φ_c. When the incidence angle φ_1 is greater than the critical angle, the condition [illegible] reflection [illegible] satisfied; that is, the light is totally reflected back into the glass with no light escaping from the glass surface. (This is an idealized situation. In practice, there is always some tunneling of optical energy through the interface. This can be explained in terms of the electromagnetic wave theory of light, which is presented in Sec. 2.[illegible]

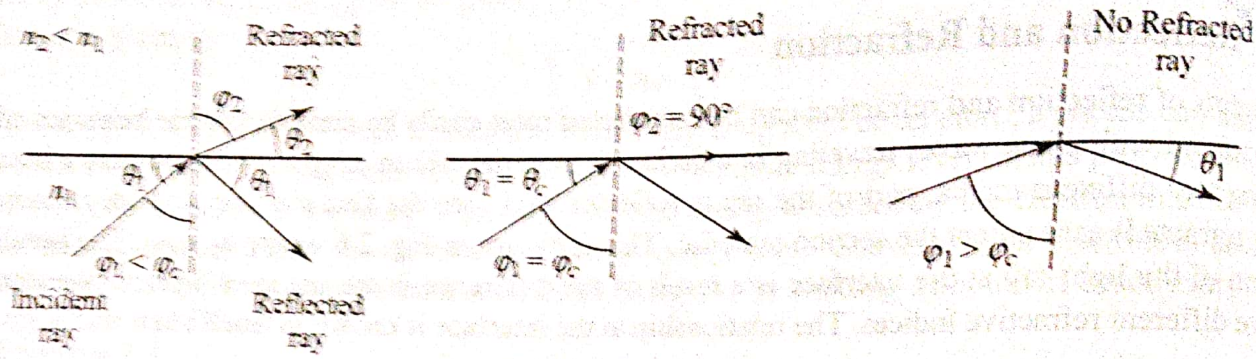

Fig. 2.7 *Representation of the critical angle and total internal reflection at a glass–air interface*

As an example, consider the glass–air interface shown in Fig. 2.7. When the light ray in air is parallel to the glass surface, then $\varphi_2 = 90°$ so that $\sin\varphi_2 = 1$. The critical angle in the glass is thus

$$\sin\varphi_c = \frac{n_2}{n_1} \qquad (2.18)$$

Example 2.2

Consider the interface between a glass slab with $n_1 = 1.48$ and air for which $n_2 = 1.00$. What is the critical angle for light traveling in the glass?

Solution:

From Eq. (2.18), for light traveling in the glass, the critical angle is

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 0.676 = 42.5°$$

Thus, any light ray traveling in the glass that is incident on the glass–air interface at a normal angle φ_1 (as shown in Fig. 2.7) greater than 42.5° is totally reflected back into the glass.

Example 2.3

A light ray traveling in air ($n_1 = 1.00$) is incident on a smooth, flat slab of crown glass, which has a refractive index $n_2 = 1.52$. If the incoming ray makes an angle of $\varphi_1 = 30.0°$ with respect to the normal, what is the angle of refraction φ_2 in the glass?

Solution:

From Snell's law given by Eq. (2.16), we find

$$\sin\varphi_2 = \frac{n_1}{n_2}\sin\varphi_1 = \frac{1.00}{1.52}\sin 30° = 0.658 \times 0.5 = 0.329$$

Solving for φ_2 then yields $\varphi_2 = \sin^{-1}(0.329) = 19.2°$.

Drill Problem 2.2

Consider the interface between a GaAs surface with a refractive index $n_1 = 3.299$ and air for which $n_2 = 1.000$. Show that the critical angle is $\varphi_c = 17.6°$.

$$n \sin \theta_{0,\max} = n \sin \theta_A = n_1 \sin \theta_c = \left(n_1^2 - n_2^2\right)^{1/2} \tag{2.22}$$

where $\theta_c = \pi/2 - \phi_c$. Thus those rays having entrance angles θ_0 less than θ_A will be totally internally reflected at the core–cladding interface. Thus θ_A defines an acceptance cone for an optical fiber.

Equation (2.22) also defines the *numerical aperture* (NA) of a step-index fiber for meridional rays:

$$\text{NA} = n \sin \theta_A = \left(n_1^2 - n_2^2\right)^{1/2} \approx n_1 \sqrt{2\Delta} \tag{2.23}$$

The approximation on the right-hand side is valid for the typical case where Δ, as defined by Eq. (2.20), is much less than 1. Since the numerical aperture is related to the acceptance angle, it is commonly used to describe the light acceptance or gathering capability of a fiber and to calculate source-to-fiber optical power coupling efficiencies. This is detailed in Chapter 5. The numerical aperture is a dimensionless quantity which is less than unity, with values normally ranging from 0.14 to 0.50.

Example 2.4

Consider a multimode silica fiber that has a core refractive index $n_1 = 1.480$ and a cladding index $n_2 = 1.460$. Find (a) the critical angle, (b) the numerical aperture, and (c) the acceptance angle.

Solution:

(a) From Eq. (2.21), the critical angle is given by

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.460}{1.480} = 80.5^\circ$$

(b) From Eq. (2.23), the numerical aperture is

$$\text{NA} = \left(n_1^2 - n_2^2\right)^{1/2} = 0.242$$

(c) From Eq. (2.22), the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1} 0.242 = 14^\circ$$

Example 2.5

Consider a multimode fiber that has a core refractive index of 1.480 and a core-cladding index difference 2.0 percent ($\Delta = 0.020$). Find the (a) numerical aperture, (b) the acceptance angle, and (c) the critical angle.

Solution:

From Eq. (2.20), the cladding index is $n_2 = n_1(1 - \Delta) = 1.480(0.980) = 1.450$.

(a) From Eq. (2.23), we find that the numerical aperture is

$$\text{NA} = n_1\sqrt{2\Delta} = 1.480(0.04)^{1/2} = 0.296$$

(b) Using Eq. (2.22), the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1} 0.296 = 17.2^\circ$$

(c) From Eq. (2.21), the critical angle at the core–cladding interface is

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 0.980 = 78.5^\circ$$

Drill Problem 2.3

Consider the interface between fiber core and cladding materials that have refractive indices of n_1 and n_2, respectively. If n_2 is smaller than n_1 by 1% and $n_1 = 1.450$, show that $n_2 = 1.435$. Show that the critical angle is $\varphi_c = 81.9^\circ$.

2.3.5 Wave Representation in a Dielectric Slab Waveguide

Referring to Fig. 2.17, the ray theory appears to allow rays at any angle ϕ greater than the critical angle ϕ_c to propagate along the fiber. However, when the interference effect due to the phase of the plane wave associated with the ray is taken into account, it is seen that only waves at certain discrete angles greater than or equal to ϕ_c are capable of propagating along the fiber.

Solving Maxwell's equations shows that, in addition to supporting a finite number of guided modes, the optical fiber waveguide has an infinite continuum of *radiation modes* that are not trapped in the core and guided by the fiber but are still solutions of the same boundary-value problem. The radiation field basically results from the optical power that is outside the fiber acceptance angle being refracted out of the core. Because of the finite radius of the cladding, some of this radiation gets trapped in the cladding, thereby causing cladding modes to appear. As the core and cladding modes propagate along the fiber, mode coupling occurs between the cladding modes and the higher-order core modes. This coupling occurs because the electric fields of the guided core modes are not completely confined to the core but extend partially into the cladding (see Fig. 2.19) and likewise for the cladding modes. A diffusion of power back and forth between the core and cladding modes thus occurs; this generally results in a loss of power from the core modes.

Guided modes in the fiber occur when the values for β satisfy the condition $n_2k < \beta < n_1k$. At the limit of propagation when $\beta = n_2k$, a mode is no longer properly guided and is called being *cutoff*. Thus unguided or radiation modes appear for frequencies below the cutoff point where $\beta < n_2k$. However, wave propagation can still occur below cutoff for those modes where some of the energy loss due to radiation is blocked by an angular momentum barrier that exists near the core-cladding interface.[17] These propagation states behave as partially confined guided modes rather than radiation modes and are called *leaky modes*.[5,6,12,13] These leaky modes can travel considerable distances along a fiber but lose power through leakage or tunneling into the cladding as they propagate.

2.4.2 Cutoff Wavelength and V Number

An important parameter connected with the cutoff condition is the *V number* defined by

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi a}{\lambda}\text{NA} \approx \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} \tag{2.27}$$

where the approximation on the right-hand side comes from Eq. (2.23).

This parameter is a dimensionless number that is related to the wavelength and the numerical aperture and determines how many modes a fiber can support. Except for the lowest-order HE_{11} mode, each mode can exist only for values of V that exceed a certain limiting value (with each mode having a different V limit). The modes are cut off when $\beta = n_2k$. As noted in Sec. 2.4.7, the wavelength at which all higher-order modes are cut off when $V \le 2.405$ is called the *cutoff wavelength* λ_c. The HE_{11} mode has no cutoff and ceases to exist only when the core diameter is zero. This is the principle on which single-mode fibers are based. Recommendation G.652 from the ITU-T states that an effective cutoff wavelength should range from 1100 to 1280 nm for single-mode fiber operation in the 1310-nm wavelength region. The mathematical details for these and other modes are given in Sec. 2.4.7.

Drill Problem 2.4

Consider a fiber that has a core refractive index of 1.480, a cladding index of 1.476, and a core radius of 4.4 μm. Using Eq. (2.27), show that the wavelength at which this fiber becomes single-mode is λ_c = 1250 nm.

Example 2.6

A step-index fiber has a normalized frequency V = 26.6 at a 1300-nm wavelength. If the core radius is 25 μm, what is the numerical aperture?

Solution:

From Eq. (2.27), the NA is

$$NA = V\frac{\lambda}{2\pi a} = 26.6\frac{1.30\,\mu\text{m}}{2\pi \times 25\,\mu\text{m}} = 0.22$$

number of modes supported in such a fiber is

$$M \approx \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2 \left(n_1^2 - n_2^2\right) = \frac{V^2}{2} \qquad (2.28)$$

Example 2.7

Consider a multimode step-index fiber with a 62.5-μm core diameter and a core–cladding index difference of 1.5 percent. If the core refractive index is 1.480, estimate the normalized frequency of the fiber and the total number of modes supported in the fiber at a wavelength of 850 nm.

Solution:

From Eq. (2.27), the normalized frequency is

$$V \approx \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi \times 31.25\,\mu\text{m} \times 1.48}{0.85\,\mu\text{m}} \sqrt{2 \times 0.015}$$
$$= 59.2$$

Using Eq. (2.28), the total number of modes is

$$M \approx \frac{V^2}{2} = 1752$$

Example 2.8

Suppose we have a multimode step-index optical fiber that has a core radius of 25 μm, a core index of 1.48, and an index difference $\Delta = 0.01$. What are the number of modes in the fiber at wavelengths 860, 1310, and 1550 nm?

Solution:

(a) First, from Eqs (2.23) and (2.28), at an operating wavelength of 860 nm, the value of V is

$$V \approx \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi \times 25\,\mu\text{m} \times 1.48}{0.86\,\mu\text{m}} \sqrt{2 \times 0.01}$$
$$= 38.2$$

Using Eq. (2.28), the total number of modes at 860 nm is

$$M \approx \frac{V^2}{2} = 729$$

(b) Similarly, at 1310 nm, we have $V = 25.1$ and $M = 315$.

(c) Finally at 1550 nm, we have $V = 21.2$ and $M = 224$.

Example 2.9

Suppose we have three multimode step-index optical fibers, each of which has a core index of 1.48 and an index difference $\Delta = 0.01$. Assume the three fibers have core diameters of 50, 62.5, and 100 μm. What are the number of modes in these fibers at a wavelength of 1550 nm?

Solution:

(a) First, from Eqs (2.23) and (2.28), at 50-μm diameter, the value of V is

$$V \approx \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi \times 25\,\mu\text{m} \times 1.48}{1.55\,\mu\text{m}} \sqrt{2 \times 0.01}$$
$$= 21.2$$

Using Eq. (2.28), the total number of modes in the 50-μm core diameter fiber is

$$M \approx \frac{V^2}{2} = 224$$

(b) Similarly, at 62.5 μm, we have $V = 26.5$ and $M = 351$.

(c) Finally, at 100 μm, we have $V = 42.4$ and $M = 898$.

Since the field of a guided mode extends partly into the cladding, as shown in Fig. 2.19, a final quantity of interest for a step-index fiber is the fractional power flow in the core and cladding for a given mode. As the V number approaches cutoff for any particular mode, more of the power of that mode is in the cladding. At the ... radiative with all the optical power of the mode residing in the cladding. Far

... in Sec. 2.4.9. Note that since M is proportional to V^2, the power flow in the cladding decreases as V increases. However, this increases the number of modes in the fiber, which is not desirable for a high-bandwidth capability.

Example 2.10

Consider a multimode step-index optical fiber that has a core radius of 25 μm, a core index of 1.48, and an index difference $\Delta = 0.01$. Find the percentage of optical power that propagates in the cladding at 840 nm.

Solution:

From Eqs (2.23) and (2.28), at an operating wavelength of 840 nm the value of V is

$$V \approx \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi \times 25\,\mu\text{m} \times 1.48}{0.84\,\mu\text{m}} \sqrt{2 \times 0.01}$$
$$= 39$$

Using Eq. (2.28), the total number of modes is

$$M \approx \frac{V^2}{2} = 760$$

From Eq. (2.29), we have

$$\frac{P_{\text{clad}}}{P} \approx \frac{4}{3\sqrt{M}} = 0.05$$

Thus approximately 5 percent of the optical power propagates in the cladding. If Δ is decreased to 0.03 in order to lower the signal dispersion (see Chapter 3), then there are 242 modes in the fiber and about 9 percent of the power propagates in the cladding.

Drill Problem 2.5

Consider a multimode step-index optical fiber that has a core diameter of 62.5 μm, a core index of 1.48, and an index difference $\Delta = 0.01$. Show that at 840 nm (a) the value of V is 49, (b) the total number of modes is 1200, and (c) the percentage of optical power that propagates in the cladding is 3.8%.

2.4.3 Maxwell's Equations*

To analyze the optical waveguide we need to consider Maxwell's equations that give the relationships between the electric and magnetic fields. Assuming a linear, isotropic dielectric material having no currents and free charges, these equations take the form[2]

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2.30a)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \qquad (2.30b)$$

$$\nabla \cdot \mathbf{D} = 0 \qquad (2.30c)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (2.30d)$$

where $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$. The parameter ϵ is the permittivity (or dielectric constant) and μ is the permeability of the medium.

A relationship defining the wave phenomena of the electromagnetic fields can be derived from Maxwell's equations. Taking the curl of Eq. (2.30a) and making use of Eq. (2.30b) yields

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu\frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\epsilon\mu\frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (2.31a)$$

2.5.1 Construction

Single-mode fibers are constructed by letting the dimensions of the core diameter be a few wavelengths (usually 8–12) and by having small index differences between the core and the cladding. From Eq. (2.27) or (2.58) with $V = 2.4$, it can be seen that single-mode propagation is possible for fairly large variations in values of the physical core size a and the core–cladding index differences Δ. However, in practical designs of single-mode fibers,[27] the core–cladding index difference varies between 0.2 and 1.0 percent, and the core diameter should be chosen to be just below the cutoff of the first higher-order mode; that is, for V slightly less than 2.4.

Example 2.13

A manufacturing engineer wants to make an optical fiber that has a core index of 1.480 and a cladding index of 1.478. What should the core size be for single-mode operation at 1550 nm?

Solution:

Using the condition that $V \le 2.405$ must be satisfied for single-mode operation, then from Eq. (2.27) we have

$$a = \frac{V\lambda}{2\pi}\frac{1}{\sqrt{n_1^2 - n_2^2}}$$

$$\le \frac{2.405 \times 1.55\ \mu\text{m}}{2\pi}\frac{1}{\sqrt{(1.480)^2 - (1.478)^2}} = 7.7\ \mu\text{m}$$

If this fiber also should be single-mode at 1310 nm, then the core radius must be less than 6.50 μm.

Example 2.14

An applications engineer has an optical fiber that has a 3.0-μm core radius and a numerical aperture of 0.1. Will this fiber exhibit single-mode operation at 800 nm?

Solution:

From Eq. (2.27)

$$V \approx \frac{2\pi a}{\lambda} NA = \frac{2\pi \times 3\ \mu\text{m}}{0.80\ \mu\text{m}} 0.10 = 2.356$$

Since $V < 2.405$, this fiber will exhibit single-mode operation at 800 nm.

2.5.2 Mode-Field Diameter

For multimode fibers the core diameter and numerical aperture are key parameters for describing the signal transmission properties. In single-mode fibers the geometric distribution of light in the propagating mode is what is needed when predicting the performance characteristics of these fibers. Thus a fundamental parameter of a single-mode fiber is the *mode-field diameter* (MFD). This parameter can be determined from the mode-field distribution of the fundamental fiber mode and is a function of the optical source wavelength, the core radius, and the refractive index profile of the fiber. The mode-field diameter is analogous to the core diameter in multimode fibers, except that in single-mode fibers not all the light that propagates through the fiber is carried in the core (see Sec. 2.4). Figure 2.28 illustrates this effect. For example, at $V = 2$ only 75 percent of the optical power is confined to the core. This percentage increases for larger values of V and is less for smaller V values.

The MFD is an important parameter for single-mode fiber because it is used to predict fiber properties such as splice loss, bending loss, cutoff wavelength, and waveguide dispersion. Chapters 3 and 5 describe these parameters and their effects on fiber performance. A variety of models have been proposed for characterizing and measuring the MFD.[30–35] These include far-field scanning, near-field scanning, transverse offset, variable aperture in the far field, knife-edge, and mask methods.[30] The main consideration of all these methods is how to approximate the optical power distribution.

A standard technique to find the MFD is to measure the far-field intensity distribution $E^2(r)$ and then calculate the MFD using the Petermann II equation[32]

2.14 (*a*) Substitute Eqs (2.35c) and (2.35d) into Eq. (2.34c), then differentiate the result and multiply by $jq^2/\varepsilon\omega$ to obtain Eq. (2.36). (*b*) Substitute Eqs. (2.35a) and (2.35b) into Eq. (2.33c), then differentiate the result and multiply by $jq^2/\varepsilon\omega$ to obtain Eq. (2.37).

2.15 Using the expressions in Eqs (2.33) and (2.34) derived from Maxwell's curl equations, derive the radial and transverse electric and magnetic field components given in Eqs (2.35*a*) to (2.35*d*). Show that these expressions lead to Eqs. (2.36) and (2.37).

2.16 Show that for $\nu = 0$, Eq. (2.55*b*) corresponds to TE_{0m} modes ($E_z = 0$) and that Eq. (2.56*b*) corresponds to TM_{0m} modes ($H_z = 0$).

2.17 Verify that $k_1^2 - k_2^2 = \beta^2$ when $\Delta \ll 1$, where k_1 and k_2 are the core and cladding propagation constants, respectively, as defined in Eq. (2.46).

2.18 A step-index multimode fiber with a numerical aperture of 0.20 supports approximately 1000 modes at an 850-nm wavelength.

(*a*) What is the diameter of its core?

(*b*) How many modes does the fiber support at 1320 nm?

(*c*) How many modes does the fiber support at 1550 nm?

2.19 (*a*) Determine the normalized frequency at 820 nm for a step-index fiber having a 25-μm core radius, $n_1 = 1.48$, and $n_2 = 1.46$.

(*b*) How many modes propagate in this fiber at 820 nm?

(*c*) How many modes propagate in this fiber at 1320 nm?

(*d*) How many modes propagate in this fiber at 1550 nm?

(*e*) What percent of the optical power flows in the cladding in each case?

2.20 Consider a fiber with a 25-μm core radius, a core index $n_1 = 1.48$, and $\Delta = 0.01$.

(*a*) If $\lambda = 1320$ nm, what is the value of V and how many modes propagate in the fiber?

(*b*) What percent of the optical power flows in the cladding?

(*c*) If the core-cladding difference is reduced to $\Delta = 0.003$, how many modes does the fiber support and what fraction of the optical power flows in the cladding?

2.21 Consider a step-index fiber that has a 5-μm core radius, an index difference $\Delta = 0.002$, and a core index $n_1 = 1.480$. (a) By calculating the V number, verify that at 1310 nm this is a single-mode fiber. (b) Verify that at 820 nm the fiber is not single-mode because $V = 3.514$. (c) From Fig. 2.24, find the LP modes that exist at 820 nm.

2.22 Consider a 62.5-μm core diameter graded-index fiber that has a parabolic index profile ($\alpha = 2$). Suppose the fiber has a numerical aperture NA = 0.275. (a) Show that the V number for this fiber at 850 nm is 63.5. (b) How many guided modes does the fiber have at 850 nm?

2.23 Consider a 50-μm core diameter graded-index fiber that has a core index $n_1 = 1.480$, a cladding index $n_2 = 1.465$. (a) Using the exact expression for the index difference Δ given in Eq. (2.79), show that $\Delta = 1.008$ percent. (b) Using the approximation for Δ given in the right-hand side of Eq. (2.79), show that $\Delta = 1.014$ percent. This shows that the approximation is quite accurate.

2.24 A graded-index fiber with a parabolic index profile ($\alpha = 2$) has a core index $n_1 = 1.480$ and index difference $\Delta = 0.010$. (a) Using Eq. (2.27), show that the maximum value of the core radius for single-mode operation at 1310 nm is 3.39 μm. (b) Show that the maximum value of the core radius for single-mode operation at 1550 nm is 4.01 μm.

2.25 Commonly available single-mode fibers have beat lengths in the range $10\text{ cm} < L_p < 2$ m. What range of birefringent refractive-index differences does this correspond to for $\lambda = 1300$ nm?

2.26 Plot the refractive-index profiles from n_1 to n_2 as a function of radial distance $r \le a$ for graded-index fibers that have α values of 1, 2, 4, 8, and ∞ (step index). Assume the fibers have a 25-μm core radius, $n_1 = 1.48$, and $\Delta = 0.01$.

2.27 Calculate the number of modes at 820 nm and 1.3 μm in a graded-index fiber having a parabolic-index profile ($\alpha = 2$), a 25-μm core