DAYANANDA SAGAR COLLEGE OF ENGINEERING

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COURSE NAME: MIMO Communication

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CONTENTS:

Space-time block and trellis codes: Transmit diversity with two antennas: The Alamouti scheme – Orthogonal and Quasi-orthogonal space-time block codes – Linear dispersion codes – Generic space-time trellis codes – Basic space-time code design principles – Representation of space-time trellis codes for PSK constellation – Performance analysis for space-time trellis codes – Comparison of space-time block and trellis codes.

Space-Time Block Codes

In the previous module, we have seen that the capacity and constrained information rates over fading channels increase significantly with the employment of multiple transmit and multiple receive antennas. In this module, we will introduce the idea of space-time block coding which is a practical means to achieve the benefits offered by MIMO systems. Space-time block coding is a simple yet very effective means of achieving transmit diversity when other forms of diversity may be limited or non-existent, e.g., for quasi-static fading channels. Such codes can be easily generalized to the case of multiple receive antennas as well, thus providing receive diversity in addition to transmit diversity. Furthermore, they can be decoded efficiently at the receiver by simple linear processing of the set of received signals at different receive antennas.

Our objective in this module is to study space-time block coding in detail. We begin with the simple case of two transmit antennas. We will detail the methods, give performance analysis results, and illustrate their performance by both analytical tools and using simulations.

The module is organized as follows. We first describe the Alamouti scheme which is a simple way of obtaining transmit diversity for the case of two transmit antennas. We then generalize this scheme to the case of more than two transmit antennas. For both approaches, we consider the optimal receiver structures, theoretical performance analysis in the presence of Rayleigh fading and performance evaluation by simulations. Furthermore, we study quasi-orthogonal space-time block codes, and linear dispersion codes. Finally, we provide our conclusions and suggestions for further reading.

Transmit Diversity with Two Antennas:

The Alamouti Scheme

As we have discussed in the previous chapter, it is relatively easy to obtain spatial diversity by employing multiple receive antennas. Consider, for instance, the uplink of a cellular telephony system that is the transmission is from a mobile to the base-station. Since the base-stations can be equipped with multiple antennas with sufficient separation easily, the signal transmitted by the mobile unit can be picked up by multiple receive antennas and they can be combined using a diversity-combining technique, e.g., maximal-ratio combining, selection combining, equal-gain combining, etc., to obtain receive diversity. However, if the situation is reversed (i.e., for downlink transmission) achieving diversity gain is not that simple due to the fact that the mobile units are typically limited in size, and it is usually difficult to place multiple antennas that are separated by sufficiently large distances for reception of multiple copies of the transmitted signal through independent channels. Therefore, it is desirable to have a scheme where the benefits of (spatial) diversity are exploited through “transmit diversity”. With this motivation, Alamouti (1998) introduced a way of obtaining transmit diversity when there are two transmit antennas.

Transmission Scheme

The Alamouti scheme is a simple transmit diversity scheme suitable for two transmit antennas. Two symbols are considered at a time, say $x1$ and $x2$, and they are transmitted in two consecutive time slots. In the first time slot, $x1$ is transmitted from the first antenna and $x2$ is transmitted from the second one. In the second time slot, $-x\ \ 2$ is transmitted from the first antenna, while $x\ \ 1$ is transmitted from the second antenna. This process is illustrated in

Figure 1. The signals $x1$ and $x2$ are picked from an arbitrary (M-ary) constellation. Since two symbols are transmitted in two time slots, the overall transmission rate is 1 symbol per channel use, or log2M bits per channel use.

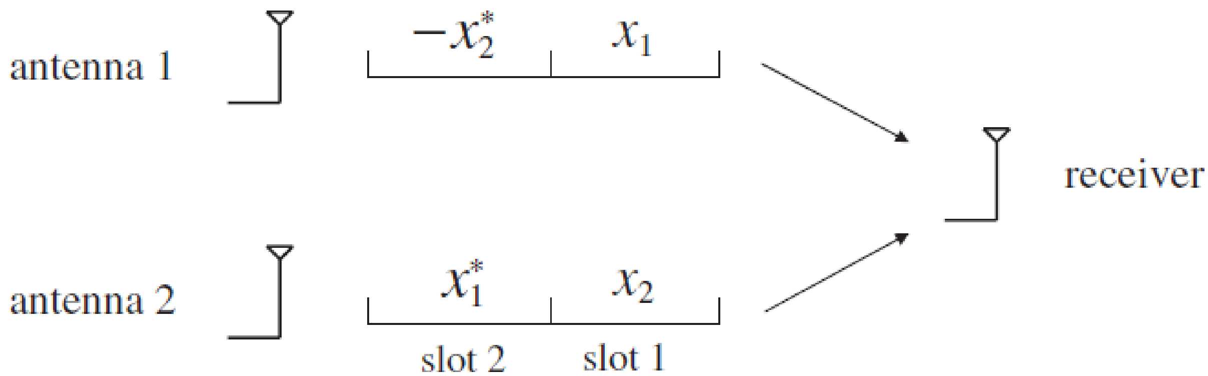


Figure 1 The Alamouti scheme

Optimal Receiver for the Alamouti Scheme

Let us now derive the optimal receiver for the Alamouti scheme. We consider two separate cases, namely, a single receive antenna and multiple receive antennas

Single Receive Antenna System

Consider the case of a single receive antenna. The received signal in the first time slot is then

$$y_1(1) = \sqrt{\rho}(h_{1,1}x_1 + h_{2,1}x_2) + n_1(1),$$

$$y_1(2) = \sqrt{\rho}(-h_{1,1}x_2^* + h_{2,1}x_1^*) + n_1(2)$$

in the second time slot. We assume that the channel is Rayleigh fading, i.e., $h1,\ 1$and $h2,1$ are zero mean complex Gaussian random variables with unit variance (i.e., with variance ½ per dimension), and they remain the same for two consecutive time intervals. We normalize the power of each constellation to be 1/2, thus the total transmit power per channel use is unity. The additive noise terms $n1(1)$ and $n1(2)$ are complex AWGN with variance ½ per dimension. With these definitions, the signal-to-noise ratio becomes *d*efine the vector of the received signals (where the second signal is conjugated) as where

$$\boldsymbol{H} = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{bmatrix}.$$

We have used the fact that $\boldsymbol{H}^H \boldsymbol{y}$ (multiplication by $\boldsymbol{H}^H$) is a one-to-one transforma
Assuming that all the input symbol pairs are equally likely, and using Bayes' rule, one
equivalently write the optimal decoded symbols as

$$(\hat{x}_1, \hat{x}_2) = \arg\max_{(x_1,x_2)} P(\boldsymbol{H}^H \boldsymbol{y} | x_1, x_2, h_{1,1}, h_{2,1}).$$

Note that

$$\boldsymbol{H}^H \boldsymbol{y} = \sqrt{\rho} \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & 0 \\ 0 & |h_{1,1}|^2 + |h_{2,1}|^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1'(1) \\ n_1'(2) \end{bmatrix},$$

where the new noise terms are given by

$$\begin{bmatrix} n_1'(1) \\ n_1'(2) \end{bmatrix} = \begin{bmatrix} h_{1,1}^* & h_{2,1} \\ h_{2,1}^* & -h_{1,1} \end{bmatrix} \begin{bmatrix} n_1(1) \\ n_1^*(2) \end{bmatrix}.$$

Clearly, being linear combinations of jointly Gaussian random variables, *n*_ 1*(*1*)* and *n*1*(*2*)* are jointly Gaussian. Since they are also uncorrelated, they are independent. Also, they both have zero mean and a variance of 12*(*|*h*1*,*1|2 + |*h*2*,*1|2*)* per dimension.

Therefore, the optimal decisions ˆ*x*1 and ˆ*x*2 decouple, and simplify to the usual minimization of the Euclidean distance between the possible transmitted symbols and the respective Components vector ***HH*** ***y***. That is,

$$\boldsymbol{y} = \begin{bmatrix} y_1(1) \\ y_1^*(2) \end{bmatrix},$$

which can be written as

$$\boldsymbol{y} = \sqrt{\rho} \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1(1) \\ n_1^*(2) \end{bmatrix}.$$

Assuming that the receiver has perfect knowledge of the channel state informatio
optimal receiver, which minimizes the probability of error, chooses $\hat{x}_1$ and $\hat{x}_2$ as foll

$$(\hat{x}_1, \hat{x}_2) = \arg\max_{(x_1,x_2)} P(x_1, x_2 | \boldsymbol{y}, h_{1,1}, h_{2,1}),$$

which can also be written as

$$(\hat{x}_1, \hat{x}_2) = \arg\max P(x_1, x_2 | \boldsymbol{H}^H \boldsymbol{y}, h_{1,1}, h_{2,1}),$$

where

$$\boldsymbol{H} = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{bmatrix}.$$

We have used the fact that $\boldsymbol{H}^H \boldsymbol{y}$ (multiplication by $\boldsymbol{H}^H$) is a one-to-one transform: Assuming that all the input symbol pairs are equally likely, and using Bayes' rule, on equivalently write the optimal decoded symbols as

$$(\hat{x}_1, \hat{x}_2) = \arg\max_{(x_1, x_2)} P(\boldsymbol{H}^H \boldsymbol{y} | x_1, x_2, h_{1,1}, h_{2,1}).$$

Note that

$$\boldsymbol{H}^H \boldsymbol{y} = \sqrt{\rho} \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & 0 \\ 0 & |h_{1,1}|^2 + |h_{2,1}|^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1'(1) \\ n_1'(2) \end{bmatrix},$$

where the new noise terms are given by

$$\begin{bmatrix} n_1'(1) \\ n_1'(2) \end{bmatrix} = \begin{bmatrix} h_{1,1}^* & h_{2,1} \\ h_{2,1}^* & -h_{1,1} \end{bmatrix} \begin{bmatrix} n_1(1) \\ n_1^*(2) \end{bmatrix}.$$

Clearly, being linear combinations of jointly Gaussian random variables, $n\ 1(1)$ and $n1(2)$ are jointly Gaussian. Since they are also uncorrelated, they are independent. Also, they both have zero mean and a variance of 12 $(|h1,1|2 + |h2,1|2)$ per dimension. Therefore, the optimal decisions ˆ$x1$ and ˆ$x2$ decouple, and simplify to the usual minimization of the Euclidean distance between the possible transmitted symbols and the respective components of the vector $\boldsymbol{H}H\,\boldsymbol{y}$.

That is,

$$\hat{x}_1 = \arg\min_{x_1} |h_{1,1}^* y_1(1) + h_{2,1} y_1^*(2) - \sqrt{\rho}(|h_{1,1}|^2 + |h_{2,1}|^2)x_1|,$$

$$\hat{x}_2 = \arg\min_{x_2} |h_{2,1}^* y_1(1) - h_{1,1} y_1^*(2) - \sqrt{\rho}(|h_{1,1}|^2 + |h_{2,1}|^2)x_2|.$$

We note that the above decoding rule is very useful and what makes the Al
scheme (and, more generally, orthogonal space-time block codes) attractive. This
cause decoupling of the optimal decisions clearly reduces the search space for the o
selection of the transmitted symbol, and thus simplifies the receiver structure consid

If we make further assumptions on the modulation schemes used, we can simpl
receiver structures even more. For instance, for the case of a constant energy constel
e.g., BPSK, QPSK, 8-PSK, the optimal decision rule can be written as the usual corr
maximization. For the special case of BPSK modulation, it reduces to

$$\hat{x}_1 = \begin{cases} 1, & \text{if Re } \{h_{1,1}^* y_1(1) + h_{2,1} y_1^*(2)\} > 0 \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\hat{x}_2 = \begin{cases} 1, & \text{if Re } \{h_{2,1}^* y_1(1) - h_{1,1} y_1^*(2)\} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

As we will discuss later, the full-diversity of the MIMO system, which is two for the case of two transmit and one receive antenna, can be obtained with the use of the Alamouti scheme. Therefore, even if the receivers are not equipped with multiple antennas, spatial diversity can still be achieved, which makes this scheme very beneficial.

Multiple Receive Antenna System:

The Alamouti scheme can easily be extended to systems with multiple receive antennas as well, resulting in receive diversity in addition to the existing transmit diversity. For this case, the available diversity order is twice the number of receive antennas, and it can also be achieved by a simple linear receiver.

Assume that the received signal during the kth time slot at the jth receive antenna is $yj\ (k)$, where $k = 1, 2, j = 1, 2, \ldots, Nr$. The channel coefficient from the ith transmit antenna to the jth receive antenna is denoted by hi,j . We can then write

$$y_j(1) = \sqrt{\rho}(h_{1,j}x_1 + h_{2,j}x_2) + n_j(1)$$

in the first time slot, and

$$y_j(2) = \sqrt{\rho}(-h_{1,j}x_2^* + h_{2,j}x_1^*) + n_j(2)$$

in the second time slot. Here, *nj (k)* is the AWGN term at time slot *k* and receive antenna *j*.

Of course, the optimal decision rule can be simplified further if a constant energy constellation, such as phase shift keying, is employed. For instance, for the case of BPSK modulation, it takes the form

$$\hat{x}_1 = \begin{cases} 1, & \text{if Re}\left\{\sum_{j=1}^{N_r} h_{1,j}^* y_j(1) + h_{2,j} y_j^*(2)\right\} > 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$\hat{x}_2 = \begin{cases} 1, & \text{if Re}\left\{\sum_{j=1}^{N_r} h_{2,j}^* y_j(1) - h_{1,j} y_j^*(2)\right\} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Performance Analysis of the Alamouti Scheme:

Consider a 2 × *Nr* MIMO system employing the Alamouti scheme. After linear processing (which is shown to be optimal), as argued in the previous section, the decision variable for the symbol *xk* is given by

$$y(k) = \sqrt{\rho}\left(\sum_{i=1}^{2}\sum_{j=1}^{N_r} |h_{i,j}|^2\right) x_k + n''(k)$$

From this expression it is clear that the average bit error probability of the Alamouti scheme when BPSK modulation is employed decays inversely with the *(2Nr)*th power of the signal-to-noise ratio. Therefore, the diversity provided is 2*Nr* (i.e., full spatial diversity) over a Rayleigh flat fading channel.

$$P_b \approx \binom{4N_r - 1}{2N_r}\left(\frac{1}{2\rho}\right)^{2N_r}$$

In this section, we present several error rate results for the Alamouti scheme used over Rayleigh flat fading channels based on both simulations and performance analysis carried out in the previous section. In Figure 2, we present the bit error rate obtained using BPSK modulation for several cases. For comparison purposes, we also show the performance of the no-diversity case, i.e., the single-input single-output system. For the two transmit and one receive antenna case, we observe that the diversity order is two, and for the two transmit and two receive antenna case, it is four. We also observe that the simulations and theoretical results agree with each other.

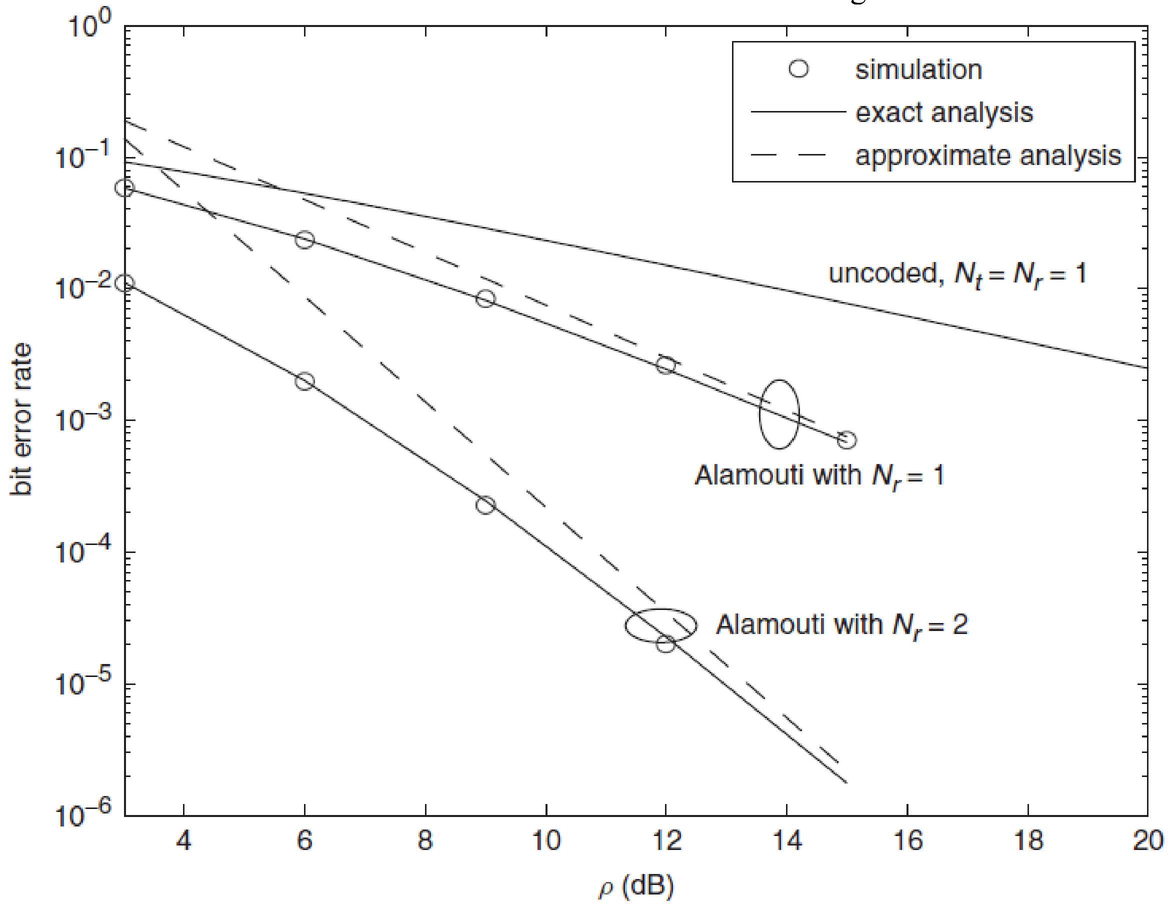
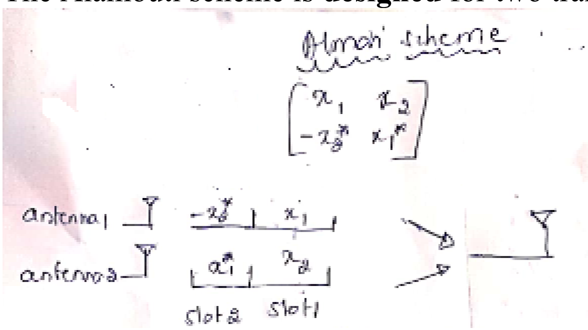


Figure 2.Bit error rate performance of the Alamouti scheme with BPSK modulation

Orthogonal Space-Time Block Codes:

The Alamouti scheme is **designed** for two transmit antennas.



A question that naturally comes to mind is: How can this transmit diversity scheme be extended to the case of more than two transmit antennas?

In this section, we describe what is known as the general **space-time block codes**

This based on the theory of orthogonal designs.

Let us present example of a space-time block code. Assume that we are to transmit the four symbols **{*x*1, *x*2, *x*3, *x*4} selected from a real signal constellation**

Consider a system with *Nt* transmit antennas.

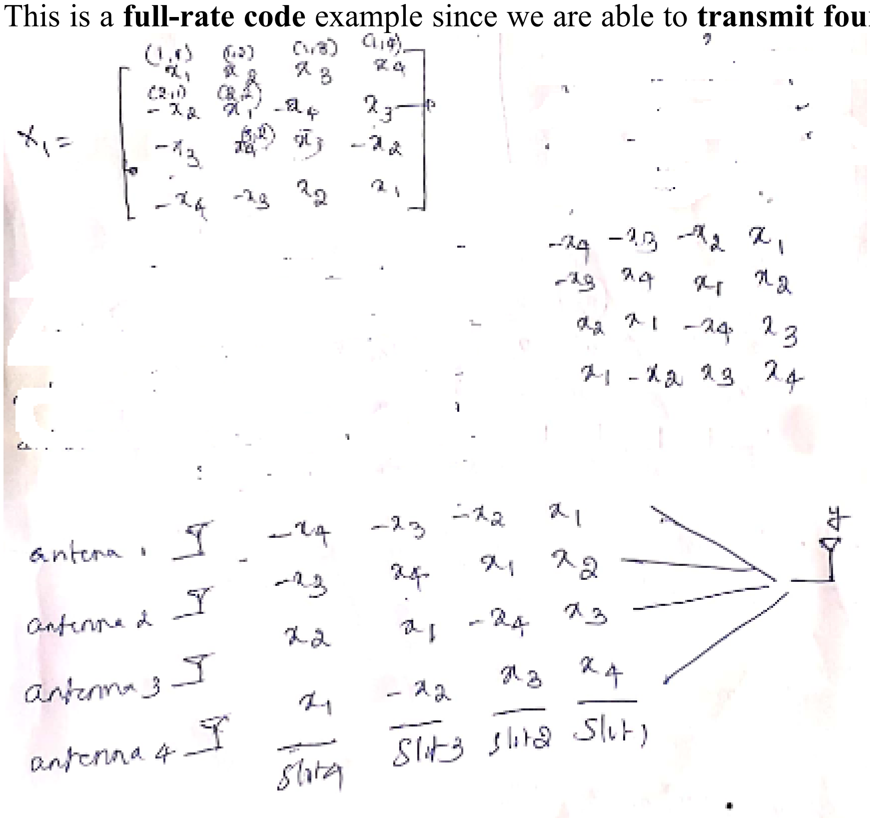
The objective is to design a set of *Nt* × *Nt* matrices

Assume that *Nt* = 4 and a.
We form the code matrix, A specific space-time block code can be described by the 4 × 4 matrix

$$X_1 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix}.$$

Clearly, the columns of *X*1 are mutually orthogonal.
*i*th row of this matrix using the four transmit antennas.
Symbol interval *i* ($1 \le i \le 4$),

That is, the *(i, j)*th element of the matrix is transmitted from antenna *j* during the *i*th time slot. This is a **full-rate code** example since we are able to **transmit four symbols in four time slots**,

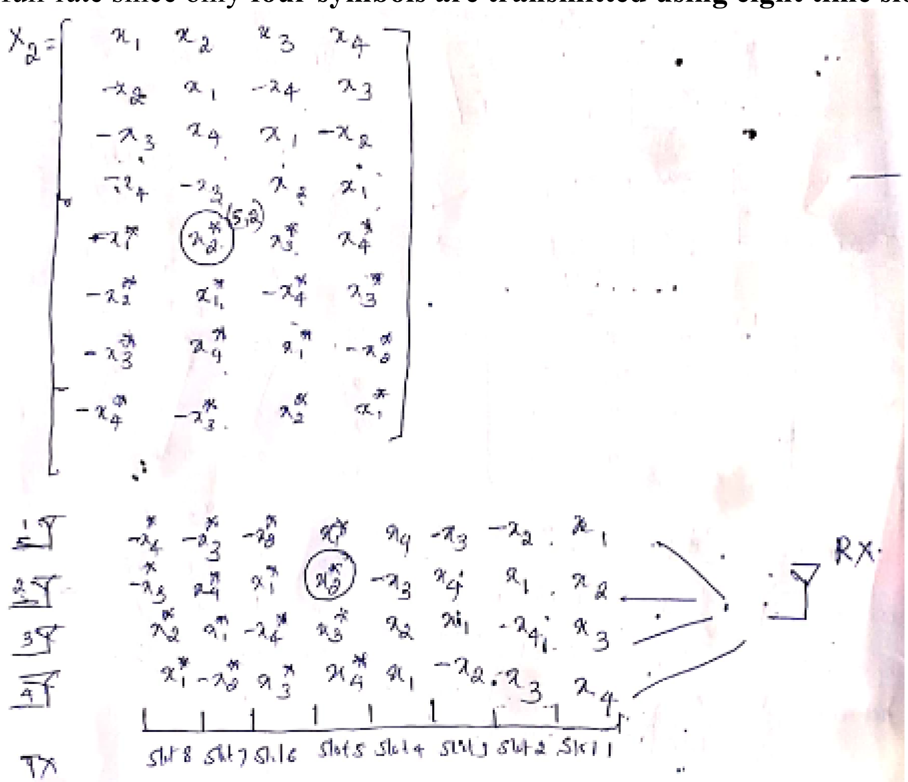


Let us present another example of a space-time block code. Assume that we are to transmit the four symbols **{*x*1, *x*2, *x*3, *x*4} selected from a complex signal constellation.**

A specific space-time block code can be described by the 8 × 4 matrix.

$$X_2 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}.$$

At time interval i ($1 \leq i \leq 8$), the ith row of $\boldsymbol{X}2$ is transmitted using the four transmit antennas. This code achieves full-diversity as well (this will be illustrated later); however, clearly, it is not full-rate since only **four symbols are transmitted using eight time slots.**



This simple example clearly illustrates that if we **relax the full-rate code design condition**, there are many **potential designs** one can employ.

In order to present **general space-time block codes** in a more concrete framework, let us describe the important sub class of linear orthogonal designs in detail.

Linear Orthogonal Designs:

Let us now describe the general class of linear orthogonal designs in a mathematical framework to combine different space-time block codes in a unified manner.

We denote the $\boldsymbol{N} \times \boldsymbol{N_t}$ code matrix that defines the space-time block code by $\boldsymbol{X}$.

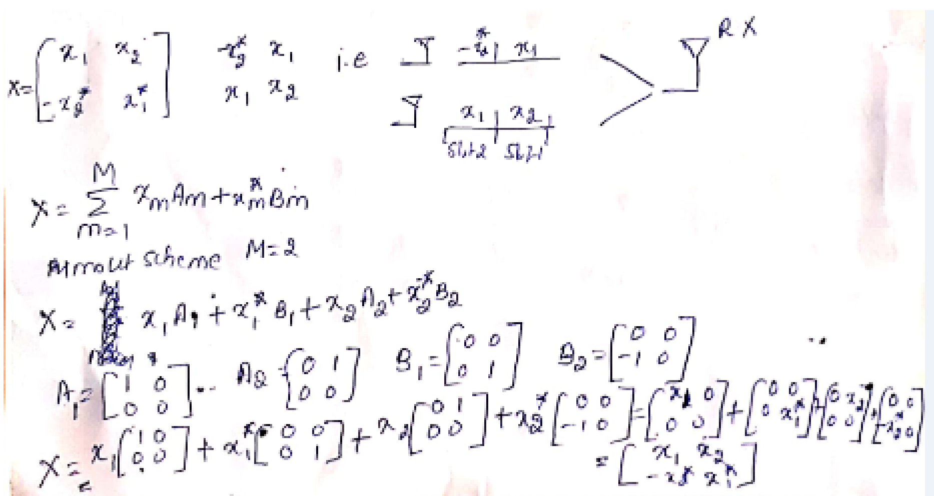
This matrix is used to transmit M symbols in N time slots (achieving a rate of M/N).

X is orthogonal.

We refer to this class of space-time block codes as linear orthogonal designs. We can equivalently write the code matrix $\boldsymbol{X}$ in the form

$$\boldsymbol{X} = \sum_{m=1}^{M} (x_m \boldsymbol{A}_m + x_m^* \boldsymbol{B}_m)$$

where $\boldsymbol{A}_m$ and $\boldsymbol{B}_m$ are $N \times N_t$ matrices, and $\{x_m\}$, M are the set of symbols being transmitted. This representation is general, and thus can be used to describe different codes studied earlier. For example, for the Alamouti code, we have $N_t = T = M = 2$ with



$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}.$$

As another example, for the code given in X_1 (4.33), we have

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix},$$

and $B_1 = B_2 = B_3 = B_4 = \mathbf{0}_4$.

For the space-time block code defined by $X2$ in expression (4.34), it is easy to see that the matrices take the form

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

It is straightforward to show that the space-time block code matrix $\boldsymbol{X}$ defines an orthogonal design if and only if the equalities

$$\mathbf{A}_i^H \mathbf{A}_j + \boldsymbol{B}_j^H \boldsymbol{B}_i = \delta_{ij} \boldsymbol{D}_i,$$

$$\mathbf{A}_i^H \boldsymbol{B}_j + \mathbf{A}_j^H \boldsymbol{B}_i = \mathbf{0},$$

are satisfied (see Xu and Kwak (2005)). Here δij is the Kronecker delta (i.e., $\delta ij = 1$ if $i = j$, and 0 otherwise), and $\boldsymbol{Di}$ is a diagonal matrix where the diagonal elements are strictly positive.

Decoding of Linear Orthogonal Designs

Let us now describe the decoding of space-time block codes based on the framework presented in the previous section. Let $\boldsymbol{yj}$ denote the $N \times 1$ received signal vector at antenna j whose kth element, $yj\ (k)$, shows the received signal at the kth time interval. We have

$$\boldsymbol{y}_j = \sqrt{\rho} \boldsymbol{X} \boldsymbol{h}_j + \boldsymbol{n}_j$$

where the $Nt \times 1$ vector $\boldsymbol{hj}$ shows the channel coefficients from all the transmit antennas to the jth receive antenna, and the vector $\boldsymbol{nj}$ is the complex Gaussian noise term. The channel gains and noise terms are normalized so that their variance per dimension is ½ and the signal constellation is scaled so that its average energy is $1/Nt$ at each transmit antenna. Therefore, ρ denotes the signal-to-noise ratio at each receive antenna.

Define the $2N \times 1$ vectors

$$\tilde{y}_j = \begin{bmatrix} y_j(1) \\ y_j(2) \\ \vdots \\ y_j(N) \\ y_j^*(1) \\ y_j^*(2) \\ \vdots \\ y_j^*(N) \end{bmatrix}, \quad \tilde{n}_j = \begin{bmatrix} n_j(1) \\ n_j(2) \\ \vdots \\ n_j(N) \\ n_j^*(1) \\ n_j^*(2) \\ \vdots \\ n_j^*(N) \end{bmatrix}.$$

Based on the structure of the space-time block codes based on the orthogonal designs discussed in the previous section, we can write the equality.

$$\tilde{y}_j = \sqrt{\rho} \sum_{m=1}^{M} \left\{ \begin{bmatrix} A_m h_j \\ B_m^* h_j^* \end{bmatrix} x_m + \begin{bmatrix} B_m h_j \\ A_m^* h_j^* \end{bmatrix} x_m^* \right\} + \tilde{n}_j$$

Since the noise is AWGN, and the channel coefficients are assumed to be known at the receiver, the optimal decision can be obtained by minimizing the squared Euclidean distance between a candidate codeword (***X(ˆx)***) and the received signal, which is given by

$$d(\hat{x}) = \sum_{j=1}^{N_r} \| y_j - \sqrt{\rho} X(\hat{x}) h_j \|^2.$$

$$v_1^2 = v_2^2 = v_3^2 = v_4^2 = \sum_{j=1}^{N_r} \sum_{i=1}^{N_t} 2|h_{i,j}|^2$$

With this set of equations, the optimal decoding rule which is simple linear combining is obtained explicitly

Performance Analysis of Space-Time Block Codes

In this section, we calculate the error rates offered by space-time block codes over Rayleigh fading channels. We again assume that the sub-channels between different antenna pairs fade independently. Assume that the sequence of symbols $\{x1, x2, \ldots, xM\}$ are being transmitted. Then, the decision variable *um* computed for *xm* at the receiver by linear processing is given by

$$u_m = \sum_{j=1}^{N_r} h_j^H A_m^H \left(\sqrt{\rho} X h_j + n_j \right) + \left(\sqrt{\rho} h_j^H X^H + n_j^H \right) B_m h_j$$

By using the properties of linear orthogonal designs given by , we can simplify this expression to

$$u_m = \sqrt{\rho} x_m \sum_{j=1}^{N_r} \sum_{i=1}^{N_t} d_{i,m} |h_{i,j}|^2 + n'_m$$

This expression can be approximated for large signal-to-noise ratios as

$$P_b \approx \binom{2N_t N_r - 1}{N_t N_r} \left(\frac{N_t}{4 d_m \rho} \right)^{N_t N_r}.$$

This expression clearly shows the diversity advantage provided by the space-time block coding schemes. The bit error rate for the BPSK modulation scheme decays inversely with the *(NrNt)*th power of the signal-to-noise ratio, i.e., the diversity order achieved is *NrNt* which is the full spatial diversity that can be obtained. Clearly this generalizes the result obtained for the case of the Alamouti scheme which was specific to the case of two transmit antennas.

$$P_{e,M-PSK} \approx 2 \binom{2N_t N_r - 1}{N_t N_r} \left(\frac{N_t}{4 d_m \sin^2(\frac{\pi}{M})} \right)^{N_t N_r} \left(\frac{1}{\rho} \right)^{N_t N_r}$$

which clearly demonstrates the diversity advantage that can be obtained

Examples

We now examine the performance of the two space-time block code examples of the previous section through simulations. We first consider the case of four transmit antennas with the code defined by ***X***1. We present the bit error rate with BPSK modulation over a Rayleigh flat fading channel in Figure 3. This is a full-rate code achieving one bit per transmission. We show both the simulation results and the theoretical results derived in the previous section. We observe that the diversity achieved is four for the case of one receive antenna, and it is eight for the case of two receive antennas as expected.

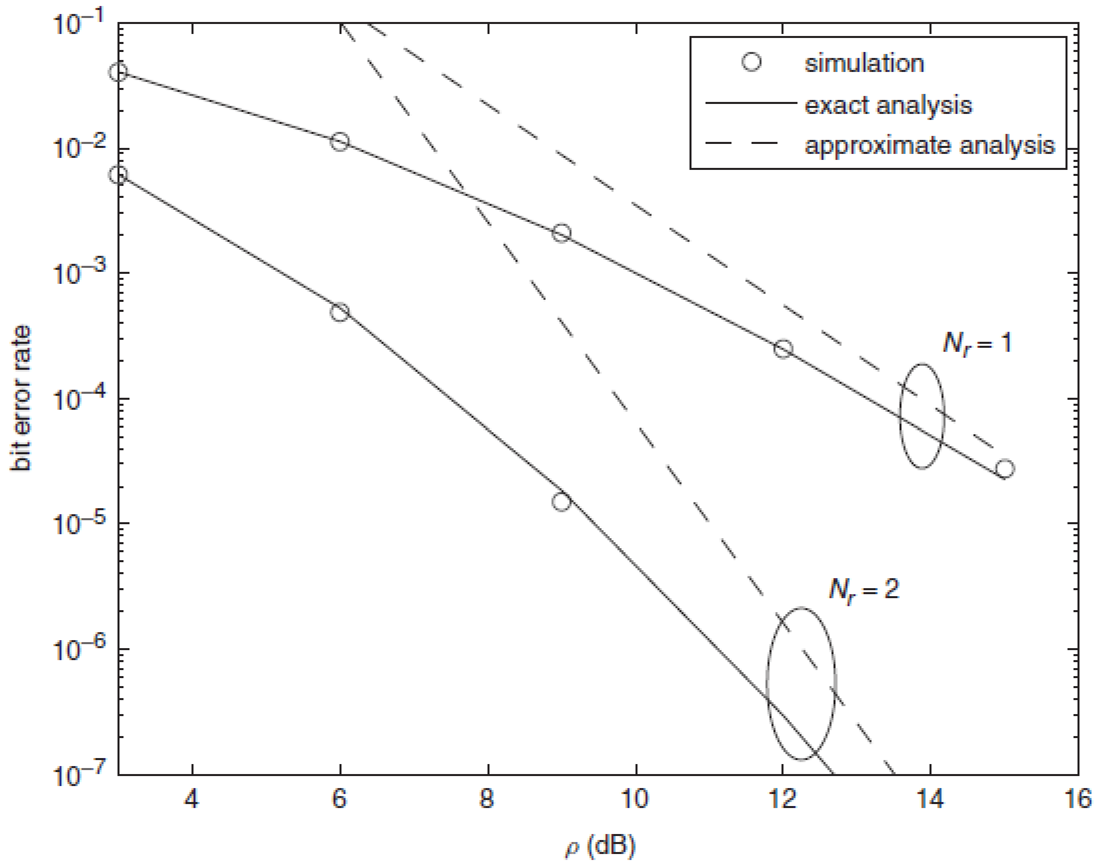


Figure 3 Bit error rate of ***X***1 with BPSK modulation

We consider the half rate space-time block code example given by ***X***2 in Figure 4. In this case there are four transmit antennas, and we assume that QPSK is employed resulting in a transmission rate of one bit per channel use. We observe that the simulation results and the theoretical expectations clearly match, and they demonstrate a diversity order of four

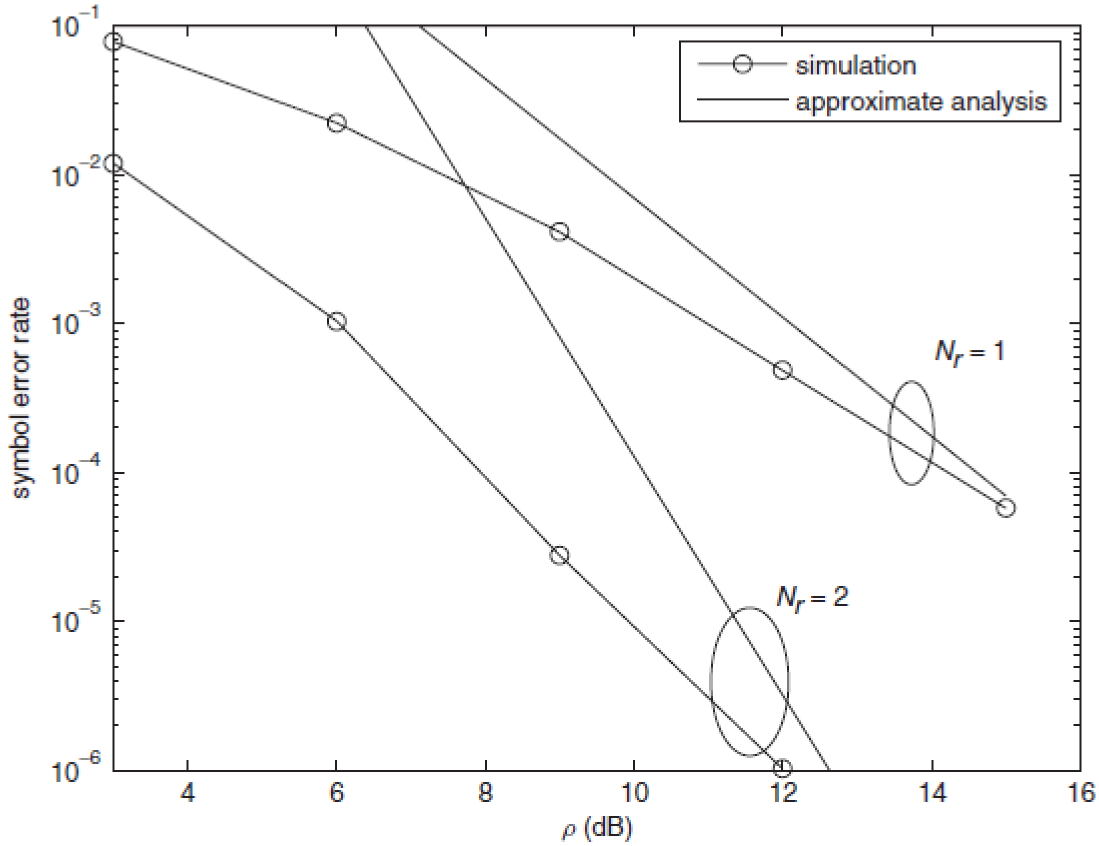


Figure 4 Symbol error rate of $\boldsymbol{X}2$ with QPSK modulation

Quasi-Orthogonal Space-Time Block Codes:

As we have seen earlier in this chapter, **there are no full-rate, full-diversity codes** (for complex constellations) other than the Alamouti scheme, and such designs are very limited for real constellations.

The quasi-orthogonal space- time block coding approach of Jafarkhani (2001) gives a way of obtaining full-rate (or increased-rate) space-time block coding designs **using smaller designs** as building blocks.

Let us illustrate the ideas via a simple example by presenting a full-rate design for the $Nt = 4$ scheme using the Alamouti code.

We consider the transmission of four symbols, say $x1$, $x2$, $x3$ and $x4$.

We use the Alamouti code to encode these symbols pair wise ie x1,x2 and x3 x4 resulting in two 2 × 2 matrices of the form

$$\boldsymbol{X}_{12} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \quad \text{and} \quad \boldsymbol{X}_{34} = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix}.$$

We then use these two matrices in another orthogonal design to obtain the 4 × 4 quasiorthogonal space-time block code matrix as where *(·)* denotes the complex conjugate of a matrix.

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_{12} & \boldsymbol{X}_{34} \\ -\boldsymbol{X}_{34}^* & \boldsymbol{X}_{12}^* \end{bmatrix}$$

Thus, we obtain

$$\boldsymbol{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}.$$

This is clearly a way of obtaining a full-rate code with complex constellations for the four transmit antenna case. However, as we know it cannot achieve full-diversity as no such full-rate full-diversity design exists. In fact, it is easy to show that the diversity order achieved with this scheme is $2N_r$.

Clearly not all columns of the code matrix are orthogonal.

To describe the decoding process, we observe that the first and fourth columns of the code matrix are orthogonal to the second and third columns, respectively. Therefore, with appropriate linear processing, the decisions for $x1$ and $x4$ can be decoupled from decisions for $x2$ and $x3$.

After that step, the pairs of codewords have to be decoded jointly, thus increasing the complexity of the decoding algorithm compared with the case of linear orthogonal designs.

We conclude this section by emphasizing that other existing smaller orthogonal designs can also be extended to a larger number of transmit antennas by a similar approach in a straightforward way, giving rise to similar conclusions.

Linear Dispersion Codes:

We now describe another class of codes, called linear dispersion codes, which generalize the idea of orthogonal and quasi-orthogonal space-time block codes.

As discussed earlier, orthogonal space-time block codes achieve full diversity over a MIMO channel. However, the rate of transmission offered by them is not very high.

To alleviate this problem, Hassibi and Hochwald (2002) introduced linear dispersion codes which basically remove the orthogonality constraint imposed before. Let us now describe this class of codes in some detail, Assume that $x1, x2, \ldots, xM$ represent M symbols to be transmitted over N time slots. The transmitted codeword is described by

$$\boldsymbol{X} = \sum_{m=1}^{M} (x_m \boldsymbol{A}_m + x_m^* \boldsymbol{B}_m)$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are arbitrary $N \times Nt$ matrices, called dispersion matrices. This description looks identical to the one in (4.35), however we are now talking about a much broader class of codes, since the condition given in (4.37) to obtain linear orthogonal designs is not imposed here.

$$\boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{A}_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

As another example, consider the scenario where the number of symbols transmitted is $M = N\,Nt$, and the space-time codeword is given by

$$X = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \\ x_{N+1} & x_{N+2} & \cdots & x_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N(N_t-1)+1} & x_{N(N_t-1)+2} & \cdots & x_{NN_t} \end{bmatrix}$$

which basically describes a very high-rate transmission scheme where at different time slots and different antennas, different symbols are transmitted. Clearly, this is not an orthogonal design, or any other design we have seen thus far. Since the special properties that were imposed in orthogonal designs are lacking, decoding or analysis may not be simple. In fact, this scheme will be referred as the vertical Bell Labs layered space-time (VBLAST) architecture.
Maximum likelihood decoding for linear dispersion codes (with parameters of interest) is out of the question due to the very high complexity requirements. Linear processing (or one of its variations) is also not feasible since there is no orthogonality constraint imposed on the dispersion matrices.

Other Things in STBC
Space–time block coding is a technique used in wireless communications to transmit multiple copies of a data stream across a number of antennas and to exploit the various received versions of the data to improve the reliability of data transfer. The fact that the transmitted signal must traverse a potentially difficult environment with scattering, reflection, refraction and so on and may then be further corrupted by thermal noise in the receiver means that some of the received copies of the data may be closer to the original signal than others. This redundancy results in a higher chance of being able to use one or more of the received copies to correctly decode the received signal. In fact, space–time coding combines all the copies of the received signal in an optimal way to extract as much information from each of them as possible.
In the case of STBC in particular, the data stream to be transmitted is encoded in blocks, which are distributed among spaced antennas and across time. While it is necessary to have multiple transmit antennas, it is not necessary to have multiple receive antennas, although to do so improves performance.

An STBC is usually represented by a matrix. Each row represents a time slot and each column represents one antenna's transmissions over time.

Only one standard STBC can achieve full-rate (rate 1) — Alamouti'scode.

$$
\begin{array}{cc}
 & \text{transmit antennas} \longrightarrow \\
\text{time-slots} \downarrow & \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n_T} \\ s_{21} & s_{22} & \cdots & s_{2n_T} \\ \vdots & \vdots & & \vdots \\ s_{T1} & s_{T2} & \cdots & s_{Tn_T} \end{bmatrix}
\end{array}
$$

Here, Sij is the modulated symbol to be transmitted in time sloti from antenna j. There are to be T time slots and T transmit antennas as well as R receive antennas. This block is usually considered to be of 'length' T

The code rate of an STBC measures how many symbols per time slot it transmits on average over the course of one block. If a block encodes k symbols, the code-rate is

$$r = \frac{k}{T}.$$

Analysis of Orthogonal STBC..

This means that the *STBC is* designed such that the vectors representing any pair *of* columns taken from the coding matrix *is orthogonal*.

Orthogonal STBC (O-STBC) has the simplest decoding complexity, as its ML decoding can be achieved by linear detection.

Orthogonality

STBCs as originally introduced, and as usually studied, are orthogonal. This means that the STBC is designed such that the vectors representing any pair of columns taken from the coding matrix is orthogonal. The result of this is simple, linear, optimal decoding at the receiver. Its most serious disadvantage is that all but one of the codes that satisfy this criterion must sacrifice some proportion of their data rate.

Call a codeword

$$\mathbf{c} = c_1^1 c_1^2 \cdots c_1^{n_T} c_2^1 c_2^2 \cdots c_2^{n_T} \cdots c_T^1 c_T^2 \cdots c_T^{n_T}$$

and call an erroneously decoded received codeword

$$\mathbf{e} = e_1^1 e_1^2 \cdots e_1^{n_T} e_2^1 e_2^2 \cdots e_2^{n_T} \cdots e_T^1 e_T^2 \cdots e_T^{n_T}.$$

Then the matrix

$$\mathbf{B}(\mathbf{c},\mathbf{e}) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_T^1 - c_T^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_T^2 - c_T^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^{n_T} - c_1^{n_T} & e_2^{n_T} - c_2^{n_T} & \cdots & e_T^{n_T} - c_T^{n_T} \end{bmatrix}$$

has to be full-rank for any pair of distinct codewords c and e to give the maximum possible diversity order of NtNr . If instead, B(c,e) has minimum rank over the set of pairs of distinct codewords, then the space–time code offers diversity order . STBCs offer only diversity gain (compared to single-antenna schemes) and not coding gain. There is no coding scheme included here — the redundancy purely provides diversity in space and time. This is contrast with space–time trellis codes which provide both diversity and coding gain since they spread a conventional trellis code over space and time.

Hypothesize Quazi Orthogonal STBC.

These codes exhibit partial orthogonality and provide only part of the diversity gain. The *quasi-orthogonal* STBCs achieve higher data rates than Orthogonal STBC at the cost *of* inter-symbol interference (ISI). Antenna system and has the coding matrix: where * denotes complex conjugate.

$$C_{4,1} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix}.$$

The orthogonality criterion only holds for columns (1 and 2), (1 and 3), (2 and 4) and (3 and 4). Crucially, however, the code is full-rate and still only requires linear processing at the receiver, although decoding is slightly more complex than for orthogonal STBCs. Results show that this Q-STBC outperforms (in a bit-error rate sense) the fully orthogonal 4-antenna STBC over a good range of signal-to-noise ratios (SNRs). At high SNRs, though (above about 22 dB in this particular case), the increased diversity offered by orthogonal STBCs yields a better BER.

Decoding of Orthogonal STBC.

One particularly attractive feature of orthogonal STBCs is that maximum likelihood decoding can be achieved at the receiver with only linear processing. In order to consider a decoding method, a model of the wireless communications system is needed.

At time t, the signal r_t^j received at antenna j is:

$$r_t^j = \sum_{i=1}^{n_T} \alpha_{ij} s_t^i + n_t^j,$$

where α_{ij} is the path gain from transmit antenna i to receive antenna j, s_t^i is the signal transmitted by transmit antenna i and n_t^j is a sample of additive white Gaussian noise (AWGN).

The maximum-likelihood detection rule[9] is to form the decision variables

$$R_i = \sum_{t=1}^{n_T} \sum_{j=1}^{n_R} r_t^j \alpha_{\epsilon_t(i)j} \delta_t(i)$$

where $\delta_k(i)$ is the sign of s_i in the k^{th} row of the coding matrix, $\epsilon_k(p) = q$ denotes that s_p is (up to a sign difference), the (k, q) element of the coding matrix, for $i = 1, 2, \ldots, n_T$ and then decide on constellation symbol s_i that satisfies

$$s_i = \arg\min_{s \in \mathcal{A}} \left(|R_i - s|^2 + \left(-1 + \sum_{k,l} |\alpha_{kl}|^2 \right) |s|^2 \right),$$

with $\mathcal{A}$ the constellation alphabet. Despite its appearance, this is a simple, linear decoding scheme that provides maximal diversity.

Space-Time Trellis Codes

Last section the space-time block coding which provided full spatial diversity for MIMO systems. If properly designed, in addition to providing full spatial diversity, space-time block codes can be **decoded efficiently** using **linear processing** at the receiver.

In this section, we study space-time trellis coding which is **another basic method of coding** for MIMO systems. This is same as that of the convolution coding. In this case trellis structure determines the symbols to be transmitted from different antennas.

Comparison: Space-time block codes do not have memory from one block to the next, Space-time trellis codes have a coding advantage, but at the cost of increased decoding complexity

The content is organized as follows.

We begin with a simple example of a space-time trellis code.

We then consider general space-time trellis codes and the corresponding suitable decoding algorithm.

We also develop the pair wise error probability expressions for Rayleigh and Rician fading channels, present basic code design principles, and give examples of good space-time trellis codes.

A Simple Space-Time Trellis Code

To illustrate the main ideas in space-time trellis coding, let us start with a simple example of a space-time trellis code. Assume that there are $N_t = 2$ transmit antennas.

Assume that a four-state trellis whose section is shown in Figure 1 is used for space-time coding.

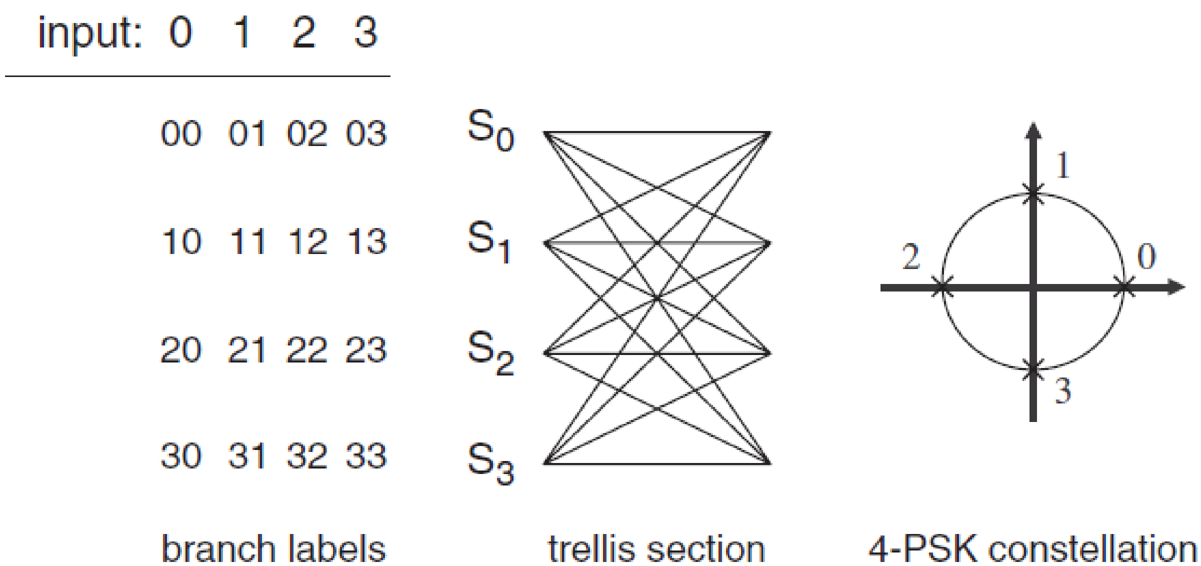


Figure 1 A four-state space-time trellis code example.

Encoding is performed as follows. The initial state of the finite state machine is assumed to be $S0$. Each pair of bits indicates the state transition.

Each pair of information bits to be encoded determines the state transition, and the two coded symbols to be transmitted from the two transmit antennas simultaneously as shown in Figure 1. For instance, the encoded sequence corresponding to the (4-ary) information sequence

$$2, 1, 2, 3, 0, 0, 1, 3, 2, \ldots$$

is simply

$$0, 2, 1, 2, 3, 0, 0, 1, 3, \ldots$$

from the first antenna, and

$$2, 1, 2, 3, 0, 0, 1, 3, 2, \ldots$$

from the second antenna. Once all the information bits are encoded,

Once all the information bits are encoded, the trellis is terminated to complete each frame of data.

We observe that for this example, two information bits are transmitted per use of the channel, thus resulting in a transmission rate of 2 bits per channel use. In general to obtain a transmission rate of R bits per channel use, we need to use a trellis with $2R$ branches emanating from each state.

This process is a simple extension of convolutional coding (more precisely, trellis coded modulation). That is, instead of using a trellis which determines only a single symbol to be transmitted for each trellis section (e.g., in the standard trellis coding), each branch is associated with multiple symbols that are to be transmitted from the multiple antenna elements.

General Space-Time Trellis Codes

In this section, we study general space-time trellis codes which were introduced through a simple example in the previous section. We consider a general mathematical description, establish notation, and derive the decoding algorithm for MIMO fading channels. Since spatial diversity through space-time coding is desirable for limited time diversity systems, we concentrate on quasi-static fading channels.

Furthermore, we assume that the channel state information is known at the receiver

Notation and Preliminaries

Consider a MIMO system with *Nt* transmit and *Nr* receive antennas.

An *ns* state space-time trellis code with transmission rate *R* is defined by an *ns* state trellis diagram where there are 2*R* branches emanating from each state. For every branch there are *Nt* branch labels chosen from a certain signal constellation,. Assuming that *xi(k)* is the transmitted signal from antenna *i* at time *k*, the received signal at the *j*th antenna corresponding to this time interval is given by

$$y_j(k) = \sqrt{\rho} \sum_{i=1}^{N_t} h_{i,j} x_i(k) + n_j(k)$$

where *i* = 1, 2, . . .,*Nt. j* = 1, 2, . . .,*Nr* , *t* = 1, 2, . . .,*N*, and *N* is the frame length; *hi,j* denotes the complex Gaussian channel coefficient between the *i*th transmit and *j*th receive antennas. Thus the channel is Rayleigh or Rician fading where the channel gains are constant over an entire frame of length *N* (i.e., quasi-static fading), and are independent across different sub-channels; *nj (k)* is the additive Gaussian noise sample at the *j*th receive antenna at time *k* The input-output relationship can be written in the matrix form as

$$\boldsymbol{Y} = \sqrt{\rho}\boldsymbol{X}\boldsymbol{H} + \boldsymbol{N}$$

where ***X*** denotes the *N* × *Nt* transmitted codeword (whose *(k, i)*th element is *xi(k)*), ***H*** is the *Nt* × *Nr* matrix of channel coefficients (whose *(i, j)*th element is *hi,j*), ***Y*** is the *N* × *Nr* received matrix consisting of *yj (k)* as its entries, and ***N*** is the *N* × *Nr* noise matrix.

Decoding of Space-Time Trellis Codes:

Assuming that the receiver has access to the channel coefficients, the optimal decision rule minimizing the probability of error is given by

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}}{\arg\min}\, P(\boldsymbol{X}|\boldsymbol{Y}, \boldsymbol{H}).$$

This is the maximum a posteriori (MAP) decoding rule. If all the symbols are equally likely, then it is equivalent to the ML decoding rule, and it is given by

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}}{\arg\min}\, P(\boldsymbol{Y}|\boldsymbol{X}, \boldsymbol{H}),$$

Also, observing that the metric is additive at each time step, it is clear that one can use the well-known Viterbi algorithm to perform the decoding very efficiently.

Basic Space-Time Code Design Principles

To design the space time code, we will require the pairwise pobality expression is required, that is considered first.

Pairwise Error Probability

Assume that two space-time trellis codeword's are given by ***X***1 and ***X***2. The pair wise error probability *P(**X***1 → ***X***2*)* is the probability that the received signal vector is closer to the erroneous codeword ***X***2 given that the codeword ***X***1 is transmitted.

Conditioned on the instantaneous channel realization, the pair wise error probability is given by

$$P(\boldsymbol{X}_1 \rightarrow \boldsymbol{X}_2|\boldsymbol{H}) = P(\|\boldsymbol{Y} - \sqrt{\rho}\boldsymbol{X}_1\boldsymbol{H}\|^2 > \|\boldsymbol{Y} - \sqrt{\rho}\boldsymbol{X}_2\boldsymbol{H}\|^2),$$

where $\boldsymbol{Y} = \sqrt{\rho}\boldsymbol{X}_1\boldsymbol{H} + \boldsymbol{N}$. This can equivalently be written as

$$P(\boldsymbol{X}_1 \rightarrow \boldsymbol{X}_2|\boldsymbol{H}) = P(\|\boldsymbol{N}\|^2 > \|\boldsymbol{N} - \sqrt{\rho}\boldsymbol{D}\|^2),$$

where $\boldsymbol{D} = (\boldsymbol{X}_2 - \boldsymbol{X}_1)\boldsymbol{H}$. Noting that the norm used in the above expression is the Frobenius norm, and writing the argument of the probability expression explicitly, we have

$$P(\boldsymbol{X}_1 \rightarrow \boldsymbol{X}_2|\boldsymbol{H}) = P\left(\sum_{k=1}^{N}\sum_{j=1}^{N_r}|n_j(k)|^2 > \sum_{k=1}^{N}\sum_{j=1}^{N_r}|n_j(k) - \sqrt{\rho}d_j(k)|^2\right),$$

where $d_j(k)$ is the (k, j)th element of the matrix $\boldsymbol{D}$.

Therefore, we obtain

$$P(\boldsymbol{X}_1 \rightarrow \boldsymbol{X}_2|\boldsymbol{H}) = Q(\sqrt{\rho/2}\|\boldsymbol{D}\|),$$

which can be simply be upper-bounded as

$$P(\boldsymbol{X}_1 \rightarrow \boldsymbol{X}_2|\boldsymbol{H}) \leq \exp\left(-\frac{\rho\|\boldsymbol{D}\|^2}{4}\right).$$

Denoting the columns of the channel coefficient matrix by $\boldsymbol{h}_j$, we can rewrite the matrix $\boldsymbol{D}$ as

$$\boldsymbol{D} = [(\boldsymbol{X}_1 - \boldsymbol{X}_2)\boldsymbol{h}_1 \quad (\boldsymbol{X}_1 - \boldsymbol{X}_2)\boldsymbol{h}_2 \quad \cdots \quad (\boldsymbol{X}_1 - \boldsymbol{X}_2)\boldsymbol{h}_{N_r}].$$

Therefore, its Frobenius norm can be written as

$$\|\boldsymbol{D}\|^2 = \sum_{j=1}^{N_r} \|(\boldsymbol{X}_1 - \boldsymbol{X}_2)\boldsymbol{h}_j\|^2,$$

$$= \sum_{j=1}^{N_r} \boldsymbol{h}_j^H (\boldsymbol{X}_1 - \boldsymbol{X}_2)^H (\boldsymbol{X}_1 - \boldsymbol{X}_2)\boldsymbol{h}_j.$$

Since the matrix $\boldsymbol{A} = (\boldsymbol{X}_1 - \boldsymbol{X}_2)^H (\boldsymbol{X}_1 - \boldsymbol{X}_2)$ is Hermitian .
it can be diagonalzed using a unitary matrix ***U***,
is a diagonal matrix whose diagonal elements are the eigenvalues of the matrix ***A***. Denoting the eigenvalues of ***A*** by λi (where $i = 1, 2, \ldots, Nt$), and defining

$$\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{N_t}\}$$

Using this expression, the conditional pairwise error probability that we require can be written as

$$P(\boldsymbol{X}_1 \to \boldsymbol{X}_2|\boldsymbol{H}) \le \exp\left(-\frac{\rho}{4}\sum_{j=1}^{N_r}\sum_{i=1}^{N_t}\lambda_i|\beta_{i,j}|^2\right),$$

$$= \prod_{j=1}^{N_r}\prod_{i=1}^{N_t}\exp\left(-\frac{\rho\lambda_i|\beta_{i,j}|^2}{4}\right).$$

Rayleigh Fading
Assuming that the wireless channel is modeled by Rayleigh fading, the random variables $\beta i,j$ are zero-mean complex Gaussian with variance 1/2 per dimension.

Thus, we can easily average over the channel variations to obtain an upper bound on the (unconditional) pair wise error probability as

$$P(\boldsymbol{X}_1 \to \boldsymbol{X}_2) \le \prod_{j=1}^{N_r}\prod_{i=1}^{N_t}\int_0^{\infty} \exp\left(-\frac{\rho\lambda_i z}{4}\right) e^{-z} dz,$$
$$= \prod_{j=1}^{N_r}\prod_{i=1}^{N_t}\frac{1}{1+\frac{\rho\lambda_i}{4}},$$
$$= \left(\frac{1}{\prod_{i=1}^{N_t}\left(1+\frac{\rho\lambda_i}{4}\right)}\right)^{N_r}.$$

Rician Fading

averaging the conditional pairwise error probability expression given by

$$P(X_1 \to X_2) \le \prod_{j=1}^{N_r}\left(\prod_{i=1}^{N_t}\frac{1}{1+\frac{\rho\lambda_i}{4}}\exp\left\{-\frac{K_{i,j}\frac{\rho}{4}\lambda_i}{1+\frac{\rho\lambda_i}{4}}\right\}\right)$$

Clearly, the earlier expression on the pairwise error probability derived for Rayleigh fading is a special case of this one for $K_{i,j} = 0$.

Space-Time Code Design Principles

In this section, we consider Rayleigh fading since it represents practically the worst case scenario among different types of fading as noted above.

Assuming that the first r of the N_t eigenvalues of the matrix $\boldsymbol{A}$ are non-zero (they have to be positive as the matrix is Hermitian), and the remaining are zeros, we can further upper-bound the pairwise error probability expression as

$$P(X_1 \to X_2) \le \left(\prod_{i=1}^{r}\lambda_i\right)^{N_r} (\rho/4)^{-rN_r}$$

which is a tight bound on the pairwise error probability for large signal-to-noise ratios In light of the above results, we now make the following observations.

The pairwise error probability decays with the inverse of the (rN_r)th power of the signal-to-noise ratio ρ. Therefore, the diversity order obtained is simply rN_r

Since $r \le N_t$, clearly the diversity order provided by the space-time code is upper-bounded by N_tN_r

We note that since there are only N_tN_r different independent fading coefficients, the maximum available diversity over this channel is N_tN_r regardless of the code used.

We also observe that the eigen values of the matrix $\boldsymbol{A}$ determine the "coding gain" that can be achieved with the space-time code, and that the larger the product of the non-zero eigenvalues, the lower the pairwise error probability at high signal-to-noise ratios.

Based on these observations, the basic code design principles for space-time codes over quasi-static Rayleigh fading channels are as follows:

1) **Rank Criterion:**

To get the full diversity the RANK of A should be full rank for two distinct pair of codeword.

The maximum diversity is achieved if the matrix

$$A = (X_1 - X_2)^H (X_1 - X_2)$$

is full rank for all the pairs of distinct codewords ***X***1 and ***X***2.

Otherwise, if the minimum rank of ***A*** among all the codeword pairs, *r*, is smaller than *Nt* , a diversity of order *rNr* is achieved.

2) **Determinant Criterion:**

To get the coding advantage the determinant of A should be minimum.

Clearly, the first criterion is the more important code design principle as it is more beneficial to get the full diversity advantage. Therefore, among the codes that satisfy the rank criterion, one can search for the codes that satisfy the determinant criterion. In other words, there is no need to try to optimize the determinant criterion without making sure that the rank criterion is satisfied.

We also note that, although we concentrate in this module on space-time trellis codes, the above design rules are general, that is, they can be used to design other coding schemes
Let us consider the Alamouti scheme as a simple example.

In this case, the pair of distinct codewords are of the form

$$X_1 = \begin{bmatrix} x_{1,1} & x_{1,2} \\ -x_{1,2}^* & x_{1,1}^* \end{bmatrix}$$

$$X_2 = \begin{bmatrix} x_{2,1} & x_{2,2} \\ -x_{2,2}^* & x_{2,1}^* \end{bmatrix}$$

therefore, the corresponding ***A*** matrix is given by

$$A = (X_1 - X_2)^H (X_1 - X_2)$$

$$= \begin{bmatrix} x_{1,1} - x_{2,1} & x_{1,2} - x_{2,2} \\ -x_{1,2}^* + x_{2,2}^* & x_{1,1}^* - x_{2,1}^* \end{bmatrix} \begin{bmatrix} x_{1,1} - x_{2,1} & x_{1,2} - x_{2,2} \\ -x_{1,2}^* + x_{2,2}^* & x_{1,1}^* - x_{2,1}^* \end{bmatrix}$$

$$\begin{bmatrix} (x_{1,1}-x_{2,1})(x_{1,1}-x_{2,1}) + (x_{1,2}-x_{2,2})(-x_{1,2}^*+x_{2,2}^*) & (x_{1,1}-x_{2,1})(x_{1,2}-x_{2,2}) + (x_{1,2}-x_{2,2})(x_{1,1}^*-x_{2,1}^*) \\ (-x_{1,2}^*+x_{2,2}^*)(x_{1,1}-x_{2,1}) + (-x_{1,1}^*+x_{2,1}^*)(-x_{1,2}^*+x_{2,2}^*) & (-x_{1,2}^*+x_{2,2}^*)(x_{1,2}-x_{2,2}) + (x_{1,1}^*-x_{2,1}^*)(x_{1,1}^*-x_{2,1}^*) \end{bmatrix}$$

$$= \begin{bmatrix} |x_{1,1}-x_{2,1}|^2 + |x_{1,2}-x_{2,2}|^2 & 0 \\ 0 & |x_{1,1}-x_{2,1}|^2 + |x_{1,2}-x_{2,2}|^2 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} |x_{1,1} - x_{2,1}|^2 + |x_{1,2} - x_{2,2}|^2 & 0 \\ 0 & |x_{1,1} - x_{2,1}|^2 + |x_{1,2} - x_{2,2}|^2 \end{bmatrix},$$

Clearly, since the codewords are distinct, the rank of the above matrix is definitely two, and full diversity is achieved.

To give another example, consider the 4 × 4 orthogonal space-time block code. In this case, the two distinct codewords are of the form

$$\boldsymbol{X}_1 = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} \\ -x_{1,2} & x_{1,1} & -x_{1,4} & x_{1,3} \\ -x_{1,3} & x_{1,4} & x_{1,1} & -x_{1,2} \\ -x_{1,4} & -x_{1,3} & x_{1,2} & x_{1,1} \end{bmatrix},$$

and

$$\boldsymbol{X}_2 = \begin{bmatrix} x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \\ -x_{2,2} & x_{2,1} & -x_{2,4} & x_{2,3} \\ -x_{2,3} & x_{2,4} & x_{2,1} & -x_{2,2} \\ -x_{2,4} & -x_{2,3} & x_{2,2} & x_{2,1} \end{bmatrix}.$$

Therefore, the corresponding $\boldsymbol{A}$ matrix is given by

$$\boldsymbol{A} = \left((x_{1,1} - x_{2,1})^2 + (x_{1,2} - x_{2,2})^2 + (x_{1,3} - x_{2,3})^2 + (x_{1,4} - x_{2,4})^2 \right) \boldsymbol{I}_4$$

The pair of symbols being transmitted are different , the rank of the matrix is four, meaning that the full diversity is achieved.

In the Alamouti example, there is no coding gain, i.e., the minimum determinant can be very small. Clearly, it is easy to use the same argument for other orthogonal space-time block codes to demonstrate that the full diversity is obtained without resorting to the more complicated analysis

Examples of Good Space-Time Codes:

Based on the rank and determinant criteria, a number of space-time trellis codes are developed. Previously explained the four states, four signal constellation is discussed which is shown in figure 2. This code is designed for transmission 2 bits per channel using the two transmit antennas.

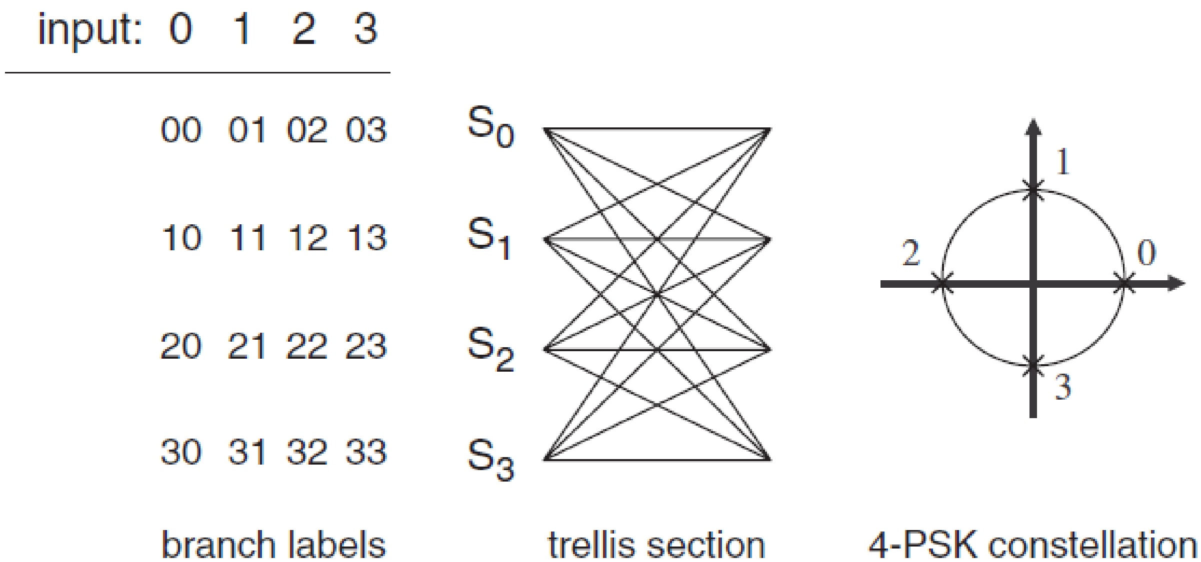


Figure 2: Simple trellis code structure

Next good space time trellis codes are

Examples of eight-state and 16-state space-time trellis codes with 2 bits per channel use transmission rate using the same number of transmit antennas and the same signal constellation are given in Figure 3

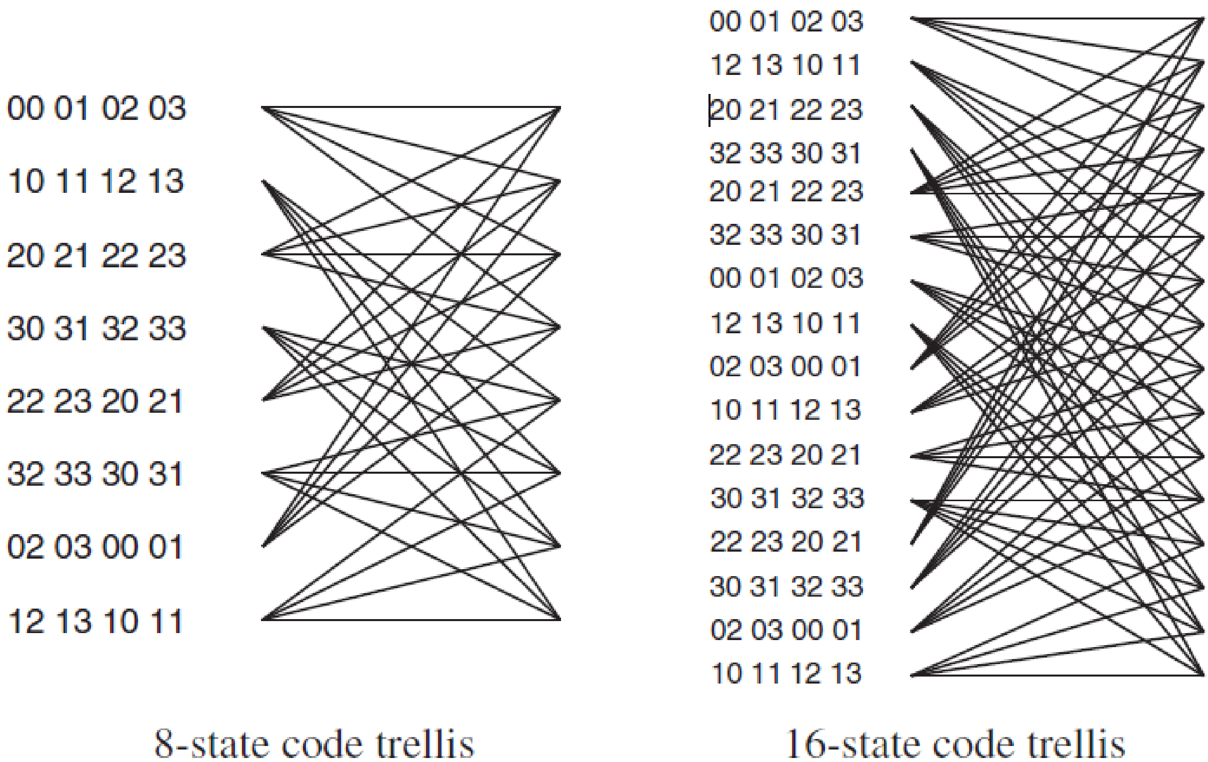


Figure 3 Eight-state and 16-state space-time trellis codes using 4-PSK modulation (2 bits per channel use)
All the three examples considered above achieve a rate of 2 bits per channel use.

An example of a code employing 8-PSK modulation that achieves a higher bandwidth efficiency is given in Figure 4. For code examples with larger number of states, i.e. higher complexity levels but with better performance

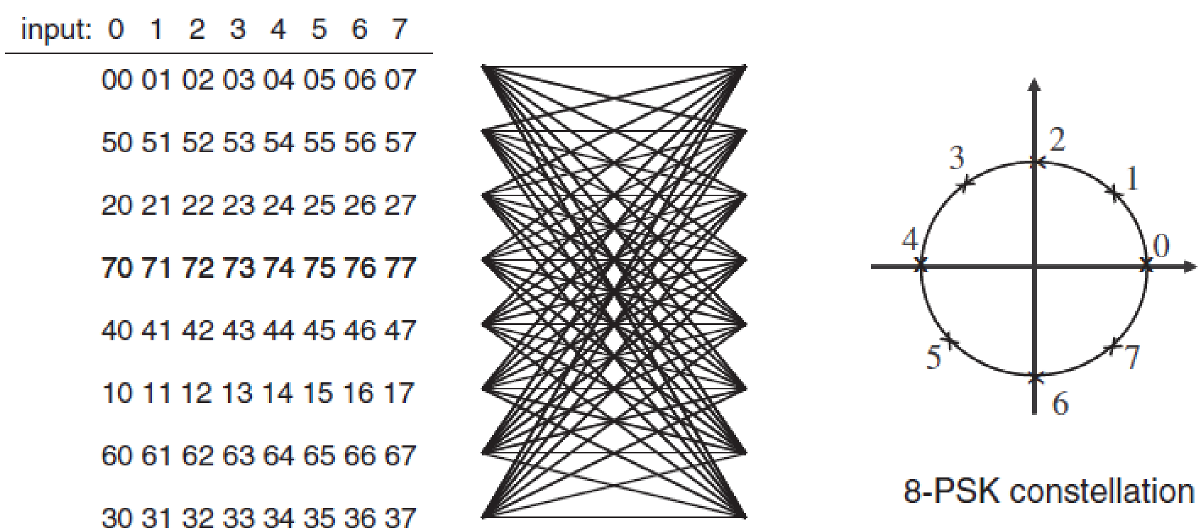


Figure 4 Eight-state space-time trellis code using 8-PSK modulation (3 bits per channel use).
It is certainly possible to design codes for rates higher than 2 bits per channel use, however, the

complexity of the decoder (implementing the Viterbi algorithm on the code trellis) is exponential in the transmission rate R.

Figure 5 illustrates the frame error rates of the four-, eight-, and 16-state space-time trellis codes that achieve a transmission rate of 2 bits per channel use when only a single receive antenna is used. We observe that all three schemes achieve a diversity order of two, i.e., full spatial diversity, as expected. We also see that the coding gain improves by using a more complicated trellis (with a larger number of states), but this comes at the expense of some increase in the decoding complexity.)

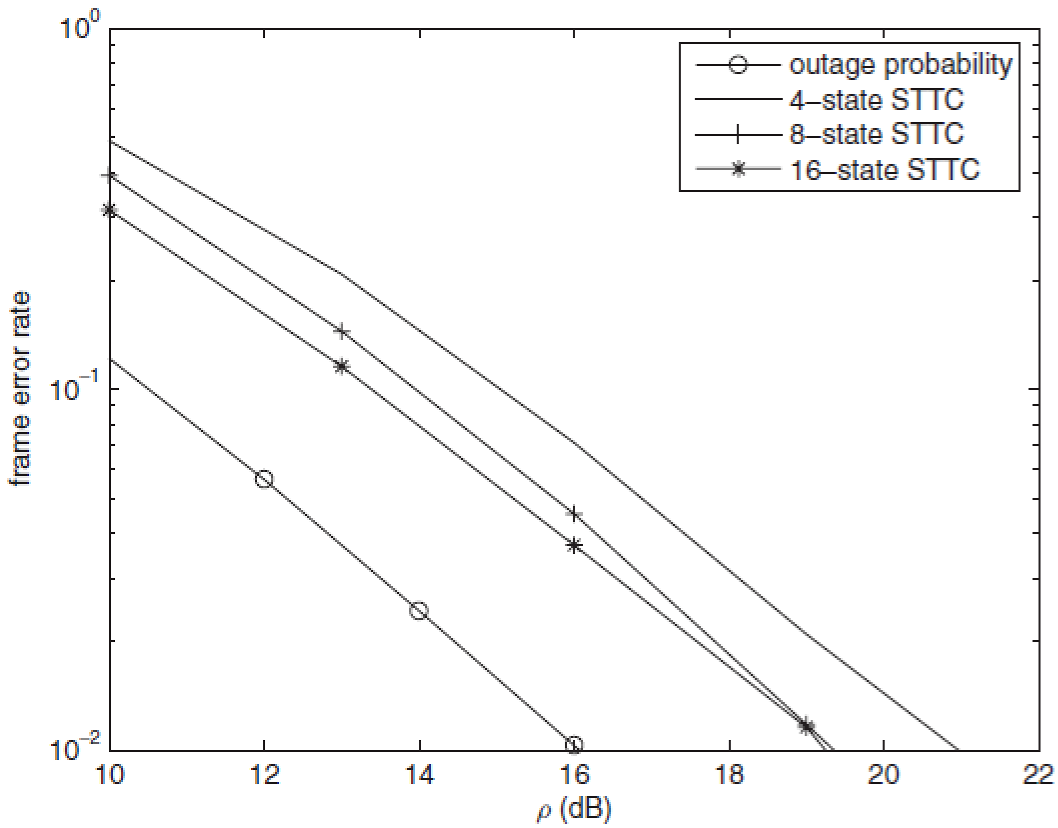


Figure 5 Outage probability and frame error rates of several space-time trellis codes with QPSK (quasi-static fading, two transmit and one receive antennas).

Representation of Space-Time Trellis Codes for PSK Constellations

Let us present a different representation of such codes which can be considered to be analogous to the generator matrix representation of convolution codes.

Generator Matrix Representation

For a code with transmission rate R and a frame length of N, there are a total of RN information bits to be transmitted in each frame of data. Let us denote these information bits by the sequence $\{un\}$.

The bits $u_{Rk+R-1}, \ldots, u_{Rk-s}$ are used to compute the kth set of transmitted symbols. In vector form, we can write this set of bits as

$$\boldsymbol{u}_k = [u_{Rk-s} \; u_{Rk-s+1} \; \cdots u_{Rk} \quad \cdots \quad u_{Rk+R-1}].$$

The coded symbols transmitted from the N_t antennas at the kth stage of the trellis can be written using matrix multiplication as

$$\boldsymbol{x}_k = [x_1(k) \; x_2(k) \; \cdots \; x_{N_t}(k)]$$

$$= \boldsymbol{u}_k \boldsymbol{G},$$

where the operation is modulo $2R$, s is a parameter that depends on the trellis memory (for instance, $s = 0$ will be the case with no memory; a simple mapping, i.e., uncoded transmission), and $\boldsymbol{G}$ is a matrix that describes the space-time trellis code. A simple example is in order. Consider the four-state space-time trellis code described in the previous section. It can be observed that this code can be generated using the parameters $R = 2$, $s = 2$ and

$$\boldsymbol{G} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

R → Transmission rate, N → frame length

Total information transmitted in each frame = RN

Let us denote these informations bit sequence U_n.

$$U_K = [U_{RK-S}\ U_{RK-S+1} \cdots U_{RK} \cdots U_{RK+R-1}]$$

K^{th} set of transmitted symbol.

The coded symbols transmitted from N_t antennas at K^{th} stage of trellis written using matrix multiplixaction as

$$x_K = [x(k)\ x_2(k) \cdots x_{N_t}(k)]$$

$$x_K = U_K G$$

where the operation is 2^R modulo

G → describes the space time trellis code

R = 2, S = 2

$$G = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

R : 2, S : 2

Sequence = {2, 1, 2, 3, 0, 0, 1, 3, 2}

Express in bits

U = {10 01 10 11 00 00 01 11 10]

K = 0

$$\therefore x_K = U_K G$$

$$x_0 = U_0 G$$

$$U_K = [U_{RK-S}\ \ U_{RK-S+1}\ \ U_{RK}\ \ U_{RK+R-1}]$$

$$U_0 = [U_{2\times0-2}\ \ U_{2\times0-2+1}\ \ U_{2\times0}\ \ U_{2\times0+2-1}]$$

$$U_0 = [U_{-2}\ \ U_{-1}\ \ U_0\ \ U_1]$$

$$U_0 = [0\ \ 0\ \ 1\ \ 0]$$

$$x_0 = [0\ \ 0\ \ 1\ \ 0] \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

$$x_0 = [0\times2 + 0\times1 + 1\times0 + 0\times0 \quad 0\times0 + 0\times0 + 1\times2 + 0\times0]$$

$$x_0 = [0\ \ 2]$$

III^{ly} $K = 1$

$x_1 = U_1 G$

$U_1 = [u_{2\times1-2} \quad u_{2\times1-2+1} \quad u_{2\times1} \quad u_{2\times1+2-1}]$

$U_1 = [u_0 \quad u_1 \quad u_2 \quad u_3]$

$$x_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+0\times1+0\times0+1\times0 & 1\times0+0\times0+0\times2+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$$x_0 = [0\ 0\ 1\ 0]G = [0\ 2],$$

$$x_1 = [1\ 0\ 0\ 1]G = [2\ 1],$$

$$x_2 = [0\ 1\ 1\ 0]G = [1\ 2],$$

$$\boldsymbol{x}_3 = [1\ 0\ 1\ 1]\boldsymbol{G} = [2\ 3],$$

$$\boldsymbol{x}_4 = [1\ 1\ 0\ 0]\boldsymbol{G} = [3\ 0],$$

$$\boldsymbol{x}_5 = [0\ 0\ 0\ 0]\boldsymbol{G} = [0\ 0],$$

$$\boldsymbol{x}_6 = [0\ 0\ 0\ 1]\boldsymbol{G} = [0\ 1],$$

$$\boldsymbol{x}_7 = [0\ 1\ 1\ 1]\boldsymbol{G} = [1\ 3],$$

$$\boldsymbol{x}_8 = [1\ 1\ 1\ 0]\boldsymbol{G} = [3\ 2],$$

which are the same as the ones produced by the trellis representation used above

Improved Space-Time Code Design

The resulting code matrices for four- (s = 1), eight- (s = 2) and 16-state (s = 2) space-time trellis codes with 4-PSK modulation are

$$\boldsymbol{G}_{4\text{-}state} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}, \quad \boldsymbol{G}_{8\text{-}state} = \begin{bmatrix} 2 & 2 \\ 0 & 1 \\ 2 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix}, \quad \boldsymbol{G}_{16\text{-}state} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 2 & 1 \\ 0 & 2 \\ 0 & 2 \\ 2 & 0 \end{bmatrix}$$

These codes provide gains over the codes given in the previous section. The gains, however, are only significant when a larger number of antennas are employed.

Comparison of Space-Time Block and Trellis Codes

It is of interest to compare and contrast the two approaches for space-time coding, namely, space-time block coding versus space-time trellis coding. Both space-time block codes and space-time trellis codes are designed to achieve full diversity advantage over the MIMO wireless channels. However, there are some basic differences in their use.

For instance, space-time block codes are very easy to encode and decode (which is achieved using the simple linear processing receivers), whereas space-time trellis codes require more complicated trellis-based decoders. Whereas space time trellis codes require more complicated trellis-based decoders.

Also, it is relatively easy to find and employ space-time block codes for more than two transmit antennas (although there may be a rate loss penalty), this is not the case for space-time trellis codes – they are most widely used for the case of two transmit antennas. These are clear advantages for space-time block coding. On the other hand, the resulting error rates of the space-time trellis codes are generally better than those of the space-time block codes.

Acknowledgment: This material is based on the text book authored by Tolga M. Duman and Ali Ghrayeb, "Coding for MIMO Communication systems", John Wiley & Sons, West Sussex, England, 2007. Some additional material are taken and/or inspired by material from various paper and / or electronic resources.

Multiple choice Questions:

a) The use of multiple transmit antennas to achieve reliability is -------------

i) Receive Diversity ii) **Transmit Diversity** iii) Flexible Diversity iv) Spatial Multiplexing

b) Receive diversity is that each element in the receive array receives an independent copy of the--------

i) Interference ii) Different Signal iii) **Same Signal** iv) Dispersion

c) In Receive Diversity probability that all signals are in deep fade simultaneously is then significantly --------------.

i) Remains same ii) Fluctuates iii) Increased iv) **Reduced**

d) Base station antenna comprises multiple elements while the mobile device has only one or two, why?

i) **Space considerations** ii) Bandwidth iii) Interference iv) No Reason

e) Multiple transmit/receive antennas should allow us to transmit ---------

i) Data Slower ii) **Data faster** iii) Same data rate iv) Less data rate

f) The capacity of the channel is defined as the maximum possible mutual information between the input (x) and --------.

i) CSI ii) input(x) iii) **output (y)** iv) CQI

g) Max Capacity is calculated by maximization of the probability distribution of the ---------

i) **Input f_x(x)** ii) output f_y(x) iii) both Input(x) &output (y) iv) f(y)/f(x)

h) $C = \max_{f_X(x)} [I(X;Y)] =$

i) $\max_{f_X(x)}$ [h(Y)-h(X)]   ii) $\max_{f_X(x)}$ [h(Y)]   **iii)** $\max_{f_X(x)} [h(Y) - h(Y/X)]$ iv) $\max_{f_X(x)}$ [h(X)]

i) h(Y) is the notation for ------------ of the output Y

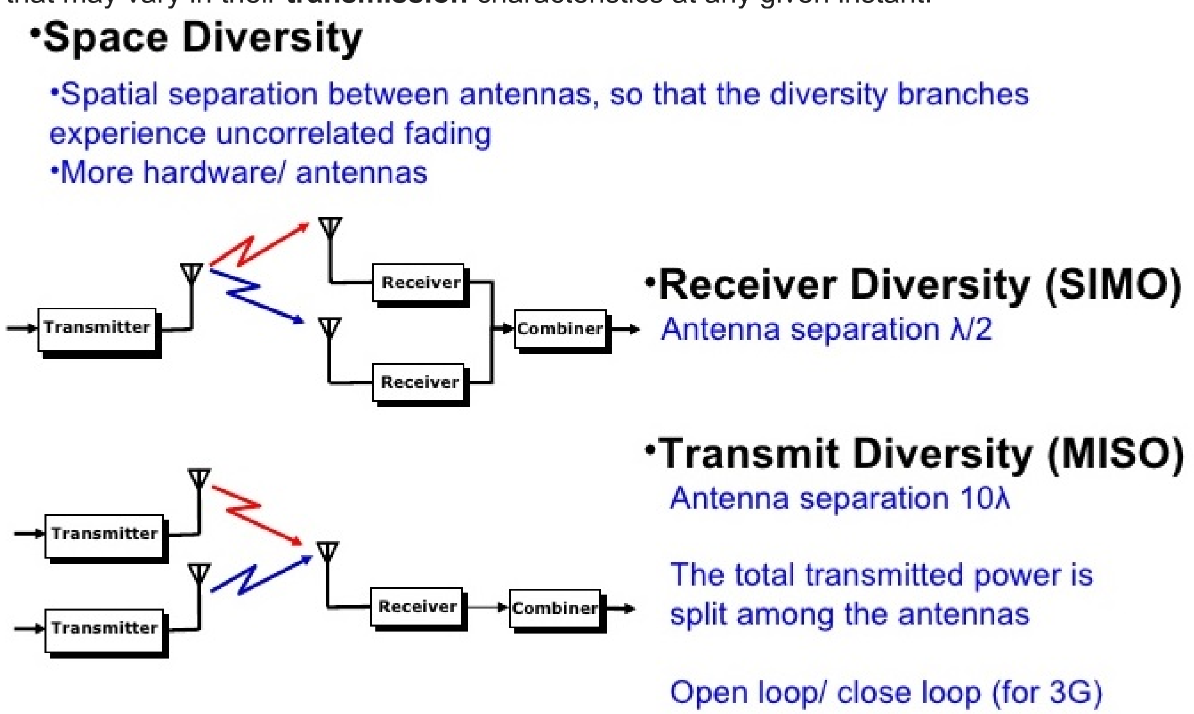
i) Time Domain Value ii) ergodicity iii) enthalpy   iv) **entropy**

j) For AWGN h(N) = log2 (πeσ2)=

i) h(Y)   **ii) h(Y /X)**   iii) 0   iv) h(X)

Q1. Explain for Transmit diversity with two antennae.

In telecommunications, a **diversity** scheme refers to a method for improving the reliability of a message signal by using two or more **communication** channels with different characteristics. ... It is based on the fact that individual channels experience different levels of fading and interference.
Transmit diversity is radio communication using signals that originate from two or more independent sources that have been modulated with identical information-bearing signals and that may vary in their **transmission** characteristics at any given instant.



Q2. Illustrate Alamouti Scheme for Transmit Diversity.

It is a complex space-time diversity technique that can be used in 2×1 MISO mode or in a 2×2 MIMO mode. The Alamouti block code is the only complex block code that has a data rate of *1* while achieving maximum diversity gain. Such performance is achieved using the following space-time block code:

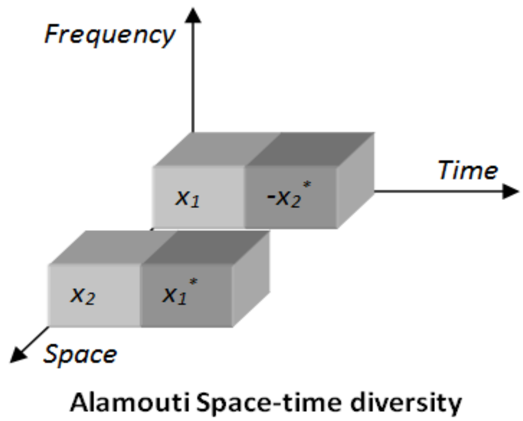


Figure 2: Alamouti space-time diversity technique

Briefly, two antennas are used, to send two OFDM symbols and their conjugate, in two time slots, which brings a diversity gain without having to compromise on the data rate. Over the air, the transmitted symbols will suffer from channel fading and at the receiver, their sum will be received. Here is the schematic diagram of an Alamouti wireless system in 2×2 MIMO mode:

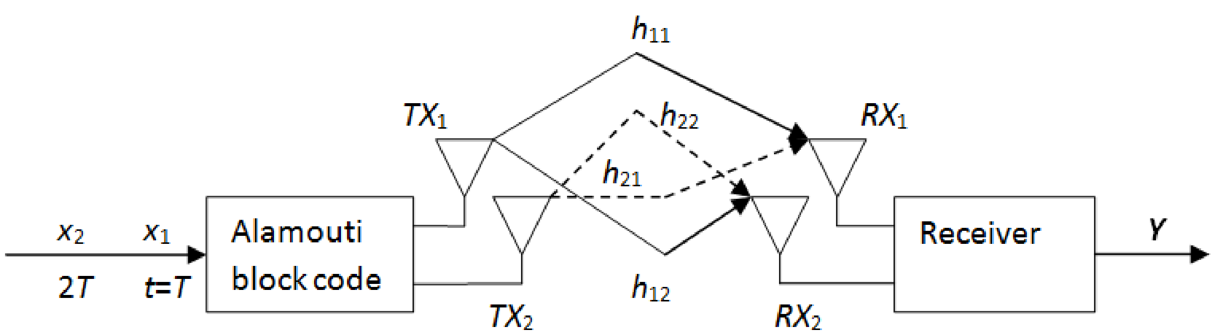


Figure 3: A 2×2 MIMO wireless system using the Alamouti block code

Since the transmission is done over two periods of time, the decoding will also be done over two periods of time. At the receiver, the received vector ***Y*** can be represented by the following equation:

$$\boldsymbol{Y} = \begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1^1 \\ n_2^1 \end{bmatrix}$$

This is for the first time period. For the second time period, the equation is as follows:

$$\boldsymbol{Y} = \begin{bmatrix} y_1^2 \\ y_2^2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + \begin{bmatrix} n_1^2 \\ n_2^2 \end{bmatrix}$$

where $\begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix}$ represents the received OFDM symbol at the first time period, for antennas 1 and 2, respectively, and where $\begin{bmatrix} y_1^2 \\ y_2^2 \end{bmatrix}$ represents the received OFDM symbol at the second time period for antennas 1 and 2, respectively. Both equations can easily be combined and arranged to produce the following result:

$$\boldsymbol{Y} = \begin{bmatrix} y_1^1 \\ y_2^1 \\ y_1^{2*} \\ y_2^{2*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1^1 \\ n_2^1 \\ n_1^{2*} \\ n_2^{2*} \end{bmatrix}$$

The next step is to find a way to isolate the transmitted symbols, x_1 and x_2. One way to reduce the number of unknowns is by using a channel estimator to estimate the channel coefficients.Channel estimation OFDM symbols are sent with each transmitted packet to enable estimating those channel coefficients at the receiver. Given the following matrix:

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix}$$

we can isolate x_1 and $x_2$by simply multiplying the matrix $\boldsymbol{Y}$ by the inverse of $\boldsymbol{H}$. However, since this matrix is not square, we need to use the Moore-Penrose pseudo-inverse $\boldsymbol{H}^+$ to solve our equations:

$$\boldsymbol{H}^+ = \left(\boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\boldsymbol{H}}$$

Using this inverse matrix expression, the noisy estimated transmitted symbols can be found using the following expression:

$$\begin{bmatrix} \widehat{x_1} \\ x_2 \end{bmatrix} = \left(\boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\boldsymbol{H}} \begin{bmatrix} y_1^1 \\ y_2^1 \\ y_1^{2*} \\ y_2^{2*} \end{bmatrix}$$

The last step would be to make a final decision on the transmitted symbols. The decision is made based on the minimum squared Euclidian distance criterion.

The Alamouti space-time block coding is a simple MIMO technique that can be used to reduce the BER of a system, at a specific SNR, without any loss on the data rate.

Define the following:

Moore-Penrose pseudo-inverse - The Moore-Penrose pseudo inverse is a generalization of the matrix inverse when the matrix may not be invertible. If A is invertible, then the Moore-Penrose pseudo inverse is equal to the matrix inverse. However, the Moore-Penrose pseudo inverse is defined even when A is not invertible. A common use of the pseudoinverse is to compute a "best fit" (least squares) solution to a system of linear equations that lacks a unique solution. Another use is to find the minimum (Euclidean) norm solution to a system of linear equations with multiple solutions.

OFDM Symbol: To complete the OFDM symbol, a 0.8 us duration Guard Interval (GI) is then added to the beginning of the OFDM waveform. This produces a "single" OFDM symbol with a time duration of 4 us in length, (3.2 us + 0.8 us). The process is repeated to create additional OFDM symbols for the remaining input data bits.

A single OFDM symbol contains 52 subcarriers; 48 are data subcarriers and 4 are pilot subcarriers. The center, "DC" or "Null", zero subcarrier is not used.

Orthogonality. Conceptually, OFDM is a specialized frequency-division multiplexing (FDM) method, with the additional constraint that all subcarrier signals within a communication channel are orthogonal to one another.

OFDM Signal: OFDM signals are made up from a sum of sinusoids, with each corresponding to a subcarrier. The baseband frequency of each subcarrier is chosen to be an integer multiple of the inverse of the symbol time, resulting in all subcarriers having an integer number of cycles per symbol.

OFDM divides a given channel into many narrower subcarriers. The spacing is such that the subcarriers are orthogonal, so they won't interfere with one another despite the lack of guard bands between them. This comes about by having the subcarrier spacing equal to the reciprocal of symbol time. Because of this relationship, the resulting sinc frequency response curves from each subcarrier create signal nulls in the adjacent subcarrier frequencies thus preventing **inter-carrier interference (ICI)**.

Subcarrier spacing is equal to the reciprocal of symbol time.

- Subcarrier spacing = 312.5 KHz
- Useful symbol duration = 3.2us IFFT
- Reciprocal = 1 cycle / 0.0000032 sec = 312,500 cycles/sec = 312.5 KHz

Since IFFT is used for modulation the spacing of the subcarriers is such that at the frequency where we evaluate the received signal (the center frequency of each subcarrier) all other signals are zero. And this in turn drives the duration of the useful symbol time and is the reason why we use 3.2us IFFT.

OFDM: Orthogonal Frequency Division Multiplexing (OFDM) is a technique for transmitting large amounts of digital data over a radio wave The technology works by splitting the radio signal into multiple smaller sub-signals that are then transmitted simultaneously at different frequencies to the receiver.

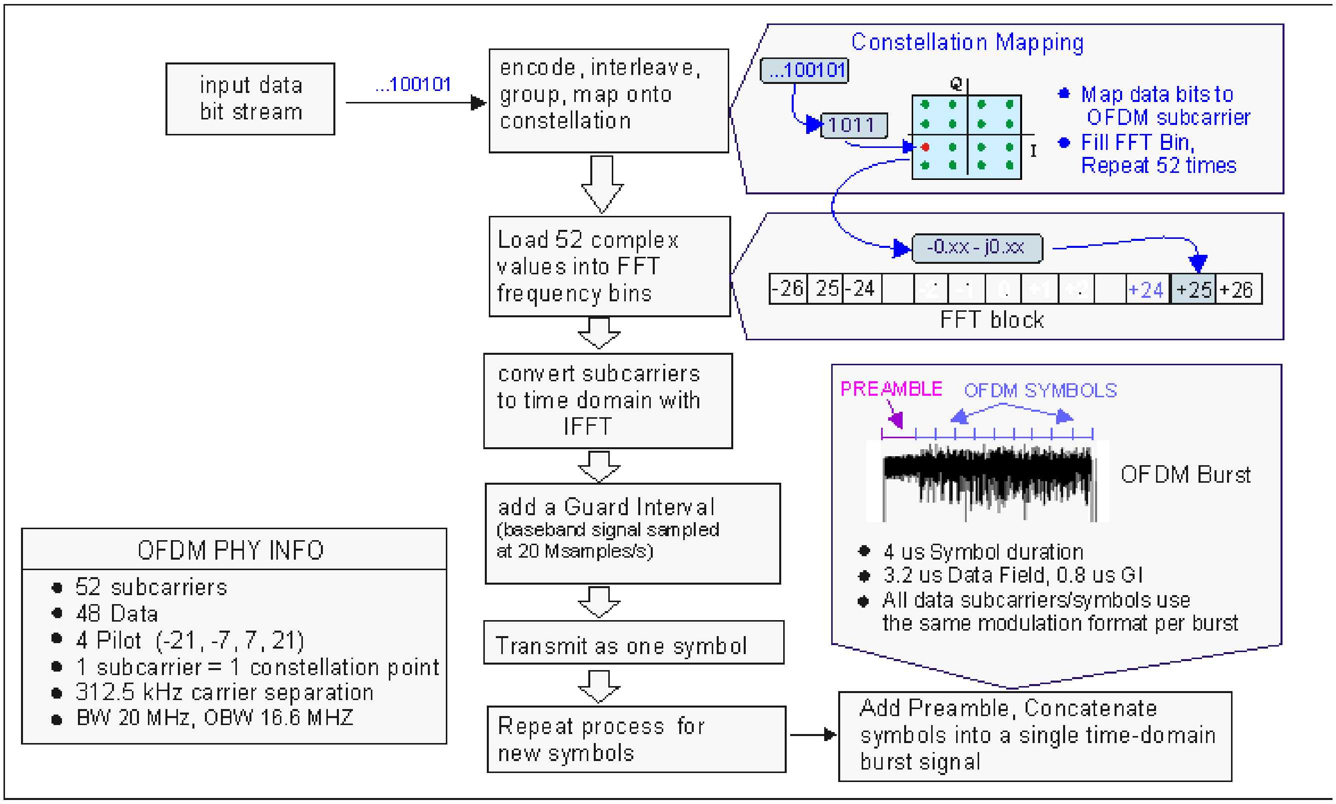
Pilot Subcarrier: Pilot subcarriers transmit with a known data sequence. This information is used to determine the difference, or error, between an ideal signal and the actual received signal. Because the data is complex, the VSA calculates phase, amplitude, and timing error data.

FFT Symbol: Since **Fast Fourier** Transform is complex Fourier Transform by nature, the **output** has real and imaginary parts for positive and negative frequencies. The input of forward transform can be real or complex. In DSP we convert a signal into its frequency components, so that we can have a better analysis of that signal. Fourier Transform (FT) is used to convert a signal into its corresponding frequency domain.

The **FFT size** defines the number of bins used for dividing the window into equal strips, or bins. Hence, a bin is a spectrum sample , and defines the frequency resolution of the window.

FFT Bin: A frequency **bin** is a segment [fl,fh] of the frequency axis that "collect" the amplitude, magnitude or energy from a small range of frequencies, often resulting from a Fourier analysis. ... The frequency **bin** can be derived for instance from the sampling frequency and the resolution of the Fourier transform.

Q3.Elaborate OFDM Signal Generation Process.



802.11a OFDM Signal Generation Process

Q4.Examine the STBC.

Space–time block coding is a technique used in wireless communications to transmit multiple copies of a data stream across a number of antennas and to exploit the various received versions of the data to improve the reliability of data transfer. The fact that the transmitted signal must traverse a potentially difficult environment with scattering, reflection, refraction and so on and may then be further corrupted by thermal noise in the receiver means that some of the received copies of the data may be closer to the original signal than others. This redundancy results in a higher chance of being able to use one or more of the received copies to correctly decode the received signal. In fact, space–time coding combines all the copies of the received signal in an optimal way to extract as much information from each of them as possible.

In the case of STBC in particular, the data stream to be transmitted is encoded in blocks, which are distributed among spaced antennas and across time. While it is necessary to have multiple transmit antennas, it is not necessary to have multiple receive antennas, although to do so improves performance.

An STBC is usually represented by a matrix. Each row represents a time slot and each column represents one antenna's transmissions over time.

Only one standard STBC can achieve full-rate (rate 1) — Alamouti's code.

$$\begin{array}{cc} & \text{transmit antennas} \longrightarrow \\ \text{time-slots} \downarrow & \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n_T} \\ s_{21} & s_{22} & \cdots & s_{2n_T} \\ \vdots & \vdots & & \vdots \\ s_{T1} & s_{T2} & \cdots & s_{Tn_T} \end{bmatrix} \end{array}$$

Here, Sij is the modulated symbol to be transmitted in time slot i from antenna j. There are to be T time slots and T transmit antennas as well as R receive antennas. This block is usually considered to be of 'length' T

The code rate of an STBC measures how many symbols per time slot it transmits on average over the course of one block. If a block encodes k symbols, the code-rate is

$$r = \frac{k}{T}.$$

Q5. Analyze Orthogonal STBC..

This means that the ***STBC is*** designed such that the vectors representing any pair ***of*** columns taken from the coding matrix ***is orthogonal***.
Orthogonal STBC (O-STBC) has the simplest decoding complexity, as its ML decoding can be achieved by linear detection.

Orthogonality

STBCs as originally introduced, and as usually studied, are orthogonal. This means that the STBC is designed such that the vectors representing any pair of columns taken from the coding matrix is orthogonal. The result of this is simple, linear, optimal decoding at the receiver. Its most serious disadvantage is that all but one of the codes that satisfy this criterion must sacrifice some proportion of their data rate.

Call a codeword

$$\mathbf{c} = c_1^1 c_1^2 \cdots c_1^{n_T} c_2^1 c_2^2 \cdots c_2^{n_T} \cdots c_T^1 c_T^2 \cdots c_T^{n_T}$$

and call an erroneously decoded received codeword

$$\mathbf{e} = e_1^1 e_1^2 \cdots e_1^{n_T} e_2^1 e_2^2 \cdots e_2^{n_T} \cdots e_T^1 e_T^2 \cdots e_T^{n_T}.$$

Then the matrix

$$\mathbf{B}(\mathbf{c},\mathbf{e}) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_T^1 - c_T^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_T^2 - c_T^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^{n_T} - c_1^{n_T} & e_2^{n_T} - c_2^{n_T} & \cdots & e_T^{n_T} - c_T^{n_T} \end{bmatrix}$$

has to be full-rank for any pair of distinct codewords c and e to give the maximum possible diversity order of NtNr . If instead, B(c,e) has minimum rank over the set of pairs of distinct codewords, then the space–time code offers diversity order . STBCs offer only diversity gain (compared to single-antenna schemes) and not coding gain. There is no coding scheme included here — the redundancy purely provides diversity in space and time. This is contrast with space–time trellis codes which provide both diversity and coding gain since they spread a conventional trellis code over space and time.

Q6. Hypothesize Quazi Orthogonal STBC.

These codes exhibit partial orthogonality and provide only part of the diversity gain.

The ***quasi-orthogonal*** STBCs achieve higher data rates than Orthogonal STBC at the cost ***of*** inter-symbol interference (ISI).

Antenna system and has the coding matrix:

where * denotes complex conjugate.

$$C_{4,1} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix}.$$

The orthogonality criterion only holds for columns (1 and 2), (1 and 3), (2 and 4) and (3 and 4). Crucially, however, the code is full-rate and still only requires linear processing at the receiver, although decoding is slightly more complex than for orthogonal STBCs. Results show that this Q-STBC outperforms (in a bit-error rate sense) the fully orthogonal 4-antenna STBC over a good range of signal-to-noise ratios (SNRs). At high SNRs, though (above about 22 dB in this particular case), the increased diversity offered by orthogonal STBCs yields a better BER.

Q7. Explain Decoding of Orthogonal STBC.

One particularly attractive feature of orthogonal STBCs is that maximum likelihood decoding can be achieved at the receiver with only linear processing. In order to consider a decoding method, a model of the wireless communications system is needed.

At time t, the signal r_t^j received at antenna j is:

$$r_t^j = \sum_{i=1}^{n_T} \alpha_{ij} s_t^i + n_t^j,$$

where α_{ij} is the path gain from transmit antenna i to receive antenna j, s_t^i is the signal transmitted by transmit antenna i and n_t^j is a sample of additive white Gaussian noise (AWGN).

The maximum-likelihood detection rule[9] is to form the decision variables

$$R_i = \sum_{t=1}^{n_T} \sum_{j=1}^{n_R} r_t^j \alpha_{\epsilon_t(i)j} \delta_t(i)$$

where $\delta_k(i)$ is the sign of s_i in the k^{th} row of the coding matrix, $\epsilon_k(p) = q$ denotes that s_p is (up to a sign difference), the (k, q) element of the coding matrix, for $i = 1, 2, \ldots, n_T$ and then decide on constellation symbol s_i that satisfies

$$s_i = \arg\min_{s \in \mathcal{A}} \left(|R_i - s|^2 + \left(-1 + \sum_{k,l} |\alpha_{kl}|^2\right) |s|^2 \right),$$

with $\mathcal{A}$ the constellation alphabet. Despite its appearance, this is a simple, linear decoding scheme that provides maximal diversity.

Q8. Evaluate Linear Dispersion Codes in Diversity techniques.

The codes are designed to optimize the mutual information between the transmitted and received signals. Because of their linear structure, the codes retain the decoding simplicity of V-BLAST, and because of their information theoretic optimality, they possess many coding advantages.
The structures of most existing ST coding designs mainly fall into two categories, either trellis structure or linear structure. ST codes with trellis structure, such as space-time trellis codes (STTCs) and space-time turbo trellis codes (ST Turbo TCs) can achieve full diversity and large coding rate. However, their computational complexity grows exponentially with respect to the number of states and transmit antennas. Also they are often designed by hand and the trellis structure is not flexible in terms of rate. ST codes with linear structure have been also proposed, such as space time block codes (STBCs) aiming at improved error performance aiming at higher data rate. However, the STBCs suffer from rate and error performance loss as the number of antennas grows. Additionally, a common drawback of early ST coding schemes is that they are not flexible in rate-performance tradeoff. As a result, the design of high-rate schemes that provide flexible tradeoff between performance and data rate has attracted a great deal of research attention. Among these, LDCs (Linear dispersion Code) possess many of the coding and diversity advantages with simple decoding and flexible linear structure. The LDC breaks the data stream into sub-streams, each substream is dispersed over space and time and then the substreams are combined linearly at the transmit antennas. Thereby, the mutual information between transmit and receive signals is maximized. The resultant codes do not necessarily yield optimal performance. Diversity gain is included in the optimization objectives in addition to the mutual information.

Q9. Verify with proof Generic space-time trellis codes.

Space–time trellis codes (STTCs) are a type of space–time code used in multiple-antenna wireless communications. This scheme transmits multiple, redundant copies of a generalised TCM signal distributed over time and a number of antennas ('space'). These multiple, 'diverse' copies of the data are used by the receiver to attempt to reconstruct the actual transmitted data. For an STC to be used, there must necessarily be multiple transmit antennas, but only a single receive antennas is required; nevertheless multiple receive antennas are often used since the performance of the system is improved by the resulting spatial diversity.

In contrast to space–time block codes (STBCs), they are able to provide both coding gain and diversity gain and have a better bit-error rate performance. In essence they marry single channel continuous time coding with the signaling protocol being used, and extend that with a multi-antenna framework. However, that also means they are more complex than STBCs to encode and decode; they rely on a Viterbi decoder at the receiver where STBCs need only linear processing. Also, whereas in a single transmitter, single receiver framework the Viterbi algorithm (or one of the sequential decoding algorithms) only has to proceed over a trellis in a single time dimension, in here the optimal decoding also has to take into consideration the number of antennas, leading to an extraneous polynomial complexity term.

TCM:- In telecommunication, trellis modulation (also known as trellis coded modulation, or simply TCM) is a modulation scheme that transmits information with high efficiency over band-limited channels such as telephone lines. TCM is a bandwidth efficient modulation based on convolutional coding. It conserves bandwidth by doubling the number of constellation points of the signal. This way the bit rate increases but the symbol rate stays the same.

Viterbi Algorithm:- The Viterbi algorithm is a dynamic programming algorithm for finding the most likely sequence of hidden states—called the Viterbi path—that results in a sequence of observed events, especially in the context of Markov information sources and hidden Markov models (HMM).

Q10. What are Basic space-time code design principles

Space–time block coding is a technique used in wireless communications to transmit multiple copies of a data stream across a number of antennas and to exploit the various received versions of the data to improve the reliability of data transfer.

A space–time code (STC) is a method employed to improve the reliability of data transmission in wireless communication systems using multiple transmit antennas.

Pairwise Error Probability Assume that two space-time trellis codewords are given by X1 and X2. The pairwise error probability P(X1 → X2) is the probability that the received signal vector is closer to the erroneous codeword X2 given that the codeword X1 is transmitted. Conditioned on the instantaneous channel realization, the pairwise error probability is given by

$P(X1 \rightarrow X2|H) = P(Y - \sqrt{\rho}X1H2 > Y - \sqrt{\rho}X2H2)$,

Rayleigh Fading Assuming that the wireless channel is modeled by Rayleigh fading, the random variables $\beta_{i,j}$ are zero-mean complex Gaussian with variance 1/2 per dimension. Therefore, their norm square is exponential with parameter 1. Thus, we can easily average over the channel variations to obtain an upper bound on the (unconditional) pairwise error probability as

$$P(X_1 \to X_2) \le \prod_{j=1}^{N_r}\prod_{i=1}^{N_t}\int_0^{\infty} \exp\left(-\frac{\rho\lambda_i z}{4}\right) e^{-z} dz,$$

$$= \prod_{j=1}^{N_r}\prod_{i=1}^{N_t} \frac{1}{1+\frac{\rho\lambda_i}{4}},$$

$$= \left(\frac{1}{\prod_{i=1}^{N_t}\left(1+\frac{\rho\lambda_i}{4}\right)}\right)^{N_r}.$$

Examining this upper bound carefully, we can develop basic code design principles for space-time codes over.

In this case, the elements βi,j have non-zero means, i.e., their norms are Rician distributed. Assuming that Ki,j =|E(βi,j)|2, by averaging the conditional pairwise error probability expression over the channel statistics, it can be shown that

$$P(X_1 \to X_2) \le \prod_{j=1}^{N_r}\left(\prod_{i=1}^{N_t} \frac{1}{1+\frac{\rho\lambda_i}{4}} \exp\left\{-\frac{K_{i,j}\frac{\rho}{4}\lambda_i}{1+\frac{\rho\lambda_i}{4}}\right\}\right).$$

The basic code design principles for space-time codes over quasi-static Rayleigh fading channels are as follows:

Rank Criterion: The maximum diversity is achieved if the matrix A = (X1 −X2)H(X1 −X2) is full rank for all the pairs of distinct codewords X1 and X2.

Otherwise, if the minimum rank of A among all the codeword pairs, r, is smaller than Nt, a diversity of order rNr is achieved. •

Determinant Criterion: Assume that full rank is obtained using the criterion in the first part – clearly, this is the more interesting case. To obtain the maximum coding advantage possible, the minimum of the product of the eigenvalues of A over all pairs of distinct codewords (i.e., the minimum determinant of possible A matrixes) should be maximized.

Q11. Represent and Evaluate of space-time trellis codes for PSK constellation.

For a code with transmission rate R and a frame length of N, there are a total of RN information bits to be transmitted in each frame of data. Let us denote these information bits by the sequence $\{u_n\}$. The bits $u_{Rk+R-1}, \ldots, u_{Rk-s}$ are used to compute the kth set of transmitted symbols. In vector form, we can write this set of bits as

$$\boldsymbol{u}_k = [u_{Rk-s}\ u_{Rk-s+1}\ \cdots u_{Rk}\ \ \cdots\ \ u_{Rk+R-1}]. \tag{5.32}$$

The coded symbols transmitted from the N_t antennas at the kth stage of the trellis can be written using matrix multiplication as

$$\boldsymbol{x}_k = [x_1(k)\ x_2(k)\ \cdots\ x_{N_t}(k)] \tag{5.33}$$

$$= \boldsymbol{u}_k \boldsymbol{G}, \tag{5.34}$$

where the operation is modulo 2^R, s is a parameter that depends on the trellis memory (for instance, $s = 0$ will be the case with no memory; a simple mapping, i.e., uncoded transmission), and $\boldsymbol{G}$ is a matrix that describes the space-time trellis code.

A simple example is in order. Consider the four-state space-time trellis code described in the previous section. It can be observed that this code can be generated using the parameters $R = 2$, $s = 2$ and

$$\boldsymbol{G} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}.$$

To see how the encoding is performed, consider the same sequence used in Section 5.1, i.e., 2, 1, 2, 3, 0, 0, 1, 3, 2. Expressing this in terms of bits, we have $\boldsymbol{u} = [100110110000011\ 110]$. Therefore, the symbols to be transmitted are given by (note that the ﬁrst symbol considers two 0's for initialization, i.e., the bits with negative indexes are taken as 0's)

$$\boldsymbol{x}_0 = [0\ 0\ 1\ 0]\boldsymbol{G} = [0\ 2],$$
$$\boldsymbol{x}_1 = [1\ 0\ 0\ 1]\boldsymbol{G} = [2\ 1],$$
$$\boldsymbol{x}_2 = [0\ 1\ 1\ 0]\boldsymbol{G} = [1\ 2],$$

$$\boldsymbol{x}_3 = [1\ 0\ 1\ 1]\boldsymbol{G} = [2\ 3],$$
$$\boldsymbol{x}_4 = [1\ 1\ 0\ 0]\boldsymbol{G} = [3\ 0],$$
$$\boldsymbol{x}_5 = [0\ 0\ 0\ 0]\boldsymbol{G} = [0\ 0],$$
$$\boldsymbol{x}_6 = [0\ 0\ 0\ 1]\boldsymbol{G} = [0\ 1],$$
$$\boldsymbol{x}_7 = [0\ 1\ 1\ 1]\boldsymbol{G} = [1\ 3],$$
$$\boldsymbol{x}_8 = [1\ 1\ 1\ 0]\boldsymbol{G} = [3\ 2],$$

which are the same as the ones produced by the trellis representation used above.

Q12. Feature the Performance analysis for space-time trellis codes.

Main challenge in wireless communication field is to achieve reliable communication with flexible data rate providing services to large capacity under the restraint of limited power and limited available power and spectrum bandwidth. Space time trellis coding is also a space time code (STC) which provides both

coding gain as well as diversity gain and has better bit error rate performance.
Diversity Gain: Diversity gain is the increase in signal-to-interference ratio due to some diversity scheme, or how much the transmission power can be reduced when a diversity scheme is introduced, without a performance loss. Diversity gain is usually expressed in decibels, and sometimes as a power ratio. An example is soft handoff gain. For selection combining N signals are received, and the strongest signal is selected. When the N signals are independent and Rayleigh distributed, the expected diversity gain has been shown to be k=>1…N,({\displaystyle \sum _{k=1}^{N}{\frac {1}{k}}}∑1/k), expressed as a power ratio
Coding Gain;- Coding gain is the measure in the difference between the signal-to-noise ratio levels between the uncoded system and coded system required to reach the same bit error rate levels when used with the error correcting code.

Q13. Compare between space-time block and trellis codes.

STBCs offer only diversity gain (compared to single-antenna schemes) and not coding gain. ... This is contrast with space–time trellis codes which provide both diversity and coding gain since they spread a conventional trellis code over space and time.
Nevertheless, performance of STTC is slightly superior to the performance of STBC under different circumstances.
If we compare the performance of STBC and STTC in terms of the frame error rate keeping the transmit power, spectral efficiency and number of trellis states fixed. We discover that a simple concatenation of space-time block codes with traditional AWGN (additive white Gaussian noise) trellis codes outperforms some of the best known space-time trellis codes at SNRs (signal to noise ratios) of interest.
Simulation results indicate that performance of STTC in Rayleigh channel is obviously improved with the increasing number of transmitting and receiving antennas, but the encoder state has little impact on the performance. In the Rayleigh channel, the performance of Alamouti code is better than that of STTC.
Nevertheless, performance of STTC is slightly superior to the performance of STBC under different circumstances.