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Review article

Influence decision models: From cooperative game theory to social network analysis

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ABSTRACT

Cooperative game theory considers simple games and influence games as essential classes of games. A simple game can be viewed as a model of voting systems in which a single alternative, such as a bill or an amendment, is pitted against the status quo. An influence game is a cooperative game in which a team of players (or coalition) succeeds if it is able to convince sufficiently many agents to participate in a task. Furthermore, influence decision models allow to represent discrete system dynamics as graphs whose nodes are activated according to an influence spread model. It let us to depth in the social network analysis. All these concepts are applied to a wide variety of disciplines, such as social sciences, economics, marketing, cognitive sciences, political science, biology, computer science, among others. In this survey we present different advances in these topics, joint work with M. Serna.

These advances include representations of simple games, the definition of influence games, and how to characterize different problems on influence games (measures, values, properties and problems for particular cases with respect to both the spread of influence and the structure of the graph). Moreover, we also present equivalent models to the simple games, the computation of satisfaction and power in collective decision-making models, and the definition of new centrality measures used for social network analysis. In addition, several interesting computational complexity results have been found.

Contents

1. Introduction

Game theory can be defined as the study of mathematical models of strategic interaction between rational decision-makers. The first known discussion of game theory occurred in a letter written by Charles Waldegrave, an active Jacobite, and uncle to James Waldegrave, a British diplomat, in 1713 [1,2]. However, modern game theory was developed extensively in the 1950s with the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games and its proof by John von Neumann. Von Neumann's original proof used the Brouwer fixed-point theorem on continuous mappings into compact convex sets, which became a standard method in game theory and mathematical economics. His paper, *Mathematical Foundations of Quantum Mechanics (1932)*, was followed by the 1944 book [3], co-written with Oskar Morgenstern, which considered cooperative games. A cooperative game is a mathematical structure formed by a set of

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players, *coalition*, that can achieve a common benefit, enforcing a cooperative behavior.

From the beginning, game theory has been growing interest among scientifics of a lot of disciplines. Today, it has applications in all fields as mathematics, computer science and social sciences. Game theory also applies to a wide range of behavioral relations, and is now an umbrella term for the science of decision-making models. This survey presents some advances, joint work with M. Serna, that goes from cooperative simple games to social networks.

In the last quarter of a century five related works with the analysis of the game theory has received the Nobel Memorial Prize in Economic Sciences. First, John F. Nash, Reinhard Selten and John Harsanyi were awarded for their pioneering analysis of equilibria in the theory of non-cooperative games (1994). Second, Thomas Schelling worked on dynamic models, early examples of evolutionary game theory. At the same time, Robert Aumann contributed to the equilibrium school. He introduced an equilibrium coarsening, correlated equilibrium, and developed an extensive formal analysis of the assumption of common knowledge and of its consequences. Both, T. Schelling and R. Aumann, won the award (2005). Leonid Hurwicz, together with Eric Maskin and Roger Myerson, were also honored for having laid the foundations of mechanism design theory (2007). Myerson's contributions included the notion of proper equilibrium, and an important graduate text about game theory and analysis of conflict. For its part, Hurwicz also introduced and formalized the concept of incentive compatibility. On the other hand, Alvin E. Roth and Lloyd S. Shapley introduced the theory of stable allocations and the practice of market design (2012). The fifth graceful was the game theorist Jean Tirole for his analysis of market power and regulation (2014).

All results presented here can be divided in four parts or sections. Section 2 introduces both some forms of representations for simple games [4] and the advances in simple games. Complexity results also appear in this section. Third section introduces a new viewpoint of simple games based on the influence spread phenomenon, the influence games. This section also characterizes different problems on these influence games (measures, values, properties and other related problems). Section 4 considers different models based on collective decision-making processes. It also studies the satisfaction and the power of an actor in these models, as well as particular models. Next section considers new centrality measures applied to social networks. Indeed, it provides two lines of inquiry. The first line dealts with power indices of simple games used as centrality measures. The second line introduces new centrality measures based on the influence spread on graphs.

Finally, some conclusions are supplied in this survey.

2. Simple games

Game theory arises in the first half of the 20th century from the need to study formally situations of conflict and cooperation between intelligent rational decision-makers [5]. It is closely related with other disciplines such as decision theory, voting theory, social choice theory, logic and threshold logic, circuit complexity, computational complexity theory, artificial intelligence, geometry, linear programming, Sperner theory, order theory, agent systems, social network analysis, etc. [6–8]

From the beginning, a relevant branch of game theory has been cooperative game theory [9]. A cooperative game is a mathematical structure formed by a set of players that by forming coalitions can achieve a common benefit, enforcing a cooperative behavior [3,10–14]. A well known subclass of cooperative games is the class of simples games [3], also called simple coalitional games, in which the benefit that a coalition may have is always binary, i.e., a coalition may be winning or losing, depending on whether the players in the coalition are able to benefit themselves from the game by achieving together some goal.

The preface of [6] starts saying "*Few structures in mathematics arise in more contexts and lend themselves to more diverse interpretations than do hypergraphs or simple games*". In fact, simple games are closely related with other mathematical and computational structures, such as self-dual hypergraphs, Sperner families, antichains, monotone Boolean functions, free distributive lattices, monotone collective decision making systems and multi-agent systems, among others [6–8]. This motivated us, joint with M. Serna, to study different forms of representation for simple games, and those for some of their subfamilies like regular games and weighted games. In the same vein, we also provided bounds on the computational resources needed to transform a game from one form of representation to another one. We finally presented the problem of enumerating the fundamental families of coalitions of a simple game.

In particular, we studied several forms of representation for simple games [4] based on the set of coalitions, binary trees, partially condensed binary trees, and binary decision diagrams. Moreover, regular games (simple games where a desirability relation among players is considered) [15–17] were also studied under the form of representation based on the so-called set of shift-minimal winning coalitions, and in fully condensed binary trees. Another important subclass of simple games is the subclass of weighted (simple) games [18,19] where each player has assigned a weight (e.g. a rational or a natural number), and a coalition is winning if and only if the sum of the weights of players of this coalition is more or equal than a fixed real number called the *quota* of the game. Thus, a weighted game can be represented by a very compact form of representation, which is just a vector that contains the quota of the game, followed by the weights of each player.

Despite the fact that weighted games are a strict subclass of simple games, it is known that every simple game can be expressed as an intersection or a union of a finite number of weighted games. The result for intersection (*dimension* concept) was first shown for hypergraphs [20], and then expressed for simple games [6,21]. After that, the result for union (*codimension* concept) was also introduced for simple games [22]. A simple game is said to be of *dimension* (*codimension*) k if and only if it can be represented as the intersection (union) of exactly k weighted games, but not as the intersection (union) of $(k-1)$ weighted games. It is known that given k weighted games, to decide whether the dimension of their intersection exactly equals k is NP-hard [23]. In this vein, some complexity results on simple games appear in [24]. In particular, this paper analyzes the complexity of changing the representation form of a simple game, the complexity of some properties on simple games (strongness, properness, weightedness, homogeneousness, decisiveness and majorityness) and natural succinct representations of simple games by means of Boolean formulas.

A generalization of games constructed through binary operators is the family of *boolean weighted* games introduced in [25]. A boolean weighted game is defined by a propositional logic formula and a finite collection of weighted games. The boolean formula determines the requirements for a coalition to be winning in the described game. When considering only monotone formulas, boolean weighted games provide another representation of simple games. Thus, a simple game can be represented in *vector-weighted representation form* by a set of weighted games.

On the other hand, we have also presented an extended concept of dimensionality of simple games in [26].

3. Influence games

One of the main results obtained together with M. Serna has been to establish a relationship between the influence spread phenomenon coming from social network analysis and the binary decision-making in voting systems [27]. This can be done through the definition of influence games, a new form of representation of simple games based on the so-called *influence graph* and an influence spread process through that graph.

An *influence graph* is a tuple (G, w, f), where $G = (V, E)$ is a weighted, labeled and directed graph (without loops). As usual V is the set of vertices or agents, E is the set of edges and $w : E \to \mathbb{N}$ is a *weight function*. Finally, $f : V \to \mathbb{N}$ is a labeling function that quantifies how influenceable each agent is. An agent $i \in V$ has *influence* over another agent $j \in V$ if and only if $(i, j) \in E$. It also includes the family of *unweighted influence graphs* (G, f) in which every edge has weight 1.

Given an influence graph (G, w, f) and an initial activation set $X \subseteq V$, the *spread of influence* of X is the set $F(X) \subseteq V$ which is formed by the agents activated through an iterative process. Originally, this influence spread process was determined by the *linear threshold model* [28,29] as follows. We denote $F_k(X)$ the set of nodes activated at step k. Initially, at step 0, only the vertices in X are activated, that is $F_0(X) = X$. The set of vertices activated at step $i > 0$ consists of all vertices for which the total weight of the edges connecting them to nodes in $F_{i-1}(X)$ meets or exceeds their labels, i.e.,

$$F_i(X) = F_{i-1}(X) \cup \left\{ v \in V \mid \sum_{\substack{u \in F_{i-1}(X) \\ (u,v) \in E}} w((u, v)) \geq f(v) \right\}.$$

The process stops when no additional activation occurs. The final set of activated nodes is denoted by $F(X)$.

Fig. 1 shows the spread of influence $F(X)$ in an unweighted influence graph (G, f) for the initial activation $X = \{2, 3\}$. In the first step we obtain $F_1(X) = \{2, 3, 6, 7\}$, and in the second step (the last one) we obtain $F(X) = F_2(x) = \{2, 3, 4, 6, 7\}$.

An *influence game* is given by a tuple (G, w, f, q, N) where (G, w, f) is an influence graph, q is an integer *quota*, $0 \leq q \leq |V|+1$, and $N \subseteq V$ is the *set of players* that can belong to the initial activation. $X \subseteq N$ is a *successful team* if and only if $|F(X)| \geq q$, otherwise X is an *unsuccessful team*. It is also possible to consider the family of *unweighted influence games* for the cases in which the corresponding influence graph is unweighted. In such a case, the influence game can be denoted by (G, f, q, N).

Consider the influence graph described in Fig. 1. It is easy to check that the influence game $(G, f, 4, \{1, 2, 3\})$ is equivalent to the simple game with players 1, 2 and 3, and with just two winning coalitions, $\{2, 3\}$ and $\{1, 2, 3\}$. In fact, note that the influence spread process is monotone, in the same way than simple games, i.e., each superset of a winning coalition is also a winning coalition, and each subset of a losing coalition is also a losing coalition. This allowed us to prove that every influence game has an associated simple game, and that every simple game can be represented by an influence game [27]. Moreover, every simple game represented by its (minimal) winning coalitions can be converted in an unweighted influence game in polynomial time. Furthermore, given any weighted game represented by its weighted representation form, then it can be converted in an influence game in polynomial time, and in an unweighted influence game in pseudo-polynomial time [27].

We have also characterized the computational complexity of various problems on influence games. On the one hand, we studied measures like the *length* (the minimum size of a successful team) and the *width* (the maximum size of a unsuccessful team); values like *Shapley–Shubik* and *Banzhaf*; and properties of games (*proper*, *strong*, *decisive*), teams (*blocking*, *swing*) and players (*dummy*, *passer*, *vetoer*, *dictator*, *critical*, *symmetric*, *blocking*, *critical*, *symmetric*). We also studied when two influence games are isomorphic or equivalent. On the other hand, we also analyzed those problems for some particular extremal cases, showing tighter complexity characterizations and even some polynomial-time algorithms. The extremal cases include, for instance, some restrictions over the label function (e.g., maximum and minimum influence) and some restrictions over the influence graph topology [30].

4. Collective decision-making models

Decision theory is the study of the decision-making processes carried out from the agents' choices. It is an interdisciplinary topic closely related to the field of game theory [31], with applications in economy, mathematics, cognitive science, political science, social sciences, computer science, among others [32]. Meanwhile, social choice theory blends elements of welfare economics and voting theory to focus on those cases in which agents are social individuals or actors capable of reaching agreements and making decisions collectively [33].

A collective decision-making model (in what follows we will simply say "decision model") considers the initial individual choices of the actors. Then, the model includes a procedure (usually based on social influence) through which these initial choices can change, generating the final decisions of the actors. Finally, the global decision of the system is determined through a collective decision function that applies some voting rule (e.g. simple majority) to the final decisions of the actor.

We noted that a decision model can be associated with an influence game [34]. Given an influence game, an initial activation can represent the set of actors whose initial choices coincide. Then, through the influence spread process we obtain the final decision of the actors. Finally, the quota determines if the coalition is winning or losing, which is the global decision of the system.

A simple but expressive class of decision models is the one of opinion leader–follower (OLF) models [35]. These models are based on a two-step decision process [36,37]. Here the actors are classified into opinion leaders (L), followers (F), and independent actors (I). The only accepted influence relationships range from opinion leaders to followers, forming a bipartite digraph. The initial decision of the opinion leaders and the independent actors never change, but the final decision of each follower may change if the opinion leaders pointing to it choose differently from she/he. Finally, the global decision is obtained by simple majority rule.

For the same graph structure of the OLF models, we defined a *generalized opinion leader–follower* (*gOLF*) represented by an influence game (G, f, q, N) where $N = \mathtt{L} \cup \mathtt{I}$ and the labeling function f is defined as

$$f(i) = \begin{cases} \lceil r \cdot \delta^-(i) \rceil & \text{if } i \in \mathtt{F} \\ 1 & \text{if } i \in \mathtt{L} \cup \mathtt{I} \end{cases}$$

where $\delta^-(i)$ is the in-degree of i (i.e., the number of opinion leaders pointing to the follower i) and $r \in [0, 1] \in \mathbb{Q}$ is the rational number of active opinion leaders necessary to active (i.e., change the opinion of) the followers. In an OLF model, $n = |V(G)|$ must be odd, $r \in [\frac{1}{2}, 1]$ and $q = \frac{n+1}{2}$ [35].

Although the gOLF models are more expressive than OLFs, their topological structure is still very restrictive. Therefore, we defined a much more general decision model. A *non-oblivious influence model* is like a gOLF model but applied on any influence graph. Here opinion leaders becomes actors who can influence

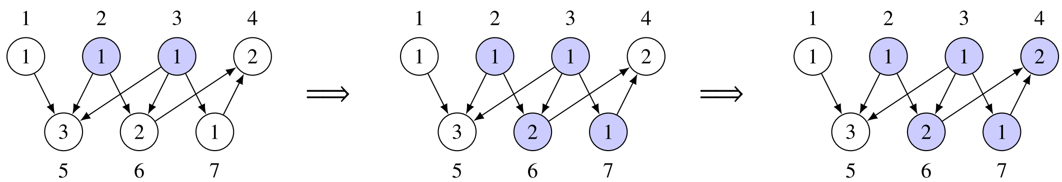

Fig. 1. Spread of influence on an influence graph, represented by coloring of nodes from the initial activation $X = \{2, 3\}$.

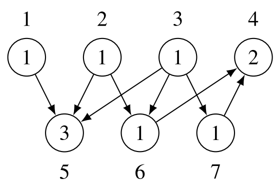

Fig. 2. Influence graph.

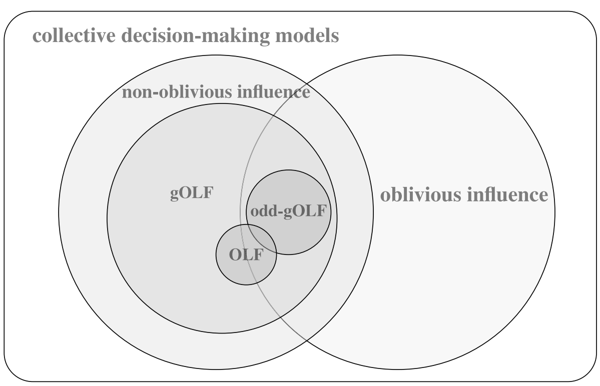

Fig. 3. Inclusion relationship between subfamilies of collective decision-making models.

others and are not influenced by anyone, while followers are actors who do not influence anyone and are influenced by others. As in the gOLF model, here the initial decision of the followers is taken into consideration when a tie arises in the global decision. Analogously, an *oblivious influence model* is like a non-oblivious influence model but where the initial decision of the followers is not taken into account, so that it is replaced by a negative initial decision.

In general, from an initial decision vector, oblivious and non-oblivious models give us different final decision vectors. Let $(G, f, q = 5, N = \{1, 2, 3\})$ be an influence game, where (G, f) is the influence graph described by Fig. 2. Consider the initial decision vector $x = (1, 0, 1, 0, 0, 0, 0)$. For the oblivious case, we have that $|F(\{1, 3\} \cap N)| = |\{1, 3, 4, 6, 7\}| \geq q$. Thus, the final decision is 1. For the non-oblivious case, we have the leaders (players 1, 2 and 3) have the final decision as the initial decision. On the other hand, players 4 and 7 will keep the final decision equal to 1, but players 5 and 6 will keep it equal to 0. So, the final decision vector $x = (1, 0, 1, 1, 0, 0, 1)$ verifies $|x| < q$, i.e., the final decision for the non-oblivious case is 0.

We know that the oblivious and non-oblivious models coincide when there are no tiebreak problems. Therefore, not all OLF and gOLF models are oblivious. However, if we restrict us to gOLF models with $r = \frac{1}{2}$ and an odd number of actors pointing to every follower, then we obtain a subfamily (the so called *odd-gOLF* models) of models that are both oblivious and non-oblivious [34]. A diagram of all the decision models seen so far is shown in Fig. 3.

Given a decision model, it is relevant to study the effects that collective decision-making can have on the actors. The *satisfaction* of an actor is the number of initial decisions for which the final collective decision coincides with the initial decision of the actor. Furthermore, the *power* of an actor is the number of initial decision for which the collective decision changes when the actor changes its initial decision [35]. We found that computing the satisfaction is equivalent to computing the Rae index [34], which is a classical power index applied to anonymous games [38] and simple games [39]. Moreover, we also proved that the power of an actor is equal to the double of its Banzhaf value, and that the satisfaction and power are closely related, in such a way that computing both measures has the same computational complexity for all the decision models considered above [40].

We proved that computing the satisfaction and the power for odd-gOLF models is #P-hard, and hence computing both measures is #P-hard for both oblivious and non-oblivious influence models [34,41]. Moreover, their computation is #P-hard for (oblivious and/or non-oblivious) *unanimous majority influence models*, which are decision models with symmetrical digraphs (i.e., undirected graphs), labels $f(i) = \delta^-(i)$ for the followers (i.e., to change the initial choice of each follower i, unanimity between all the actors pointing to i is required) and simple majority rule for the global decision.

Although computing satisfaction and power is hard even for very restricted influence models, we have defined subfamilies for which the computation becomes polynomial. That is the case of (oblivious and non-oblivious) *strong hierarchical influence models* and *star influence models*. The topological structure of both models allows to represent organizational situations in which besides independent actors, followers and opinion leaders, an additional type of actors, namely the *mediators*, can also be recognized to intermediate between opinion leaders and followers [34,41].

5. Centrality in social network analysis

The collective decision-making processes seen in Section 4 are usually used to study networks with a relatively small number of actors. However, the influence spread phenomenon of influence games can also be used to understand the dynamics of large social networks. The influence spread on social networks is a well known phenomenon that has been applied in viral marketing, information propagation, search strategies, expertise recommendation, community systems, management, percolation theory, among others [42]. Therefore, it is quite natural to study social networks as influence graphs, where an influence spread model is explicitly used.

In social network analysis, we have focused on studying the centrality on social networks, one of the most studied problems in the discipline. A centrality measure allows to rank the different actors of a network according to their relevance (i.e., influence, popularity, activity, etc.), assigning each one a numerical value dependent on several quantifiable factors. The most known centrality measures were formally defined in the 1970s [43], although the problem comes at least since the 1940s [44]. These classical measures have to do, for each actor, with their number of neighbors (degree), the sum of their shortest paths to all other actors in the network (closeness), and the number of existing paths that pass through the actor (betweenness). Many centrality measures are based on the PageRank, which is based in turn on the eigenvector centrality [45]. Nowadays, there are hundreds of different centrality measures, that differ in their computing techniques as well as in their application context [46].

Our first contribution in the area was to apply the classic power indices of simple game theory as centrality measures for social networks [47]. The underlying idea is simple. Briefly speaking, the power indices are measures used to rank the different players of a game according to their ability to form winning coalitions. Hence, power indices can be used as centrality measures for simple games represented as graphs. Although this idea is not new [48], our model of influence games (which is as expressive as the whole family of simple games [27]) allowed us to apply power indices on any social network represented as an influence graph with an associated quota. Given the large number of existing power indices (Banzhaf index, Shapley–Shubik index, Deegan–Packel index, Holler index, Coleman indices, Johnston index, Chow parameters, among others [49]), this result considerably increases the variety of existing centrality measures.

Power indices have continued to be used as centrality measures [50]. In addition, we also proposed as centrality measure the satisfaction measure mentioned in Section 4 [47]. Of course, the power measure also works. In general, any measure that behaves like a function $f : V \to \mathbb{Q}$, with V a set of players or actors, can be applied as a centrality measure. However, as we have already said, there are hundreds of centrality measures, so the problem is not to define new measures, but to propose useful and functional measures that provide relevant information for different case studies. Thus, let (G, w, f, q, N) be an influence game (for simplicity we can assume $N = V$), we proposed the *effort centrality* $C_E(i) = (f(V) - \texttt{Effort}(i))/f(V)$ as a centrality measure created specifically for influence games, where $\texttt{Effort}(i) = \min\{f(X) \mid |F(X \cup \{i\})| \geq q\}$ is the (minimum) effort required by the network to choose a winning coalition that contains actor i [47].

All the previous centrality measures (power indices, satisfaction, power and effort) are useful and descriptive for influence games with a small number of nodes. However, they are not useful for social networks with many actors. Indeed, we have already shown that the computation of power indices, satisfaction and power in influence games is #P-hard [27,34,41]. Besides, to compute the effort centrality it is necessary to make exhaustive searches between the $2^{|V|}$ possible coalitions. On the contrary, all the classic centrality measures are efficient, so they can be used on social networks with thousands of actors.

The above brings us to our second contribution in this area, namely the definition of efficient centrality measures for social networks represented as influence graphs [42]. The *Linear Threshold Rank* (*LTR*) of an actor i is defined as $\mathrm{LTR}(i) = |F(\{i\} \cup \mathrm{neighbors}(i))|/n$, where $\mathrm{neighbors}(i)$ are all the actors connected with i by some edge, and F is the spread of influence process following the linear threshold model. This measure can be computed in polynomial time, since F can be computed in polynomial time [27]. We do not use other influence spread models like the independent cascade model, because they depend on randomness and therefore each execution can lead to different results. The LTR measure represents how much an actor can spread his influence within the network, investing resources outside the formal network to be able to convince his immediate neighbors.

This new centrality measure was applied in both directed and undirected social networks of hundreds of thousands of nodes, and compared with other centrality measures based on influence criteria, such as the Katz centrality, the PageRank, and the Independent Cascade Rank (ICR). We proved that the correlation between these measures is low, so that their criteria are different, and thus they can be used in a complementary way to obtain different results. Moreover, the actor rankings provided by the LTR and the ICR measures presented the highest standard deviations, and of both, the first one returned the larger number of different values. Therefore, the LTR measure was shown to be capable for ranking actors in a more distinguishable way [42]. Furthermore, a generalization of LTR measure was proposed, such that the neighborhood level is changed by $\mathrm{neighbors}(i, \ell) = \{j \in V \mid \mathrm{dist}(i, j) \leq \ell\}$, where $\mathrm{dist}(i, j)$ denotes the length of the shortest undirected path between nodes i and j. It was also shown that using neighbors at distance larger than 2, the rankings distinguish better the influential actors [51].

Finally, our third contribution in the area has been the definition of a centralization measure for social networks represented as influence graphs. Unlike centrality measures, the centralization measures aim to determine to what extent an entire network has a centralized structure. A well known example of centralization measure is the *Average Clustering Coefficient* (*ACC*) [52], which is the average of the local clustering coefficients of all the nodes in a network. Analogously to the LTR measure, we defined the *Linear Threshold Centralization (LTC)* of a network G as $\mathrm{LTC}(G) = |F(C(G))|/n$ where $C(G)$ contains all the actors that belong to the main core of the network. The idea behind this measure is the following. As the actors outside the k-shell have a degree smaller than the actors inside of it, then the first ones are more able to be influenced by the second ones.

Like the LTR measure, the LTC measure can also be applied to social networks with hundreds of thousands of actors. This centralization measure was compared to the ACC measure, concluding that both measures can be computed in polynomial time and provide different information about the network.

Conclusions

There are several ways to represent simple games. Each form of representation opens new possibilities to understand the properties of games from the context of cooperative game theory, but also the possibility of applying these games in other contexts. The definition of influence games has opened a variety of possible applications, e.g., in the study of collective decision-making models, as well as in social network analysis. However, the foregoing has not remained there. Recently, the influence graphs have even been applied to multimodal learning analytics, specifically, to model collaborative work teams on which cooperation, activity and ability to influence are measured [53].

With M. Serna we have worked together for a decade. Since then we have been able to show several interesting results, although each result usually opens new questions and problems. The open problems are many. Just to mention a few, regarding computational complexity, it remains open to find the complexity of the satisfaction and power for OLF models. Besides, it would be interesting to study influence games under other influence spread models, such as the independent cascade model. This could lead us to other applications more related to complex systems with random phenomena, studies for example in neuroscience. Finally, the LTR and LTC measures can be studied more thoroughly, in order to better understand the importance of the spread of influence on the variation of the centrality of the actors in dynamic social networks. Regarding the latter, currently we are working on a generalized LTR measure that explores the sensitivity of the original LTR, with respect to the distance of the neighbors included in the initial activation set.

Regarding the latter, we conjecture that using neighborhoods at a greater distance (that is, including not only immediate neighbors, but also actors achievable through paths of a specific length) can generate more distinguishable rankings, that is, with greater different results and a greater standard deviation. However, if the distance of the neighborhoods is very high, then on the contrary, there is a risk that the initial activations already contain all the actors of the network, which loses interest. Thus, the initial distance of the neighborhood is a critical parameter to measure the centrality of an actor under the influence spread phenomenon.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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