DAYANANDA SAGAR COLLEGE OF ENGINEERING

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CONTENTS:

Concatenated Codes and Iterative Decoding

In this module, we study various code concatenation schemes for MIMO systems. Since most of the existing concatenation schemes borrow ideas from turbo codes, either in terms of encoding, iterative decoding or both, we introduce turbo codes in detail.

In particular, we give details about the encoding, iterative decoding and performance analysis of such codes over AWGN channels. e extend the use of these codes to fading channels and see how they can be combined with space-time codes to provide additional diversity and coding gains.

Before proceeding to the introduction of turbo codes, it is instructive to give a brief summary of the development of the major concatenated coding.

Development of Concatenated Codes

Concatenated codes were first introduced by Forney (1966), where he proposed a scheme that involves concatenating two single codes in a serial fashion. The inner code is a convolutional code and the outer code is a high-rate algebraic Reed–Solomon (RS) code which has a powerful error correction capability. The performance improvements achieved by this concatenated coding scheme were very promising and opened the door for further developments in this area.

Motivated by the work of Forney on concatenated codes and Deng and Costello (1989) proposed an alternative concatenation scheme, which involves using a TCM(trellis coded modulation (TCM)) scheme as an inner code and an RS code as an outer code, The inner code comprises a convolutional code combined with a higher order modulation scheme such as 8-PSK.

This concatenated code can be decoded in two different ways. First, the inner code is decoded using a soft decision Viterbi algorithm (VA) that provides hard decisions to the outer RS decoder.

Another concatenation scheme which involves serial concatenation of two convolutional codes separated by a pseudo-random interleaver was proposed by Hagenauer and Hoeher (1989).

The inner code is decoded using a soft-output VA (SOVA) that provides to the outer Viterbi decoder soft information on the received sequence. The function of the interleaver is to separate the bursts of errors produced by the inner decoder in order to help the outer decoder to have a better error correction capability.

The discovery of turbo codes by Berrou et al. (1993) marks one of the most important breakthroughs in the history of coding theory, and changed the way concatenated codes are exploited to achieve further improvements. Turbo codes represent a different way of concatenating two simple codes to obtain an overall powerful code. A typical turbo code comprises two parallel concatenated convolutional codes separated by an interleaver, and is decoded using iterative decoding techniques.

Inspired by turbo codes, a new concatenation scheme was proposed by Benedetto et al. (1998) which involves the serial concatenation of two convolutional codes separated by an interleaver and decoded using iterative decoding techniques. These codes were introduced as alternatives to turbo codes as, for some applications, they are less complex with performance comparable to or better than that of turbo codes.

Most of the above concatenated coding schemes and their variants have been developed and optimized for AWGN channels. They have also been successfully applied to wireless communication systems.

In fact, many coding schemes have been introduced in the literature which involve the concatenation of classical channel codes, such as turbo and convolutional codes, with space-time codes. Such code concatenation schemes have been shown to be very effective in terms of providing performance improvement, and they motivate the material in this chapter.

As described earlier shown in figure 1, fading channels exhibit correlation in time, that is, in a typical wireless channel, consecutive bits normally see very similar channel fades, depending on the coherence time of the channel. However, in order to obtain diversity using a channel code, different parts of the codeword should undergo different channel fades. Therefore, the use of an interleaver, whose role is to scramble the coded bits (or, symbols) before transmission, is necessary. Clearly, a de-interleaver whose function is exactly the opposite is needed at the receiver before channel decoding can be performed.

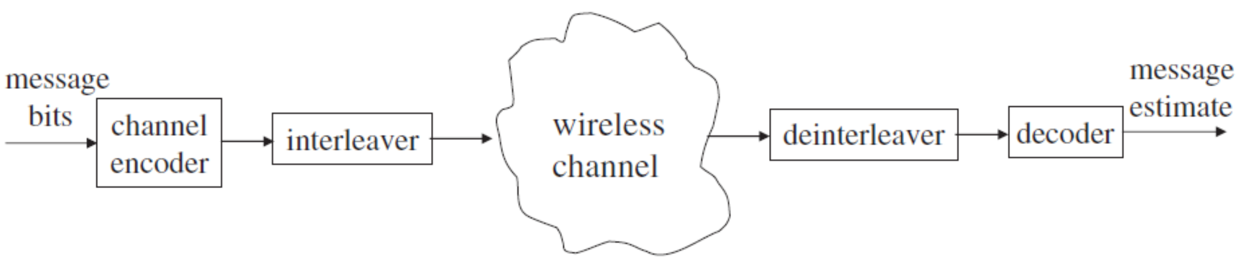

Figure 1: Coding over wireless channel

Convolutional codes

A second major class of channel codes is known as convolutional coding. The convolutional code can operate on a continuous string of data, whereas block codes operated on words. Convolutional codes also have memory—the behavior of the code depends on previous data.

Turbo codes are used in 3G/4G mobile communications (e.g., in UMTS and LTE) and in (deep space) satellite communications as well as other applications where designers seek to achieve reliable information transfer over bandwidth- or latency-constrained communication links in the presence of data-corrupting noise.

Turbo codes are error-correcting codes with performance close to the Shannon theoretical limit [SHA]. The encoder shown in figure 2 is formed by the parallel concatenation of two convolutional codes separated by an interleaver or permuter. An iterative process through the two corresponding decoders is used to decode the data received from the channel.

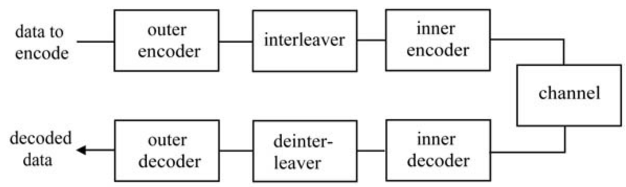

Figure 2. Turbo code

The turbo code is the name of a class of convolutional codes which developed by parallel concatenating two convolutional code blocks which are identical. This new class of channel error control code is well known for its high performance at low to moderate signal to noise ratio

Concatenated Codes for AWGN Channels

In this section, we consider parallel and serial concatenated codes for AWGN channels. In particular, we introduce the encoder and iterative decoder structures for such codes, and present their performance analysis.

Encoder Structures

The idea in turbo coding is to concatenate two recursive systematic convolutional (RSC) codes in parallel via an interleaver, as shown in Figure 3. Each RSC encoder is normally described by two polynomials, namely $(g_1(D), g_2(D))$, where $g_1(D)$ is the feedforward polynomial and $g_2(D)$ is the feedback polynomial. We henceforth refer to this scheme as a parallel concatenated convolutional code (PCCC).

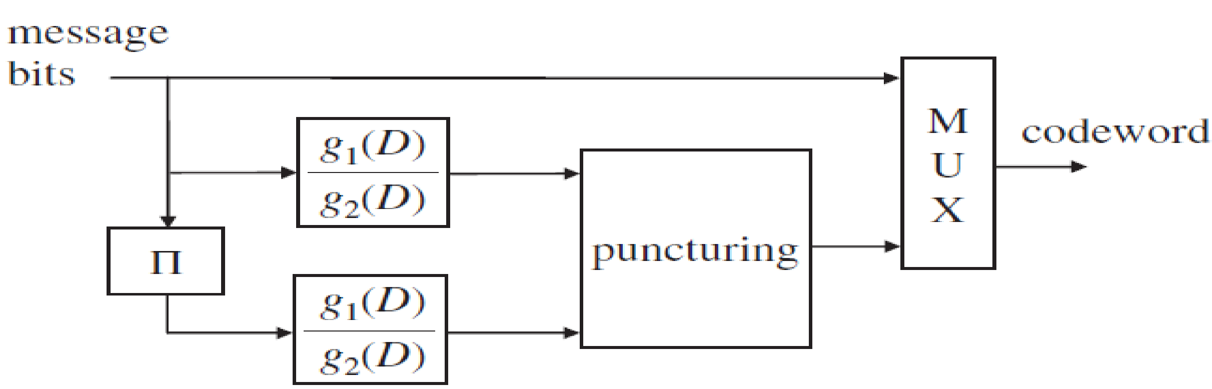

Figure 3 PCCC encoder structure.

The encoding process proceeds as follows.

The information sequence at the input of the turbo encoder is divided into blocks of a certain length, and these blocks are encoded sequentially.

The input to the first encoder is an information block, whereas the input of the second encoder is an interleaved version of this information block.

The encoded sequence (codeword) corresponding to that information block is then the information block itself, the first parity block and the second parity block.

Thus, the natural code rate of a turbo code is 1/3.

When higher code rates are desired, puncturing of parity bits may be used, as shown in the figure where the punctured bits are omitted, i.e., not transmitted.
The other ingredient of the PCCC scheme, namely the interleaver, can be chosen pseudo-randomly to obtain a good performance.

There are some obvious generalizations of the standard turbo coding scheme. The most prominent generalization is the serial concatenated convolutional code (SCCC), which comprises of two serial concatenated convolutional codes separated by an interleaver. The SCCC encoder is shown in Figure 4.

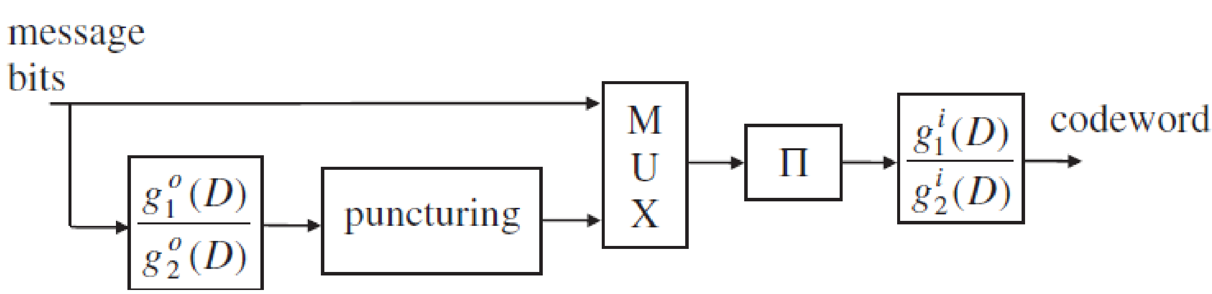

Figure 4 A typical SCCC encoder structure.

In the figure, the generator polynomials of the outer RSC encoder are $go^1(D)$, $go^2(D)$ and the polynomials of the inner RSC encoder are $gi^1(D)$, $gi^2(D)$

The overall rate of the SCCC code is $Rc = Ro \cdot Ri$, where Ro is the rate of the outer code and Ri is the rate of the inner code.

The SOVA Decoder

In this section, we present the SOVA algorithm as an alternative to the APP algorithm. This is motivated by the fact that the complexity of the SOVA algorithm is roughly half that of the APP algorithm. This advantage comes at the expense of a degradation in the performance, as compared with the APP algorithm, by about one dB over AWGN channels and about two dBs over fading channels.

Performance with Maximum Likelihood Decoding

The optimal decoder for a concatenated code is the maximum likelihood sequence detector (MLSD). However, the MLSD is very complex to realize because of the complexity associated with the interleaver, as noted before. Therefore, as an alternative, the decoding of concatenated codes is achieved using a sub-optimal iterative decoding algorithm. Simulations have shown that the performance of PCCCs and SCCCs is very closely approximated by their ML performance.

Additionally, since these codes are linear (the all-zero codeword can be used as a reference), their ML performance is very easy to derive compared with the performance of the iterative decoder. Therefore, we will derive the ML performance of these concatenated codes over AWGN channels. We begin with the PCCC case and later extend our derivation to the SCCC case. We begin with the PCCC case and later extend our derivation to the SCCC case

Parallel Concatenated Codes

The ML analysis of concatenated codes involves invoking the union bound technique. Let us assume that the all-zero codeword was transmitted. Let the data block size be *N*. Assuming antipodal signaling, the number of possible codewords is then equal to 2*N*. The pairwise error probability (i.e., the decoder picking codeword, ***xi*** , over the all-zero codeword, ***x***0) is given by *P(**x***0 → ***xi*** *)* = Pr choosing ***xi*** for *i* 1, 2*, . . . ,* 2*N* – 1 | given that ***x***0 is transmitted*)*. Let *di* denote the Hamming distance between the two codewords. When the signalling scheme is BPSK with equiprobable signals, the pairwise error probability is therefore equal to

$$P(\boldsymbol{x}_0 \rightarrow \boldsymbol{x}_i) = Q\left(\sqrt{2d_i\rho}\right)$$

where ρ represents the signal-to-noise ratio at the receiver. Assuming that all possible information sequences are equally likely to be transmitted and ML soft decision decoding is employed, by invoking the union bound argument, the error probability can be upper bounded by

$$P_e \le \sum_{i=1}^{2^N-1} Q\left(\sqrt{2d_i\rho}\right)$$

where the sum runs over all non-zero codewords. The probability of bit error *Pb* can then be upper bounded by

$$P_b \le \sum_{i=1}^{2^N-1} \frac{w_i}{N} Q\left(\sqrt{2d_i\rho}\right)$$

$$= \sum_{w=1}^{N} \sum_{v=1}^{\binom{N}{w}} \frac{w}{N} Q\left(\sqrt{2d_{wv}\rho}\right).$$

where *dwv* is the total weight of the *v*th codeword with information weight *w*.

When operating at medium to high signal-to-noise ratios, which is typical in real-life applications, only the first few terms in above equation will contribute to the sum. This is because *dwv* typically increases as *w* increases. Thus, the last line of equation can be approximated as

$$P_b \approx \sum_{w=2}^{t} \frac{w n_w}{N} Q\left(\sqrt{2 d_{w,\min}^{PCCC} \rho}\right)$$

where *nw* is the number of weight *w* input information sequences that result in the lowest weight codeword, *dPCCC w,*min .

Serial Concatenated Codes

The derivation of the ML performance of the SCCC scheme is similar to that of the PCCC scheme. The reason is that the component encoders making up the SCCC encoder are also linear. Therefore, bit error rate in the floor region can be approximated as

$$P_b \approx \sum_{w=2}^{t} \frac{w n_w}{N} Q\left(\sqrt{2 d_{w,\min}^{SCCC} \rho}\right)$$

Concatenated Codes for MIMO Channels

In this and subsequent sections, we discuss how one can combine channel coding and space-time coding for MIMO channels in an effort to achieve further performance improvements. One of the major advantages of using channel coding in conjunction with space-time coding is to achieve time diversity, especially for block fading channels.

As mentioned before, time diversity is achieved by transmitting the signal components carrying the same information in multiple time intervals, provided that these time intervals are mutually separated by at least the coherence time of the channel. To use the available bandwidth more efficiently, while achieving additional diversity, one may use a more sophisticated coding scheme such as convolutional code or TCM schemes in conjunction with interleaving. Several code concatenation schemes have been developed for MIMO systems

Concatenated Space-Time Turbo Coding Scheme

Two concatenation schemes have been introduced by Liu et al. (2001a), one derives its structure from parallel concatenated codes as shown in Figure 6, whereas the other one derives its structure from serial concatenated codes as shown in Figure 7.

As shown in the figure 6, the parallel concatenated code is designed for two transmit antennas. Puncturing is used to allow for flexible code rates. The channel interleaving and multiplexing are intended to take advantage of both time and space diversity. For this scheme to achieve full-rate, the number of bits fed at a time into each RSC encoder and the overall code rate are dictated by the modulation scheme used. For example, if QPSK is employed, one may use a rate 2/4 turbo encoder to achieve 2 bits/symbol. Although there is no guarantee that this code will achieve full diversity, if the interleavers (that of the channel and turbo encoder) are selected randomly, full diversity will be achieved with high probability. The decoder for this concatenation scheme is iterative and is based on belief propagation on Bayesian networks.

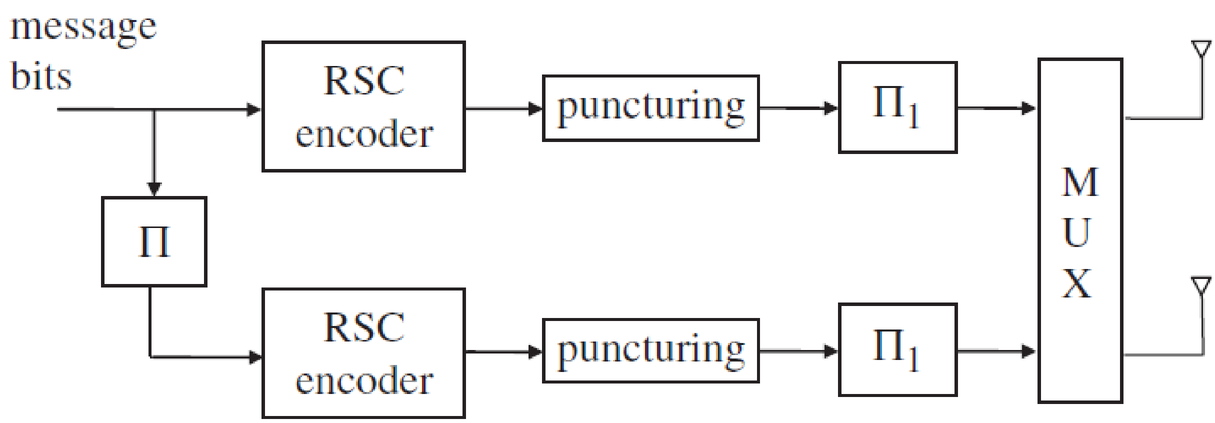

Figure 6 Parallel concatenated space-time turbo code encoder (for two transmit antennas).

It is required for the serial concatenated code that the convolutional codes be identical, recursive and of rate one. The latter requirement is to ensure full-rate. It is also required that the numerator of the transfer function be one to avoid catastrophic codes. As for the interleavers, they must be identical. The decoder for this code resembles that of a standard serial concatenated code except that the inner decoder jointly decodes the recursive convolutional codes to ensure that all available diversity available in the channel is captured. In a standard serial concatenated code, the recursive convolutional codes are decoded separately.

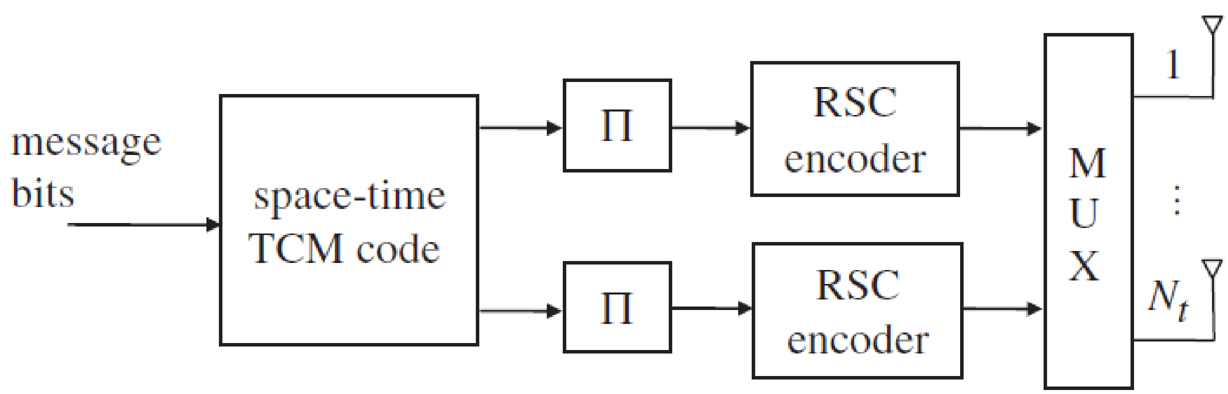

Figure 7. Encoder structure for the serial concatenated space-time turbo code.

Turbo Space-Time Trellis Coding Scheme

This concatenation scheme which borrows ideas from turbo codes and STTCs has been introduced by Gulati and Narayanan (2003). The encoder structure is depicted in Figure 8. The outer code is a binary code, whereas the inner code is a recursive STTC. The recursive nature of the inner code enhances the overall minimum Hamming distance of the combined code, thereby improving the available diversity. It is easy to design the inner code to achieve full diversity.

This stems from the fact that any non-recursive space-time code that achieves full diversity, i.e. full rank, can be transformed easily into a recursive, full rank code. However, it does not achieve full-rate because of the presence of the outer code. This concatenation scheme is decoded iteratively where the front end receiver generates log-likelihoods for the *mNt* bits which are deinterleaved and passed to the outer decoder. The outer decoder computes its own extrinsic information and passes them back to the inner detector to refine its log-likelihoods for the next iteration. This repeats for a number of iterations before final decisions are made.

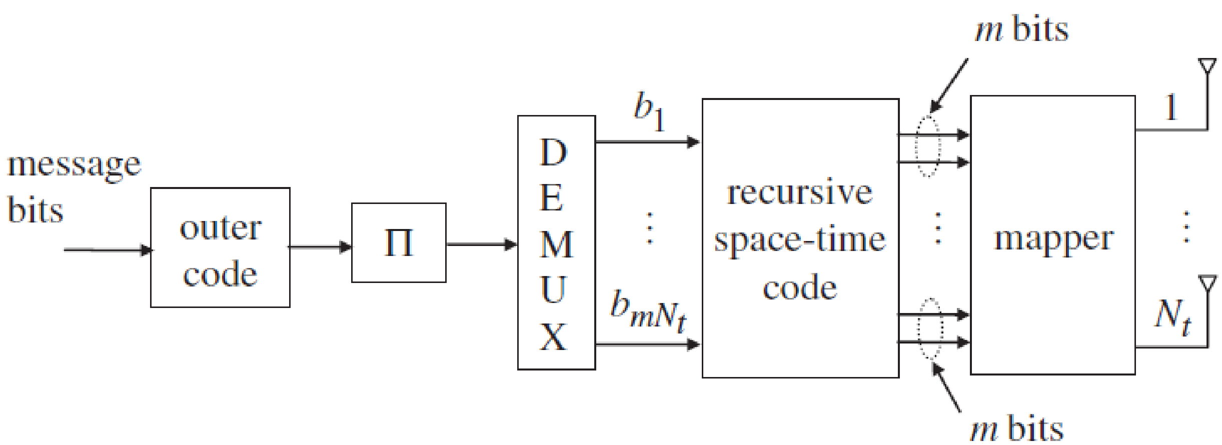

Figure 8 Encoder structure for the serial concatenation of an outer channel code and an inner recursive space-time code.

Turbo Space-Time Coding Scheme

This scheme was introduced by Cui and Haimovich (2001) and is referred to as turbo spacetime coded modulation (turbo-STCM). The encoder structure is depicted in Figure 6

As shown in the figure 9, the code consists of two systematic (recursive) STTCs concatenated in a parallel fashion. One of the two antennas is permanently connected to the systematic part of the output of the top encoder and the second antenna is connected to the parity of the two STTC encoders. To achieve full-rate, puncturing must be used. In this case, puncturing is achieved by keeping one parity symbol for each systematic symbol and discarding the other parity symbol. Therefore, this scheme is guaranteed to achieve full-rate; however, it may achieve full diversity but this is not guaranteed because of puncturing.

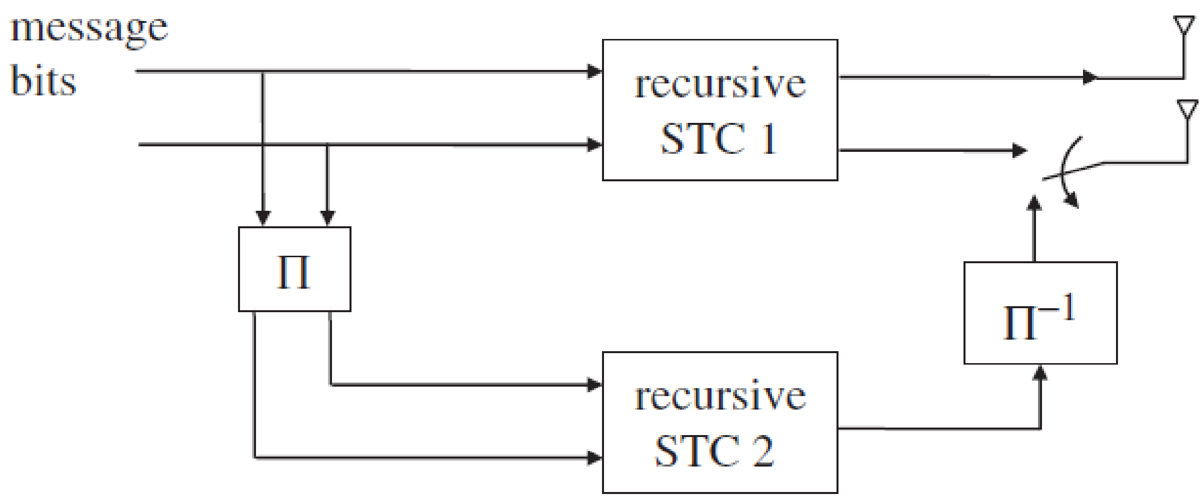

Figure 9 Encoder structure for the turbo-TCM scheme (_ denotes a symbol interleaver).

Consequently, the interleaver operates on symbols. The symbol deinterleaver at the output of the lower STTC encoder is used to ensure that the systematic data at the output is identical to that of the upper encoder. The decoder for this scheme resembles that of an iterative turbo decoder.

In the next sections, we describe in detail two code concatenation schemes developed for MIMO systems, namely, turbo-coded modulation and combined channel coding and space-time block coding

Turbo-Coded Modulation for MIMO Channels

The concatenation scheme we consider here involves the serial concatenation of a turbo code and a mapper with an interleaver separating them, which was introduced by Stefanov and Duman (2001c). In this scheme, the mapper is flexible in the sense that it can be an STTC, STBC or BLAST-like scheme. We refer to this scheme as *turbo-coded modulation* (TuCM).

A block diagram of the TuCM encoder is shown in Figure 10. As shown in the figure, the primitive data is first encoded by a turbo encoder. The turbo-coded sequence is interleaved so that bursts of errors resulting from deep fades are distributed, which makes it easier to deal with such errors. The interleaved sequence is demultiplexed into a number of parallel substreams. In this case we assume there are *mNt* substreams.

In general, the number of substreams depends on the type of modulation and mapping used. These *mNt* streams are fed into a mapper. The function of the mapper is to map the incoming bits to a particular signal constellation. Specifically, to simultaneously transmit *Nt* symbols from the available *Nt* transmit antennas, the mapper maps every set of *m* coded bits to one symbol from a signal constellation of size 2*m*.

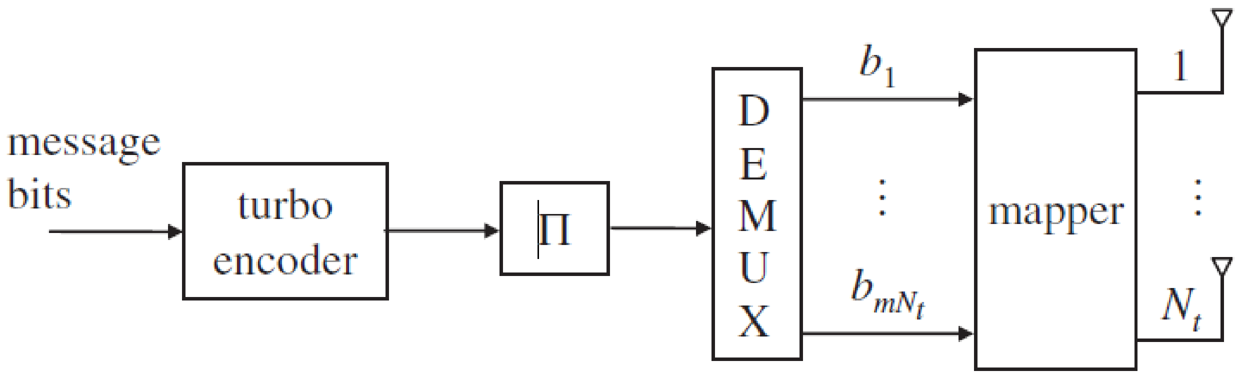

Figure 10 Encoder structure for the turbo-coded modulation scheme

It is assumed here that the underlying channel is block fading and coding is done across a number of consecutive differently faded blocks so as to achieve time diversity. Since block fading is assumed, the interleaver size is chosen in such a way that it is large enough to span a number of differently faded blocks to break the correlation between adjacent bits in an effort to achieve additional (time) diversity.

Now define $\boldsymbol{b}$ to be a vector of coded bits of length mN_t, which is the input to the mapper at any given time, i.e.,

$$\boldsymbol{b} \triangleq \left[b_1, \ldots, b_m, \ldots, b_{m+1}, \ldots, b_{mN_t}\right].$$

Let the corresponding vector of N_t symbols be $\boldsymbol{x}$, i.e.,

$$\boldsymbol{x} \triangleq \left[x_1, x_2, \ldots, x_{N_t}\right].$$

The components of $\boldsymbol{x}$ are then transmitted simultaneously from the available N_t transmit antennas. Let $y_j(k)$ denote the signal received by antenna j at time k, which can be expressed as

$$y_j(k) = \sqrt{\rho} \sum_{i=1}^{N_t} x_i(k) h_{i,j}(k) + n_j(k),$$

where $h_{i,j}(k)$ denotes the fading coefficient between the ith transmit and jth receive antenna at time k, and $n_j(k)$ is the complex Gaussian noise sample at time k corresponding to receive antenna j. Both the fading coefficients and AWGN samples are assumed to be independent and $\mathcal{CN}(0, 1)$ distributed. For notational convenience, we drop the time index k. With this, the signals received by all receive antennas can be expressed in a compact form as

$$\boldsymbol{y} = \sqrt{\rho} \boldsymbol{x} \boldsymbol{H} + \boldsymbol{n},$$

where $\boldsymbol{H}$ is the $N_t \times N_r$ channel matrix, $\boldsymbol{y} = [y_1, y_2, \ldots, y_{N_r}]$ and $\boldsymbol{n} = [n_1, n_2, \ldots, n_{N_r}]$.

Concatenated Space-Time Block Coding

Encompasses a wide variety of concatenation schemes since there is no restriction on its mapper. Among these schemes is the concatenation of an outer channel code and an inner orthogonal STBC, where in this case the STBCsimply replaces the mapper as shown in Figure 11.

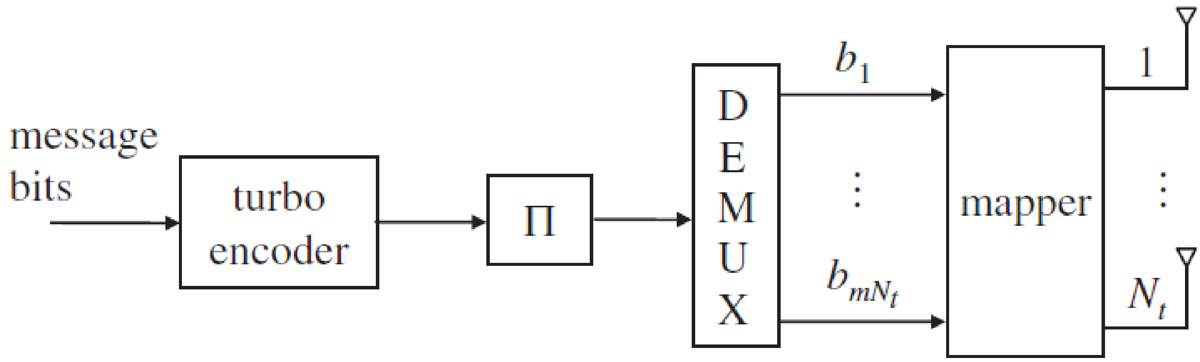

Figure 11 Encoder structure for the turbo-coded modulation scheme.

It is shown that the coded STBC scheme gives the best performance–complexity trade-off among other concatenation schemes when the outer code is a convolutional code or a turbo code. It has been included as an option in several standards for future wireless communication systems. The coded STBC scheme owes its popularity to its simplicity and flexibility. For example, the complexity of the STBC decoder is much less than that of the TuCM scheme and it is more stable. The reason is that the STBC decoder exploits the orthogonality structure of the STBC to decouple the received signals. In addition, it is easier to obtain performance bounds for the coded STBC scheme for various outer channel codes.

Encoder Structure

The encoder of the coded STBC scheme is shown in Figure 12

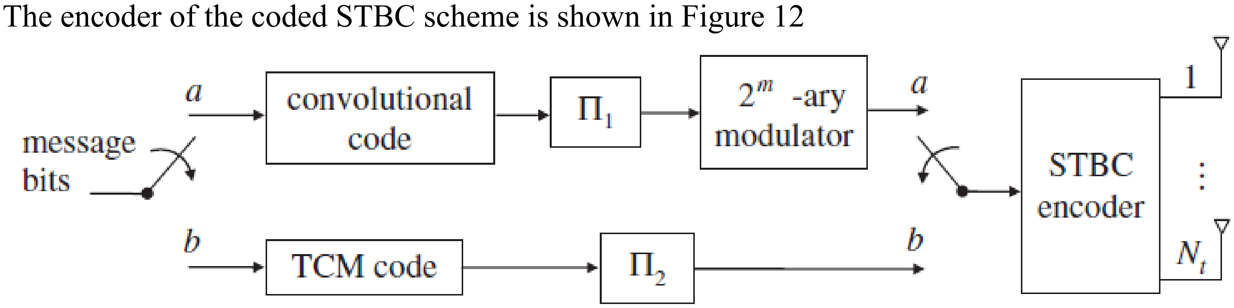

Figure 12 Encoder structure for the coded STBC scheme. (_1 denotes a bit interleaver whereas _2 denotes a symbol interleaver.)

In the figure 12, the primitive data stream is first encoded by an outer channel encoder. There is no restriction on the type of channel code employed, but the focus here is on convolutional codes, turbo codes and TCM codes because they are widely used. In addition, we only consider binary codes. The coded sequence is interleaved and multiplexed. Each set of *m* bits at the output of the demultiplexer is mapped onto a symbol taken from a 2*m*-ary signal constellation. The output of the modulator is then fed into the STBC encoder, which groups every *Nt consecutive* symbols and transmits them from the available *Nt* antennas according to the STBC encoding principles, assuming that the underlying STBC is full rate. As such, the transmission rate achieved is $R = mRc$ bits per channel use, where Rc is the rate of the outer channel code.

Decoder Structure

The decoder structure for the coded STBC scheme is depicted in Figure 9.

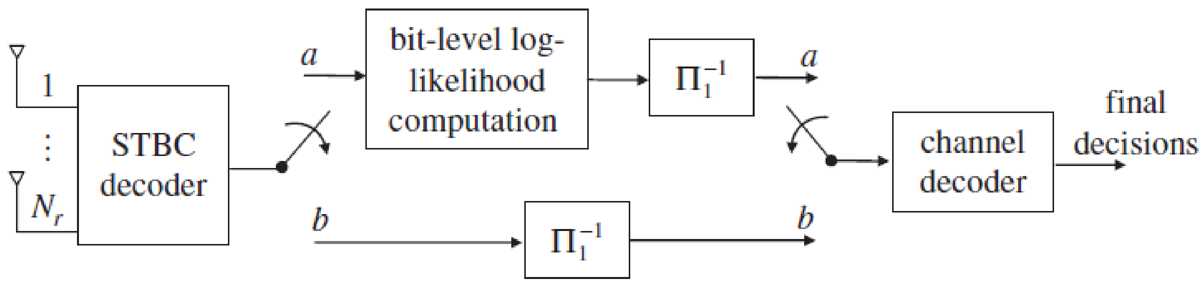

Figure 13 Decoder structure for the coded STBC scheme.

In the figure 13, when the switch is in position '*a*', the resulting decoder corresponds to the convolutional (or turbo) coded system. In this case, the output of the STBC decoder is fed into the loglikelihood computation module, which computes the log-likelihoods for the bits comprising the corresponding symbols. These log-likelihoods are then deinterleaved and passed to the channel decoder.

The type of channel decoder employed is dictated by the channel encoder employed at the transmitter. For example, when a convolutional code is used, the corresponding decoder would be the Viterbi decoder. Whereas, when a turbo code is used, the corresponding decoder would consist of two log-APP or SOVA decoders working together iteratively.

When the switch is in position '*b*', the resulting decoder corresponds to the TCM coded system. In this case, the sequence of the decoupled symbols at the output of the STBC decoder is deinterleaved and passed to the channel decoder, which is the Viterbi decoder.

Acknowledgment: This material is based on the text book authored by Tolga M. Duman and Ali Ghrayeb, "Coding for MIMO Communication systems", John Wiley & Sons, West Sussex, England, 2007. Some additional material are taken and/or inspired by material from various paper and / or electronic resources.

Objective Questions.

a) Concatenated codes are ---- ---
   i) Compression Code **ii) error-correcting codes** iii) Source Code iv) none

b) Concatenated codes are constructed from----------Codes
   i) **2 or more** ii) single code iii) hundreds of iv) none

c) Concatenated codes are having --------- performance and reasonable complexity
   i) Bad ii) Worst **iii) Good** iv) none

d) The --------- decoding principles have found widespread applications not only in error control, but in detection, interference suppression and equalization.
i) Reed Solomon Code ii) BCH Code iii) Hamming Code iv) **Turbo Code**
e) --------is the process of removing some of the parity bits after encoding with an error-correction code.
i) puncturing ii) equalization iii) Source coding iv) decoding

f) The recursive systematic convolutional (RSC) encoder is obtained from the nonrecursive nonsystematic (conventional) convolutional encoder by feeding back one of its ---- outputs to its input.
i) Decoded **ii) Encoded** iii) compressed iv) expanded

g) Codes with output symbols that do not include the input data are called ---------- Code.
i) Compression ii) Systematic **iii) non-systematic** iv) Puncture

h) Turbo codes are ---- ---
i) Convolution Code ii) FEC codes iii) Channel Code iv) **all of mentioned**

i) -------, a technique for making forward error correction more robust with respect to burst errors
i) Interleaving ii) puncturing iii) equalization iv) source coding

j) Trellis termination is an important method for improving performance of --------- by periodically adding tail bits into information sequence.
i) Reed Solomon Code ii) BCH Code iii) Hamming Code iv) **Turbo Code**

Q1. What are concatenated codes for MIMO Diversity?
The uses of space-time code (STC) and iterative processing have enabled robust communications over fading channels at previously unachievable signal-to-noise ratios. Maintaining desired transmission rate while improving the diversity from STC is challenging, and the performance of the STC suffers considerably due to lack of channel state information (CSI).
First, we concatenate space-time block code (STBC) with Channel Coding for improving diversity gain as well as coding gain. Proper soft-information sharing is indispensable to the iterative decoding process.
Traditionally, the performance of STBC schemes has been evaluated under perfect channel estimation. For fast time-varying channel, obtaining the CSI is tedious if not impossible.
The encoder of STTC, which is generally decoded using Viterbi like algorithm, is based on a trellis structure. This trellis structure provides an inherent advantage for the STTC scheme that an iterative decoding is feasible with the minimal addition computational complexity.
Concatenated codes were first introduced by Forney (1966), where he proposed a scheme that involves concatenating two single codes in a serial fashion. The inner code is a convolutional code and the outer code is a high-rate algebraic Reed–Solomon (RS) code which has a powerful error correction capability. The performance improvements achieved by this concatenated coding scheme were very promising and opened the door for further developments in this area.
Q2. Write a Note on Development of Concatenated Notes.
Motivated by the work of Forney on concatenated codes and that of Ungerboeck on trellis coded

modulation (TCM) (Ungerboeck (1982)), Deng and Costello (1989) proposed an alternative concatenation scheme, which involves using a TCM scheme as an inner code and an RS code as an outer code. The inner code comprises a convolutional code combined with a higher order modulation scheme such as 8-PSK. This concatenated code can be decoded in two different ways. First, the inner code is decoded using a soft- decision Viterbi algorithm (VA) that provides hard decisions to the outer RS decoder. Second, the VA is modified such that it provides, in addition to hard decisions, reliability information to the outer decoder in the form of symbol erasures. The RS decoder is then modified in such a way that it uses the reliability information as side information to make better decisions. The latter decoding technique achieves better performance than the former one.

Another concatenation scheme which involves serial concatenation of two convolutional codes separated by a pseudo-random interleaver was proposed by Hagenauer and Hoeher (1989). The inner code is decoded using a soft-output VA (SOVA) that provides to the outer Viterbi decoder soft information on the received sequence. The function of the interleaver is to separate the bursts of errors produced by the inner decoder in order to help the outer decoder to have a better error correction capability.

The discovery of turbo codes by Berrou et al. (1993) marks one of the most important breakthroughs in the history of coding theory, and changed the way concatenated codes are exploited to achieve further improvements. Turbo codes represent a different way of concatenating two simple codes to obtain an overall powerful code. A typical turbo code comprises two parallel concatenated convolutional codes separated by an interleaver, and is decoded using iterative decoding techniques. Since their invention, turbo codes have generated an abundance of literature, mainly because of their exceptional performance for very low signal-to-noise ratios. To give a simple example, for a rate 1/2 turbo code with an interleaver of size 65,536 and memory-4 component codes, it was demonstrated by Berrou et al. (1993) that a bit error probability of $10{-}5$ at a signal-to-noise ratio per bit of 0.7 dB over an AWGN channel is possible. For an AWGN channel, the capacity is equal to the transmission rate, i.e., 1/2, when the signal-to-noise ratio is 0 dB. Therefore, if we consider $10{-}5$ probability of error as the goal, the performance is only 0.7 dB away from the channel capacity, which is remarkable.

Inspired by turbo codes, a new concatenation scheme was proposed by Benedetto et al. (1998) which involves the serial concatenation of two convolutional codes separated by an interleaver and decoded using iterative decoding techniques. These codes were introduced as alternatives to turbo codes as, for some applications, they are less complex with performance comparable to or better than that of turbo codes. Another concatenated coding scheme called hybrid concatenated codes has been introduced by Divsalar and Pollara (2000). A hybrid concatenated code is a parallel concatenation of a serial concatenated code and a single convolutional code separated by two interleavers, one for the serial concatenated code and the other for the combined code.

Another major development in capacity achieving coding has been the resurrection of low-density parity check (LDPC) codes. These codes were first proposed by Gallager (1962). They were generalized by Tanner (1981) where a graph representation for these codes is also proposed. Since their introduction, LDPC codes did not receive much at- tention from the coding community until MacKay and others (MacKay and Neal (1995), MacKay (1999), Alon and Luby (1996)) showed their great potential. In general, LDPC codes are a class of block codes generated using large, sparse (low-density) generator matrices, resulting in a large minimum distance. They achieve a near-capacity perfor- mance, and they are decoded via the so-called message passing or belief propagation algorithm.

Most of the above concatenated coding schemes and their variants have been developed and optimized for AWGN channels. They have also been successfully applied to wireless communication systems. In fact, many coding schemes have been introduced in the literature which involve the concatenation of classical channel codes, such as turbo and convolutional codes,with space-time codes.

Q3.Explain about Concatenated codes for AWGN and MIMO channels.

AWGN:

Most of the above concatenated coding schemes and their variants have been developed and optimized for AWGN channels. They have also been successfully applied to wireless communication systems. In fact, many coding schemes have been introduced in the literature which involve the concatenation of classical channel codes, such as turbo and convolutional codes,with space-time codes.

MIMO Channels:

One of the major advantages of using channel coding in conjunction with space-time coding is to achieve time diversity, especially for block fading channels. As mentioned before, time diversity is achieved by transmitting the signal components carrying the same information in multiple time intervals, provided that these time intervals are mutually separated by at least the coherence time of the channel.

For a given coded MIMO system employing Nt transmit and Nr receive antennas with i.i.d. channel fading coefficients (in time and space), the maximum diversity that can be achievedis NtNrdH min where dH min denotes the minimum Hamming distance of the outer code, provided that coherent soft decision decoding is employed. This is a significant enhancement in the diversity order.

When the channel is quasi-static fading where interleaving is not useful, the outer channel code provides only coding gain, whereas when the channel is block fading, one may still use interleaving to achieve some increase in diversity. The level of diversity gain depends on the interleaving depth and the coherence time of the channel.

The advantages of employing an outer channel code obviously come at the expense of reducing the bandwidth efficiency, and this may become challenging in situations when the available bandwidth is scarce. To overcome this problem one may use higher order modulation schemes such as M-PSK and M-QAM to achieve a good use of the total available bandwidth. The results mentioned above concerning diversity and coding gains still hold for these modulation schemes.

Q4. Illustrate Concatenated STB Coding.

The coded STBC scheme owes its popularity to its simplicity and flexibility. For example, the complexity of the STBC decoder is much less than that of the TuCM scheme and it is more stable. The reason is that the STBC decoder exploits the orthogonality structure of the STBC to decouple the received signals. In addition, it is easier to ob- tain performance bounds for the coded STBC scheme for various outer channel codes.

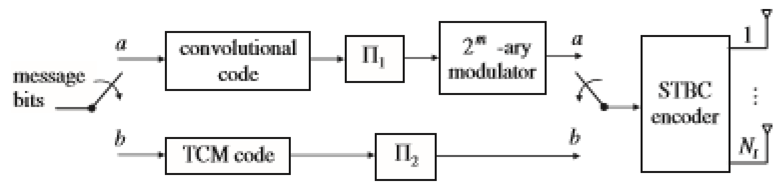

Figure 7.21 Encoder structure for the coded STBC scheme. (Π_1 denotes a bit interleaver whereas Π_2 denotes a symbol interleaver.)

Encoder: There is no restriction on the type of channel code employed, but the focus here is on convolutional codes, turbo codes and TCM codes because they are widely used. In addition, we only consider binary codes. The coded sequence is interleaved and demultiplexed. Each set of m bits at the output of the demultiplexer is mapped onto a symbol taken from a 2m-ary signal constellation. The output of the modulator is then fed into the STBC encoder, which groups every Nt consecutive symbols and transmits them from the available Nt antennas according to the STBC encoding principles, assuming that the underlying STBC is full rate. As such, the transmission rate achieved is R = mRc bits per channel use, where Rc is the rate of the outer channel code.

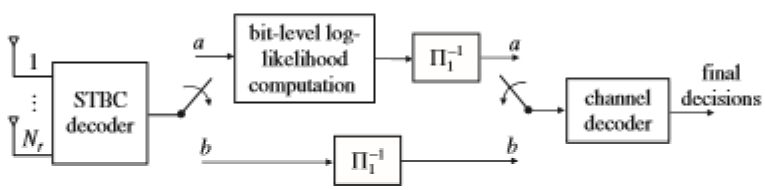

Figure 7.22 Decoder structure for the coded STBC scheme.

Decoder: The output of the STBC decoder is fed into the log- likelihood computation module, which computes the log-likelihoods for the bits comprising the corresponding symbols. These log-likelihoods are then deinterleaved and passed to the channel decoder. The type of channel decoder employed is dictated by the channel encoder employed at the transmitter. For example, when a convolutional code is used, the corresponding decoder would be the Viterbi decoder. Whereas, when a turbo code is used, the corresponding decoder would consist of two log-APP or SOVA decoders working together iteratively. When the switch is in position ‘b’, the resulting decoder corresponds to the TCM coded system. In this case, the sequence of the decoupled symbols at the output of the STBC decoder is deinterleaved and passed to the channel decoder, which is the Viterbi decoder.

Q5. Elaborate on Turbo Coded Modulation for MIMO Channels.

The concatenation scheme we consider here involves the serial concatenation of a turbo code and a mapper with an interleaver separating them, which was introduced by Stefanov and Duman (2001c).

The turbo-coded sequence is interleaved so that bursts of errors resulting from deep fades are distributed, which makes it easier to deal with such errors. The interleaved sequence is demultiplexed into a number of parallel substreams. In this case we assume there are mNt substreams. In general, the number of substreams depends on the type of modulation and mapping used. These mNt streams are fed into a mapper. The function of the mapper is to map the incoming bits to a particular signal constellation. Specifically, to simultaneously transmit Nt symbols from the available Nt transmit antennas, the mapper maps every set of m coded bits to one symbol from a signal constellation of size 2^m.

Encoder:

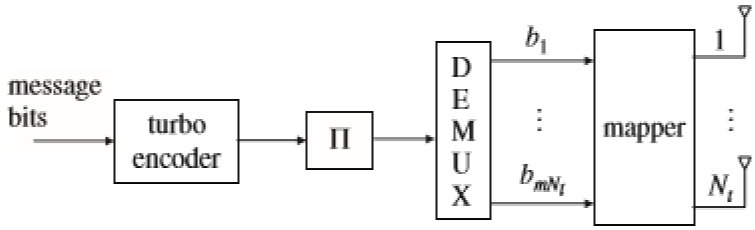

Figure 7.17 Encoder structure for the turbo-coded modulation scheme.

It is assumed here that the underlying channel is block fading and coding is done across a number of consecutive differently faded blocks so as to achieve time diversity. Since block fading is assumed, the interleaver size is chosen in such a way that it is large enough to span a number of differently faded blocks to break the correlation between adjacent bits in an effort to achieve additional (time) diversity.

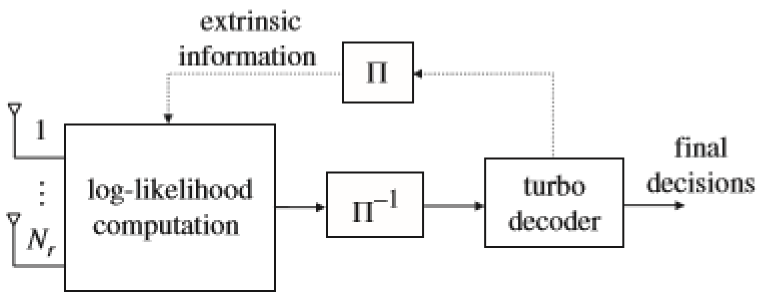

Figure 7.18 Iterative decoder structure for the turbo-coded modulation scheme.

Decoder: The turbo decoder consists of two SISO decoders that work iteratively between themselves, where each SISO decoder is matched to one of the component encoders of the turbo code. The channel detector and turbo decoder can work iteratively or non-iteratively, depending on the performance/complexity requirements. In the former case, the turbo decoder feeds back its extrinsic information to the channel detector after the first iteration so that refined log-likelihoods are computed. In the latter case, the log-likelihoods are computed only once and the turbo decoder runs for a few times after which final decisions are made.

Q6. List out the features briefly for Concatenated Space Time Turbo Coding.

It is required for the serial concatenated code that the convolutional codes be identical, recursive and of rate one. The latter requirement is to ensure full-rate. It is also required that the numerator of the transfer function be one to avoid catastrophic codes. As for the interleavers, they must be identical. The decoder for this code resembles that of a standard serial concatenated code except that the inner decoder jointly decodes the recursive convolutional codes to ensure that all available diversity available in the channel is captured. In a standard serial concatenated code, the recursive convolutional codes are decoded separately.

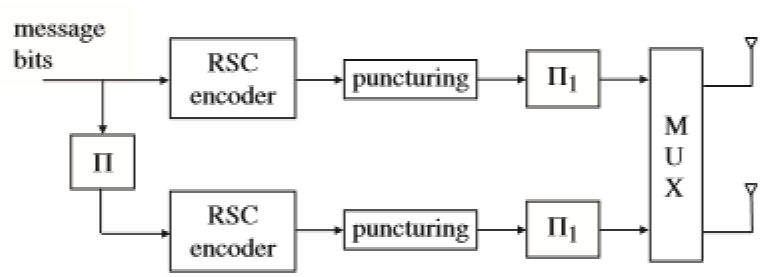

Figure 7.13 Parallel concatenated space-time turbo code encoder (for two transmit antennas).

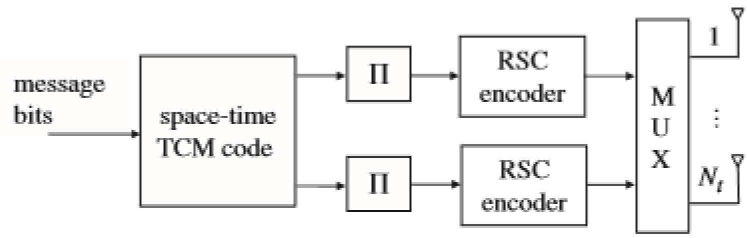

Figure 7.14 Encoder structure for the serial concatenated space-time turbo code.

Q7. Analyze the Turbo Space Time Trellis coding scheme.

The outer code is a binary code, whereas the inner code is a recursive STTC. The recursive nature of the inner code enhances the overall minimum Hamming distance of the combined code, thereby improving the available diversity. It is easy to design the inner code to achieve full diversity. This stems from the fact that any non-recursive space-time code that achieves full diversity, i.e. full rank, can be transformed easily into a recursive, full rank code. However, it does not achieve full-rate because of the presence of the outer code. This concatenation scheme is decoded iteratively where the front end receiver generates log-likelihoods for the mNt bits which are deinterleaved and passed to the outer decoder. The outer decoder computes its own extrinsic information and passes them back to the inner detector to refine its log-likelihoods for the next iteration. This repeats for a number of iterations before final decisions are made.

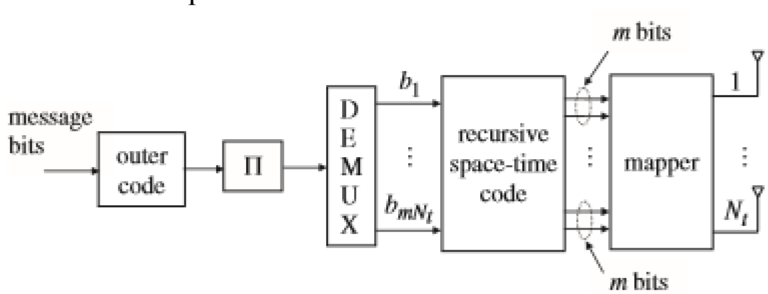

Figure 7.15 Encoder structure for the serial concatenation of an outer channel code and an inner recursive space-time code.

Q8. Discuss about the Performance of Maximum Likelihood Decoding.

The challenge in finding performance bounds for concatenated codes is due to the presence of the interleaver, and particularly so since it is random. To overcome this hurdle, one may use one

of the following two approaches, both of which invoke the union bound technique. In thefirst approach,the interleaver isassumedto be uniform, that is,it canbe anyone of the possible deterministic interleavers with equal probability. By this, the union bound becomes interleaver-independent since it is averaged with respect to all possible interleavers.
The second approach is based on deriving the union bound under the assumption that the interleaver is fixed. This involves finding, for a fixed code and interleaver, the most likely error events that impact the performance at high signal-to-noise ratios.
The optimal decoder for a concatenated code is the maximum likelihood sequence de- tector (MLSD). However, the MLSD is very complex to realize because of the complexity associated with the interleaver, as noted before. Therefore, as an alternative, the decoding of concatenated codes is achieved using a sub-optimal iterative decoding algorithm.
Q9. Explain APP Algorithm for decoding.
The posterior probability is calculated by updating the prior probability using Bayes' theorem. In statistical terms, the posterior probability is the probability of event A occurring given that event B has occurred.

A common modeling problem involves how to estimate a joint probability distribution for a dataset.

For example, given a sample of observation (*X*) from a domain (*x1, x2, x3, …, xn*), where each observation is drawn independently from the domain with the same probability distribution (so-called independent and identically distributed, i.i.d., or close to it).
Density estimation involves selecting a probability distribution function and the parameters of that distribution that best explains the joint probability distribution of the observed data (*X*).
Often estimating the density is too challenging; instead, we are happy with a point estimate from the target distribution, such as the mean.

There are many techniques for solving this problem, although two common approaches are:

- Maximum a Posteriori (MAP), a Bayesian method.
- Maximum Likelihood Estimation (MLE), a frequentist method.

Both approaches frame the problem as optimization and involve searching for a distribution and set of parameters for the distribution that best describes the observed data.

Q10. Write about SOVA Decoder.

SOVA – Soft Output Viterbi Algorithm

The Soft-Output Viterbi Algorithm (SOVA) is a variation of the Viterbi algorithm. This algorithm has two modifications over the classical Viterbi algorithm. First, the path metrics used to select the maximum likelihood path through the trellis are modified to take account of a-priori information. Second, the algorithm is modified to provide a soft output for each decoded bit.
Consider the operation of a Viterbi algorithm. At some time t, each surviving path in the trellis denotes a series of add/compare/select operations, each resulting in the selection of a value

for an information bit or symbol. Hagenauer and Hoeher noted that the probability that a given value is correct is proportional to how close the algorithm came to selecting the other value (or values). Let's assume here that the code in question is binary and the decoding depth for the decoder is D. The SOVA algorithm proceeds as follows.

The algorithm traverses the entire trellis, thereby tracing the ML path (the most probable).

Next the algorithm traces back from time t along the ML path, noting all path metric comparisons that could have changed the ML information bit value selected at time t-D. From among these comparisons that comparison is selected in which the difference between the compared partial path metrics is the smallest one. Thus the minimization is carried out only for those paths merging with the ML path which would have given a different value for the bit at time t-D if they had been selected as the survivor. Let Δ_{min} be the difference between these two partial path metrics. Likelihood of the decision can be expressed as $P(u_{t-D};O) \cong C \mid \Delta_{min} \mid$. We have to normalize the probabilities so that (in binary case) P(1;O)+P(0;O)=1.

Operation of the SOVA algorithm is illustrated in Fig. 1.

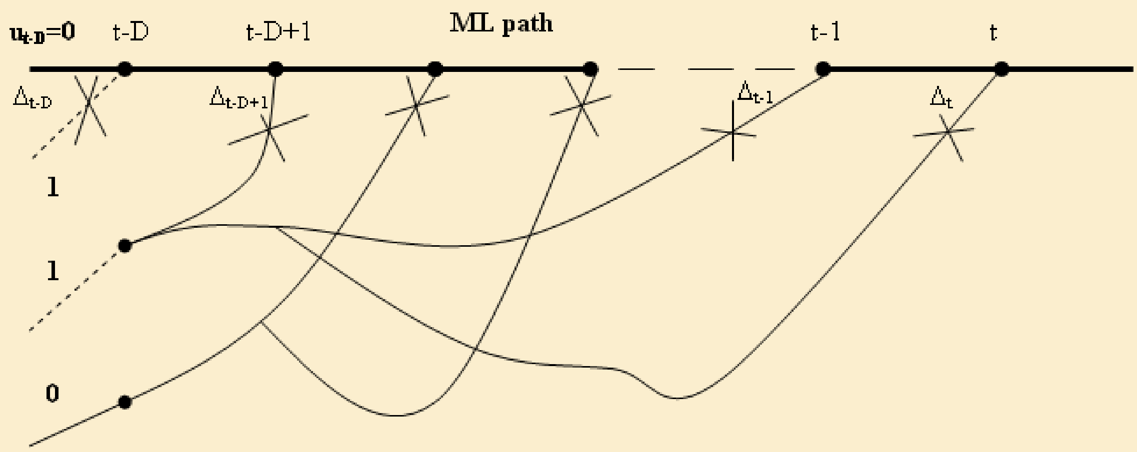

Fig. 1. Example of the trellis for a binary code with SOVA decoding

The SOVA algorithm selects minimum among the following values $\{\Delta_{t-D}, \Delta_{t-D+1}, \ldots, \Delta_{t-1}, \Delta_t\}$. The paths merging the ML path and rejected at time instants *t-D, t-D+1, ..., t-1, t* have the information bit u_{t-D} equal 1.

The SOVA approximation of the probability P(u;O) is quite good except at very low SNR's, at which point the estimate becomes too optimistic.

Q11. What are PCCC and SCCC in Decoder.

The PCCC interleaver is a 1024 bit pseudo random interleaver with no constraints added (e.g., no V random constraint). The SCCC interleaver is a 1152 bit pseudo random interleaver with no constraints added.

PCCC (Parallel concatenated convolutional codes) design consists of two convolutional encoders which are joined in parallel but one of them accepts the same primary data through an interleaver. In each case, decoder's design is different and more complicated than encoder's section.

Serial concatenated convolutional codes (SCCC) are a class of forward error correction (FEC) codes highly suitable for turbo (iteractive) decoding. ... The recursive inner code provides the 'interleaver gain' for the SCCC, which is the source of the excellent performance of these codes.

Q12. List the features of Iterative decoding.

The decoding operation is based on either a maximum a posteriori (MAP) algorithm or a Viterbi algorithm generating a weighted soft estimate of the input sequence. The iterative algorithm performs the information exchange between the two component decoders.

The performance gain of the MAP algorithm over the Viterbi algorithm at low SNR leads to a slight performance advantage.

The MAP algorithm is computationally much more complex than the Viterbi algorithm. The operations in the MAP algorithm are multiplications and exponentiations while in the Viterbi algorithm they are simple add, compare and select operations.