

Financial Mathematics 33000

Lecture 8

Roger Lee

2022 November 30

The implied volatility skew

Realized Volatility

Realized variance of S , sampled at interval Δt , from time 0 to time T can be defined, using log-returns by letting $t_n = n\Delta t$ and $T = t_N$ and

$$RVar = \frac{1}{T} \sum_{n=0}^{N-1} \left(\log \frac{S_{t_{n+1}}}{S_{t_n}} \right)^2$$

Alternatively could use simple returns, letting $\Delta S = S_{t_{n+1}} - S_{t_n}$ and

$$RVar = \frac{1}{T} \sum_{n=0}^{N-1} \left(\frac{\Delta S}{S_{t_n}} \right)^2$$

Realized volatility of S is

$$RVol = \sqrt{RVar}$$

If $dS_t = \mu S_t dt + \sigma S_t dW_t$, then $RVol \rightarrow \sigma$ as $\Delta t \rightarrow 0$.

Implied Volatility

Given a time- t price C for a given call option (K, T) on an underlying S_t assuming interest rate r , the **implied volatility** is the σ such that

$$C = C^{BS}(S_t, t, K, T, r, \sigma)$$

where C^{BS} is the Black-Scholes formula.

- ▶ This exists and is unique, if
- ▶ The bigger the dollar price C , the bigger the implied vol σ_{imp}
- ▶ Gives is another way quoting an option price. Instead of saying the option is trading at \$x.xx, can say it's trading at yy% vol.

Interpretations

Interpretations of time- t implied volatility

- ▶ Intuitively, implied vol is in some sense the market's forward-looking expectation of "realized volatility" from time t until T , along paths that go near K .
- ▶ A language / a metric / a scale in which to quote option prices. Instead of quoting in dollars, can quote in vol points.
(Use of this language does not mean we actually believe the Black-Scholes assumptions!)

Volatility smile/skew

If the underlying truly has GBM dynamics with volatility σ then

$$C(K, T) = C^{BS}(K, T, \sigma) \quad \text{for all } (K, T)$$

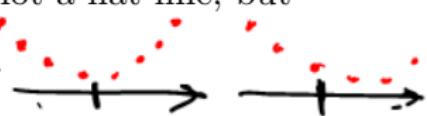
Hence

$$\sigma_{imp}(K, T) = \sigma \quad \text{for all } (K, T)$$



However, empirically,

- ▶ Plotting σ_{imp} against K , you typically see not a flat line, but rather a volatility *smile* or a volatility *skew*.
- ▶ Also, σ_{imp} varies wrt T ; implied volatility has a *term structure*.
- ▶ The function $\sigma_{imp}(K, T)$ is called the implied vol *surface* or *skew*.



Volatility surface: SPX, by strike

SPX Index		Asset		Actions		Views		Settings		Volatility Surface													
S&P 500 INDEX		4720.29 USD	Bloomberg	• Mid	As of	<12-Jan-2022	15:35	•															
Vol Table		3D Surface	3 Term	4 Skew	9 Dividends	8 Prices																	
Strike	• Listed	• Edit	<input checked="" type="checkbox"/> Fwd	<input checked="" type="checkbox"/> Strikes																			
Exp Date	ImpFwd	4670.0	4675.0	4680.0	4685.0	4690.0	4695.0	4700.0	4705.0	4710.0	4715.0	4720.0	4725.0	4730.0	4735.0	4740.0	4745.0	4750.0	4755.0	4760.0	4765.0	4770.0	4775.0
		4670.0	4675.0	4680.0	4685.0	4690.0	4695.0	4700.0	4705.0	4710.0	4715.0	4720.0	4725.0	4730.0	4735.0	4740.0	4745.0	4750.0	4755.0	4760.0	4765.0	4770.0	4775.0
14 Jan 2022	4719.93	18.92	18.66	18.40	18.14	17.88	17.61	17.35	17.07	16.79	16.49	16.18	15.89	15.67	15.53	15.41	15.29	15.16	15.04	14.92	14.82	14.73	14.66
18 Jan 2022	4719.83	13.84	13.68	13.52	13.36	13.19	13.01	12.84	12.65	12.47	12.27	12.07	11.88	11.70	11.57	11.46	11.35	11.24	11.14	11.04	10.95	10.87	10.80
19 Jan 2022	4719.86	14.40	14.24	14.07	13.90	13.73	13.54	13.36	13.17	12.98	12.78	12.58	12.37	12.18	12.01	11.87	11.74	11.62	11.50	11.38	11.27	11.17	11.08
21 Jan 2022	4719.69	14.83	14.64	14.46	14.27	14.08	13.89	13.70	13.51	13.32	13.14	12.95	12.77	12.60	12.43	12.27	12.12	11.97	11.83	11.70	11.58	11.47	11.37
24 Jan 2022	4719.61	14.62	14.45	14.29	14.12	13.95	13.78	13.61	13.44	13.28	13.11	12.95	12.78	12.62	12.47	12.31	12.16	12.02	11.88	11.75	11.62	11.50	11.38
26 Jan 2022	4719.58	15.66	15.49	15.33	15.16	14.99	14.83	14.66	14.49	14.33	14.16	13.99	13.83	13.67	13.51	13.35	13.20	13.04	12.90	12.75	12.61	12.48	12.35
28 Jan 2022	4719.11	16.16	16.00	15.84	15.67	15.51	15.34	15.17	15.01	14.84	14.67	14.51	14.34	14.18	14.01	13.85	13.70	13.54	13.39	13.24	13.09	12.95	12.81
31 Jan 2022	4718.92	15.55	15.40	15.25	15.09	14.94	14.78	14.63	14.47	14.32	14.16	14.00	13.85	13.70	13.54	13.39	13.24	13.10	12.95	12.81	12.67	12.54	12.41
2 Feb 2022	4718.98	15.76	15.61	15.46	15.31	15.15	15.00	14.85	14.70	14.54	14.39	14.24	14.09	13.93	13.79	13.64	13.49	13.35	13.20	13.06	12.93	12.79	12.66
4 Feb 2022	4718.25	15.92	15.78	15.63	15.48	15.33	15.18	15.04	14.89	14.74	14.59	14.44	14.29	14.15	14.00	13.86	13.72	13.57	13.44	13.30	13.16	13.03	12.90
7 Feb 2022	4718.32	15.46	15.32	15.18	15.04	14.90	14.76	14.62	14.48	14.34	14.20	14.06	13.92	13.78	13.64	13.50	13.37	13.23	13.10	12.97	12.84	12.72	12.59
9 Feb 2022	4717.54	15.59	15.45	15.32	15.18	15.04	14.90	14.76	14.62	14.48	14.34	14.20	14.07	13.93	13.79	13.66	13.52	13.39	13.26	13.13	13.01	12.88	12.76
11 Feb 2022	4717.17	15.79	15.66	15.52	15.39	15.25	15.12	14.98	14.85	14.71	14.57	14.44	14.31	14.17	14.04	13.91	13.78	13.65	13.52	13.39	13.27	13.14	13.02
18 Feb 2022	4715.36	15.70	15.58	15.45	15.33	15.20	15.07	14.95	14.82	14.69	14.57	14.44	14.32	14.19	14.07	13.95	13.82	13.70	13.58	13.46	13.35	13.23	13.12
25 Feb 2022	4714.54	15.80	15.68	15.57	15.45	15.33	15.22	15.10	14.99	14.87	14.75	14.64	14.52	14.41	14.29	14.18	14.07	13.95	13.84	13.73	13.62	13.52	13.41
28 Feb 2022	4714.09	15.63	15.51	15.40	15.29	15.17	15.06	14.95	14.83	14.72	14.61	14.50	14.38	14.27	14.16	14.05	13.94	13.83	13.72	13.61	13.51	13.40	13.30
18 Mar 2022	4711.30	16.36	16.26	16.16	16.06	15.96	15.86	15.76	15.66	15.56	15.46	15.36	15.27	15.17	15.07	14.97	14.87	14.77	14.68	14.58	14.49	14.39	14.30
31 Mar 2022	4710.35	16.73	16.63	16.54	16.45	16.36	16.26	16.17	16.08	15.98	15.88	15.80	15.71	15.61	15.52	15.43	15.34	15.24	15.15	15.06	14.97	14.88	14.79
14 Apr 2022	4709.51	17.03	16.94	16.85	16.77	16.68	16.59	16.50	16.42	16.33	16.24	16.15	16.07	15.98	15.89	15.81	15.72	15.63	15.55	15.46	15.38	15.30	15.21
29 Apr 2022	4709.62	17.39	17.31	17.23	17.15	17.07	16.99	16.91	16.83	16.75	16.67	16.59	16.51	16.43	16.35	16.27	16.19	16.11	16.03	15.95	15.87	15.79	15.72
20 May 2022	4706.54	17.83	17.75	17.68	17.61	17.53	17.46	17.39	17.31	17.24	17.16	17.09	17.02	16.94	16.87	16.80	16.72	16.65	16.58	16.50	16.43	16.36	16.29
31 May 2022	4705.83	17.94	17.86	17.79	17.72	17.65	17.58	17.51	17.44	17.37	17.29	17.22	17.15	17.08	17.01	16.94	16.87	16.80	16.73	16.66	16.59	16.52	16.45
17 Jun 2022	4703.73	18.31	18.24	18.17	18.11	18.04	17.97	17.90	17.83	17.77	17.70	17.63	17.56	17.50	17.43	17.36	17.30	17.23	17.16	17.10	17.03	16.96	16.90
30 Jun 2022	4703.82	18.46	18.40	18.33	18.27	18.20	18.14	18.07	18.00	17.94	17.87	17.81	17.74	17.68	17.61	17.55	17.48	17.42	17.35	17.28	17.22	17.16	17.09
15 Jul 2022	4703.38	18.62	18.56	18.49	18.43	18.36	18.30	18.23	18.17	18.10	18.04	17.97	17.91	17.84	17.78	17.72	17.65	17.59	17.53	17.46	17.40	17.34	17.28
19 Aug 2022	4701.68	19.08	19.02	18.96	18.90	18.84	18.78	18.72	18.66	18.60	18.54	18.48	18.42	18.36	18.30	18.24	18.18	18.12	18.06	18.00	17.94	17.89	17.83
16 Sep 2022	4699.69	19.34	19.28	19.23	19.17	19.11	19.05	19.00	18.94	18.88	18.83	18.77	18.71	18.66	18.60	18.54	18.49	18.43	18.37	18.32	18.26	18.21	18.15
30 Sep 2022	4700.41	19.52	19.47	19.41	19.36	19.30	19.25	19.19	19.14	19.08	19.03	18.98	18.92	18.87	18.81	18.76	18.70	18.65	18.59	18.54	18.49	18.43	18.38
21 Oct 2022	4700.74	19.67	19.62	19.57	19.51	19.46	19.40	19.35	19.30	19.24	19.19	19.08	19.03	18.97	18.92	18.87	18.81	18.76	18.71	18.66	18.60	18.55	

Option Pricing (OVNE) Legend

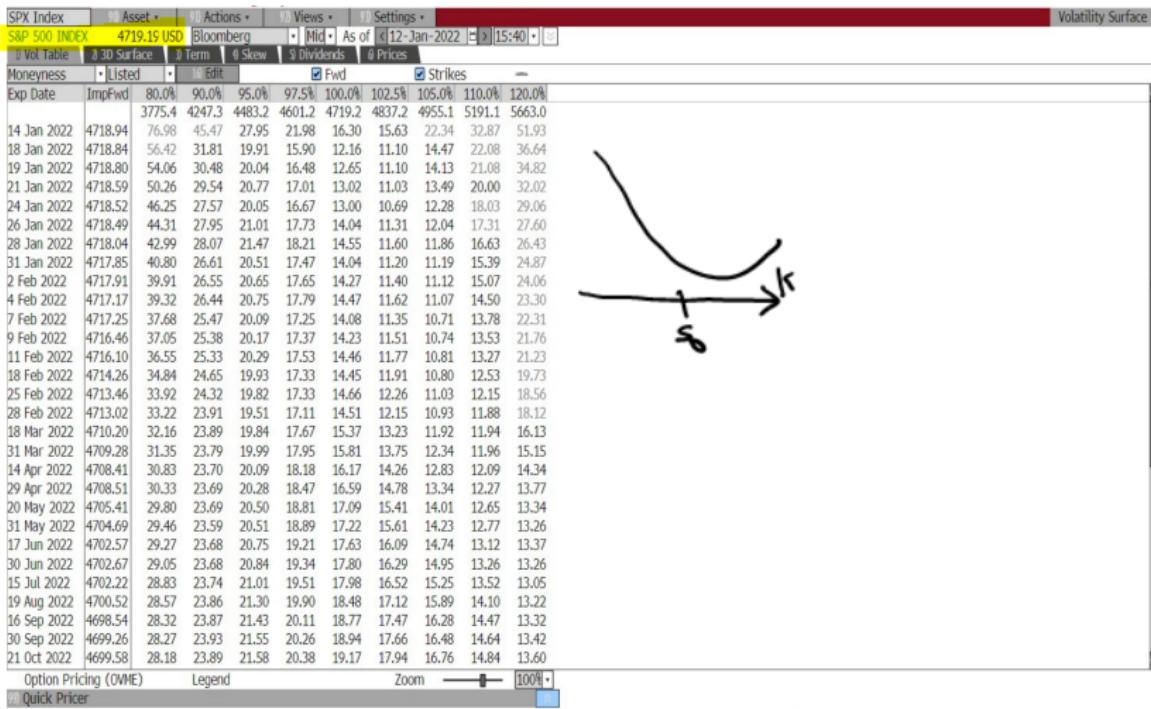
Zoom 100%

SN 417340 EST GMT-5:00 G721-6557-166 12-Jan-2022 15:56:46

The implied volatility skew

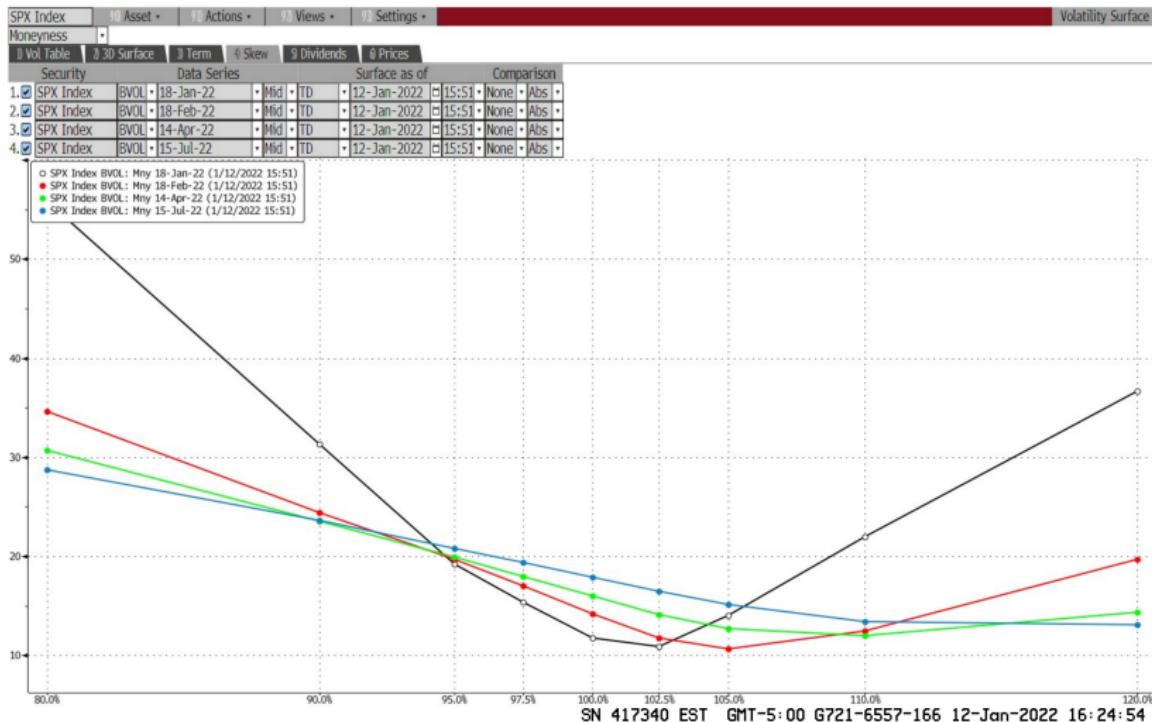
oooooooooooooooo

Volatility surface: SPX, by moneyness



SN 417340 EST GMT-5:00 G721-6557-166 12-Jan-2022 15:58:14

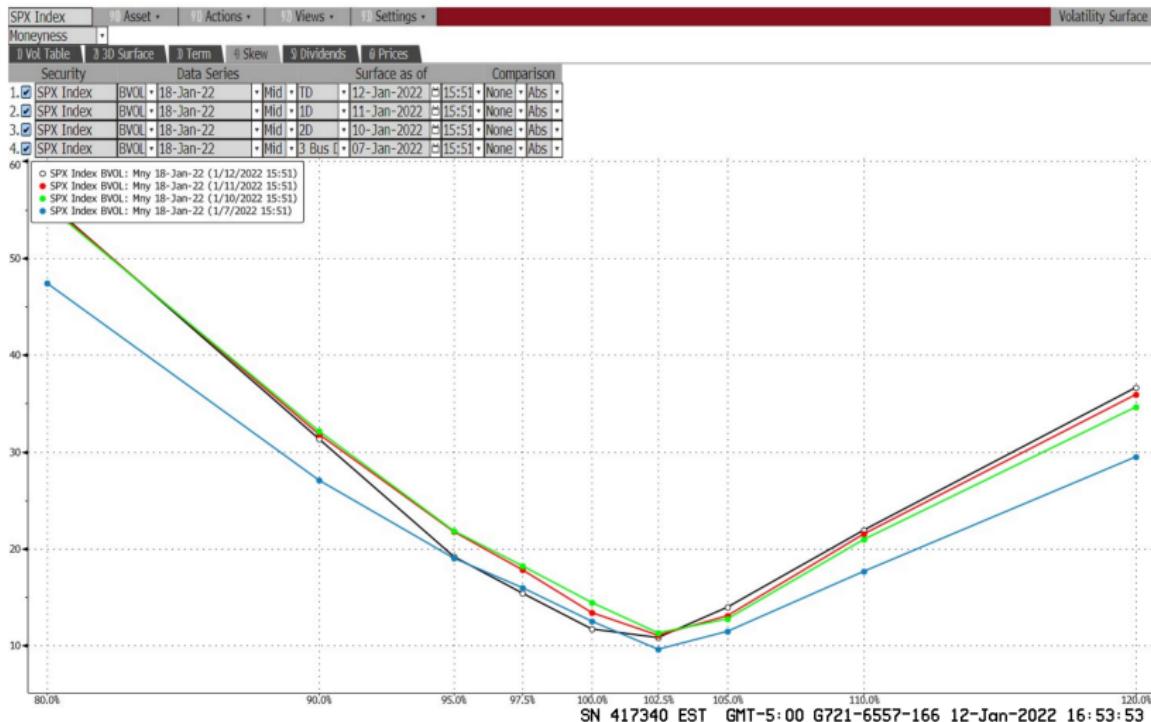
Volatility skews: SPX, by moneyness, 4 expirations



The implied volatility skew



Volatility skews: SPX, by moneyness, 4 observations



The implied volatility skew



Volatility smile/skew: EBAY

lower bound: $S - K e^{-r(T-t)}$

Apr 1

Buy call

short stock.

Collect

\$39.91

↓ July

\$40.XX

EBAY US \$ * ! 74.61 -1.59 B 1s										Equity OCM							
DELAY 12:26 Vol 5,752,860 Op 76.6 Q Hi 76.68 Q Lo 74.07 Q ValTrd 433.120m				OPTION MONITOR 1 COMP Center: 74.12 1 <GO> to Edit Spreadsheet													
BID	dASK	divBD	IVAS	IVMD	OPIN d	EBAY				BID	dASK	divBD	IVAS	IVMD	OPIN d		
Bid	Ask	Imp	Imp	Imp		EBAY				Bid	Ask	Imp	Imp	Imp			
Price	Price	Volat	Volat	Volat	Open	<-CALLS Bid Ask Volat Volat Open				Price	Price	Volat	Volat	Open			
Price	Price	Bid	Ask	Mid	Intrst	<-PUTS-> Price Price Bid Ask Mid Intrst				Price	Price	Bid	Ask	Mid	Intrst		
74.61074.620	74.61074.620					QXB JUL4 74.61074.620											
44.60	44.80	N.A.	69.07	N.A.		335	130	16		.05	N.A.	74.86	69.02	478			
39.60	39.80	N.A.	70.07	54.16		305	235	17		.05	N.A.	62.87	57.97	453			
34.70	34.80	N.A.	57.73	36.43		378	340	18		.05	N.A.	52.53	48.44	1,169			
29.70	29.90	N.A.	52.91	46.90		355	445	19	.05	.10	43.44	47.80	45.89	735			
24.80	25.00	37.23	46.40	42.65		514	550	20	.15	.20	41.55	43.68	42.61	1,559			
20.00	20.20	36.56	41.46	39.18		889	655	21	.30	.35	38.88	40.05	39.50	2,205			
						XBA JUL4											
15.40	15.50	34.85	37.76	36.34		1,931	760	22	.65	.75	35.56	37.06	36.31	4,757			
11.10	11.30	33.40	34.36	33.88		3,851	865	23	1.40	1.45	33.46	34.45	33.95	9,285			
7.40	7.60	31.33	32.79	32.06		3,313	970	24	2.65	2.75	31.84	32.56	32.20	4,905			
4.50	4.60	29.70	31.03	30.37		7,000	1075	25	4.70	4.80	30.15	30.81	30.48	2,415			
2.45	2.55	29.16	29.88	29.52		2,166	11080	26	7.60	7.80	28.98	30.39	29.68	619			
1.20	1.30	28.78	29.25	29.02		1,429	12085	27	11.40	11.60	27.94	29.89	28.92	209			
.55	.65	28.82	29.50	29.16		1,115	13090	28	15.70	15.90	27.27	30.24	28.80				
.20	.30	27.69	30.02	28.92		16	1495	29	20.40	20.70	N.A.	32.77	29.60				
.10	.15	28.66	30.60	29.69		305	15100	30	25.30	25.50	N.A.	32.83	30.88				

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
 Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2004 Bloomberg L.P.
 G263-945-0 13-Apr-04 12:41:14

Volatility smile/skew: GOOG

skip

GOOG US \$	↑ 555.40	+ .50		Q555.20 / 555.37Q	1x1																																																																																																																																																																																																																																																																																																																																																																																																																																																																		
.... At 12:19 d Vol 1,878,854 0 559.62Q H 562.00P L 552.95P Val 1.048B																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
GOOG US Equity 95) Templates 90) Actions 97) Expiry Option Monitor: Option Monitor																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
.... GOOGLE INC-C 1555.4001 .5001 .0901% 555.20 / 555.3701 Hi 562.00 Lo 552.95 Voln 1878854 HV.00																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
<input type="checkbox"/> Calc Mode Center 555.00 Strikes 19 Exch US Composit	92) Next Earnings(EM) 04/16/14 C																																																																																																																																																																																																																																																																																																																																																																																																																																																																						
81) Center Strike 82) Calls/Puts 83) Calls 84) Puts 85) Term Structure																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
<table border="1"> <thead> <tr> <th colspan="6">Calls</th> <th colspan="6">Puts</th> </tr> <tr> <th>Ticker</th><th>Bid</th><th>Ask</th><th>Last</th><th>IVM</th><th>DM</th><th>Volm</th><th>OInt</th> <th>Ticker</th><th>Bid</th><th>Ask</th><th>Last</th><th>IVM</th><th>DM</th><th>Volm</th><th>OInt</th> </tr> </thead> <tbody> <tr> <td>19 Apr 14 (10d); CSize 100; IDiv .77; R .12; IFwd 556.55</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> <td>19 Apr 14 (10d); CSize 100; IDiv .77; R .12; IFwd 556.55</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>1) GOOG 4 CS10 48.20 50.80 50.00 58.97 .84 2 34</td><td>510.00</td><td>58) GOOG 4 P510</td><td>3.20</td><td>3.60</td><td>3.08</td><td>55.30</td><td>-.14</td> <td>69 203</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>2) GOOG 4 CS15 43.70 46.80 43.00 57.94 .82</td><td>37</td><td>515.00</td><td>59) GOOG 4 P515</td><td>4.10</td><td>4.40</td><td>4.24</td><td>55.20</td><td>-.17</td><td>28 304</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>3) GOOG 4 CS20 39.90 40.60 44.40 53.44 .81</td><td>11 35</td><td>520.00</td><td>60) GOOG 4 P520</td><td>4.90</td><td>5.30</td><td>4.90</td><td>54.18</td><td>-.20</td><td>13 485</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>4) GOOG 4 CS25 36.20 37.70 40.00 55.38 .76</td><td>1 58</td><td>525.00</td><td>61) GOOG 4 P525</td><td>6.00</td><td>6.40</td><td>6.14</td><td>53.70</td><td>-.23</td><td>27 163</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>5) GOOG 4 CS30 32.40 33.50 34.00 54.21 .73</td><td>1 76</td><td>530.00</td><td>62) GOOG 4 P530</td><td>7.10</td><td>7.70</td><td>7.05</td><td>52.96</td><td>-.27</td><td>14 183</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>6) GOOG 4 CS35 28.90 29.80 30.56 53.31 .69</td><td>45 73</td><td>535.00</td><td>63) GOOG 4 P535</td><td>8.70</td><td>9.10</td><td>7.80</td><td>52.84</td><td>-.30</td><td>45 348</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>7) GOOG 4 CS40 26.50 26.50 27.52 53.30 .65</td><td>12 214</td><td>540.00</td><td>64) GOOG 4 P540</td><td>10.40</td><td>10.80</td><td>9.91</td><td>52.60</td><td>-.35</td><td>26 249</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>8) GOOG 4 CS45 22.50 23.30 23.70 52.81 .61</td><td>18 168</td><td>545.00</td><td>65) GOOG 4 P545</td><td>12.10</td><td>12.70</td><td>12.30</td><td>51.82</td><td>-.39</td><td>17 148</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>9) GOOG 4 CS50 19.60 20.20 20.00 52.13 .56</td><td>168 358</td><td>550.00</td><td>66) GOOG 4 P550</td><td>14.30</td><td>14.80</td><td>13.80</td><td>51.63</td><td>-.44</td><td>84 155</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>10) GOOG 4 CS55 17.00 17.50 17.95 51.84 .52</td><td>192 256</td><td>555.00</td><td>67) GOOG 4 P555</td><td>16.60</td><td>17.10</td><td>16.50</td><td>51.18</td><td>-.48</td><td>41 159</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>11) GOOG 4 CS60 14.60 15.00 15.41 51.29 .47</td><td>472 694</td><td>560.00</td><td>68) GOOG 4 P560</td><td>19.20</td><td>19.70</td><td>18.61</td><td>51.09</td><td>-.53</td><td>101 113</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>12) GOOG 4 CS65 12.40 13.00 13.14 51.28 .43</td><td>115 321</td><td>565.00</td><td>69) GOOG 4 P565</td><td>21.80</td><td>22.50</td><td>20.50</td><td>50.64</td><td>-.58</td><td>1 29</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>13) GOOG 4 CS70 10.50 10.70 10.51 50.74 .38</td><td>154 652</td><td>570.00</td><td>70) GOOG 4 P570</td><td>24.90</td><td>25.70</td><td>23.82</td><td>50.26</td><td>-.62</td><td>1 51</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>14) GOOG 4 CS75 8.80 9.20 9.20 50.26 .34</td><td>244 317</td><td>575.00</td><td>71) GOOG 4 P575</td><td>28.00</td><td>28.80</td><td>31.60y</td><td>49.87</td><td>-.67</td><td>8</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>15) GOOG 4 CS80 7.30 7.80 7.50 50.28 .30</td><td>124 431</td><td>580.00</td><td>72) GOOG 4 P580</td><td>31.40</td><td>32.30</td><td>29.10</td><td>49.19</td><td>-.71</td><td>11 47</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>16) GOOG 4 CS85 5.90 6.30 6.43 50.02 .25</td><td>28 211</td><td>585.00</td><td>73) GOOG 4 P585</td><td>34.80</td><td>36.00</td><td>38.00y</td><td>48.78</td><td>-.75</td><td>11</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>17) GOOG 4 CS90 4.80 5.30 5.07 50.20 .22</td><td>71 333</td><td>590.00</td><td>74) GOOG 4 P590</td><td>37.70</td><td>40.20</td><td>36.23</td><td>46.97</td><td>-.80</td><td>8 21</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>18) GOOG 4 CS95 3.90 4.40 4.15 50.05 .19</td><td>91 136</td><td>595.00</td><td>75) GOOG 4 P595</td><td>41.60</td><td>44.10</td><td>40.26</td><td>44.85</td><td>-.84</td><td>8 13</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>19) GOOG 4 C600 3.20 3.60 3.39 49.94 .16</td><td>80 837</td><td>600.00</td><td>76) GOOG 4 P600</td><td>45.90</td><td>48.30</td><td>63.95y</td><td>44.80</td><td>-.87</td><td>35</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>17 May 14 (38d); CSize 100; IDiv .23; R .16; IFwd 556.62</td><td>19</td><td colspan="5">17 May 14 (38d); CSize 100; IDiv .23; R .16; IFwd 556.62</td><td colspan="5"></td><td colspan="5"></td></tr> <tr> <td>20) GOOG 5 CS10 52.10 54.80 51.30y 36.10 .79</td><td>15 510.00</td><td>77) GOOG 5 P510</td><td>6.70</td><td>7.20</td><td>5.80</td><td>33.61</td><td>-.20</td><td>6 323</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr> <td>93) Default color legend</td><td colspan="5"></td><td>Zoom</td><td colspan="5"></td><td>90%</td><td colspan="5"></td></tr> <tr> <td>Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000</td><td colspan="5"></td><td>Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000</td><td colspan="5"></td><td>Copyright 2014 Bloomberg Finance L.P.</td><td colspan="5"></td></tr> <tr> <td>SN 785509 EDT GMT-4:00 H177-3818-0 09-Apr-2014 12:34:21</td><td colspan="5"></td><td colspan="5"></td><td colspan="5"></td><td colspan="2"></td></tr> </tbody> </table>	Calls						Puts						Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	19 Apr 14 (10d); CSize 100; IDiv .77; R .12; IFwd 556.55								19 Apr 14 (10d); CSize 100; IDiv .77; R .12; IFwd 556.55								1) GOOG 4 CS10 48.20 50.80 50.00 58.97 .84 2 34	510.00	58) GOOG 4 P510	3.20	3.60	3.08	55.30	-.14	69 203									2) GOOG 4 CS15 43.70 46.80 43.00 57.94 .82	37	515.00	59) GOOG 4 P515	4.10	4.40	4.24	55.20	-.17	28 304								3) GOOG 4 CS20 39.90 40.60 44.40 53.44 .81	11 35	520.00	60) GOOG 4 P520	4.90	5.30	4.90	54.18	-.20	13 485								4) GOOG 4 CS25 36.20 37.70 40.00 55.38 .76	1 58	525.00	61) GOOG 4 P525	6.00	6.40	6.14	53.70	-.23	27 163								5) GOOG 4 CS30 32.40 33.50 34.00 54.21 .73	1 76	530.00	62) GOOG 4 P530	7.10	7.70	7.05	52.96	-.27	14 183								6) GOOG 4 CS35 28.90 29.80 30.56 53.31 .69	45 73	535.00	63) GOOG 4 P535	8.70	9.10	7.80	52.84	-.30	45 348								7) GOOG 4 CS40 26.50 26.50 27.52 53.30 .65	12 214	540.00	64) GOOG 4 P540	10.40	10.80	9.91	52.60	-.35	26 249								8) GOOG 4 CS45 22.50 23.30 23.70 52.81 .61	18 168	545.00	65) GOOG 4 P545	12.10	12.70	12.30	51.82	-.39	17 148								9) GOOG 4 CS50 19.60 20.20 20.00 52.13 .56	168 358	550.00	66) GOOG 4 P550	14.30	14.80	13.80	51.63	-.44	84 155								10) GOOG 4 CS55 17.00 17.50 17.95 51.84 .52	192 256	555.00	67) GOOG 4 P555	16.60	17.10	16.50	51.18	-.48	41 159								11) GOOG 4 CS60 14.60 15.00 15.41 51.29 .47	472 694	560.00	68) GOOG 4 P560	19.20	19.70	18.61	51.09	-.53	101 113								12) GOOG 4 CS65 12.40 13.00 13.14 51.28 .43	115 321	565.00	69) GOOG 4 P565	21.80	22.50	20.50	50.64	-.58	1 29								13) GOOG 4 CS70 10.50 10.70 10.51 50.74 .38	154 652	570.00	70) GOOG 4 P570	24.90	25.70	23.82	50.26	-.62	1 51								14) GOOG 4 CS75 8.80 9.20 9.20 50.26 .34	244 317	575.00	71) GOOG 4 P575	28.00	28.80	31.60y	49.87	-.67	8								15) GOOG 4 CS80 7.30 7.80 7.50 50.28 .30	124 431	580.00	72) GOOG 4 P580	31.40	32.30	29.10	49.19	-.71	11 47								16) GOOG 4 CS85 5.90 6.30 6.43 50.02 .25	28 211	585.00	73) GOOG 4 P585	34.80	36.00	38.00y	48.78	-.75	11								17) GOOG 4 CS90 4.80 5.30 5.07 50.20 .22	71 333	590.00	74) GOOG 4 P590	37.70	40.20	36.23	46.97	-.80	8 21								18) GOOG 4 CS95 3.90 4.40 4.15 50.05 .19	91 136	595.00	75) GOOG 4 P595	41.60	44.10	40.26	44.85	-.84	8 13								19) GOOG 4 C600 3.20 3.60 3.39 49.94 .16	80 837	600.00	76) GOOG 4 P600	45.90	48.30	63.95y	44.80	-.87	35								17 May 14 (38d); CSize 100; IDiv .23; R .16; IFwd 556.62	19	17 May 14 (38d); CSize 100; IDiv .23; R .16; IFwd 556.62															20) GOOG 5 CS10 52.10 54.80 51.30y 36.10 .79	15 510.00	77) GOOG 5 P510	6.70	7.20	5.80	33.61	-.20	6 323									93) Default color legend						Zoom						90%						Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000						Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000						Copyright 2014 Bloomberg Finance L.P.						SN 785509 EDT GMT-4:00 H177-3818-0 09-Apr-2014 12:34:21																	
Calls						Puts																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt	Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt																																																																																																																																																																																																																																																																																																																																																																																																																																																								
19 Apr 14 (10d); CSize 100; IDiv .77; R .12; IFwd 556.55								19 Apr 14 (10d); CSize 100; IDiv .77; R .12; IFwd 556.55																																																																																																																																																																																																																																																																																																																																																																																																																																																															
1) GOOG 4 CS10 48.20 50.80 50.00 58.97 .84 2 34	510.00	58) GOOG 4 P510	3.20	3.60	3.08	55.30	-.14	69 203																																																																																																																																																																																																																																																																																																																																																																																																																																																															
2) GOOG 4 CS15 43.70 46.80 43.00 57.94 .82	37	515.00	59) GOOG 4 P515	4.10	4.40	4.24	55.20	-.17	28 304																																																																																																																																																																																																																																																																																																																																																																																																																																																														
3) GOOG 4 CS20 39.90 40.60 44.40 53.44 .81	11 35	520.00	60) GOOG 4 P520	4.90	5.30	4.90	54.18	-.20	13 485																																																																																																																																																																																																																																																																																																																																																																																																																																																														
4) GOOG 4 CS25 36.20 37.70 40.00 55.38 .76	1 58	525.00	61) GOOG 4 P525	6.00	6.40	6.14	53.70	-.23	27 163																																																																																																																																																																																																																																																																																																																																																																																																																																																														
5) GOOG 4 CS30 32.40 33.50 34.00 54.21 .73	1 76	530.00	62) GOOG 4 P530	7.10	7.70	7.05	52.96	-.27	14 183																																																																																																																																																																																																																																																																																																																																																																																																																																																														
6) GOOG 4 CS35 28.90 29.80 30.56 53.31 .69	45 73	535.00	63) GOOG 4 P535	8.70	9.10	7.80	52.84	-.30	45 348																																																																																																																																																																																																																																																																																																																																																																																																																																																														
7) GOOG 4 CS40 26.50 26.50 27.52 53.30 .65	12 214	540.00	64) GOOG 4 P540	10.40	10.80	9.91	52.60	-.35	26 249																																																																																																																																																																																																																																																																																																																																																																																																																																																														
8) GOOG 4 CS45 22.50 23.30 23.70 52.81 .61	18 168	545.00	65) GOOG 4 P545	12.10	12.70	12.30	51.82	-.39	17 148																																																																																																																																																																																																																																																																																																																																																																																																																																																														
9) GOOG 4 CS50 19.60 20.20 20.00 52.13 .56	168 358	550.00	66) GOOG 4 P550	14.30	14.80	13.80	51.63	-.44	84 155																																																																																																																																																																																																																																																																																																																																																																																																																																																														
10) GOOG 4 CS55 17.00 17.50 17.95 51.84 .52	192 256	555.00	67) GOOG 4 P555	16.60	17.10	16.50	51.18	-.48	41 159																																																																																																																																																																																																																																																																																																																																																																																																																																																														
11) GOOG 4 CS60 14.60 15.00 15.41 51.29 .47	472 694	560.00	68) GOOG 4 P560	19.20	19.70	18.61	51.09	-.53	101 113																																																																																																																																																																																																																																																																																																																																																																																																																																																														
12) GOOG 4 CS65 12.40 13.00 13.14 51.28 .43	115 321	565.00	69) GOOG 4 P565	21.80	22.50	20.50	50.64	-.58	1 29																																																																																																																																																																																																																																																																																																																																																																																																																																																														
13) GOOG 4 CS70 10.50 10.70 10.51 50.74 .38	154 652	570.00	70) GOOG 4 P570	24.90	25.70	23.82	50.26	-.62	1 51																																																																																																																																																																																																																																																																																																																																																																																																																																																														
14) GOOG 4 CS75 8.80 9.20 9.20 50.26 .34	244 317	575.00	71) GOOG 4 P575	28.00	28.80	31.60y	49.87	-.67	8																																																																																																																																																																																																																																																																																																																																																																																																																																																														
15) GOOG 4 CS80 7.30 7.80 7.50 50.28 .30	124 431	580.00	72) GOOG 4 P580	31.40	32.30	29.10	49.19	-.71	11 47																																																																																																																																																																																																																																																																																																																																																																																																																																																														
16) GOOG 4 CS85 5.90 6.30 6.43 50.02 .25	28 211	585.00	73) GOOG 4 P585	34.80	36.00	38.00y	48.78	-.75	11																																																																																																																																																																																																																																																																																																																																																																																																																																																														
17) GOOG 4 CS90 4.80 5.30 5.07 50.20 .22	71 333	590.00	74) GOOG 4 P590	37.70	40.20	36.23	46.97	-.80	8 21																																																																																																																																																																																																																																																																																																																																																																																																																																																														
18) GOOG 4 CS95 3.90 4.40 4.15 50.05 .19	91 136	595.00	75) GOOG 4 P595	41.60	44.10	40.26	44.85	-.84	8 13																																																																																																																																																																																																																																																																																																																																																																																																																																																														
19) GOOG 4 C600 3.20 3.60 3.39 49.94 .16	80 837	600.00	76) GOOG 4 P600	45.90	48.30	63.95y	44.80	-.87	35																																																																																																																																																																																																																																																																																																																																																																																																																																																														
17 May 14 (38d); CSize 100; IDiv .23; R .16; IFwd 556.62	19	17 May 14 (38d); CSize 100; IDiv .23; R .16; IFwd 556.62																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
20) GOOG 5 CS10 52.10 54.80 51.30y 36.10 .79	15 510.00	77) GOOG 5 P510	6.70	7.20	5.80	33.61	-.20	6 323																																																																																																																																																																																																																																																																																																																																																																																																																																																															
93) Default color legend						Zoom						90%																																																																																																																																																																																																																																																																																																																																																																																																																																																											
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000						Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000						Copyright 2014 Bloomberg Finance L.P.																																																																																																																																																																																																																																																																																																																																																																																																																																																											
SN 785509 EDT GMT-4:00 H177-3818-0 09-Apr-2014 12:34:21																																																																																																																																																																																																																																																																																																																																																																																																																																																																							

The implied volatility skew

oooooooooooo●ooooo

Volatility surface: GOOG

GOOG US \$ ↓ 556.80 +1.90 P556.65/556.84Q 2x1
... At 12:32 d Vol 1,926,365 0 559.62Q H 562.00P L 552.95P Val 1.075B
GOOG US Equity 95) Templates | 90) Actions | 97) Expiry | Option Monitor: Option Monitor
... GOOGLE INC-C ↓ 556.80 1.90 .3424% 556.6501 / 556.8401 Hi 562.00 Lo 552.95 Volm 1926365 HV.00
 Calc Mode Center 555.00 Strikes 19 Exch US Composit
 81) Center Strike 82) Calls/Puts 83) Calls 84) Puts 85) Term Structure

Expiry		19 Apr 2014				17 May 2014				21 Jun 2014				20 Sep 2014				17 Jan 2015			
Calls/Puts		Calls	Puts	Calls	Puts																
Strike	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM	DM	IVM			
505.00	.91	51.46	-.11	56.24	.83	33.50	-.17	34.07	.80	28.63	-.20	28.79	.74	26.46	-.26	26.02	.71	25.49	-.29	25.08	
510.00	.88	50.89	-.14	55.57	.81	33.13	-.20	34.25	.77	28.55	-.23	28.69	.72	26.38	-.28	26.03	.70	25.20	-.30	24.56	
515.00	.85	51.33	-.16	55.34	.79	32.84	-.22	33.86	.75	28.26	-.25	28.51	.70	26.34	-.29	25.96	.68	25.16	-.32	24.68	
520.00	.82	51.42	-.19	54.24	.76	33.23	-.24	33.63	.73	28.38	-.27	28.43	.68	26.45	-.31	25.86	.66	25.12	-.33	24.60	
525.00	.79	51.53	-.22	54.26	.73	33.14	-.27	33.50	.70	28.74	-.30	28.37	.66	26.29	-.33	25.68	.65	25.21	-.35	24.43	
530.00	.75	51.68	-.26	53.59	.70	33.56	-.30	33.37	.67	28.54	-.32	28.27	.64	26.18	-.35	25.61	.63	24.98	-.37	24.60	
535.00	.71	52.68	-.29	53.02	.67	33.28	-.33	33.16	.65	28.54	-.35	28.08	.62	26.24	-.37	25.49	.62	25.14	-.38	24.31	
540.00	.67	52.50	-.33	52.64	.63	33.20	-.37	32.79	.62	28.27	-.38	27.94	.60	26.08	-.39	25.46	.60	25.08	-.40	24.43	
545.00	.62	52.27	-.38	52.45	.60	32.95	-.40	32.84	.59	28.20	-.41	27.78	.58	26.14	-.42	25.45	.58	25.15	-.42	24.24	
550.00	.58	51.82	-.42	51.92	.57	32.76	-.43	32.58	.56	28.04	-.44	27.75	.56	25.98	-.44	25.37	.57	24.96	-.43	24.37	
555.00	.53	51.92	-.47	51.67	.53	32.61	-.47	32.44	.53	27.99	-.46	27.64	.54	26.03	-.46	25.25	.55	25.07	-.45	24.23	
560.00	.48	50.99	-.52	51.18	.50	32.35	-.50	32.40	.51	27.99	-.49	27.55	.52	25.97	-.48	25.30	.53	25.01	-.47	24.27	
565.00	.44	50.88	-.56	50.90	.46	32.34	-.54	32.05	.48	27.74	-.52	27.41	.50	25.95	-.50	25.28	.52	25.00	-.48	24.20	
570.00	.39	50.26	-.61	50.53	.43	32.02	-.57	32.02	.45	27.70	-.55	27.48	.48	25.86	-.52	25.22	.50	24.83	-.50	24.31	
575.00	.35	49.95	-.65	50.23	.39	31.85	-.61	31.75	.42	27.58	-.58	27.31	.46	25.80	-.54	25.13	.49	24.90	-.52	24.28	
580.00	.30	49.76	-.69	50.05	.36	31.95	-.64	31.76	.39	27.49	-.61	27.20	.40	25.71	-.56	25.17	.47	24.86	-.53	24.03	
585.00	.26	49.38	-.73	50.06	.33	31.86	-.67	31.66	.36	27.40	-.64	27.10	.42	25.63	-.58	25.28	.45	24.83	-.55	24.02	
590.00	.23	49.41	-.76	51.33	.30	31.71	-.70	31.47	.34	27.28	-.66	27.28	.40	25.76	-.60	25.23	.44	24.78	-.57	23.95	
595.00	.19	49.62	-.79	51.70	.27	31.30	-.73	31.21	.31	27.35	-.69	27.23	.38	25.89	-.62	25.18	.42	24.39	-.58	23.86	
600.00	.16	49.15	-.82	51.71	.24	31.34	-.76	31.59	.29	27.29	-.71	27.10	.37	25.86	-.64	25.20	.41	24.59	-.60	23.87	
605.00	.14	49.64	-.85	51.82	.22	31.06	-.78	31.40	.27	27.28	-.74	26.76	.35	25.87	-.66	25.24	.39	24.74	-.61	23.91	
610.00	.12	49.72	-.88	50.09	.19	31.02	-.80	31.89	.24	27.21	-.76	27.17	.33	25.94	-.68	25.11	.38	24.65	-.63	23.79	

83) Default color legend Zoom 85%
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2014 Bloomberg Finance L.P.
SN 785509 EDT GMT-4:00 H177-3818-0 09-Apr-2014 12:47:46

The implied volatility skew

oooooooooooo●oooo

Volatility surface: AMZN

AMZN US \$ ↓ **326.48** -.59 P326.36/326.51P 2x3
 At 13:08 d Vol 2,908,935 0 328.47Q H 328.49D L 322.50Q Val 947.826M
 AMZN US Equity 95) Templates | 96) Actions | 97) Expiry | Option Monitor: Option Monitor
 AMAZON.COM INC ↓326.48 -.59 -.180% 326.360 /326.51 Hi 328.49 Lo 322.50 Volm 2908935 HV 24.76
 Calc Mode Center 326.66 Strikes 5 Exch US Composit
 81) Center Strike 82) Calls/Puts 83) Calls 84) Puts 85) Term Structure

Expiry	19 Apr 2014				17 May 2014				21 Jun 2014				19 Jul 2014				18 Oct 2014			
Calls/Puts	Calls	Puts	Calls	Puts																
Strike	DM	IVM	DM	IVM																
280.00	.98	55.66	-.02	53.40	.86	48.25	-.14	46.67	.83	40.35	-.16	38.61	.81	36.77	-.18	35.63	.77	34.48	-.22	33.08
285.00	.98	47.38	-.03	50.74	.83	48.31	-.16	45.71	.81	39.40	-.19	38.04	.79	36.61	-.20	35.10	.75	34.06	-.24	32.75
290.00	.94	53.36	-.05	48.05	.81	47.17	-.19	44.94	.78	39.05	-.21	37.48	.76	36.34	-.23	34.44	.73	33.43	-.27	32.50
295.00	.93	47.70	-.06	45.69	.78	45.17	-.22	44.18	.75	38.71	-.24	37.02	.74	35.31	-.26	34.08	.71	32.87	-.29	32.11
300.00	.89	47.88	-.09	43.23	.75	44.56	-.25	43.42	.72	38.21	-.27	36.51	.71	34.87	-.29	34.00	.68	32.81	-.31	31.80
305.00	.85	45.62	-.13	41.09	.71	43.74	-.29	42.74	.69	37.40	-.31	36.07	.68	34.48	-.32	33.54	.66	32.30	-.34	31.68
310.00	.82	39.98	-.18	39.65	.67	43.80	-.33	42.16	.65	37.05	-.34	35.56	.65	33.86	-.35	33.14	.63	32.05	-.37	31.33
315.00	.74	38.38	-.26	38.31	.63	42.99	-.37	41.56	.61	36.59	-.38	35.35	.62	33.54	-.38	32.92	.61	32.05	-.39	31.12
320.00	.65	37.84	-.35	37.21	.58	42.54	-.42	41.19	.58	36.16	-.42	34.67	.58	33.56	-.42	32.66	.58	31.64	-.42	30.93
325.00	.54	36.78	-.46	36.31	.54	41.87	-.46	40.53	.54	35.78	-.46	34.46	.54	33.18	-.46	32.43	.55	31.49	-.45	30.69
330.00	.43	35.57	-.57	34.99	.49	41.35	-.51	40.00	.50	35.45	-.50	33.89	.51	33.00	-.49	32.16	.53	31.53	-.47	30.65
335.00	.32	34.82	-.69	34.19	.44	40.81	-.56	39.45	.46	35.01	-.54	33.67	.47	32.67	-.53	31.98	.50	31.29	-.50	30.59
340.00	.22	34.59	-.78	33.80	.40	40.58	-.60	39.57	.42	34.77	-.58	33.26	.44	32.63	-.57	31.62	.47	31.21	-.53	30.29
345.00	.15	34.47	-.85	34.14	.35	40.18	-.65	38.82	.39	34.63	-.61	33.73	.40	32.40	-.60	31.50	.45	31.09	-.56	30.05
350.00	.09	34.92	-.91	33.55	.31	40.07	-.70	38.37	.35	34.56	-.65	33.51	.37	32.30	-.63	31.43	.42	30.80	-.58	30.03
355.00	.06	35.90	-.97	28.83	.27	39.71	-.73	38.86	.31	34.42	-.70	32.51	.34	32.17	-.67	31.27	.40	30.93	-.61	30.04
360.00	.04	37.29	-.99	29.45	.24	39.64	-.77	38.67	.28	34.01	-.72	33.19	.31	32.09	-.70	31.04	.37	30.75	-.63	29.83
365.00	.03	38.73			.20	39.53	-.80	38.60	.25	33.81	-.75	32.76	.28	31.82	-.73	30.85	.35	30.49	-.66	29.60
370.00	.02	40.89	-1.0		.17	39.55	-.84	37.03	.22	33.96	-.79	31.97	.25	31.75	-.76	30.72	.32	30.37	-.68	29.49

83) Default color legend Zoom 100%

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2014 Bloomberg Finance L.P.
 SN 785509 EDT GMT-4:00 H177-3818-0 09-Apr-2014 13:23:54

The implied volatility skew

oooooooooooo●oooo

Volatility smile: possible explanations

skip

Why does the smile exist?

- ▶ The market prices options using a risk-neutral distribution of log-returns with *fatter tails* than the Normal (*excess kurtosis*).

What contributes to those fat tails?

- ▶ Clustering of volatility.
- ▶ Jumps introduce extreme outcomes into return distributions, especially short-term.

Volatility skew: possible explanations in equity markets

Why's the smile skewed? (Why fatter tails on *left*?) Physical causes:

- ▶ Negative correlation between price and instantaneous volatility.

Empirically, instantaneous volatility increases as price decreases.

This fattens the left tail of the distribution (negative skewness).

- ▶ Possibility of big down-jumps (crashes) also fattens the left tail.
- ▶ “The bull walks up the stairs, the bear jumps out the window.”

Combined with risk preferences:

- ▶ “Supply and demand”: Fear of downside leads to demand for protection/insurance, driving up the prices (hence implied vols) of OTM puts. And willingness to sell part of the upside leads (covered-) call writers to supply OTM calls, driving prices down.

Volatility skew: possible explanations in equity markets

Why is the smile skewed? Another way to think about it:

- ▶ Implied volatility at strike K depends on the option price, which depends on expected future volatility – specifically, on expected future volatility *along paths near K* . Why? Future volatility along paths away from K does not help the option holder, because linear payoffs (e.g. zero, forward) are insensitive to vol. Only the *convex* part of the payoff brings benefits from volatility.
- ▶ Future volatility along paths near downside strikes is likely higher than future volatility along paths near upside strikes.
So by the above logic, downside-strike options should trade higher above their intrinsic lower bound than upside-strike options do.