भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Matrix Assignment - Circle

Surajit Sarkar

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I. PROBLEM

Consider a family of circles passing through two fixed points A(3,7) and B(6,5) show that the chords in which the circle $x^2+y^2-4x-6y-3=0$ cuts the members of the family are concurrent at a point.Find the coordinates of this point ?

II. SOLUTION

$$\mathbf{x^2+y^2-9x-12y+53=0} \tag{1}$$

$$\mathbf{x^\top V_1 x+2u_1^\top x+f_1} \tag{2}$$

$$\mathbf{x^\top}\begin{pmatrix}1&0\\0&1\end{pmatrix}\mathbf{x}+2\left(\tfrac{-9}{2}\right)-6\mathbf{x}+53=0 \tag{3}$$

Where

$$\mathbf{V_1}=\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{4}$$

$$\mathbf{u_1}=\left(\tfrac{-9}{2}-6\right) \tag{5}$$

$$f_1=53 \tag{6}$$

Equation of circle with A and B as diameter
Equation of line passing through A and B

Direction vector

$$\mathbf{m=A-B} \tag{7}$$

Normal vector

$$\mathbf{n=R_{\frac{\pi}{2}}\left(m\right)} \tag{8}$$

where

$$\mathbf{R}_{\frac{\pi}{2}}=\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix} \tag{9}$$

Equation of L_1

$$\mathbf{n}^T\left(\mathbf{x-A}\right)=0 \tag{10}$$

$$\mathbf{n^Tx-n^TA}=0 \tag{11}$$

Given circle

$$\mathbf{x^2+y^2-4x-6y-3}=0 \tag{12}$$

$$\mathbf{x^\top V_2 x+2u_2^T x}+f_2 \tag{13}$$

$$\mathbf{x^\top}\begin{pmatrix}1&0\\0&1\end{pmatrix}\mathbf{x}+2\begin{pmatrix}-2&-3\end{pmatrix}\mathbf{x}-3=0 \tag{14}$$

Where

$$\mathbf{V}_2=\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{15}$$

$$\mathbf{u}_2=\begin{pmatrix}-2&-3\end{pmatrix} \tag{16}$$

$$f_2=-3 \tag{17}$$

Common chord is given by

$$\mathbf{c}_1-\mathbf{c}_2+\lambda L_1 \tag{18}$$

Where
c_1 is circle having A and B as diameter
c_2 is given circle

$$\begin{pmatrix}-5&-6\end{pmatrix}\mathbf{x}+56+\lambda L_1 \tag{19}$$

$$\begin{pmatrix}5&6\end{pmatrix}\mathbf{x}=56---\left(L_2\right) \tag{20}$$

Using python we get the L_1 and intersection point

$$\begin{pmatrix}2&3\end{pmatrix}\mathbf{x}=27---\left(L_1\right) \tag{21}$$

$$\begin{pmatrix}5&6\\2&3\end{pmatrix}\mathbf{x}=\begin{pmatrix}56\\27\end{pmatrix} \tag{22}$$

$$\mathbf{x}=\left(2,7.667\right) \tag{23}$$

III. CODE LINK

https://github.com/sssurajit/fwc/blob/main/matrix/circle/codes/circle.py

Execute the code by using the command
python3 circle.py

IV. FIGURE

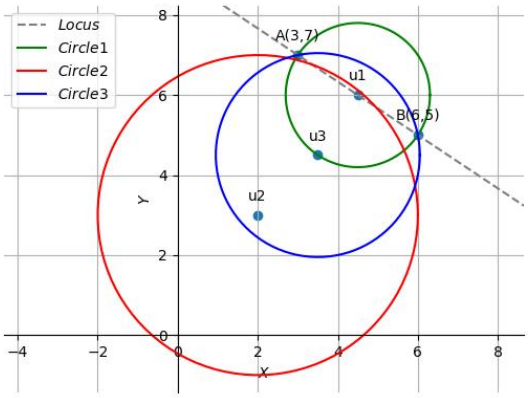

Fig. 1.