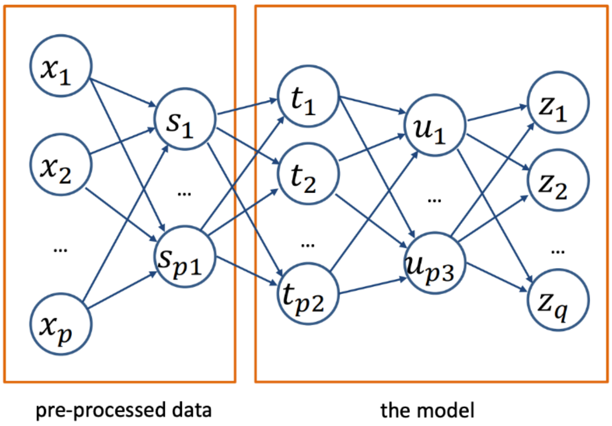
Lecture 8: Deep learning, CNN, Autoencoders

Deep learning

- ANNs with a single hidden layer are **universal approximators**
  - E.g. OR, AND, NOT
- It's efficient to **stack** several hidden layers $\rightarrow$ Deep neural networks

Representation learning

- Consecutive layers form **representations of the input** of increasing complexity
- ANN using complex non-linear representation
- A hidden layer can be though of as the *transformed feature space* (e.g. $\mathbf{u} = \phi(\mathbf{x})$)
- Parameters of such a transformation are learned from data
- ANN layers as data transformation:

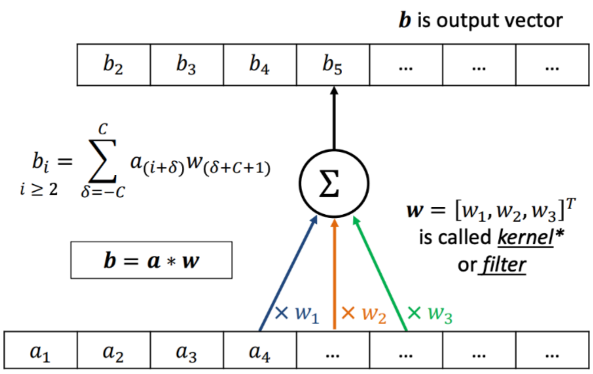


Depth v.s. width

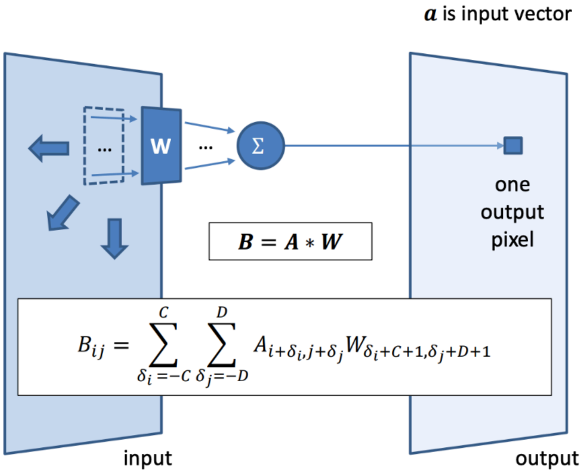
- Width: single infinitely wide layer gives a universal approximator
- Depth: yields more accurate models
- Seek to mimic layered complexity in a network
- However **vanishing gradient problem** affects learning with very deep models

Convolutional Neural Networks

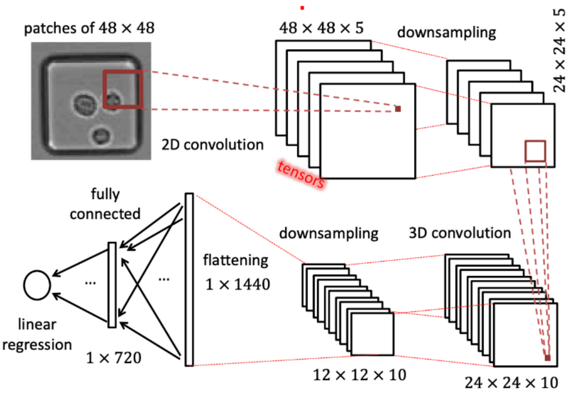
- In this example
  - C = 1, (2C+1) is the filter size



- Stride = 1



- For 2D images
- For computer vision
  - Use 5 filters for 2D convolution
  - Downsampling could be Max Pooling
  - Use 10 filters for 3D convolution



Components of CNN

- Convolutional layers
  - Complex input representations based on convolution operation
  - Weights of the filters are learned from training data
- Downsampling
  - Re-scales to smaller resolution, imits parameter explosion
  - Usually via Max Pooling
- Fully connected parts and output layer
  - Merges representations together

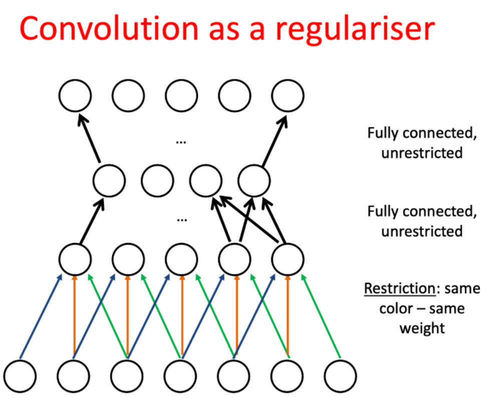
Downsampling via max pooling

- Special type of processing layer. For $m \times m$ patch

$$v = max(u_{11}, u_{12}, ..., u_{mm})$$

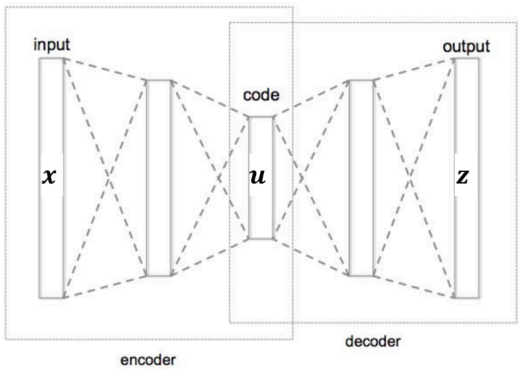
- Strictly speaking, not everywhere differentiable (pooling layers not differentiable). Instead, gradient is defined according to **"sub-gradient"**
- Max pooling:
  - Tiny changes in values of u_{ij} that is not max do not change v
  - If u_{ij} is max value, tiny changes in that value change v linearly
  - Use $\frac{\partial v}{\partial u_{ij}} = 1$ if $u_{ij} = v$, and $\frac{\partial v}{\partial u_{ij}}$ otherwise
- **Forward pass** records maximising element, which is then used in the backward pass during back-propagation

Convolution as a regulariser



Autoencoder

- Given data without labels $x_1, \ldots, x_n$
  - Set $y_i \equiv x_i$ (target/output = input)
  - train an ANN to predict $z(x_i) \approx x_i$ (approximate input)



- Set bottleneck layer (**representation**) u in middle "thinner" than input

Bottleneck

- Manage to train a network that gives a good **restoration** of the original signal $z(x_i) \approx x_i$
- That means that the data structure can be effectively **described (encoded) by a lower dimensional representation u**

Dimensionality reduction

- Autoencoders can used for **compression** and **dimensionality reduction** via a non-linear **transformation**
- If you use **linear activation functions** and **only one hidden layer**, then the setup becomes almost that of **PCA**

Lecture 9: Support Vector Machine

Linear hard-margin SVM

- **Binary classifier**
  - $s = b + \sum_{i=1}^{m} x_i w_i$
  - Predict class A if $s \geq 0$
  - Predict class B if $s < 0$
- **Linear classifier**
  - s is a linear function of inputs, and the **separating boundary** is linear
- Model the data as **linearly separable**
  - There exists a hyperplane perfectly separating the classes
- Training using all data at once

SVM vs. Perceptron

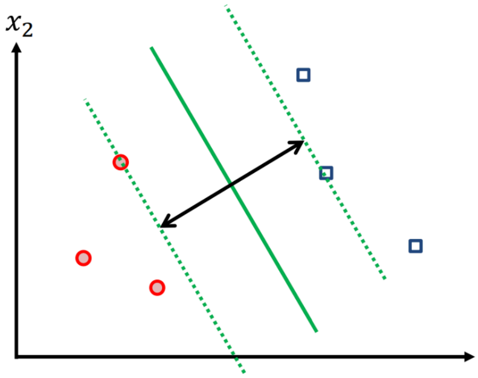
- Given learned parameter value, an **SVM makes predictions exactly like a perceptron**
- Different ways to learn parameter
  - SVM: maximise margin
  - Perceptron: min perceptron loss

Separation boundary

- Choosing parameters means choosing a separating boundary (hyperplane)
- **For perceptron,** all boundaries that separates classes perfectly are equally good
  - since perceptron loss is 0 for all of them
- **For SVM,** it aim to find the separation boundary that maximises the margin between the classes
- **Margin** is defined by the **location(b) and orientation(w)** of the separating boundary, which are defined by SVM parameters
  - Space between two dashed lines

Margin width

- Distance between the hyperplane and the nearest data points
  - therefore, **distance to the nearest red and blue points are the same**
- Points on margin boundaries called **support vectors**



SVM Parameters

- The separation boundary is defined by parameters $\mathbf{w}$ and b
  - $\mathbf{w}$: vetor normal (perpendicular) to the boundary
  - b: bias / intercept
- For a point X, let X_p denote the **projection** of X onto the hyperplane
- Distance from point to hyperplane
  - $||r|| = \pm \frac{w'x+b}{||w||}$
- In training data, y_i corresponds to binary label (-1 or 1)
  - y_i encode the side of the boundary each x_i is on
- Thus, **distance from i-th point** to a perfect hyperplane:
  - $||r_i|| = \frac{y_i(w'x_i+b)}{||w||}$

SVM: Finding separating boundary

- Margin width = distance from the hyperplane to the closest point
- **SVM Objective:**
  - Maximise ($\min_{i=1,...,n} \frac{y_i(w'x_i+b)}{||w||}$) as a function of $\mathbf{w}$ and b
  - Problem: non-unique representation of separating boundary (hyperplane)
    - i.e. can use any $\alpha(w'x+b) = \tilde{w}'x + \tilde{b} = 0$
    - Infinite number of solutions
  - Possible solution to **resolve ambiguity**:
  - Measure the distance to the closest point (i^*) and rescale param s.t.
    - margin: $\frac{y_{i^*}(w'x_{i^*}+b)}{||w||} = \frac{1}{||w||}$
    - (Arbitrary set the numerator to 1 to get unique values of $\mathbf{w}$ and b)
- **SVM Objective with extra requirement:**
  - Extra requirement: Set margin width = $\frac{y_{i^*}(w'x_{i^*}+b)}{||w||} = \frac{1}{||w||}$
  - i^* denotes index of closest example to boundary
- Therefore, (hard margin) SVM aims to find:
  - $\arg\min_w ||w||$ s.t. $y_i(w'x_i+b) \geq 1$ for $i = 1, ..., n$
  - Minimum $||w||$ => maximise margin
  - Constraint: perfect separation of points

SVM as regularised ERM

- SVM objective
  - $\arg\min_w ||w||$: data-independent regularisation term
  - $y_i(w'x_i+b) \geq 1$ for $i = 1, ..., n$: constraints as data-dependent training error
    - Can be interpreted as loss
      - $l_\infty = 0$ if prediction correct
      - $l_\infty = \infty$ if prediction wrong (give infinite loss to make perfect separation)