Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

Let P $(a\sec\theta, b\tan\theta)$ and Q $(a\sec\phi, b\tan\phi)$,where $\theta+\phi=\frac{\pi}{2}$,be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.if (h,k) is the point pf intersection of normals at P and Q,then k is equal to

II. SOLUTION

Equation of normal at Q for a conic

$$\mathbf{x}^T\mathbf{V}\mathbf{x}+2\mathbf{u}^T\mathbf{x}+f=0 \tag{1}$$

where

$$\mathbf{n}=\mathbf{V}_{\mathbf{q}}+\mathbf{u} \tag{2}$$

$$\mathbf{m}=\mathbf{R}_{\frac{\pi}{2}}\mathbf{n} \tag{3}$$

$$\mathbf{m}^T\left(\mathbf{x}-\mathbf{q}\right)=0 \tag{4}$$

normal at

$$\mathbf{k}=\mathbf{q} \tag{5}$$

For L_1

$$\mathbf{m}_1^T\left(\mathbf{x}-\mathbf{q}_1\right)=0 \tag{6}$$

For L_2

$$\mathbf{m}_2^T\left(\mathbf{x}-\mathbf{q}_2\right)=0 \tag{7}$$

Point of intersection

$$\begin{pmatrix}\mathbf{m}_1 & \mathbf{m}_2\end{pmatrix}\mathbf{R}=\begin{pmatrix}\mathbf{m}_1 & \mathbf{q}_1\\ \mathbf{m}_2 & \mathbf{q}_2\end{pmatrix} \tag{8}$$

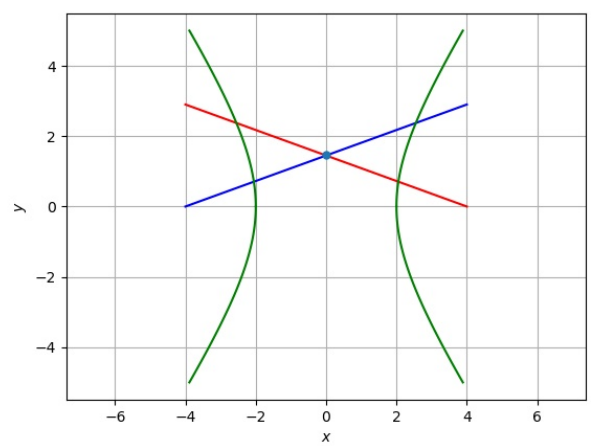
$$\mathbf{R}=\mathbf{m}^{-1}\begin{pmatrix}\mathbf{m}_1^T & \mathbf{q}_1\\ \mathbf{m}_2^T & \mathbf{q}_2\end{pmatrix} \tag{9}$$

Using python we get the value of k

$$\mathbf{k}=-\left(\frac{a^2+b^2}{b}\right) \tag{10}$$

https://github.com/sssurajit/fwc/blob/main/matrix/conics/codes/sconic.py

III. FIGURE



IV. CODE LINK

https://github.com/sssurajit/fwc/blob/main/matrix/conics/codes/conic.py

Execute the code by using the command
python3 conic.py