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DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

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Department of Telecommunication Engineering

Online Continuous Internal Assessment Test - II

Course:**MIMO Communication**
Course Code: **TE814**
Semester:**VIII 'A' &'B'**

Date: 21/04/2020
Maximum marks: **50**
Duration: **90 Min**

	Note: Answer 5 full questions.	**Marks**
1	a) Shannon's Capacity gives the theoretical maximum data rate or capacity of a ----- i) Signaling Channel ii) Imperfect Channel iii) Perfect Channel iv) Noisy Channel b) The main benefit of space-time trellis coding is its -___________provided over the approach of space-time block coding, which comes at the cost of ___________ (i) coding advantage, increased decoding complexity.(ii) coding disadvantage, increased decoding complexity (iii) coding disadvantage, decreased decoding complexity (iv) coding disadvantage, decreased decoding complexity c) More certain or deterministic the event is, the less ---------- it will contain. i) BW ii) information iii) Energy iv) invalid data d) Information entropy tells how much --------- there is in an event i) BW ii) information iii) Energy iv) invalid data e) The symbol for entropy is an ----- i) Erlang ii) K iii) S iv) N f) Receive diversity is that each element in the receive array receives an independent copy of the-------- i) Interference ii) Different Signal iii) Same Signal iv) Dispersion g) With the ordering of the equivalent parallel channels from good to bad (i.e., with $\lambda_1 \geq \lambda_2 \geq \cdot \cdot \cdot \geq \lambda_v$), the transmitter may choose to allocate all its power to the first channel and no power to the remaining links, resulting in a capacity ___________ i) $C = \log(1 + \rho\lambda_1)$ ii) $C = \log(\rho\lambda_1)$ iii) both i)& ii) iv) None of the above h) Base station antenna comprises multiple elements while the mobile device has only one or two, why? i) Space considerations ii) Bandwidth iii) Interference iv) No Reason i) Multiple transmit/receive antennas should allow us to transmit --------- i) Data Slower ii) Data faster iii) Same data rate iv) Less data rate	1x10

	j) The capacity of the channel is defined as the maximum possible mutual information between the input (x) and --------. i) CSI ii) input(x) iii) output (y) iv) CQI	
2	Estimate the capacity of MIMO Channels	10
3	Explain for Transmit diversity with two antennae(Alamouti scheme)	10
4	With example describe the simple space time trellis code.	10
	(OR)	
5	Investigate on Deterministic MIMO channel.	10
6	With example discuss the Orthogonal Space-Time Block Codes	10
	(OR)	
7	Hypothesize Quazi Orthogonal STBC	10

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No. of copies: 120

CIE

Solution Scheme

CIE-II Date 21/04/2020

Marks 50 prepared by Dr. Sayed Abdulhayan

Q1 (a) → Noisy channel — (iv)

(b) → Coding advantage, increased decoding complexity — (i)

(c) → Information — (ii)

(d) → Information — (ii)

(e) → S — (iii)

(f) → Same signal — (iii)

(g) → $C = \log(1 + \rho\lambda_1)$ — (i)

(h) → Space consideration — (i)

(i) → Data faster — (ii)

(j) → output (y) — (iii)

Q2. Capacity of MIMO Channel

$N_t \rightarrow$ no of tx antenna

$N_r \rightarrow$ no of Rx antenna

$$y = \sqrt{\rho}\, x H + n$$

Assume no ISI i.e subchannels are flat fading

$x = 1 \times N_t$ vector of transmitted signal

$H = N_t \times N_r$ matrix of channel gains.

n — $1 \times N_r$ vector of independent comp. Gaussian noise terms.

The signal vector satisfies the power constrain

$$E[x x^H] \leq 1$$

$$H = U \Sigma V^H$$

$U \rightarrow N_t \times N_t$ unitary matrix

$V \rightarrow N_r \times N_r$ unitary matrix

$$U^H U = I_{N_t}, \quad V^H V = I_{N_r}$$

$\Sigma = N_t \times N_r$ non negative diagonal matrix whose diagonal elements are singular values of matrix H.

$\sigma_1 \ \sigma_2 \ \cdots \ \sigma_Y \rightarrow$ singular values

$$\tilde{y} = \sqrt{P}\, \tilde{x} \Sigma + \tilde{n}$$

$$\tilde{x} = xU \qquad \tilde{y} = yV \qquad \tilde{n} = nV$$

U & V are invertible matrices

$$E[\tilde{x} \tilde{x}^H] = E[x U U^H x^H] = E[x x^H] \leq 1$$

$$\tilde{y}_1 = \sqrt{P}\, \sigma_1 \tilde{x}_1 + \tilde{n}_1$$

$$\tilde{y}_2 = \sqrt{P}\, \sigma_2 \tilde{x}_2 + \tilde{n}_2$$

$$\vdots$$

$$\tilde{y}_V = \sqrt{P} \cdot \sigma_Y \tilde{x}_V + \tilde{n}_V$$

$$\tilde{y}_{V+1} = \tilde{n}_{V+1}$$

$$\tilde{y}_{N_r} = \tilde{n}_{N_r}$$

For equal transmit power allocation

$$C = \max_{p(x)} H(Y) - H(N)$$

$$H(N) = N_r \log(\pi e)$$

$$H(Y) \leq \log(\det(\pi e R_y))$$

R_y = covariance matrix

$$R_y = E[y^H y] = \rho H^H R_x H + I_{Nr}$$

$$C = \max_{R_x} \log\det(\rho H^H R_x H + I_{Nr})$$

$$E\{x x^H\} = \text{trace}(R_x) \leq 1$$

$$R_x = \frac{1}{N_t} I_{Nt}$$

$$C = \log\det\left(I_{Nr} + \frac{\rho}{N_t} H^H H\right)$$

$H^H H \rightarrow$ +ve semi definite with +ve eigen values $\lambda_1\ \lambda_2 \ldots \lambda_v$

$$\lambda_1 = \sigma_1^2$$

$$\lambda_2 = \sigma_2^2$$

$$\lambda_v = \sigma_v^2$$

W = unitary matrix

diagonalization $H^H H = W \Lambda W^H$

Λ = diagonal matrix consisting of $\lambda_1\ \lambda_2 \ldots \lambda_v$

$$\log\det\left(I_{Nr} + \frac{\rho}{N_t} H^H H\right)$$

$$= \log\det\left(I_{N_r} + \frac{\rho}{N_t} W \Lambda W^H\right)$$

$$= \log\det\left(W\left(I_{N_r} + \frac{\rho}{N_t}\Lambda\right)W^H\right)$$

$$C = \sum_{i=1}^{v} \log\left(1 + \frac{\rho}{N_t}\lambda_i\right)$$

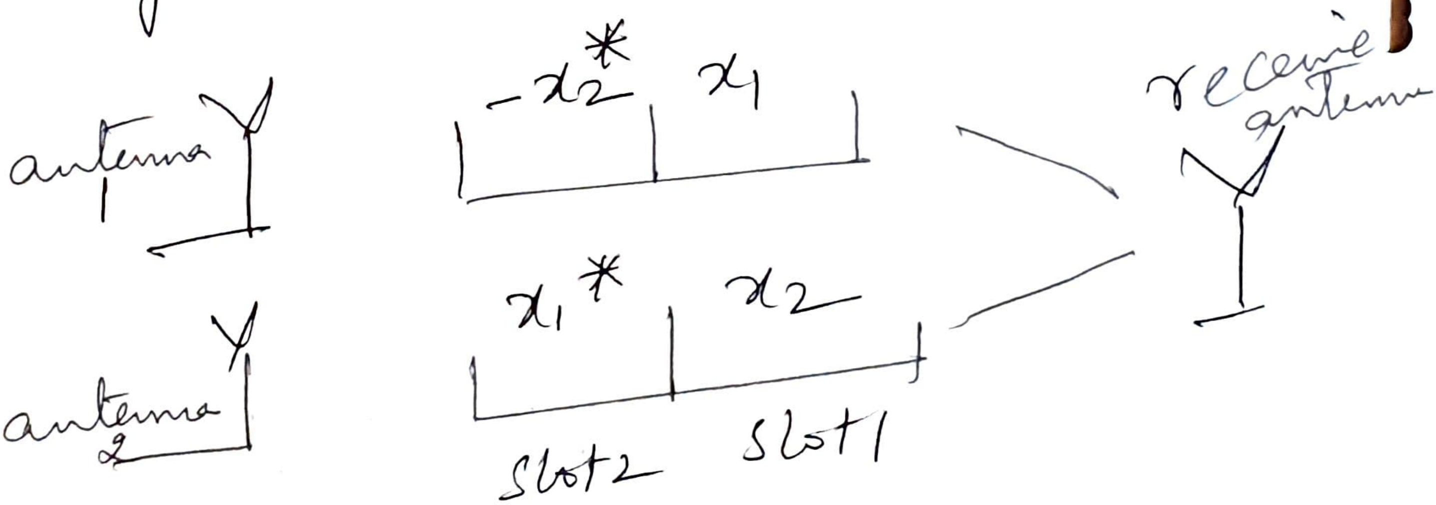
overall capacity is simply the sum of the capacities of these parallel channels.

Q3. Alamouti Scheme

- Only 1 or 2 transmit antennas
- 1 to many receive antennas.

$$y_1(1) = \sqrt{\rho}\,(h_{11}x_1 + h_{21}x_2) + n_1(1)$$

$$y_1(2) = \sqrt{\rho}\,(-h_{11}x_2^* + h_{21}x_1^*) + n_1(2)$$



$$y = \begin{bmatrix} y_1(1) \\ y_1^*(2) \end{bmatrix}$$

$$y = \sqrt{P}\begin{bmatrix} |h_{11}|^2+|h_{21}|^2 & 0 \\ 0 & |h_{11}|^2+|h_{21}|^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1'(1) \\ n_1'(2) \end{bmatrix}$$

$$\hat{x}_1 = \begin{cases} 1 & \text{if } \mathrm{Re}\{h_{11}^* y_1(1) + h_{21} y_1^*(2)\} > 0 \\ 0 & \text{— otherwise} \end{cases}$$

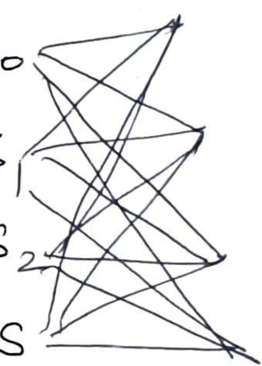
$$\hat{x}_2 = \begin{cases} 1 & \text{if } \mathrm{Re}\{h_{21}^* y_1(1) - h_{11} y_1^*(2)\} > 0 \\ 0 & \text{— otherwise} \end{cases}$$

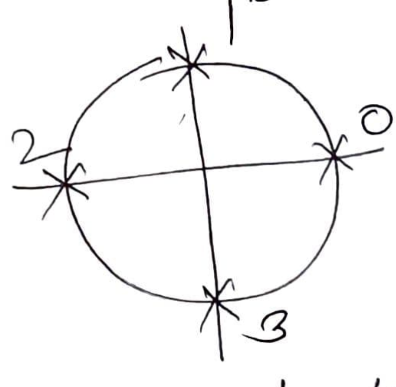
Q4. Simple Space time trellis code

Assume $N_t = 2$

input	0	1	2	3	
	00	01	02	03	— S_0
	10	11	12	13	— S_1
	20	21	22	23	— S_2
	30	31	32	33	— S_3

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{branch labels}}$





Assume 4 state trellis
Initial state is S_0.

Each pair of information bits to be encoded determines the state transition and two coded symbols to be transmitted from two transmit antennas

Encoded Sequence corresponding to the 4 ary information

4 any information sequence W

2 1 2 3 0 0 1 3 2

0 2 1 2 3 0 0 1 3 → 1st antenna

2 1 2 3 0 0 1 3 2 → 2nd antenna

Once all information bits are encoded the trellis is terminated to complete each frame of data

To obtain transmission rate of R bps R bits per channel use we need trellis with 2^R branches ~~emmanting~~ emmanating from each slot.

Instead of using trellis which determines only a single symbol to be transmitted for each trellis section each branch is associated with multiple symbols that are to be transmitted from the multiple antenna system.

Q5. Deterministic MIMO channel

H = channel gain matrix is fixed

Instance for fixed wireless links environment negligible variation in

$H = U \Sigma V^H$

$\tilde{y} = \sqrt{P}\,\tilde{x}\,\Sigma + \tilde{n}$

$$E[\tilde{x}\tilde{x}^H] = E[xUU^Hx^H] \leq E[xx^H] \leq 1$$

$$\tilde{y}_1 = \sqrt{P}\,\sigma_1 \tilde{x}_1 + \tilde{n}_1$$

$$\tilde{y}_2 = \sqrt{P}\,\sigma_2 \tilde{x}_2 + \tilde{n}_2$$

power constraint on input is

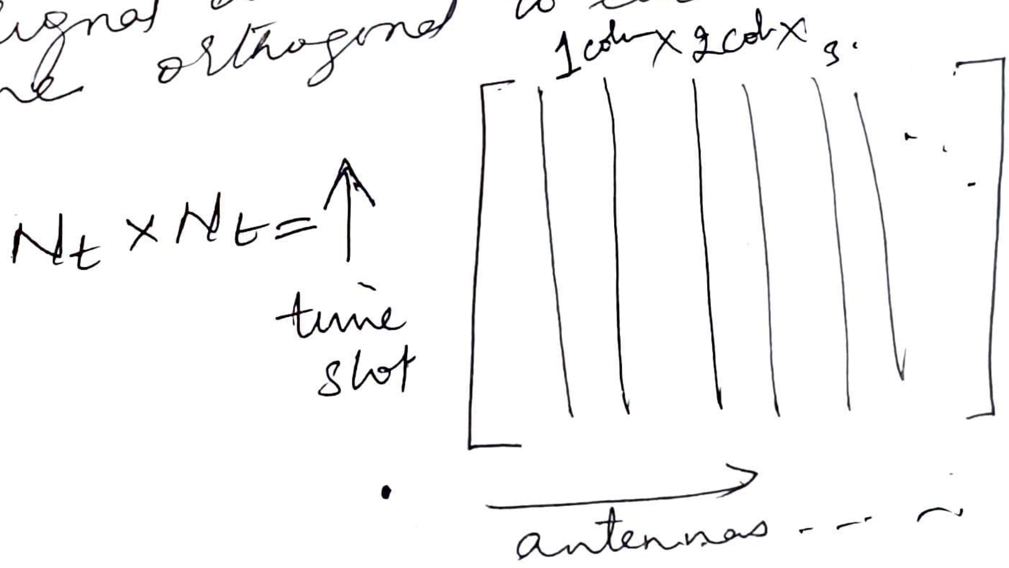
$$\sum_{i=1}^{N_t} E[|\tilde{x}_i|^2] \leq 1$$

and noise terms are independent complex Gaussian with variance 1/2 per dimension

Q6. <u>Orthogonal Space time Block code</u>

Here transmit diversity can be extended beyond 2 antennas.

We design a set of $N_t \times N_t$ matrices with elements from a desired signal constellation whose columns are orthogonal to each other



It is not full Rate

column 1 orthogonal to all other columns

Similarly all columns are orthogonal to each other

n — $1 \times N_r$ vector

Q7) Quasi orthogonal STBC

This is not full rate, full diversity code of complex installation other than alamouti code scheme and such designs are very limited for real constellations.

QOSTBC developed by Jafarkhani full rate space time block code using smaller design building blocks

$N_t = 4$, Symbols $= 4 = x_1, x_2, x_3, x_4$

Alamouti code to encode these symbols pairwise resulting in 2×2 matrices of form.

$$X_{12} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \quad X_{34} = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix}$$

Use these matrices in another orthogonal design to obtain 4×4 quasi orthogonal STBC

$$X = \begin{bmatrix} X_{12} & X_{34} \\ -X_{34}^* & X_{12}^* \end{bmatrix} = X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & x_2 & x_1 \end{bmatrix}$$

→ It is way of obtaining full-rate code with complex constellations for 4 transmit antenna

→ It cannot achieve full diversity

→ Not all collumns of code matrix are orthogonal

END