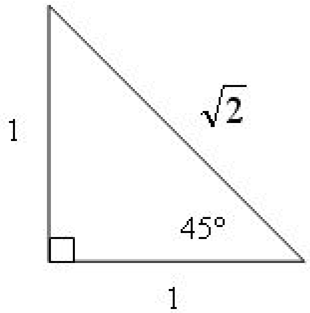
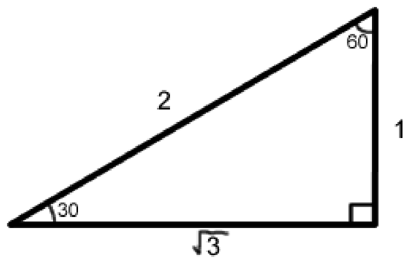
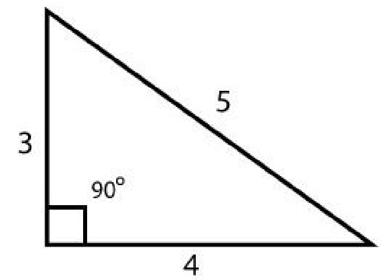






Triangles

Whenever 2 legs of a right triangle are congruent, the hypotenuse is $x\sqrt{2}$ (root 2 times the leg length). This can also be applied to the diagonals of a square.

The Pythagorean Triplets (groups of whole numbers which form right triangles) are

3, 4, 5
5, 12, 13
6, 8, 10
7, 24, 25
8, 15, 17

Don’t forget you can multiply basic triplets by a whole number to find other triples (e.g. 2(3, 4, 5)= 6, 8, 10)
Method for finding triplets:

<u>Even:</u>　　　　　　　　　　<u>Odd:</u>

$\frac{x^2}{4}+1, \frac{x^2}{4}-1$ $\frac{x^2-1}{2}, \frac{x^2-1}{2}+1$ (where x is a whole number)

Pythagorean theorem: $a^2+b^2=c^2$

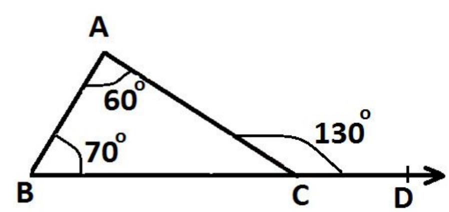
Area of an equilateral triangle: $\frac{x^2\sqrt{3}}{4}$ where x= the side length

A 5, 5, 10 triangle does not exist!

Angles sum to 180°

Length of one side must be greater than the positive difference and less than the sum of the other two sides.

To find the exterior angle of a triangle, add the measure of the non-adjacent angles.

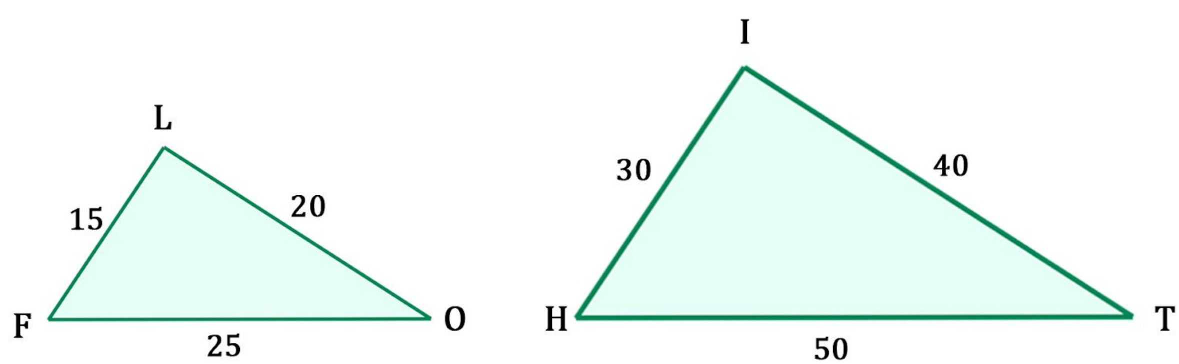


Ex: 60+70=130

Area= $\frac{bh}{2}$

Perimeter= a+b+c

Similar triangles have their angle measures equal and their side lengths proportional.



Polygons

Area of:

Rectangle: bh

Square: x^2 or diagonal squared over two

Rhombus: $\frac{d_1 \times d_2}{2}$ d= diagonal
Trapezoid: $\frac{(b_1+b_2)h}{2}$ b=base h=height

Angle Measures for Polygons

3 sides: 180° average is 60°
4 sides: 360°average is 90°
5 sides: 540° average is 108 °
6 sides: 720° average is 120°
7 sides: 900° average is unimportant
8 sides: 1080° average is 135°
9 sides: 1260° average is 140°
10 sides: 1440° average is 144°
Formula for finding total angle measure is 180(n-2)
Average is $\frac{180(n-2)}{n}$
Angle measure for regular polygons: $180 - \frac{360}{n}$
Exterior angle in polygons: $\frac{360}{n}$
Area of a circle is πr^2 Circumference is is $2\pi r$
Perimeter of a shape is side lengths added together

Volume

Rectangular Prism: lwh
You can multiply each one of the surfaces together and find the square root to find the volume as well ($\sqrt{lwlhwh} = \sqrt{l^2 w^2 h^2} = lwh$)
Cylinder: $\pi r^2 h$ (bh)
Cone: $\frac{\pi r^2 h}{3}$
Sphere: $\frac{4}{3}\pi r^3$
Cube: x^3
Pyramid: $\frac{Bh}{3}$ Where B=area of the base

Recurring Decimals to Fractions:
If all digits are the same (e.g $.\overline{111}$) you put the recurring part over the amount of places that repeats as nines ($.\overline{111}$ has one number repeating (1), so you would put 1 over one nine to get $\frac{1}{9}$)
If one a part of the number repeats (such as $2.5\overline{3}$) then you separate the number into the non-repeating section and repeating section ($2.5 + .0\overline{3}$) do the same thing for normal repeating numbers, except with adding a zero for each decimal place skipped (so $2.5 + \frac{3}{90}$) and simplify the answer.
Difference in Age
Ex: Jane is 48. Bob is 12. When will Bob be half as old as Jane? Older-2*younger= $48 - 2(12) = 24$
If the ratio is not two, divide the answer by one higher than the desired ratio (e.x. If the problem called for ⅓ of the age you would do $\frac{48-2(12)}{2}$ to get 12)
Surface area of cylinders: $2\pi r^2 + 2\pi rh$
Cube: $6x^2$ The side length value for which the surface area and volume of a cube are equivalent is 6. The diagonal of a cube is $\sqrt{3}$ times the side length.

Sums:

First 4 whole numbers: 10
First 5 whole numbers: 15
First 9 whole numbers: 45
First 10 whole numbers: 55
First 20 whole numbers: 210

First 100 whole numbers: 5050

Formula for the sum of whole numbers 1-n: $\frac{n^2+n}{2}$

Formula for the sum of the squares of whole numbers 1-n: $\frac{n(n+1)(2n+1)}{6}$

Formula for the sum of the squares of whole numbers 1-n: $(\frac{n^2+n}{2})^2$

Average of numbers with a common difference: $\frac{lowest+highest}{2}$

Percentage Change: $\frac{new-old}{old} \times 100$ Ex: 100 to 120 is $\frac{120-100}{100} \times 100 = \frac{20}{100} \times 100 = 20$ so +20% from original number

Factorials:

1!=1
2!=2
3!=6
4!=24
5!=120
6!=720
7!=5040
8!=40320

Don’t forget to simplify when doing factorials! (Don’t do $\frac{7!}{5!}$ do 7*6)

Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34…

Formula for arithmetic sequences: $a_n = a_1 + (n-1)d$

Where n is the term number and d is the common difference

For a geometric sequence: $a_n = a_1 + r^{(n-1)}$ Where n is the term number, r is the common ratio, and a_1 is the first term.

To find how many factors a number has:
1) Find the prime factorization
2) Simplify
3) Take the exponents of each factor, add 1 to each, and multiply them.
12 becomes 3×2^2, (1+1)(2+1)= 6 factors
To square numbers ending in 5:
Split number into 5 and the number before 5 (25 becomes 2 and 5)
Take number before five and multiply it by one plus itself (2*(2+1)=2*3=6)
Put a 25 after that number (6 $\rightarrow$ 625)
To square a number one higher than a known square:
Take known square
Add two times square root of the square
Add one
$20^2 = 400$ $21^2 =?$ $21^2 = 400 + 2(20) + 1 = 400 + 40 + 1 = 441$
This comes from $(a+b)^2 = a^2 + 2ab + b^2$, so we do $(20+1)^2$ to get 400+40+1
You can also do this for numbers larger than 1, just adjust the equation accordingly.
To find which fraction is greater:
Cross multiply them and find which one is greater of those numbers
$\frac{1}{2} ? \frac{3}{4} \rightarrow \frac{1}{2} \Leftrightarrow \frac{3}{4} \rightarrow 4 < 6$, so $\frac{3}{4}$ is greater
To find which radical is greater:
Square both and compare. Number that is greater is the corresponding square root which is greater.
Ex. $2\sqrt{5} ? 5\sqrt{2} \rightarrow 4 \times 5 ? 25 \times 2 \rightarrow 20 < 50$ so $5\sqrt{2}$ is greater

If you increase a value by $x\%$ and lower it by $x\%$, the new number will always be lower (the percentage change is $(-\frac{x^2}{1000})\%$ and to find the new number the formula is (- $\frac{x^2}{100}$)

To find if a number is prime:

Find closest whole number square root of the number

Divide the original number by every prime number under and including that square root.

If the original isn't divisible by any of those numbers, it is a prime. $101 \to \sqrt{101} \approx \sqrt{100} = 10$ $101/7$ isn't a whole number, $101/5$ isn't a whole number, $101/3$ isn't a whole number, and neither is $101/2$, so 101 is a prime number.

There are 25 primes under 100.

Divisibility Rules:

2: Last digit is divisible by two

3: Sum of digits is divisible by three

4: last two digits are divisible by four

5: ends in a 5 or 0

6: divisible by two and three

7: double last digit and subtract it from the rest of the number, and if that is divisible by 7, the original number is.

8: last three digits are divisible by 8

9: sum of the digits is divisible by nine

10: ends in zero

11: If the sum of the alternating digits minus the sum of the other alternating digits is divisible by 11, the entire number is. Ex. 1364 1+6=7 3+4=7 7-7=0, so yes.

12: is divisible by 3 and 4

For any number n that is a power of two, the last x digits must be divisible by that number where $x = 2^x = n$
Angles of a Triangle in an Arithmetic Progression:
60, 60, 60
Take away an amount from one of the 60's and add it to a different one. Ex. 60-30=30 60+30=90 so 30, 60, 90.
When you have a problem with a ratio and a known value:
Ex. 1:2:3:4 (ratios of angles in a quadrilateral)
1+2+3+4=10
360/10=36 so the angle measures are 3, 72, 108, and 144.
A circle has 360 degrees.
To find the area of the section of a circle with a known angle measure: $\frac{x}{360}\pi r^2$ where x is the angle measure

Clock Problems

The angle measure between two consecutive numbers (e.g. 12 and 1) on a clock is 30°
The hour hand moves at $\frac{1}{12}$ the speed of the minute hand.
Don't forget the hour hand moves too in clock problems.
The largest area that can be contained within a definite perimeter but an unknown number of sides will always be contained in a circle. As the number of sides goes up, the area goes up.

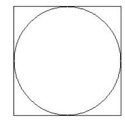
Mixture Problems:

How much of 10% solution has to be added to a liter of 20% solution to get 15% solution?
$\frac{1+x}{5+10x} = \frac{3}{20}$ (at the top is the numerator for the known amount and numerator of variable amount, and the denominator is the denominator for both added together)

To find the product of numbers with a midpoint such as 16 and 18:
Put in difference from endpoints into (x+y)(x-y) as the y and put in the midpoint as the x. (17+1)(17-1)
Do square of first minus square of that difference

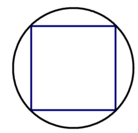
$$17^2 - 1 = 289 - 1 = 288$$

Ratio of a circle inscribed in a square:

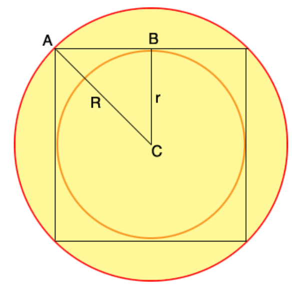


$\pi : 4$

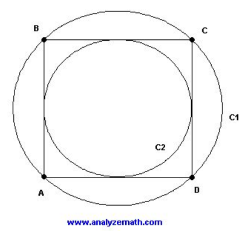
Square inscribed in a circle:



$2 : \pi$



Square inscribed in a circle inscribed in a square or circle inscribed in a square inscribed in a circle:
2:1



Squares
1,4,9,16,25,36,49,64,81,100,121,144,169,196,225,256

Cubes
1,8,27,69,125,216,512

Smallest numbers that are squares and cubes
1,64, 729, 4096
(In general, find the LCM of two factors and raise small numbers to that power.)

2^n where n =0,1,2,...

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

2^{11} =2048

$2^{12} = 4096$

2^{20} = 1,048,576

Important fractions

$\frac{1}{2}$ =0.5

$\frac{1}{4}$ =0.25

$\frac{1}{5}$ =0.2

$\frac{1}{6}$ =0.1$\overline{6}$

$\frac{1}{8}$ =0.125

$\frac{1}{3}$ = .$\overline{3}$

Keep thing as fractions to make simplication easier.

Algebra tips:

Quadratic Equation

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Graphing Quadratic Equations

$y = ax^2 + bx + c$

a>0 , the parabola will open up

a<0 , the parabola will open down

x= $\frac{-b}{2a}$ vertex

$\frac{-b}{a}$ = sum of roots

$\frac{c}{a}$ = product of roots

If you are doing an optimization problem, find the equation you can use to apply the vertex formula to.

What is the largest area you can enclose 100 feet of fencing: 2(l+w)=100
l+w=50, l=50-w
Area=w(50-w)
$-w^2 + 50w \rightarrow \frac{-50}{-2} \rightarrow 25 = w$
$25^2 = 625$ is the final answer

Difference of Squares
$x^2 - y^{2=}(x+y)(x-y)$

- To find nth digit of repeating series
  E.g. In 123, what is 216th ?
  1. Find the last digit. In 123 is **3**
  2. Find its place. In 123, 3 is in the third place.
  3. Divide 216/3=71 R2, so 2 is the answer

  Last digit of powers
  3: 3,9,7,1
  2: 2,4,8,6
  4: 4,6
  7:7,9,3,1
  8:8,4,6
  9:9,1

Average Speed
speed= $\frac{distance}{time}$

Formula for average speed given two speeds used for the same distance:

$\frac{2ab}{a+b}$

If my dad drives the 15 miles to the school at 30 mph, and returns the 15 miles home driving 60 mph, what is his average speed for the entire trip? (40 mph)

The average speed for three speeds is $\frac{3abc}{ab+bc+ca}$

If the middle number with three average speeds the average speed will be the average speed between the other two (e.g. for 30, 40, 60 the average speed is 40)

Shared Labor:

$\frac{ab}{a+b}$ where a and b is the time taken to complete a task

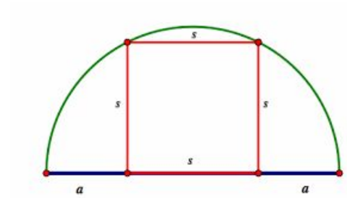
For three people the formula is $\frac{abc}{ab+bc+ac}$

Don't forget if you know the time taken by two people you can use that time as one time in in the initial equation.

$$x\% \text{ of } y = y\% \text{ of } x$$

Ratio of a square inscribed in a semicircle:

$4 : 5\pi$



Probability:

Dice:

36 different outcomes when rolling two dice

Freq.

2: 1

3: 2

4: 3

5: 4

6: 5

7: 6

8: 5

9: 4
10: 3
11: 2
12: 1
7 has the highest probability of being rolled as the sum of two dice.
Permutations:
Take number of items you want to find how many different ways can arrange and find the factorial of that number.
If the group is circular as in sitting around a table, do the same with n-1 instead of n.
Remember: If you can write the different combinations as a, "Yes/No," statement (e.g., pick and choose), you can power set
(Power setting is 2^x with x = to the number of items.)

Example: How many different 3-digit numbers can you create using only "4" and "5"?

Since the digits can be 4 <u>OR</u> 5, you can power set.
4 = Yes   5 = No
Since 3 different places the numbers can occupy, 2^3 or 8.

Diagonals Formula: $\frac{n(n-3)}{2}$ Hardshakes $\frac{n(n-1)}{2}$
(In a Polygon)
For Triangles: nt n = # of sides
(Amount of)   t= Triangles that can

be made at one point
[t = (n-2)]

★Remember that squares are an exception with 4 triangles!!!

Combinations:

$$\frac{xy...}{n}$$

- xy are terms in series, counting down from top of series.
- n= # of terms

★There are 26 letters in the English alphabet with 5 vowels and 21 consonants.

Example: How many ways can you choose three people to be President, Vice President, and Secretary in a class of 10 Students?

Starts at top

$$\frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3}$$

# of terms

${}_{y}C_{n}$ is equal to ${}_{y}C_{(y-n)}$

because n and y - n add up to y

When you are having a special requirement with a permutation problem (where you have to make it so that two or more are next to each other) find the different positions for the requirements and find the permutations between each group and multiply it.

Find permutations between other numbers, take that permutation and multiply it by the number of different positions and permutations between the groups next to each other to get you final answer.

To find permutations of a group with repeating numbers (e.g., EMERGE) do the same process but divide the final answer by the number of times a letter is repeated factorial. If multiple letters are repeated, find the factorials for the number of times each number in each group of letters is repeated and multiply those numbers together.

If you are trying to find the number of routes on a board:
$\frac{(x+y)!}{x! \times y!}$ where x=the number of times north and y=the number of times east
Minimum number of draws from a standard deck of 52 cards to be sure that you have:
2 of any color: 3
2 reds: 28
A card of each color: 27
Card of each suit: 40
Two of any card: 14
Two of any specific card: 50
$8^2 + 6^2 = 100$
When you compare ratios, find the LCM and then compare them. 2:3 and 4:5 becomes 8:12:15 goes to 8:15
Distance formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
When:
a+b=x
a+c=y
b+c=z
Subtract twice of each group to and divide by two to find each variable.
E.g. a+b=10, a+c=14, b+c=20
20+10+14=44 (2a+2b+2c)
44-20=24/2=12=c
44-28=16/2=8=b
44-40=4/2=2=a
Or you can substitute to find the rest.

GOOD LUCK!!!!

To find the chance of at least one person in a group of people solving a problem:
Find the complement of each probability of someone solving the problem, multiply the complements, and subtract from one (find the chance of them not solving it and subtract from one).
Ex: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ is $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$, 1- $\frac{1}{4} = \frac{3}{4}$ chance of one of them solving it