Gray-box Monitorability of Hyperproperties

The Case of Data Minimality

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RV Lectures @ ICTAC'20 – 30 Nov & 1 Dec 2020

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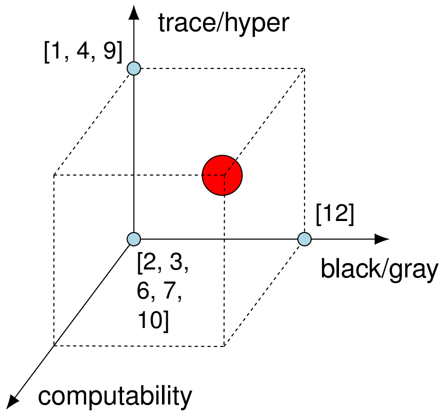
The monitorability cube



Motivation: distributed data minimality

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  - privacy property (GDPR)
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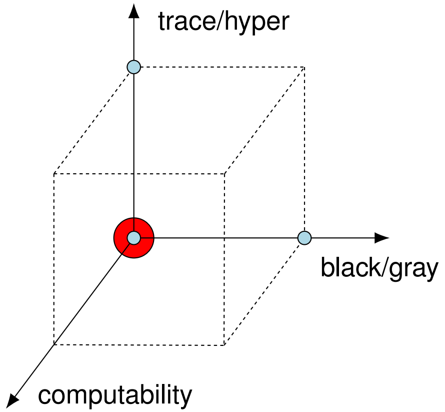
Yet, we have a monitor…

what's going on here?

Trace properties – LTL



Monitoring LTL

$\varphi_s = \Box$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \Box\Diamond$☕

Monitoring LTL

$\varphi_s = \Box$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \Box\Diamond$☕

- Observation: the world today at 10am

$u_{10} =$ ☕☕☕☕

Monitoring LTL

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- Observation: the world today at 10am

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- Update: the world at 11am

$u_{11} =$ ☕☕☕☕☕◯◯◯

Monitoring LTL

$\varphi_s = \square$☕ $\varphi_l = \lozenge$☕ $\varphi_r = \square\lozenge$☕

- Observation: the world today at 10am

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φ_s Is there always coffee?

Monitoring LTL

$\varphi_s = \Box$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \Box\Diamond$☕

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φ_s Is there always coffee? $u_{10} \rightarrow$ **?**

Monitoring LTL

$\varphi_s = \Box$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \Box\Diamond$☕

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φ_s Is there always coffee? $u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗

Monitoring LTL

$\varphi_s = \Box$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \Box\Diamond$☕

- Observation: the world today at 10am

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  $u_{11} =$ ☕☕☕☕☕⚪⚪⚪

φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓

Monitoring LTL

$\varphi_s = \Box$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \Box\Diamond$☕

- Observation: the world today at 10am

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φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓
φ_r	Is there always eventually coffee?	$u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ **?**

Monitoring LTL

$$\varphi_s = \Box ☕ \qquad \varphi_l = \Diamond ☕ \qquad \varphi_r = \Box\Diamond ☕$$

- Observation: the world today at 10am

  $u_{10} =$ ☕☕☕☕

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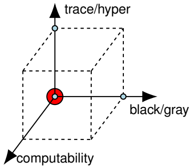
φ_s	Is there always coffee?	$u_{10} \to$ **?**, $u_{11} \to$ ✗
φ_l	Is there eventually coffee?	$u_{10} \to$ ✓, $u_{11} \to$ ✓
φ_r	Is there always eventually coffee?	$u_{10} \to$ **?**, $u_{11} \to$ **?**

A monitor for a property φ is a computable function $M_\varphi \colon \Sigma^* \to \{$✓, ✗, **?**$\}$ that decides whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

LTL – Summary



- Properties defined over individual traces.
  $\Rightarrow$ Properties describe sets of traces.
- Perfect monitors can be constructed for any formula.
- Not every formula is monitorable. For example,
  - safety and liveness properties are monitorable,
  - recurrence properties ($\square\diamond$) are not.

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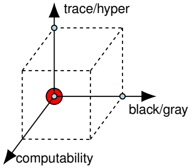

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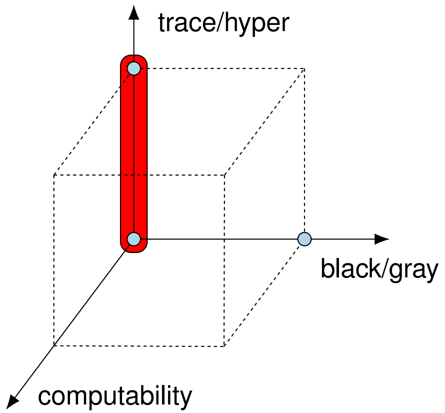
[6] Y. Falcone, J-C. Fernandez, and L. Mounier. *What can you verify and enforce at runtime?*, STTT 14(3), 2012.

[9] K. Havelund and D. Peled. *Runtime Verification: From Propositional to First-Order Temporal Logic.* RV'18, Springer, 2018.

... and many more!

Hyperproperties – HyperLTL



Hyperproperties – HyperLTL

$\varphi_u = \forall \pi . \forall \tau . \Box (☕_\pi \rightarrow ☕_\tau)$ $\qquad$ $\varphi_a = \forall \pi . \exists \tau . \Box (🍵_\pi \rightarrow ☕_\tau)$

Hyperproperties – HyperLTL

$$\varphi_u = \forall\pi.\forall\tau.\square(\text{☕}_\pi \to \text{☕}_\tau) \qquad \varphi_a = \forall\pi.\exists\tau.\square(\text{○}_\pi \to \text{☕}_\tau)$$

$$T_1 = \{\text{☕☕☕☕☕☕}\cdots\}$$

$$T_2 = \{\text{☕☕○○○○}\cdots,\ \text{○○☕☕☕☕}\cdots\}$$

$$T_3 = \{\text{☕○○○○○}\cdots,\ \text{☕☕○○○○}\cdots,\ \text{☕☕☕○○○}\cdots,\ \text{☕☕☕☕○○}\cdots,\ldots\}$$

$T_1 \models \varphi_u$	$T_1 \models \varphi_a$
$T_2 \not\models \varphi_u$	$T_2 \models \varphi_a$
$T_3 \not\models \varphi_u$	$T_3 \not\models \varphi_a$

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$$\varphi ::= \forall\pi.\varphi \mid \exists\pi.\varphi \mid \psi \qquad \psi ::= a_\pi \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc\psi \mid \psi\,\mathcal{U}\,\psi$$

$\Pi \models a_\pi$	iff	$a \in \Pi(\pi)[0]$
$\Pi \models \psi_1 \vee \psi_2$	iff	$\Pi \models \psi_1$ or $\Pi \models \psi_2$
$\Pi \models \neg\psi$	iff	$\Pi \not\models \psi$
$\Pi \models \bigcirc\psi$	iff	$\Pi[1..] \models \psi$
$\Pi \models \psi_1\,\mathcal{U}\,\psi_2$	iff	for some i, $\Pi[i,..] \models \psi_2$, and for all $j < i$ $T, \Pi[j,..] \models \psi_1$

$T, \Pi \models \forall\pi.\varphi$	iff	$T, \Pi[\pi \mapsto t] \models \varphi$ for all $t \in T$
$T, \Pi \models \exists\pi.\varphi$	iff	$T, \Pi[\pi \mapsto t] \models \varphi$ for some $t \in T$
$T, \Pi \models \psi$	iff	$\Pi \models \psi$

Hyperproperties – Relational HyperLTL

The temperature difference between two sensors never exceeds 5 °C.

$$\varphi_t = \forall\pi.\, \forall\tau.\, \Box\big(|t_\pi - t_\tau| \leq 5\big)$$

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A binary trace predicate that cannot be expressed using atomic propositions.

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A binary trace predicate that cannot be expressed using atomic propositions.

Non-interference:
Low-equivalent inputs evaluate to low-equivalent outputs.

$$\varphi_n = \forall \pi_1 . \forall \pi_2 . \left(\mathrm{in}(\pi_1) =_L \mathrm{in}(\pi_2) \rightarrow \mathrm{out}(\pi_1) =_L \mathrm{out}(\pi_2) \right)$$

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Given a signature $\sigma = (S, \mathrm{ar})$, for $r \in S$,

$$\varphi ::= \forall\pi.\varphi \mid \exists\pi.\varphi \mid \psi \qquad \psi ::= r(e,\ldots,e) \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc\psi \mid \psi\,\mathcal{U}\,\psi \qquad e ::= x_\pi$$

Given a σ-structure $\mathcal{A} = (|\mathcal{A}|, I)$,

$\Pi \models r(e_1,\ldots,e_n)$	iff	$I_{\mathcal{A}}(r)([\![e_1]\!]_\Pi,\ldots,[\![e_n]\!]_\Pi)$
$\Pi \models \psi_1 \vee \psi_2$	iff	$\Pi \models \psi_1$ or $\Pi \models \psi_2$
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$[\![x_\pi]\!]_\Pi$	$=$	$\Pi(\pi)[0](x)$
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Monitoring HyperLTL

$\varphi_u = \forall \pi . \forall \tau . \square (\text{☕}_\pi \rightarrow \text{☕}_\tau)$ $\varphi_a = \forall \pi . \exists \tau . \square (\text{◡}_\pi \rightarrow \text{☕}_\tau)$

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- Observation: the world today at 10am

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- Update: the world at 11am

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φ_u Is there always coffee everywhere at the same time?

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φ_u Is there always coffee everywhere at the same time? $U_{10} \to$ **?**, $U_{11} \to$ ✗

φ_a Is there always coffee somewhere? $U_{10} \to$ **?**, $U_{11} \to$ **?**

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Definition

A finite observation $U \in \mathcal{P}_{\textit{fin}}(\Sigma^*)$ permanently satisfies (resp. violates) φ, if every infinite extension of U satisfies (resp. violates) φ:

$$\begin{array}{rcl} U \text{ perm. satisfies } \varphi & \text{iff} & \text{all } T \in \mathcal{P}(\Sigma^\omega) \text{ such that } U \preceq T \text{ satisfy } \varphi \\ U \text{ perm. violates } \varphi & \text{iff} & \text{all } T \in \mathcal{P}(\Sigma^\omega) \text{ such that } U \preceq T \text{ violate } \varphi \end{array}$$

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$$U_{11} = \{\text{☕☕☕☕☕○○○},\ \text{○○○○☕☕☕☕},\ \text{☕☕○}\}$$

$$\begin{array}{lr} U_{11} & \text{doesn't perm. satisfy } \forall\pi.\forall\tau.\Box(\text{☕}_\pi \rightarrow \text{☕}_\tau) \\ U_{11} & \text{perm. violates } \forall\pi.\forall\tau.\Box(\text{☕}_\pi \rightarrow \text{☕}_\tau) \\ U_{11} & \text{neither perm. satisfies nor violates } \forall\pi.\exists\tau.\Box(\text{○}_\pi \rightarrow \text{☕}_\tau) \end{array}$$

Monitors for HyperLTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation U.

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The monitor M_φ is perfect if, additionally,

$M_\varphi(u) = ✓$ if U perm. satisfies φ, $M_\varphi(u) = ✗$ if U perm. violates φ,

$M_\varphi(u) =$ **?** o/w.

Monitorability of HyperLTL formulas

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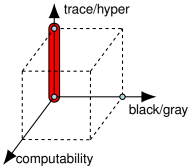
Definition (Agrawal & Bonakdarpour 2016)

A formula φ is *(semantically) monitorable* if every observation U has an extension $V \succeq U$, such that V perm. satisfies φ or V perm. violates φ.

HyperLTL – Summary



- Properties defined over sets of traces.
  $\Rightarrow$ Properties describe sets of sets of traces.
- Perfect monitors can be constructed for some formulas.
  - For example, for formulas without quantifier alternations.
  - But what about formulas with alternations?
- Most formulas are not monitorable.
  - For example, $\forall^+\exists^+$-properties are not!

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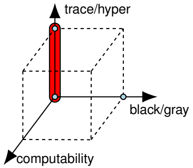

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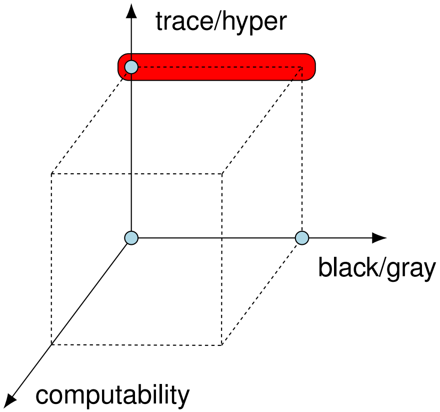
[1] S. Agrawal and B. Bonakdarpour. *Runtime Verification of k-Safety Hyperproperties in HyperLTL.* CSF'16, IEEE CS Press, 2016.

[9] K. Havelund and D. Peled. *Runtime Verification: From Propositional to First-Order Temporal Logic.* RV'18, Springer, 2018.

[8] C. Hahn. *Algorithms for Monitoring Hyperproperties.* RV'19, Springer, 2019.

Gray-box monitoring (of hyperproperties)



Why is φ_a not monitorable?

Theorem

No finite U permanently satisfies or violates $\varphi_a = \forall\pi.\exists\tau.\square(🍵_\pi \rightarrow ☕_\tau)$.

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Proof. Given any $U \in \mathcal{P}_{fin}(\Sigma^*)$,

U doesn't perm. violate φ_a $U \preceq \Sigma^\omega$, and $\Sigma^\omega \models \varphi_a$ because ☕☕☕$\cdots \in \Sigma^\omega$;

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then $U \preceq T$ and T violates φ_a. □

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This theorem can be generalized to all formulas $\varphi = \forall\pi.\exists\tau.\Box P(\pi, \tau)$ where P is

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OK, but let's have a closer look at this proof…

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When monitoring hyperproperties, we'd like to take into account
some information about the system
(gray-box monitoring).

Gray-box monitoring of HyperLTL properties

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation U.

Definition

A finite observation $U \in \mathcal{P}_{\mathit{fin}}(\Sigma^*)$ permanently satisfies (resp. violates) φ,
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$$\mathcal{S} = \{T \in \mathcal{P}(\Sigma^\omega) \mid |T| = 3\} \qquad U = \{\text{☕☕◯◯◯},\ \text{◯◯☕☕◯},\ \text{☕☕☕☕◯}\}$$

$$U \text{ doesn't perm. satisfy } \forall\pi.\exists\tau.\square(\text{◯}_\pi \to \text{☕}_\tau)$$

$$U \text{ perm. violates } \forall\pi.\exists\tau.\square(\text{◯}_\pi \to \text{☕}_\tau)$$

Gray-box monitoring in general

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation O of a system $\mathcal{S}$.

Definition

Given a set of system behaviors $\mathcal{S} \subseteq \mathcal{B}$,
a finite observation $O \in \mathcal{O}$ permanently satisfies (resp. violates) φ,
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A formula φ is semantically gray-box monitorable for a system $\mathcal{S}$ if every observation O has an extension $P \succeq O$ in $\mathcal{S}$, such that P perm. satisfies φ in $\mathcal{S}$ or P perm. violates φ in $\mathcal{S}$.

Gray-box monitors for $\forall^+\exists^+$-properties

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Assuming $\varphi = \forall\pi.\exists\tau.\psi(\pi,\tau)$, and a sufficiently restrictive $\mathcal{S}$, we may be able to statically prove that all extensions $T \succeq U$ of a given U permanently violate φ.

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Example: $\varphi_a = \forall\pi.\exists\tau.\Box(\text{🍵}_\pi \to \text{☕}_\tau)$ $\qquad$ $\mathcal{S} = \{T \in \mathcal{P}(\Sigma^\omega) \mid |T| = 3\}$

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Negate φ_a: $\neg\varphi_a = \exists\pi.\neg\exists\tau.\Box(\text{🍵}_\pi \to \text{☕}_\tau)$

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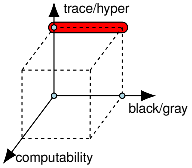
$\{☕◯,\ ◯◯☕\} \quad \mapsto \quad \{☕◯\cdots,\ ◯◯☕\cdots,\ \underline{☕☕☕\cdots}\}$ $\quad$ **?**

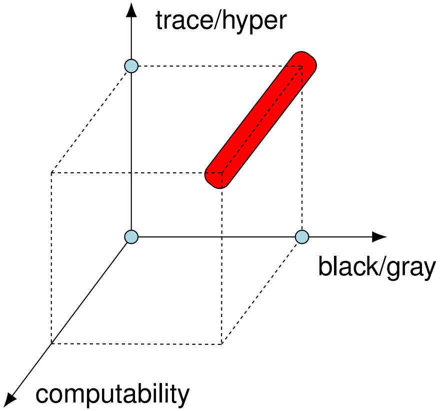
$\{☕☕\underline{◯}◯,\ ◯◯\underline{◯}☕,\ ☕◯\underline{◯}☕\} \quad \mapsto \quad \varnothing$ $\quad$ ✗

Gray-box monitoring – Summary

- Properties defined over observations (e.g. traces or sets of traces).
  $\Rightarrow$ Properties describe sets of observations.
- Perfect monitors can be constructed for some formulas.
  - For example, for formulas without quantifier alternations (as for black-box).
  - But also for $\forall^+\exists^+$-formulas when $\mathcal{S}$ imposes enough constraints.
- Monitorability of formulas depends on set of valid system behaviors $\mathcal{S}$.
  - For example, $\forall^+\exists^+$-properties are monitorable for some choices of $\mathcal{S}$.
  - We will see a more interesting example later...



Undecidable hyperproperties



Monitorability is not existence of monitors

A formula φ is semantically gray-box monitorable in $\mathcal{S}$ if every observation O has an extension $P \succeq O$ that permanently satisfies or violates φ in $\mathcal{S}$.

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Example: Let T be some Turing machine.

$$\mathcal{S} = \{t \in \Sigma^\omega \mid t_i = \text{ the state of } T \text{ after } i \text{ steps}\}, \qquad \varphi = \Diamond\text{halt}.$$

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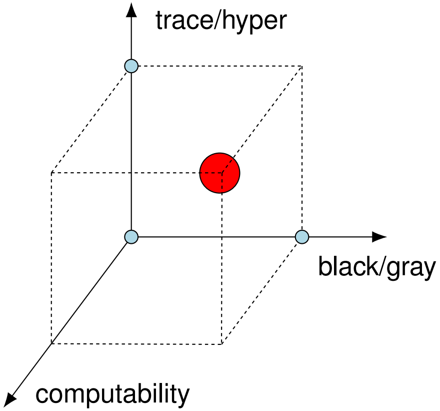
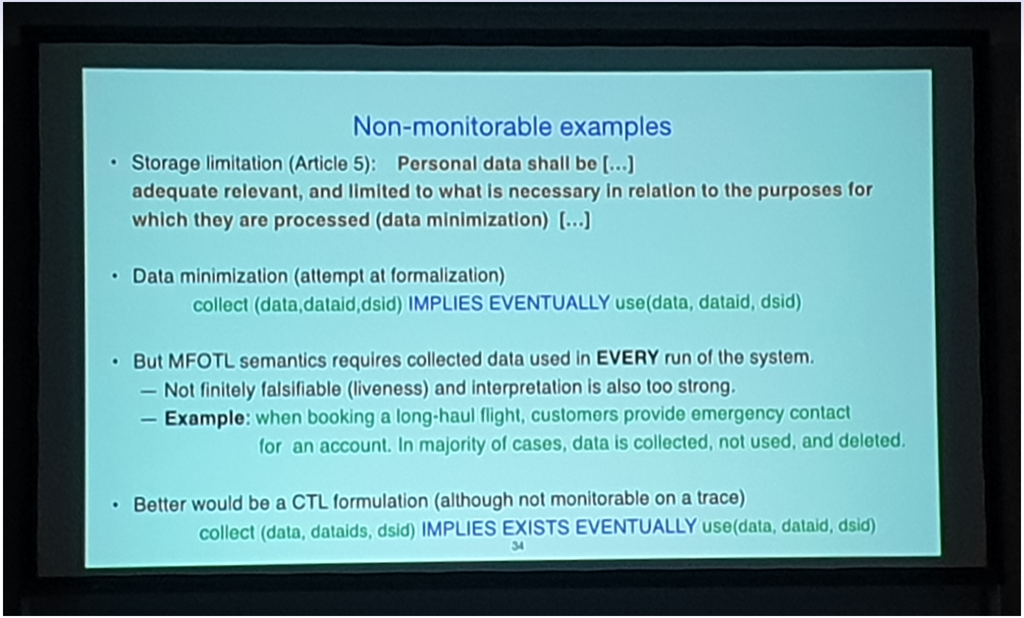
$\Rightarrow$ *there is a sound monitor $M_{\varphi,\mathcal{S}}$ that only answers ✓ or **?**!*

Case study: distributed data minimality





Non-monitorable examples

- Storage limitation (Article 5): **Personal data shall be [...] adequate relevant, and limited to what is necessary in relation to the purposes for which they are processed (data minimization) [...]**

- Data minimization (attempt at formalization)

  collect (data,dataid,dsid) IMPLIES EVENTUALLY use(data, dataid, dsid)

- But MFOTL semantics requires collected data used in **EVERY** run of the system.
  - Not finitely falsifiable (liveness) and interpretation is also too strong.
  - **Example**: when booking a long-haul flight, customers provide emergency contact for an account. In majority of cases, data is collected, not used, and deleted.

- Better would be a CTL formulation (although not monitorable on a trace)

  collect (data, dataids, dsid) IMPLIES EXISTS EVENTUALLY use(data, dataid, dsid)

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Slide by David Basin, *Can we Verify GDPR Compliance?*, RV'19 keynote.

Case study: distributed data minimality

Distributed data minimality (DDM)

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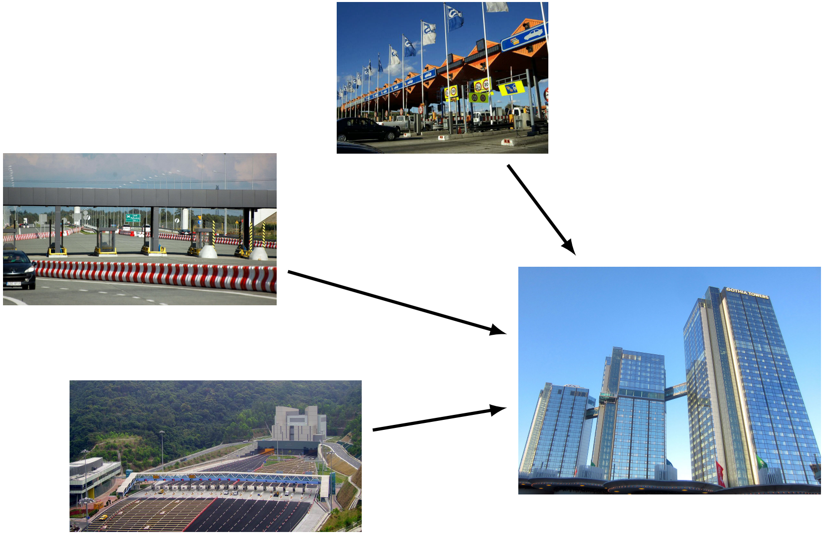
DDM example: toll road



Photos by Rauenstein, Radosław Drożdżewski, Chong Fat, Hesekiel (Wikipedia).

DDM example: toll road

```
class Toll {
  int rate(int hour, int passengers) {
    int r;                                         // standard rates:
    if (hour >= 9 && hour <= 17) { r = 90; }   //   - daytime
    else                         { r = 70; }   //   - nighttime
    if (passengers > 2) { r = r - (r / 5); }   // carpool: 20% off
    return r;
  }

  int fee(int t1, int t2, int t3, int p) {
    int r1 = rate(t1, p);              // rates at each toll station
    int r2 = rate(t2, p);
    int r3 = rate(t3, p);
    int f1 = max(r1, r2) * 4;          // fees per road section
    int f2 = max(r2, r3) * 7;
    return f1 + f2;                    // total fee
  }
}
```

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- Distributed data minimality (DDM)
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  - ∀∀∃∃-hyperproperty

$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\,\neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \begin{pmatrix} \mathrm{same}_i(\pi,\tau) \wedge \mathrm{same}_i(\pi',\tau') \,\wedge \\ \mathrm{almost}_i(\tau,\tau') \wedge \neg\,\mathrm{output}(\tau,\tau') \end{pmatrix}$$

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Yet, we have a monitor [11]. . .

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Yet, we have a monitor [11]. . .

here's how. . .

Distributed data minimality

Definition (Antignac, Sands & Schneider, 2017)

A function f is distributed data-minimal (DDM) if, for all input positions k and all $x, y \in I_k$ such that $x \neq y$, there is some $z \in I$, such that $f(z[k \mapsto x]) \neq f(z[k \mapsto y])$.

Distributed data minimality

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$$\varphi_{\textsf{dm}} \;=\; \bigwedge_{i=1}^{n} \varphi_i, \qquad \Sigma_f^{\#} = \{(x,y) \mid f(x) = y\}, \qquad \mathcal{S}_f = \mathcal{P}(\Sigma_f^{\#})$$

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- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.

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Using the generalized framework

- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.
- Not black-box monitorable, but gray-box monitorable (thanks to $\mathcal{S}$).

A sound monitor for distributed data minimality

$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\,\neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \left(\begin{array}{r}\mathrm{same}_i(\pi,\tau) \wedge \mathrm{same}_i(\pi',\tau') \;\wedge\\ \mathrm{almost}_i(\tau,\tau') \wedge \neg\,\mathrm{output}(\tau,\tau')\end{array}\right)$$

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We build a monitor

$$M_{\text{dm}}(U) = \begin{cases} \textbf{?} & \text{if } f(u_{\mathit{in}}) \neq u_{\mathit{out}} \text{ for some } u \in U, \\ \textbf{?} & \text{if } \bigwedge_{i=1}^{n} \bigwedge_{u,u' \in U} N_{f,i}(\mathrm{proj}_i(u_{\mathit{in}}), \mathrm{proj}_i(u'_{\mathit{in}})) \neq ✗, \\ ✗ & \text{otherwise.} \end{cases}$$

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using an oracle $N_{f,i}(x,y)$ (implemented as symbolic execution + SMT solver):

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The monitor is sound but not perfect.

Please do try this at home!

```
https://github.com/sstucki/minion/
```

Thank you!

Coauthors

- César Sánchez, IMDEA SW
- Borzoo Bonakdarpour, ISU
- Gerardo Schneider, GU/Chalmers

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institute imdea software

IOWA STATE UNIVERSITY

Checkout the `minion` monitor for data minimality

`https://github.com/sstucki/minion/`

Backup slides

Trace properties – LTL

$\varphi_s = \Box$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \Box\Diamond$☕

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$t_1 =$ ☕☕☕☕☕☕ $\cdots$	$t_1 \models \varphi_s$	$t_1 \models \varphi_l$	$t_1 \models \varphi_r$
$t_2 =$ ☕☕◯◯◯◯ $\cdots$	$t_2 \not\models \varphi_s$	$t_2 \models \varphi_l$	$t_2 \not\models \varphi_r$
$t_3 =$ ☕◯☕◯☕◯ $\cdots$	$t_3 \not\models \varphi_s$	$t_3 \models \varphi_l$	$t_3 \models \varphi_r$

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$\varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\varphi$ $\qquad$ $\Diamond\varphi \equiv$ true $\mathcal{U}\,\varphi$ $\qquad$ $\Box\varphi \equiv \neg\Diamond\neg\varphi$

$t \models p$	iff	$p \in t[0]$
$t \models \neg\varphi$	iff	$t \not\models \varphi$
$t \models \varphi_1 \vee \varphi_2$	iff	$t \models \varphi_1$ or $t \models \varphi_2$
$t \models \bigcirc\varphi$	iff	$t[1,..] \models \varphi$
$t \models \varphi_1\,\mathcal{U}\,\varphi_2$	iff	for some i, $t[i,..] \models \varphi_2$ and for all $j < i$, $t[j,..] \models \varphi_1$

Monitoring LTL

$\varphi_s = \Box$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \Box\Diamond$☕

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- Observation: the world today at 10am

  $u_{10} =$ ☕☕☕☕

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- Update: the world at 11am

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φ_s Is there always coffee?

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φ_s Is there always coffee? $u_{10} \rightarrow$ **?**

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φ_s Is there always coffee? $\qquad u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ ✗

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φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓

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φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓
φ_r	Is there always eventually coffee?	$u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ **?**

Monitoring LTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), at runtime.

Monitoring LTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

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A finite observation u permanently satisfies (resp. violates) φ, if every infinite extension of u satisfies (resp. violates) φ:

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$u_{11} =$ ☕☕☕☕☕○○○

u_{11} doesn't perm. satisfy $\Box$☕ $\quad$ u_{11} perm. violates $\Box$☕

u_{11} perm. satisfies $\Diamond$☕ $\quad$ u_{11} doesn't perm. violate $\Diamond$☕

u_{11} neither perm. satisfies nor violates $\Box\Diamond$☕

Monitors for LTL

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Monitors for LTL

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Fact: every LTL formula has a perfect monitor.

Monitorability of LTL formulas

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$\varphi_r = \square\Diamond$☕ $\qquad u_{11} =$ ☕☕☕☕☕◯◯◯

u_{11} doesn't perm. satisfy φ_r $\qquad u_{11}$ doesn't perm. violate φ_r

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Observation: There is no u that permanently satisfies or violates φ_r.

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Definition (Pnueli & Zaks 2006)

A formula φ is *(semantically) monitorable* if every observation u has an extension $v \succeq u$, such that either v perm. satisfies φ or v perm. violates φ.

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