A simple optimization problem using OPTAX

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1 Problem Statement

Given a force F:

$$\begin{aligned} F &= K(x)x + \frac{\partial K}{\partial x}x, \\ K(x) &= Ax^2 + Bx + C \end{aligned} \tag{1}$$

subject to the following constraints:

$$F = F_0 = 10, \text{ when } x = x_0 = 0.2 \tag{2}$$

Find the values of $A, B,$ and C such that Eq.1 holds true for Eq.2.

2 Solution

We begin by converting the given problem into an optimization problem. Let us define the parameters of interest as $\boldsymbol{\theta} = \{A, B, C\}$. Further let us define the loss function as a mean squared error based on the constraints given by Eq.2:

$$L(\boldsymbol{\theta}) = \left(F_0 - \left(F(\boldsymbol{\theta}, \boldsymbol{x_0})x_0 + \frac{\partial F(\boldsymbol{\theta}, x)}{\partial x}\bigg|_{x=x_0} \right) \right)^2. \tag{3}$$

Now our problem has been converted to an optimization problem. From an intial guess for $\boldsymbol{\theta}$ we can use gradient descent and/or its variations to minimize $L(\boldsymbol{\theta})$.

2.1 Gradient descent

2.2 Code

2.3 Results