Solution Scheme - CIE - 1

SUBJECT : MIMO Technologies
Code : 17TE7DCMTN
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Q1

(a) — Multipath - (ii)
(b) — FEC — (ii)
(c) — Selection — ~~(i)~~ - (IV)
(d) — Single — (ii)
(e) — Diversity — (iii)
(f) — Signal levels — (iv)
(g) — Reduce — (i)
(h) — Perfect channel — (iii)
(i) — Noisy channel — (iv)
(j) — Increase — (i)

Q2. Multipath delay spread: (Time domain)

Delay spread is a measure of multipath profile of a mobile communication channel. It is generally defined as the difference between the time of arrival of the earliest component (e.g LOS wave if it exists) and the time of arrival of the latest multipath component. 1

Multipath propogation, an inherent feature of a mobile communication channel, results in received signal that is dispersed in time.

Each path has its own delay and the time dispersion leads to a form of Intersymbol Interference.

1.5

(b) Doppler Spread (Frequency domain)

It refers to the widening of the Spectrum of a narrow band signal transmitted through a multipath propagation channel.

It is due to the different Doppler shift frequencies associated with the multiple propagation paths when there is relative motion between the transmitter and the receiver

max doppler shift is $f_m = v/\lambda$

f_d = doppler shift $= (v/\lambda) \cos\theta$

θ = direction of angle between the direction of motion and direction of Signal arrival.

f_c = transmitted pure frequency

Doppler spectrum range = $f_c + f_d$ to $f_c - f_d$.

c) Coherence time :-

In communication Systems, a communication channel may change with time.
Coherence time is the time duration over which the channel impulse response is considered to be not varying. Such channel variation is much more significant in wireless communication systems, due to Doppler effect.

$$\boxed{T_c = \frac{\lambda^2}{c\Delta\lambda}}$$

$T_c \rightarrow$ coherence time
$\lambda \rightarrow$ central wavelength
$\Delta\lambda \rightarrow$ spectral width of the source.
$c \rightarrow$ velocity of light in vacuum.

d) Coherence Bandwidth

Coherence bandwidth is the statistical measure of the frequency range over which the channel is considered to be flat, which means the signals with frequencies in the range of coherence bandwidth will most likely experience correlated amplitude fading.

Coherence Bandwidth is related to the inverse of delay Spread
Shorter the delay spread (D) the larger is the coherence bandwidth. It has impact on ISI

Q3. Ergodic Channel Capacity (for MIMO) when only receiver has access to the CSI.

· Channel Matrix H is random. Here optimal signaling uses spatially and temporally independent complex Gaussian input with equal power.

$\therefore$ Channel Capacity $C = E\left[\log\det\left(I_{N_r} + \frac{\rho}{N_t} H^H H\right)\right]$

The receiver using its knowledge of chan CSI classifies the observations corresponding to specific realizations of the channel coefficient. The corresponding inputs are still independent Gaussian hence for each state we can achieve a "log det" capacity

ρ = average SNR. $m = \min\{N_r, N_t\}$
$n = \max\{N_r, N_t\}$

Ergodic Capacity C =

$$= \int_0^\infty \log\left(1 + \frac{\rho\lambda}{N_t}\right) \sum_{k=0}^{m-1} \frac{k!}{(k+n-m)!} \left[L_k^{n-m}(\lambda)\right]^2 \lambda^{n-m} e^{-\lambda} d\lambda$$

$$L_K^{n-m}(x) = \frac{1}{(K)!} e^{x} x^{m-n} \frac{d^k}{dx^k}\left(e^{-x} x^{n-m+k}\right)$$

is associated with Laguerre polynomial of order k.

$xy'' + (1-x)y' + ny = 0$

→ Laguerre polynomials are solutions to Laguerre's equation which is 2nd order Linear differential equation. This equation has non singular solutions only if n is non-negative integer

→ Singular Solution : are solution of differential equations that cannot be obtained from general solution by usual method of solving the differential equation.

→ Here initial value problem fails to have a unique solution at some point on the solution.

→ $xy'' + (\alpha+1-x)y' + ny = 0$ solution to the

→ Laguerre polynomials

n	$L_n(x)$
0	1
1	$-x+1$
2	$\frac{1}{2}(x^2-4x+2)$
3	$\frac{1}{6}(-x^3+9x^2-18x+6)$

Single transmit antenna

$C = \log(1+\rho N_r)$

Single Receive antenna

$$C = \log(1+P) = \text{constant}$$

Equal number of tx & Rx antenna

$$C \simeq n \int_0^\infty \log(1+Pu) \sum_{k=0}^{n-1} L_k(nu)^2 e^{-nu}\, du$$

$$C \propto n \text{ (linearly)}$$

$$n = N_r = N_t$$

Q4. Coherent Max Likihihood Receiver for CSI available at receiver

A coherent receiver mixes the incoming signal with a local oscillator, thereby shifting any phase and amplitude fluctuation on the optical carrier to a carrier at an electronic frequency.

Coherent systems need carrier phase information at the receiver and they use matched filters to detect and decide what data was sent.

Noncoherent systems do not need carrier phase information and use methods like square law to recover the data.

→ In Statistics MLE (Estimation) is a method of estimating the parameters of a probability distribution by maximizing the likelihood function so that under the assumed statistical model the observed data is most probable.

→ Likelihood function measures the goodness of fit of a statistical model to a sample of data for a given values of unknown parameters.

→ Probability may also be described as the likelihood of an event occurring divided by the number of expected outcomes of the event. With multiple events, probability is found by breaking down each probability into separate, single calculations and then multiply each result together to achieve a single possible outcome.

Likelihood means a state of being likely or probable.

properties – ① Consistency
② Functional invariance

$$n = x_1 + x_2 + \cdots + x_m = n$$

$$p_1 + p_2 + \cdots + p_m = 1$$

$$f(x_1, x_2, \ldots x_m \mid p_1, p_2 \ldots p_m)$$

$$\hat{p}_i = \frac{x_i}{n}$$

Maximum likelihood estimation is a method that determines values for the parameters of a model. The parameter values are found such that they maximise the likelihood that the process described by the model produced the data that were actually observed.

$$f_1 \sim N(10, 2.25) \quad f_2 \sim N(10, 9)$$

(mean) variance
(μ) (σ)

probability density $P(x;\mu,\sigma)$ for gaussian distribution

$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

joint probability density of 3 data points:

$$P(9, 9.5, 11; \mu, \sigma) = P_1(x_1;\mu,\sigma) * P(x_2;\mu,\sigma) * P(x_3;\mu,\sigma)$$

$$\mu = \frac{9+9.5+11}{3} = 9.833$$

→ SD = Standard deviation = σ^2 = by how much the members of a group differ from mean value for a group.

Q5 ⓐ optimum decision rule at receiver.

optimal decision rule linearly combines the received signals through different diversity branches after cophasing and weighting them with their respective channel gains. The branches that have more channel gains are more than others. This is intuitive since received signals through better channels are more reliable and thus provide us with more accurate information

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Q5 (b) <u>Probability of error with selection combining</u>

Here at any given transmission interval, the branch with the largest SNR is used in demodulation

$$y = \left(\max_{j=1,2..L} |h_j|\right) x + n'$$

$n' \rightarrow$ complex Gaussian random variable

Effective instantaneous SNR after combining

$$\rho_{sc,eff} = \left(\max_{j=1,2..L} |h_j|^2\right) \rho$$

P.d.f of this is $\frac{L}{\rho} e^{-\frac{u}{\rho}} \left(1 - e^{-\frac{u}{\rho}}\right)^{L-1}$

for DPSK error rate in presence of Gaussian noise is given by

$$\boxed{P_b(\rho) = \frac{1}{2} e^{-\rho}}$$

$$P_{b,\,DPSK} = \int_0^\infty \frac{1}{2} e^{-u} \frac{L}{\rho} e^{-\frac{u}{\rho}} \left(1-e^{-\frac{u}{\rho}}\right)^{L-1} du$$

$$= \frac{L}{2} \sum_{l=0}^{L-1} \binom{L-1}{l} (-1)^l \frac{1}{1+l+\rho'}$$

which also behaves like $\sim \frac{1}{\rho^L}$ for $\rho \gg 1$

Thus a diversity of order L is achieved. (PDF)

Q6 For deterministic MIMO.

$$H = U \Sigma V^H$$

U = $N_t \times N_t$ unitary matrix $N_t = 3$

V = $N_r \times N_r$ unitary matrix $N_r = 4$

$\tilde{x} = xU \quad \tilde{y} = yV$

$$H_1 = \begin{bmatrix} 0.5+0.5j & j & 1 & -0.5-j \\ -0.7 & 0.3+0.5j & 1.2-0.5j & 0.8 \\ 0.6j & -0.8 & -0.2+0.9j & 0.4j \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 & 0 & 0 \\ 1 & 3 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 14 & 0 & 0 \\ 14 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = W$$

$$Wx = \lambda x \Rightarrow (W - \lambda I)x = 0$$

$$\begin{bmatrix} 20-\lambda & 14 & 0 & 0 \\ 14 & 10-\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{bmatrix} x = 0$$

$$(W - \lambda I) = 0$$

$$\underbrace{\lambda = 29.883}\qquad \lambda = \underbrace{0.117}$$

after processing complex values.

$$19.883x_1 + 14x_2 = 0$$

$$14\ x_1 + 9.883x_2 = 0$$

$$x_3 = 0 \qquad x_4 = 0$$

$$x_1 = 0.82 \qquad x_2 = -0.58$$

$$U = \begin{bmatrix} 0.82 & -0.58 & 0 & 0 \\ 0.58 & 0.82 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\lambda_1 = \sigma_1^2 \qquad \lambda_2 = \sigma_2^2 \qquad \lambda_3 = \sigma_3^2$$

W = unitary Matrix

$$H^H H = W \Lambda W^H$$

$$C = \sum_{i=1}^{r} \log\left(1 + \frac{\rho}{N_t}\lambda_i\right)$$

$H^H H \rightarrow$ positive semidefinite with +ve eigen values

$W \rightarrow$ diagonilization ~~~~ of $H^H H$ is carried out by using unitary matrix W.

$\Lambda =$ diagonal matrix containing $\lambda_1, \lambda_2 \ldots$

Q7. $N_t \times 1$ MIMO.

quasi static Rayleigh Fading

$K =$ no of available antenna used.

capacity $C(P) = \log\det\left(I_{Nr} + \rho H^H R_x(K) H\right)$

where $R_x(k) = \frac{1}{K}$ diagonal $\{\underbrace{1, 1 \ldots 1}_{K \text{ many}}, \underbrace{0.0 \ldots 0}_{(N_t - k) \text{ many}}\}$

$R_x(K) =$ covariance matrix

$$P_{out} = \min_{K = 1, 2 \ldots N_t} P\left(\log\det\left(I_{Nr} + \rho H^H R_x(k) H\right) \leq R\right)$$

for Transmit diversity

$N_r = 1$ when channels are independent Rayleigh fading.

$$P_{out} = \min_{k=1,2..N_t} \frac{1}{(k-1)!}\gamma\left(k, k\frac{2^R-1}{P}\right)$$

where $\gamma(a,x) = \int_0^x u^{a-1} e^{-u} du$ is the incomplete gamma function.

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