Assignment - 12.10.3.5

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CONTENTS

I. PROBLEM

Show that each of the given three vectors is a unit vector: $\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right), \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right), \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$ Also, Show that they are mutually perpendicular to eatch other.

II. SOLUTION

Given

$$\mathbf{A} = \begin{pmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \end{pmatrix} \tag{1}$$
$$\mathbf{B} = \begin{pmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \end{pmatrix} \tag{2}$$
$$\mathbf{C} = \begin{pmatrix} \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix} \tag{3}$$

$$\|\mathbf{A}\| = \mathbf{A}^\top\mathbf{A} \tag{4}$$
$$= \sqrt{\begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \end{pmatrix}} \tag{5}$$
$$= \sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}} \tag{6}$$
$$= \sqrt{\frac{49}{49}} \tag{7}$$
$$= 1 \tag{8}$$

$$\|\mathbf{B}\| = \mathbf{B}^\top\mathbf{B} \tag{9}$$
$$= \sqrt{\begin{pmatrix} \frac{3}{7} \\ -\frac{6}{7} \\ \frac{2}{7} \end{pmatrix} \begin{pmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \end{pmatrix}} \tag{10}$$
$$= \sqrt{\frac{9}{49}+\frac{36}{49}+\frac{4}{49}} \tag{11}$$
$$= \sqrt{\frac{49}{49}} \tag{12}$$
$$= 1 \tag{13}$$

$$\|\mathbf{C}\| = \mathbf{C}^\top\mathbf{C} \tag{14}$$
$$= \sqrt{\begin{pmatrix} \frac{6}{7} \\ \frac{2}{7} \\ -\frac{3}{7} \end{pmatrix} \begin{pmatrix} \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix}} \tag{15}$$
$$= \sqrt{\frac{36}{49}+\frac{4}{49}+\frac{9}{49}} \tag{16}$$
$$= \sqrt{\frac{49}{49}} \tag{17}$$
$$= 1 \tag{18}$$

Now, we need to show that they are mutually perprndicular to eatch other.

$$\mathbf{A}^\top\mathbf{B} = \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix} \begin{pmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \end{pmatrix} \tag{19}$$
$$= \frac{6}{49}-\frac{18}{49}+\frac{12}{49} \tag{20}$$
$$= 0 \tag{21}$$

$$\mathbf{B}^\top\mathbf{C} = \begin{pmatrix} \frac{3}{7} \\ -\frac{6}{7} \\ \frac{2}{7} \end{pmatrix} \begin{pmatrix} \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix} \tag{22}$$
$$= \frac{18}{49}-\frac{12}{49}-\frac{6}{49} \tag{23}$$
$$= 0 \tag{24}$$

$$\mathbf{C}^\top\mathbf{A} = \begin{pmatrix} \frac{6}{7} \\ \frac{2}{7} \\ -\frac{3}{7} \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \end{pmatrix} \tag{25}$$
$$= \frac{12}{49}+\frac{6}{49}-\frac{18}{49} \tag{26}$$
$$= 0 \tag{27}$$

So,

$$\mathbf{A}^\top\mathbf{B} = \mathbf{B}^\top\mathbf{C} = \mathbf{C}^\top\mathbf{A} = 0$$

Thus, they are mutully perpendiculars to eatch other