Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

Consider a family of circles passing through two fixed points A(3,7) and B(6,5) show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point.Find the coordinates of this point ?

II. SOLUTION

$$\mathbf{x^2} + \mathbf{y^2} - 9\mathbf{x} - 12\mathbf{y} + 53 = 0 \tag{1}$$

$$\mathbf{x}^T\mathbf{V}_1\mathbf{x} + 2\mathbf{u}_1^T\mathbf{x} + f_1 \tag{2}$$

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}\frac{-9}{2} - 6\end{pmatrix}\mathbf{x} + 53 = 0 \tag{3}$$

Where

$$\mathbf{V_1} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{4}$$

$$\mathbf{u_1} = \begin{pmatrix}\frac{-9}{2} - 6\end{pmatrix} \tag{5}$$

$$f_1 = 53 \tag{6}$$

Equation of circle with A and B as diameter
Equation of line passing through A and B

Direction vector

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \tag{7}$$

Normal vector

$$\mathbf{n} = \mathbf{R}_{\frac{\pi}{2}}\left(\mathbf{m}\right) \tag{8}$$

where

$$\mathbf{R}_{\frac{\pi}{2}} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

Equation of L_1

$$\mathbf{n}^T\left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{9}$$

Given circle

$$\mathbf{x^2} + \mathbf{y^2} - 4\mathbf{x} - 6\mathbf{y} - 3 = 0 \tag{10}$$

$$\mathbf{x}^T\mathbf{V}_2\mathbf{x} + 2\mathbf{u}_2^T\mathbf{x} + f_2 \tag{11}$$

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-2 & -3\end{pmatrix}\mathbf{x} - 3 = 0 \tag{12}$$

Where

$$\mathbf{V}_2 = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{13}$$

$$\mathbf{u}_2 = \begin{pmatrix}-2 & -3\end{pmatrix} \tag{14}$$

$$f_2 = -3 \tag{15}$$

Common chord is given by

$$\mathbf{c}_1 - \mathbf{c}_2 + \lambda L_1 \tag{16}$$

Where
c_1 is circle having A and B as diameter
c_2 is given circle

$$\begin{pmatrix}-5 - 6\end{pmatrix}\mathbf{x} + 56 + \lambda L_1 \tag{17}$$

$$\begin{pmatrix}5 & 6\end{pmatrix}\mathbf{x} = 56 \tag{18}$$

$$\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} = 27 \tag{19}$$

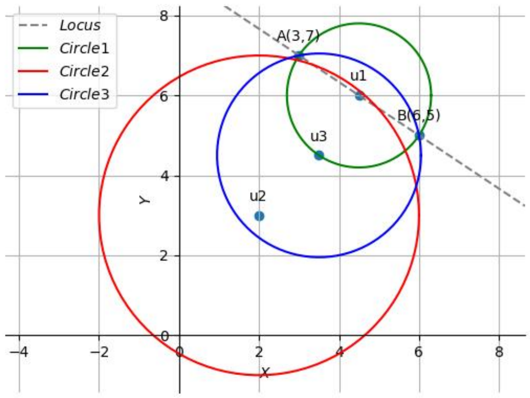
$$\begin{pmatrix}5 & 6\\2 & 3\end{pmatrix}\mathbf{x} = \begin{pmatrix}56\\27\end{pmatrix} \tag{20}$$

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{21}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} \tag{22}$$

$$\mathbf{x} = \begin{pmatrix}2, 7.667\end{pmatrix} \tag{23}$$

III. FIGURE



IV. CODE LINK

https://github.com/sssurajit/fwc/blob/main/line/codes/circle.py

Execute the code by using the command
python3 circle.py