Indian Institute of Technology Hyderabad

Matrix Assignment - Circle

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I. PROBLEM

Consider a family of circles passing through two fixed points A(3,7) and B(6,5) show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point?

II. SOLUTION

$$x^{2} + y^{2} - 9x - 12y + 53 = 0$$
 (1)

$$\mathbf{x}^T \mathbf{V}_1 \mathbf{x} + 2\mathbf{u}_1^T \mathbf{x} + f_1$$

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\left(\frac{-9}{2} - 6\right) \mathbf{x} + 53 = 0 \qquad (3)$$

Where

$$\mathbf{V_1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{u_1} = \left(\frac{-9}{2} - 6\right) \tag{5}$$

$$f_1 = 53 \tag{6}$$

Equation of circle with A and B as diameter Equation of line passing through A and B

Direction vector

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \tag{7}$$

Normal vector

$$\mathbf{n} = \mathbf{R}_{\frac{\pi}{2}} \left(\mathbf{m} \right) \tag{8}$$

where

$$\mathbf{R}_{\frac{\pi}{2}} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Equation of L_1

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \tag{9}$$

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$$\mathbf{n}^{\mathbf{T}}\mathbf{x} - \mathbf{n}^{\mathbf{T}}\mathbf{A} = 0 \tag{10}$$

Given circle

$$x^{2} + y^{2} - 4x - 6y - 3 = 0$$
 (11)

$$\mathbf{x}^T \mathbf{V}_2 \mathbf{x} + 2\mathbf{u}_2^T \mathbf{x} + f_2 \tag{12}$$

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2(-2 & -3)\mathbf{x} - 3 = 0 \tag{13}$$

Where

$$\mathbf{V}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{14}$$

$$\mathbf{u}_2 = \begin{pmatrix} -2 & -3 \end{pmatrix} \tag{15}$$

$$f_2 = -3 \tag{16}$$

(2) Common chord is given by

$$\mathbf{c}_1 - \mathbf{c}_2 + \lambda L_1 \tag{17}$$

Where

 c_1 is circle having A and B as diameter

 c_2 is given circle

$$(-5-6)\mathbf{x} + 56 + \lambda L_1$$
 (18)

$$(5 6)\mathbf{x} = 56 - - (L_2)$$
 (19)

Using python we get the L_1 and intersection point

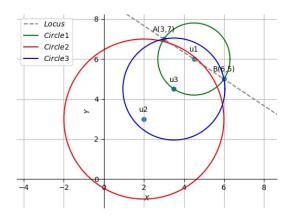
$$(2 \ 3)\mathbf{x} = 27 - - - (L_1)$$
 (20)

$$\begin{pmatrix} 5 & 6 \\ 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 56 \\ 27 \end{pmatrix}$$
 (21)

$$\mathbf{x} = (2, 7.667) \tag{22}$$

https://github.com/sssurajit/fwc/blob/main/matrix/circle/codes/scircle.py

III. FIGURE



IV. CODE LINK

https://github.com/sssurajit/fwc/blob/main/matrix/circle/codes/circle.py

Execute the code by using the command python3 circle.py