Bottle Rocket Estimations

SD2900 Fundamentals of Spaceflight

Group 5: Final countdown
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1 Method of estimation

The initial velocity of the rocket is approximated in subsection A.1 to

$$u_0 = \sqrt{\frac{2P_0\big(V_s - V_p\big)^{\gamma}\left((V_s - V_p)^{1-\gamma} - V_s^{1-\gamma}\right)}{m_s + \frac{m_s^2}{m_p}}}. \tag{1}$$

Here it is assumed that the ΔV is given to the rocket instantly. In a Cartesian coordinate system with $\boldsymbol{e}_y$ parallel with $-\boldsymbol{g}$, the direction of gravity, and origin at the rocket's starting position and velocity coordinates are

$$\begin{aligned} x_0 &= 0 \\ y_0 &= 0 \\ u_{x,0} &= \sin\gamma_0 \\ u_{y,0} &= \cos\gamma_0 \end{aligned}$$

Where γ_0 is the angle of the launch apparatus. The horizontal flight distance of the rocket is estimated by iterating the flight path using the Euler method [1]. At each time-step $\mathrm{d}t$ the position and velocity coordinates are updated according to:

$$a_{x,i} = -\frac{1}{m_s}ku_{x_i}u_i \tag{2}$$

$$a_{y,i} = -g - \frac{1}{m_s}ku_{y_i}u_i \tag{3}$$

$$u_{x,i+1} = u_{x,i} + a_x\mathrm{d}t \tag{4}$$

$$u_{y,i+1} = u_{y,i} + a_y\mathrm{d}t \tag{5}$$

$$x_{x,i+1} = x_{x,i+1} + u_{x,i+1}\mathrm{d}t \tag{6}$$

$$x_{y,i+1} = x_{y,i+1} + u_{y,i+1}\mathrm{d}t \tag{7}$$

Where $u_i = \sqrt{u_{x,i}^2 u_{y,i}^2}$. Further explanation can be seen in subsection A.1.

2 Measurements

The rocket was built with the following goals:

1. Increase mass on top to move the center of mass forward in the rocket.
2. Place fins in the back of the rocket to put the mean air drag force behind the center of mass, which will apply a torque to the bottle when going off course. Assuring the direction of the rocket always remaining parallel with the velocity relative to the air.

3. Make the rocket aerodynamic by molding a pointy top and putting the fins close to the body of the rocket.

The resulting bottle rocket can be seen in Figure 1. The mass, volume and area of the rocket were measured. The given water volume was .56 l and launch angle 46°. The drag coefficient was approximated to .3. Given by data of a spherical cone followed by a cylinder in [2] and further .3 is used in [3].

Figure 1: Constructed bottle rocket which flight distance is estimated.

Property	Value	Description
V_s	1.5E−3 m^3	Internal volume
m_s	.186 kg	Structure mass
V_p	*optimize*	Water volume
m_p	.56 kg	Water mass
γ_0	*optimize*	Launch angle
A_s	6.5E−3 m^3	Area of structure
C_d	.3	Drag coefficient
ρ_{air}	1.26 kg/m^3	Density of air

Table 1: Measured, given and estimated parameters as well as the parameters that are to be optimized for the specific rocket.

3 Result

A program is written to approximate the flight distance using the equations in section 1 and parameters in section 2. Further see subsection A.2 for more details. A range of γ_0 ($1° \leq \gamma_0 \leq 89°$) is simulated as well as a range of V_0 (0.1 l ≤ 1.4 l). The parameters that gives the longest ground distance are $V_0 = .91$ l, $\gamma_0 = 46°$. The angle is reasonable but in our research we find the optimal water volume for our rocket to be a third (.5 l) [3]. Thus we'll use .5 l water in the rocket and simulation as well. The resulting trajectory of the rocket be seen in Figure 2. The optimized values together with the predicted ground distance can be seen in Table 2.

Property	Value
γ_0	46°
$p_0 - p_{\text{atm}}$	7.0 bar
V_0	.50 l
X_{pred}	77 m

Table 2: Optimized values and predicted ground distance.

References

[1] Lakoba, Taras I., *Simple Euler method and its modifications*, University of Vermont, retrieved 29 February 2012.

[2] Hoerner SF. *Fluid Dynamic Drag*, Hoerner Fluid Dynamics, 1965.

[3] Air command rockets. *How much water?*. Available on `http://www.aircommandrockets.com/water.htm`.

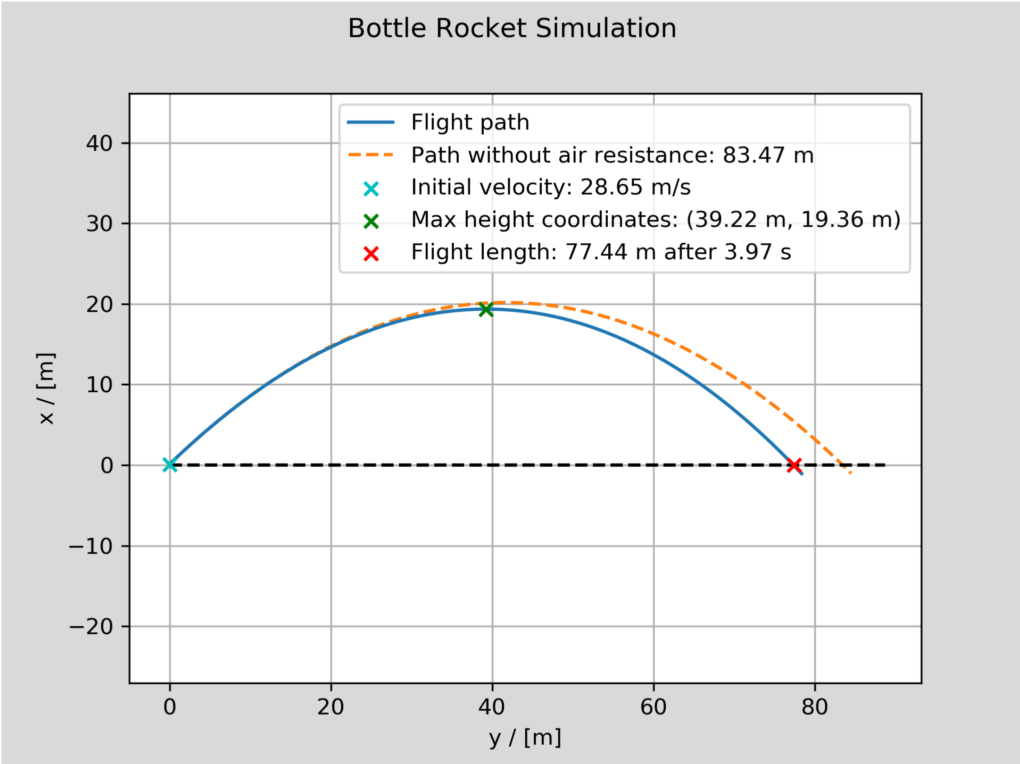

Figure 2: The approximated flight path of the bottle rocket.

A Appendix

A.1 Calculations

Let W be the *work* applied to the water by the expanding air:

$$W = \int F \mathrm{d}s = \int PA \mathrm{d}s \,.$$

This expansion is adiabatic which implies

$$PV^\gamma = P_0 V_0^\gamma \,.$$

Where $\gamma \approx 1.4$ for air. Let $P(V)$ the the momentary pressure of the air as a function of the current volume of air, V, in the bottle:

$$P(V) = P_0 \left(\frac{V_0}{V}\right)^\gamma \,. \tag{8}$$

Integrate with respect to V as the air volume increases:

$$W = \int_{V_s - V_p}^{V_s} P_0 \left(\frac{V_s - V_p}{V}\right)^\gamma \mathrm{d}V \tag{9}$$

$$= P_0 (V_s - V_p)^\gamma \left[V^{1-\gamma}\right]_{V_s - V_p}^{V_s} \tag{10}$$

$$= P_0 (V_s - V_p)^\gamma \left((V_s - V_p)^{1-\gamma} - V_s^{1-\gamma}\right) \,. \tag{11}$$

This work applied to the water is now assumed to be converted to kinetic energy for the coherent water mass and the ejected bottle. The most of the energy loss to heat has been taken account for in the adiabatic expansion.

$$W = \frac{1}{2} m_s u_s + \frac{1}{2} m_p u_p \,. \tag{12}$$

Since the momentum is initially zero:

$$0 = m_s u_s + m_p u_p \,. \tag{13}$$

(13) solved for u_p in (12):

$$W = \frac{1}{2}m_s u_s^2 + \frac{1}{2}\frac{m_s^2}{m_p}u_s \tag{14}$$

$$\implies \tag{15}$$

$$u_s = \sqrt{\frac{2W}{m_s + \frac{m_s^2}{m_p}}} \tag{16}$$

$$= \sqrt{\frac{2P_0\big(V_s - V_p\big)^{\gamma}\Big((V_s - V_p)^{1-\gamma} - V_s^{1-\gamma}\Big)}{m_s + \frac{m_s^2}{m_p}}}\,. \tag{17}$$

This $\Delta V = u_s$ is assumed to be given the bottle instantly. The bottle then follows a path affected by gravity and air drag. The force of gravity is

$$\boldsymbol{F}_g = -mg\boldsymbol{e_y}$$

and the force from air drag is

$$\boldsymbol{F}_d = -\frac{1}{2}\rho_{air}C_dA_su^2\boldsymbol{e_t}\,.$$

Let $k = \frac{1}{2}\rho_{air}C_dA_s$. Combined with Newton's second the equations of motion are

$$a_x = -\frac{1}{m_s}ku^2\sin(\gamma)$$
$$a_y = -g - \frac{1}{m_s}ku^2\cos(\gamma)$$

which can be simplified to

$$a_x = -\frac{1}{m_s}ku_xu$$
$$a_y = -g - \frac{1}{m_s}ku_yu$$

A.2 Code

Please see

`https://github.com/axelstr/BottleRocketSimulator` .