} + \frac{12}{49} \quad (18)$$

$$= 0 \quad (19)$$

$$\mathbf{B}^T \mathbf{C} = \begin{pmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \end{pmatrix} \begin{pmatrix} \frac{6}{7} \\ \frac{2}{7} \\ -\frac{3}{7} \end{pmatrix} \quad (20)$$

$$= \frac{18}{49} - \frac{12}{49} - \frac{6}{49} \quad (21)$$

$$= 0 \quad (22)$$

$$\mathbf{C}^T \mathbf{A} = \begin{pmatrix} \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix} \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix} \quad (23)$$

$$= \frac{12}{49} + \frac{6}{49} - \frac{18}{49} \quad (24)$$

$$= 0 \quad (25)$$

So,

$$\mathbf{A}^T \mathbf{B} = \mathbf{B}^T \mathbf{C} = \mathbf{C}^T \mathbf{A} = 0$$

Thus, they are mutually perpendicular to each other

<https://github.com/sssurajit/fwc/blob/main/vectors/12.10.3.5/codes/code.py>

Execute the code by using the command

python3 code.py