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DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

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Department of Telecommunication Engineering

Online Continuous Internal Assessment Test - III

Course: **MIMO Technologies** | Date: **05/01/2021**
Course Code: **17TE7DCMTN** | Maximum marks: **50**
Semester: **VII - 'A' &'B'** | Duration: **90 Min**

	Note: Answer 5 full questions.	Marks
1	a) Turbo codes are ---- --- i) Convolution Code ii) FEC codes iii) Channel Code iv) all of mentioned b) -------, a technique for making forward error correction more robust with respect to burst errors i) Interleaving ii) puncturing iii) equalization iv) source coding c) Trellis termination is an important method for improving performance of --------- by periodically adding tail bits into information sequence. i) Reed Solomon Code ii) BCH Code iii) Hamming Code iv) Turbo Code d) --------- is where the fading process is approximately constant for a number of symbol intervals. i) Block fading ii) Flat fading iii) FS fading iv) Rayleigh Fading e)A channel can be ------ block-fading' when it is block fading in both the time and frequency domains. i) octople ii) single iii) quadruple iv) double f) Channel Tap is certain delay on delay line on -------. i) Time Axis ii) Frequency axis iii) Fourier Axis iv) Complex axis g) -------- is the time duration over which the channel impulse response is considered to be not varying i) Channel Time ii) Coherence time iii) Equalization Time iv) Interference Time h) The ------ algorithm is an algorithm for maximum a posteriori decoding of error correcting codes defined on trellises i) BCJR ii) Viterbi iii) MAP iv) Priori	1x10

	i) -------is process of adjusting the spatial attribute of a sound in order to perceive desired 3D sound sensation i) Spatial Equalization ii) Temporal Equalization iii) ISI iv) ISI-Tap j) -------is transceiver architecture for offering spatial multiplexing over multiple-antenna wireless communication systems i) D Blast ii) BLAST iii) V Blast iv) K-Blast	
2	Write about SOVA Decoder in Concatenated STBC.	10
3	Verify the Frequency Selective Frequency Channel Information Rates with Gaussian Inputs.	10
4	Elaborate APP Decoder for Concatenated STBC	10
	(OR)	
5	Evaluate Full Diversity Code for MIMO FS Channels.	10
6	Demonstrate Detection Algorithms for Spatial Multiplexing Systems for Threaded STC.	10
	(OR)	
7	Verify Diversity/Multiplexing Gain Trade-off with plots and examples.	10

Faculty: Dr. SAYED ABDULHAYAN No. of copies: 85

CIE-III Solution Scheme

Solution Scheme
Subject: MIMO Technologies Date: 05/01/2021
Course code: 17TE7DCMTN
Sem: 7th 'A' & 'B'

Q1. (a) All of the mentioned — (iv)
(b) Interleaving — (i)
(c) Turbo Code — (iv)
(d) Block fading — (i)
(e) double — (iv)
(f) Time axis — (i)
(g) Coherence time — (ii)
(h) BCJR — (i)
(i) Spatial Equalization — (i)
(j) BLAST — (ii)

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(2) SOVA Decoder in Concatenated STBC

Viterbi algorithm is a dynamic programming algorithm for finding the most likely sequence of hidden states — called the viterbi path — that results in a sequence of observed events, especially in the context of Markov

in the context of Markov informat[illegible]
sources and hidden Markov models

Markov Model

In probability Model, a Markov model is a stochastic model used to model randomly changing systems. It is assumed that future states depend only on the current state, not on the events that occurred before.

Markov models can be used to recognise patterns, make predictions and to learn the statistics of sequential data. Markov model is a state machine with the state changes being probabilities. In hidden Markov model, we don't know the probabilities but we know outcomes.

HMM (Hidden Markov Models) in general are both Supervised and unsupervised are heavily applied to model sequences of data.
HMM are a class of probabilistic graphical model that allows us to predict a sequence of unknown (hidden) variables from a set of observed variables.
Eg predicting weather (hidden) based on type of cloths someone wears.

Markov chains is a discrete-time process for which the future behaviour given the past and and present, only depends on the present and not on the past.

A Markov process is continuous-time version of a Markov chain.

SOVA is alternative to APP. complexity is $\frac{1}{2}$ that of APP. at the expense of degradation of performance.

cumulative metric $m_k(s)$ for state s at time k is updated according to

$$m_s(k) = \max\{\lambda(s',s) + m_{k-1}(s'), \lambda(s'',s) + m_{k-1}(s'')\}$$

$\lambda(s',s) \rightarrow$ branch metric of transition from s' to s state

In iterative decoding, branch metric when incorporating the a priori information

$$\lambda(s',s) = \frac{1}{2} L^e(b_k) b_k + 2\sqrt{P} y_k^b b_k + 2\sqrt{P} y_k^p P_k$$

$L^e(b_k) \rightarrow$ extrinsic information could be $L^e_{21}(b_k)$ or $L^e_{12}(b_k)$

$\Delta_k \rightarrow$ difference metric for state s at time k

$$\Delta_k = |(m_{k-1}(s') + \lambda(s',s)) - (m_{k-1}(s'') + \lambda(s'',s))|$$

$$\Delta_k \approx \log \frac{P(\text{correct})}{1 - P(\text{correct})}$$

P(correct) → probability of path decision of survivor at time k was correct

∴ Δ_k → reliability that the path ending at state s at time k was correct.

$$\Delta_k^* = \min\{\Delta_k, \Delta_{k+1} \cdots \Delta_{k+\delta}\}$$

δ → delay → decoding depth.
→ min is taken only over the nonsurviving paths within the time window $[k, k+\delta]$ that would lead to different decision $\hat{b}_k$.

$$L(b_k) \approx \hat{b}_k \cdot \Delta_k^*$$

Given $\hat{b}_k$ and Δ_k^* the soft output for bit b_k is approximated by

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Q3. Frequency Selective Channel Information Rates with Gaussain Inputs

System with N_t → transmit antennas
N_r → Receiving antennas

channel has L ~~Taps~~. ISI taps.
at j^{th} receive antenna at k^{th} tap is given by

$$y_j(k) = \sqrt{P} \sum_{l=0}^{L-1} \sum_{i=0}^{N_t} h_{ij}^{(l)} \cancel{x(k)}\, x_i(k-l) + n_j(k)$$

$$I_{Gauss} = \int_0^\infty \log\left(\det\left(I_{Nr} + \frac{P}{N_t}\tilde{H}_f^T \tilde{H}_f^H\right)\right) df$$

$$\tilde{H}_f = \sum_{l=0}^{L-1} H_l \exp(-j2\pi f l)$$

$$H_l = \begin{bmatrix} h_{11}^{(l)} & h_{12}^{(l)} & \cdots & h_{1N_r}^{(l)} \\ h_{21}^{(l)} & h_{22}^{(l)} & \cdots & h_{2N_r}^{(l)} \\ \vdots & & & \\ h_{N_t,1}^{(l)} & \cdots & - & h_{N_t N_r}^{(l)} \end{bmatrix}$$

where the time dependence of the fading coefficients is dropped since they are constants.

$$I = \lim_{N\to\infty} \frac{1}{N} I(X(1), X(2) \ldots X(N); Y(1) \ldots Y(2) \ldots Y(N))$$

$X(1), X(2) \cdots X(N) \longrightarrow$ Sequence of channel inputs

$Y(1), Y(2) \ldots Y(N) \rightarrow$ Set of channel outputs

$$\frac{1}{N} H(Y(1) \ldots Y(N)) = -\frac{1}{N} E\left[\log\left(P(Y(1) \ldots Y(N))\right)\right] -$$

$$y(k) = [y_1(k), y_2(k) \ldots y_{Nr}(k)]^T$$

$$P(y(1), y(2) \ldots y(N)) = \sum_{m=0}^{n_t-1} \alpha_N(m)$$

Q4 APP decoder for Concatenated STBC.

APP Algorithm

1/2 RSC code

(N) $b = [b_1\ b_2 \ldots b_N]$. Input Sequence.

(N) $P = [P_1\ P_2 \ldots P_N]$ parity Sequence

(2N) $x = [x_1^b\ x_1^p \ldots x_N^b\ x_N^p]$ codeword sequence multiplexed.

(2N) $y = [y_1^b\ y_1^p \ldots y_N^b\ y_N^p]$ received sequence

(2N) $n = [n_1^b\ n_1^p \ldots n_N^b\ n_N^p]$ noise sequence

decision $\rightarrow \hat{b}_K$ in logarithmic domain

Log posteriori probability $= L(b_K)$

$$L(b_K) \triangleq \log \left[\frac{P(b_K = +1 | y)}{P(b_K = -1 | y)} \right] \quad \text{for BPSK}$$

Forward Recursion $\tilde{\alpha}_K(S) \triangleq P(S_K = S, y_1^K)$ $\rightarrow (b_1^K, P_1^K)$

Path branch metric $\gamma_K(S', S) \triangleq P(S_K = S, y_K | S_{K-1} = S')$ $\rightarrow (b_K, P_K)$

Backward recursion $\tilde{\beta}_K(S) \triangleq P(y_{K+1}^N | S_K = S)$

(b_{K+1}^N, P_{K+1}^N)

$$\tilde{\alpha}_K(S) = \sum_{S'} \gamma_K(S', S) \cdot \tilde{\alpha}_{K-1}(S')$$

valid state transition $S' \rightarrow S$

$$\tilde{\gamma}_k(s', s) = P(b_k)\, p(y_k|b_k)$$

$$= \begin{cases} \frac{P(b_k)}{\pi} \exp\left(-(y_{2k-1} - \sqrt{P}\, b_k)^2 - (y_{2k} - \sqrt{P}\, p_k)^2\right) & \text{for valid } s' \to s \\ 0 & \text{for invalid } s' \to s \end{cases}$$

$$\tilde{\beta}_{k-1}(s') = \sum_{s} \tilde{\beta}_k(s)\, \tilde{\gamma}_k(s', s)$$

Once $\tilde{\alpha}_k(s)$ $\tilde{\gamma}_k(s', s)$ and $\tilde{\beta}_k(s)$ are computed one can easily compute

$$L(b_k) = \frac{\sum_{T+} p(s_{k-1} = s'\ s_k = s, y)}{\sum_{T-} p(s_{k-1} = s'\ s_k = s, y)}$$

$$= \frac{\sum_{T+} \tilde{\alpha}_{k-1}(s')\, \tilde{\gamma}_k(s', s)\, \tilde{\beta}_k(s)}{\sum_{T-} \tilde{\alpha}_{k-1}(s')\, \tilde{\gamma}_k(s', s)\, \tilde{\beta}_k(s)}$$

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(5) Full Diversity Code for MIMO FS Channels

Space time codeword matrix for flat fading

$$\begin{bmatrix} x(1) & 0 & 0 & \cdots & 0 \\ x(2) & x(1) & 0 & \cdots & 0 \\ x(3) & x(2) & x(1) & \cdots & 0 \\ \vdots & & & & \\ x(N_t) & x(N_t-1) & x(N_t-2) & \cdots & x(1) \\ x(N_t+1) & x(N_t) & x(N_t-1) & \cdots & x(2) \\ \vdots & & & & \\ x(N) & x(N-1) & \cdots & & x(N-N_t+1) \end{bmatrix}$$

Flat fading

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N_t transmit antenna System 26

uses delay diversity code is obtained

simply delayed versions of sequence

in Flat fading

~~for~~ FS Fading delaying by single

symbol for each transmit antenna will

not work because some columns of

ST Codeword matrix are identical. Hence

code will not be full rank.

If we delay the transmitted symbols

of 1st antenna by number of ISI tap

L instead of a single symbol we will

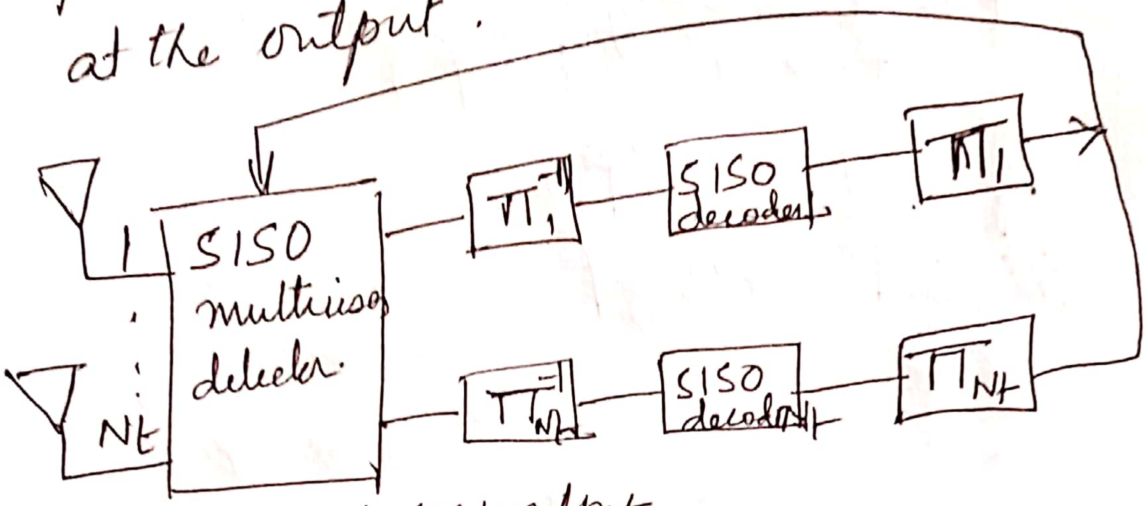
achieve Full diversity code of order $N_t N_r L$

$$\begin{bmatrix} x(1) & 0 & 0 & 0 & \cdots & 0 \\ x(2) & x(1) & 0 & 0 & \cdots & 0 \\ x(3) & x(2) & x(1) & \cdots & \cdots & 0 \\ x(N_t L) & x(N_t L-1) & x(N_t L-2) & \cdots & \cdots & x(1) \\ x(N_t L+1) & x(N_t L) & x(N_t L-1) & \cdots & \cdots & x(2) \\ \\ x(N) & x(N-1) & x(N-2) & & & x(N-N_t L+1) \end{bmatrix}$$

Spacetime codeword matrix for Frequency selective Fading.

Q6. Detection Algorithm for Spatial Multiplexing for Threaded STC.

A Thread is a layer that extends over full spatial span N_t and full temporal span N, where $N \rightarrow$ codewords length at the output.



Soft input soft output

Explanation

1. Greed Algorithm (Log likelihood in greedy fashion)
2. Belief Propagation
3. Turbo-Blast Detection
4. Reduced Complexity ZF/MMSE Detection (W_{opt}^H is computed for every detected layer)
5. SPACE decoding.

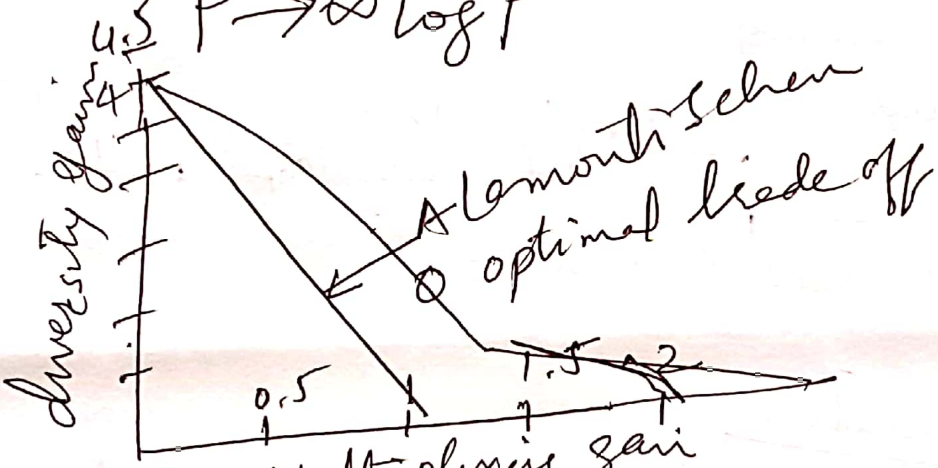
Q7. Diversity/Multiplexing Gain trade off.

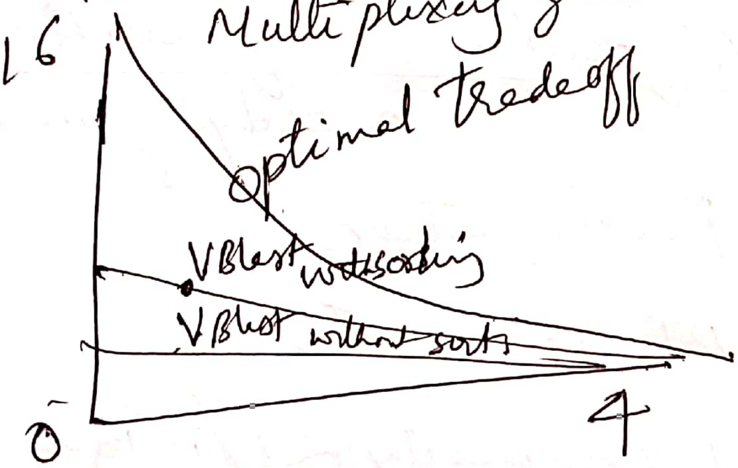
(d) Diversity gain → negative asymptotic slope of error rate curve as function of SNR in log-log scale

Multiplexing gain (r)

$$d = \lim_{P \to \infty} \frac{-\log P_e(P)}{\log(P)}$$

$$r = \lim_{P \to \infty} \frac{R(P)}{\log P}$$





Explanation