In this activity you will reflect a point and compare the motion of the point to the motion of its reflected image.

Reflect a Point

Begin by reflecting a point and describing how the reflection behaves.

1. Open geometricfunctions.org/links/reflection-family/. Read the Learning Goals and then go to page 2.

2. Use the **x** tool to construct independent variable *x.*

Finalize the *x* tool by tapping or dragging the glowing point *x*.

3. Drag *x* around the screen.

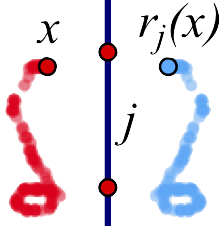
4. Construct a mirror *j*. Drag the points defining the mirror to make the mirror vertical.

5. Construct the reflection of point x across the mirror.

Match the tool’s glowing point *x* to independent variable *x*.

The resulting point, *rj*(*x*), is the dependent variable. You can read its name as “the reflection across line *j* of independent variable *x*.”

**Q1** Make sure the mirror is vertical, and then drag point *x* up. Which way does *rj*(*x*) go? Drag *x* left. Which way does *rj*(*x*) go?

6. Turn on tracing for points *x* and *rj*(*x*).Deselect both points and then drag independent point *x* to trace out an interesting shape.

**Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing showing your shapes.

7. Erase the traces and trace a new shape that goes across the mirror.

**Q3** What happens when you drag *x* across the mirror? Describe the traced shapes, and include a drawing on your paper.

Match the Traces

8. On page 3, construct a segment and construct point *x*, attaching it to the segment (but not to either endpoint or the midpoint).

When you attach independent variable *x* to a segment, the segment is called the *domain* of *x*.

9. Construct a mirror, reflect point x, and make a button to animate point x.

When x moves along its domain, the path of the dependent variable *rj*(*x*) is called the range.

**Q4** Each picture below was made by animating point *x* with traces turned on. Arrange your segment and mirror so that animating *x* produces traces like the ones in the pictures. On each picture, draw a line to show where you decided to put the mirror.

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**Q5** Describe the method(s) you used to place the mirrors. Did you develop new methods, or figure out any tricks, as you did these five challenges?

Restrict the Independent Variable to a Polygon

10. On page 4, construct a polygon to be the domain, and construct point *x* on the polygon. Then construct a mirror and reflect independent variable *x*.

**Q6** Experiment with the polygon and the mirror so that the traces make interesting shapes when you drag or animate independent variable *x.*

**Q7** A *fixed point* of a function is a point where the independent and dependent variables meet. Arrange the domain (the polygon) and the mirror so your reflection function has two fixed points. Draw a sketch of the arrangement you used.

**Q8** Arrange the domain so that there’s one side that makes both variables move in the same direction, one side that makes them move in opposite directions, and one side that makes them move in directions that are perpendicular. Draw a sketch of the arrangement you used.

|  |  |  |
| --- | --- | --- |
| same | opposite | perpendicular |

**Q9** How does the range compare to the domain? What features are similar, and what features are different? How do these features relate to the relative motions?

Reflection Challenges

**Q10** On page 5, locate the mirror that reflects the red domain to the blue range. Draw your solution, and use the widget in the corner of the sketch to measure the angle of the mirror you used.

**Q11** On pages 6 and 7, there are two more find-the-hidden-mirror challenges. Solve them, and describe your methods. What makes one challenge harder than another?

**Q12** On page 8, find the mirror that reflects independent variable *x* to dependent variable *f*(*x*). mirror. Draw your solution, and use the widget in the corner of the sketch to measure the angle of the mirror you used.

The Reflection Family websketch is here: http://geometricfunctions.org/curriculum/reflection-family/

**Q1** Make sure the mirror is vertical, and then drag point *x* up. Which way does *rj*(*x*) go? Drag x left. Which way does *rj*(*x*) go?

**Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. Make a drawing showing your shapes.

**Q3** What happens when you drag *x* across the mirror? Describe the traced shapes, and make a drawing of them.

**Q4** Each picture below was made by animating point *x* with traces turned on. Arrange your segment and mirror so that animating *x* produces traces like the ones in the pictures. On each picture, draw a line to show where you decided to put the mirror.

|  |  |  |  |  |
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|  |  |  |  |  |

**Q5** Describe the method(s) you used to place the mirrors. Did you develop new methods, or figure out any tricks, as you did these five challenges?

**Q6** Experiment with the polygon and the mirror so that the traces make interesting shapes when you drag or animate independent variable x.

**Q7** A *fixed point* of a function is a point where the independent and dependent variables meet. Arrange the domain (the polygon) and the mirror so your reflection function has two fixed points. Draw a sketch of the arrangement you used.

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**1.** Describe one important thing you learned today about the reflection function family.

**2.** Describe one thing about the reflection function family that seems confusing to you.