Simulation of Autopilot

by

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This project, presented by **Syed Siraj Ul Haque** under the direction of their project advisor and approved by the project examination committee, has been presented to and accepted by the Hamdard Institute of Information Technology, in partial fulfillment of the requirements for the degree of **BE – IT.**

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This Report is dedicated to

Shaheed Hakim Muhammad Saeed,

ACKNOWLEDGEMENT

Acknowledgement is due to Hamdard Institute of Information Technology for support of this Project. I wish to express my appreciation to Professor Tarique Ahmed who served as my major advisor. I also wish to thank the other members of my Evaluation Committee _______ and __________.

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Abstraction

The goal of this project is to present an integrated treatment of basic elements of aircraft stability, flight control and autopilot. An understanding of flight stability and control played an important role in the ultimate success of the earliest aircraft designs. In later years the design of automatic control ushered in the rapid development of commercial and military aircraft. Today both military and civilian aircraft rely heavily on automatic control systems to provide artificial stabilization and autopilot to aid pilots in navigating and landing their aircraft in adverse weather condition.

This project is small part of real time flight simulator it deals with control surfaces of airplane and their automatic control. It includes simulation of controllers like pitch, roll heading altitude and velocity controllers. These controllers are implemented using both classical control theory (standalone controllers) and modern control theory (in stability augmentation system). For implementing classical control theory based transfer function and state vectors for stability augmentation system based on modern control theory derivatives of stability coefficients are required. User of this software can feed directly derivatives of stability coefficients or user can also feed raw data so that derivatives of stability coefficients can derived. And yes for some components of controllers like rate gyro and vertical gyro transfer function are feed directly. For test purpose we have feed raw data for STOL and derivatives of stability coefficients for NAVION and F104A airplane.

This report is divided into four parts first deals with stability (both static and dynamic) and control surfaces and their contribution in aircraft dynamic. Second part deals with development of aircraft equations of motion, aerodynamic modeling of aircraft, quality of flight and motion due to control input. Third part is devoted to aircraft autopilot. Autopilot is designed both with classical and modern control theory. The fourth part is of

source code and here we have included the code of our project. The code for this project is written in MATLAB, rightly called the language of technical computing.

This project named as “Simulation of Autopilot” is presented here as our final year project, when we selected this project we never knew that it would be so difficult be to work on a project whose field was totally new for us and we had no prior knowledge of the subject. We had to start work on this project from the basic level, starting with the basic understanding of aircraft and flight controls and then stability we had to study a lot of material to build our concepts about the topic from zero level to quite an advance level. As much as we moved forward and progressed, the subject became more and more interesting and fascinating. Now when we have finished this project and look at its final form we feel very much satisfied on what we had achieved and feel proud by the encouragement and appreciation of our project advisor Mr. Tarique Ahmed, who helped us throughout the project, he was always kind in explaining the problems and he always maintained good and cordial relations with us and without his support we might not have been able to complete this project.

CHAPTER 1

INTRODUCTION

1.1 Aircraft Stability and Control

How well an airplane flies and how easily it can be controlled are subjects studied in aircraft stability and control. In the study of airplane stability and control, we are interested in what makes an airplane stable, how to design the control systems, and what conditions are necessary for good handling.

1.2 Aerodynamic Nomenclature

To describe the motion of an airplane it is necessary to define a suitable coordinate system for the formulation of the equations of motion. For most problems dealing with aircraft motion, two coordinate systems are used. One coordinate system is fixed to the Earth and may be considered for the purpose of aircraft motion analysis to be an inertial coordinate system. The other coordinate system is fixed to the airplane and is referred to as a body coordinate system. Figure 1.1 shows the two right-handed coordinate systems.

The forces acting on an airplane in flight consist of aerodynamic, thrust, and gravitational forces. These forces can be resolved along an axis system fixed to the airplane's center of gravity, as illustrated in Figure 1.2. The force components are denoted X, Y, and Z; T_x, T_y *and* T_z; and W_x, W_y, and W_z, for the aerodynamic, thrust, and gravitational force components along the x, y, and z axes, respectively.

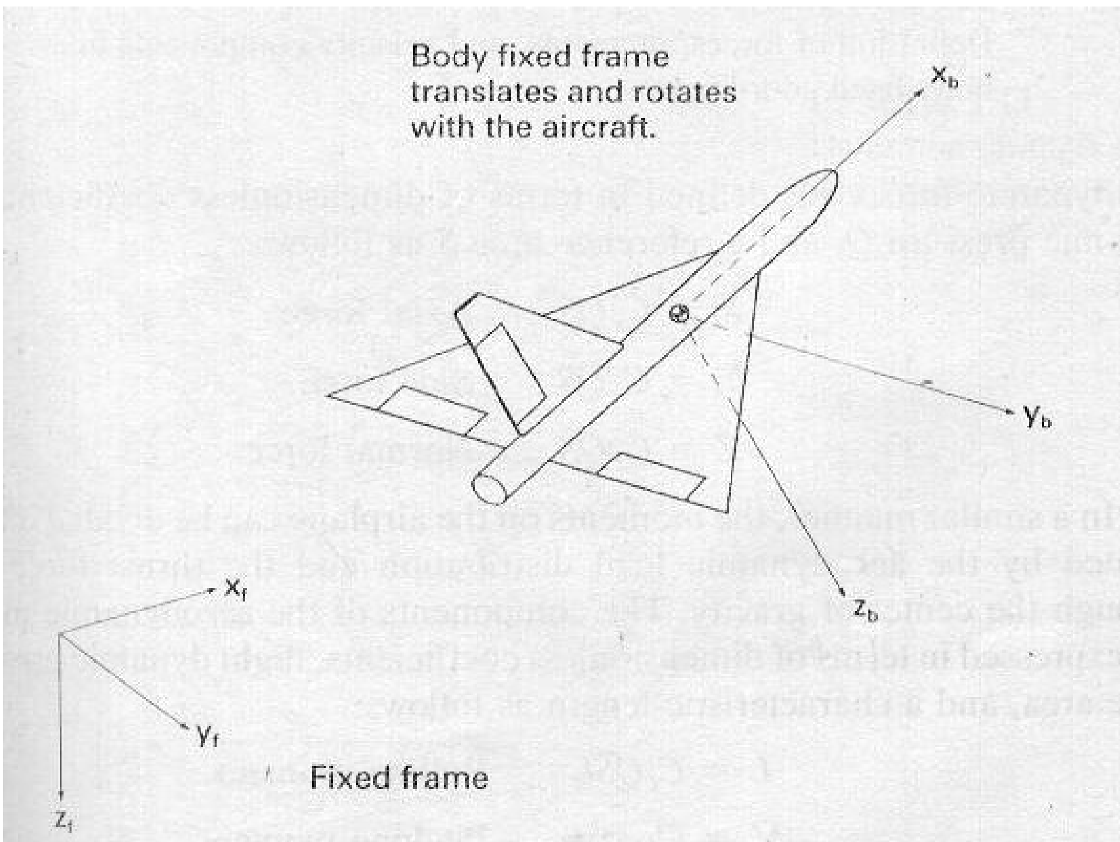


Figure 1.1 Basic axis coordinate system

The aerodynamic forces are defined in terms of dimensionless coefficients, the flight dynamic pressure Q, and a reference area S as follows:

$$X = C_x QS \qquad \text{Axial force} \tag{1.1}$$

$$Y = C_y QS \qquad \text{Side force} \tag{1.2}$$

$$Z = C_z QS \qquad \text{Normal force} \tag{1.3}$$

In a similar manner, the moments on the airplane can be divided into moments created by the aerodynamic load distribution and the thrust force not acting through the center of gravity. The components of the aerodynamic moment also are expressed in terms of dimensionless coefficients, flight dynamic pressure, reference area, and a characteristic length as follows:

$$L = C_l QSl \qquad \text{Rolling moment} \tag{1.4}$$

$$M = C_m QSl \qquad \text{Pitching moment} \tag{1.5}$$

$$N = C_n QSl \qquad \text{Yawing moment} \tag{1.6}$$

For airplanes, the reference area S is taken as the wing platform area and the characteristic length l is taken as the wingspan for the rolling and yawing moment and the mean chord for the pitching moment. For rockets and missiles, the reference area is usually taken as the maximum cross-sectional area, and the characteristic length is taken as the maximum diameter.

The aerodynamic coefficients C_x , C_y, C_z , C_l , C_m , and C_n primarily are a function of the Mach number, Reynolds number, angle of attack, and sideslip angle; they are secondary functions of the time rate of change of angle of attack and sideslip, and the angular velocity of the airplane.

The aerodynamic force and moment acting on the airplane and its angular and translational velocity are illustrated in Figure 1.2. The x and z axes are in the plane of symmetry, with the x axis pointing along the fuselage and the positive v axis along the right wing. The resultant force and moment, as well as the airplane's velocity, can be resolved along these axes.

The angle of attack and sideslip can be defined in terms of the velocity components as illustrated in Figure 1.3. The equations for α and β follow:

$$\alpha = \tan^{-1}\frac{w}{u} \qquad 1.7$$

$$\beta = \sin^{-1}\frac{v}{V} \qquad 1.8$$

Where

$$V = \left(u^2 + v^2 + w^2\right)^{1/2} \qquad 1.9$$

If the angle of attack and sideslip are small, that is, < 15° then

$$\alpha = \frac{w}{u} \qquad 1.10$$

$$\beta = \frac{v}{u} \qquad 1.11$$

Where α and β are in radians.

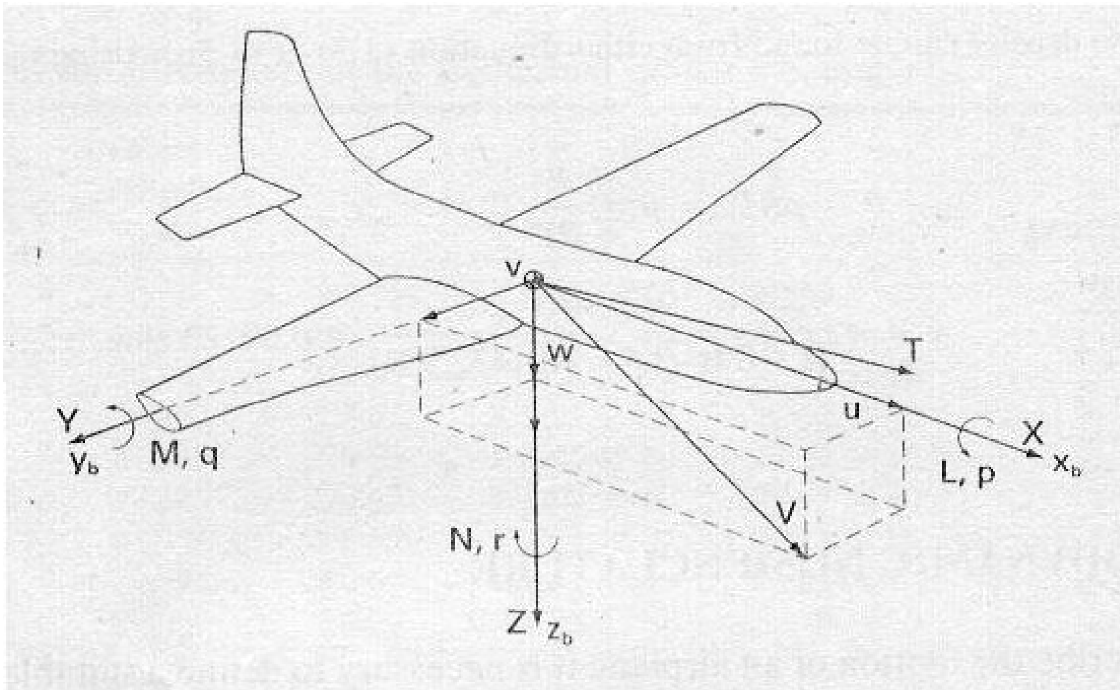


Figure 1.2 Forces, moments and velocity components in a body fixed coordinate

Table 1.1 Definition of forces, moments and velocity components in a body fixed coordinate

	Roll Axis x_b	Pitch Axis y_b	Yaw Axis z_b
Angular rates	p	Q	R
Velocity components	u	V	W
Aerodynamic forces components	X	Y	Z
Aerodynamic moment components	L	M	N
Moment of inertia about each axis	I_x	I_y	I_z
Product of inertia	I_{yz}	I_{xz}	I_{xy}

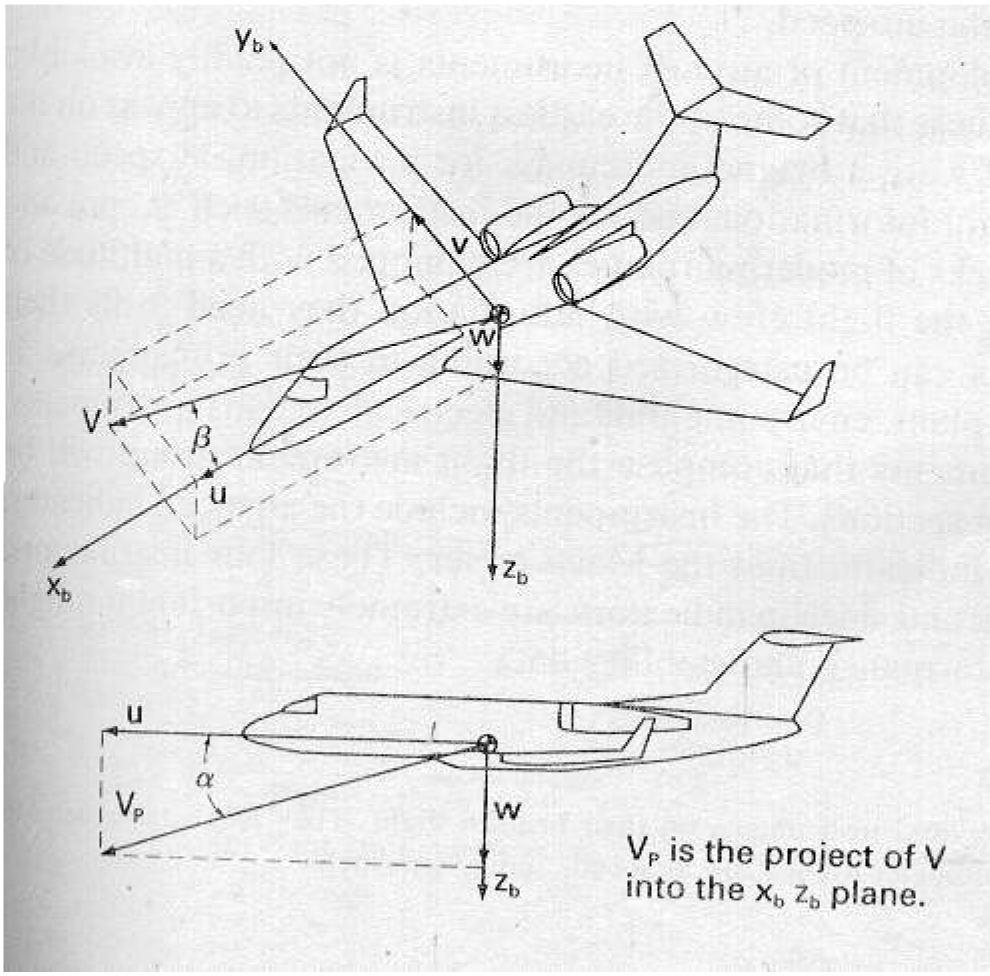


Figure 1.3 Definition of angle of attack and sideslip

CHAPTER 2

FORCES OF FLIGHT

The first requirement of flight is to overcome gravity (commonly referred to by fighter pilots as "God's G"). To overcome gravity (or the WEIGHT of the aircraft), we need to produce LIFT. Lift will provide the force necessary to get the aircraft off the ground. To produce the necessary lift, speed is required. THRUST from the engine will produce the required velocity. The forward velocity from thrust will produce DRAG. In other words following are the forces, which acts on the airplane. See Figure 2.1

- Weight
- Thrust
- Lift
- Drag

2.1 Weight (Gravitational force)

Weight is simply the force of gravity acting on the aircraft. Regardless of the aircraft's attitude, weight always acts toward the center of the earth. When an aircraft is flying straight and level, weight opposes lift. If the aircraft's wing is not producing enough lift to overcome gravity, the aircraft will be forced to descend. If the amount of lift exceeds the force of gravity, the aircraft will be able to climb.

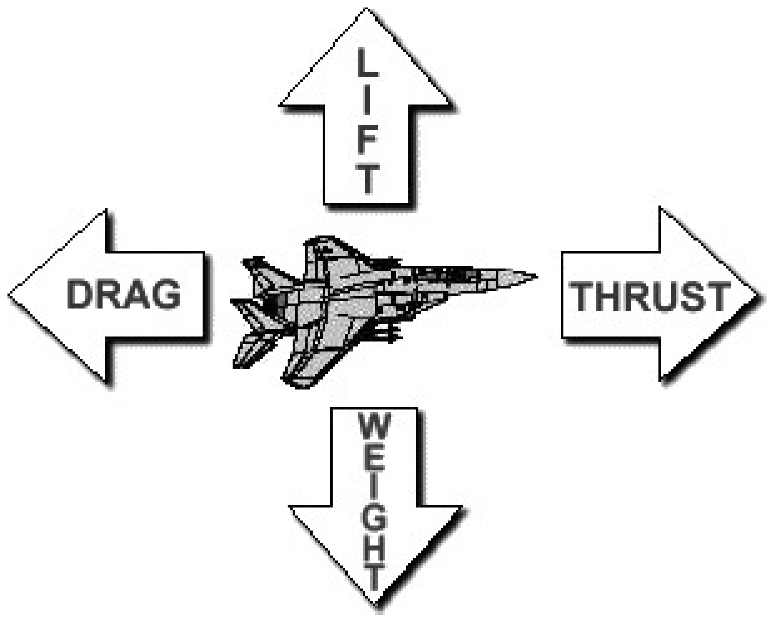


Figure 2.1 Forces acting on aircraft

The gravitational force acting on the airplane acts through the center of gravity of the airplane. Because the body axis system is fixed to the center of gravity, the gravitational force will not produce any moments. It will contribute to the external force acting on the airplane, however, and have components along the respective body axes. Figure 2.2 shows that the gravitational force components acting along the body axis are a function of the airplane's orientation in space. The gravitational force components along the *x, y,* and z-axes can be easily shown to be

$$(F_x)_{gravity} = -mg\sin\theta \tag{2.1}$$

$$(F_y)_{gravity} = mg\cos\theta\sin\Phi \tag{2.2}$$

$$(F_z)_{gravity} = mg\cos\theta\cos\Phi \tag{2.3}$$

2.2 Lift

Lift opposes weight and will be perpendicular (90 degrees) to the relative wind. See Figure 2.3. Lift is represented by the following formula:

Lift=½ (air density) x (velocity squared) x (coefficient of lift) x (wing surface)

Most aircraft cannot alter the wing surface area. Thus, to increase lift the pilot can

1) Fly at lower altitudes (denser air).
2) Increase the True Airspeed
3) Increase the coefficient of lift (the angle of attack) up to the angle for maximum lift.

2.2.1 The Airfoil

An airfoil is any shape designed to produce lift. A conventional airfoil will consist of the leading edge, trailing edge, camber, and chord.

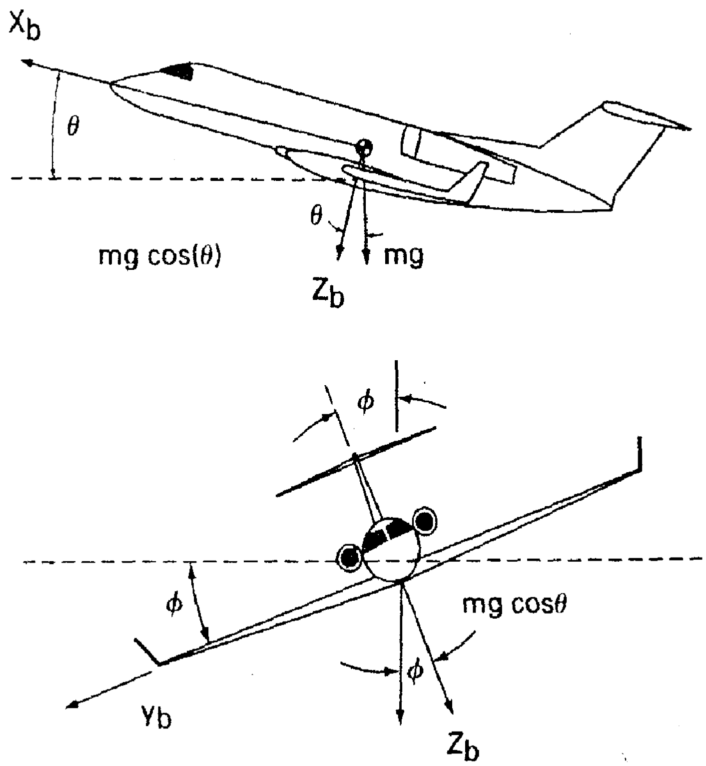


Figure 2.2 Components of gravitation forces acting along the body axis

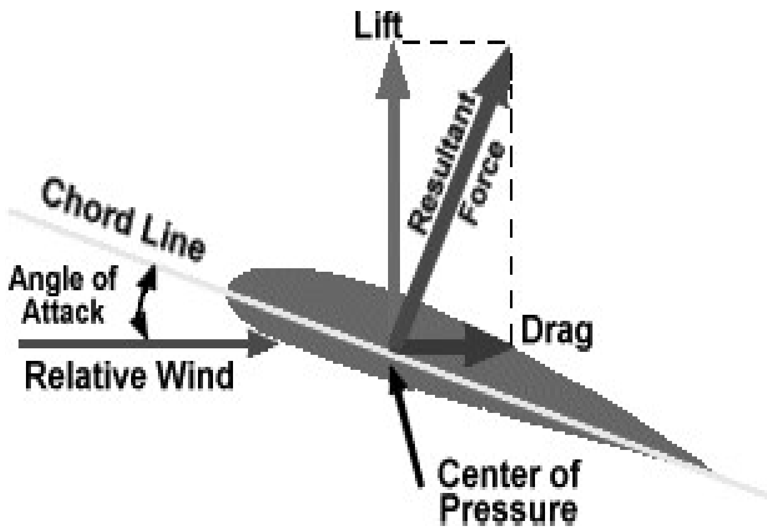


Figure 2.3 Lift

2.2.2 Leading and Trailing Edges

The leading edge is the front of the wing. It will be the first art of the wing to come into contact with the oncoming air. The trailing edge is where the airflow from the top of the wing meets the air flowing below the wing.

2.2.3 Camber

Camber will be divided into the lower and upper sections. The upper camber will be curved more than the lower camber - this will cause the air across the top of the wing to move faster than the air on the bottom of the wing (i.e. LIFT!). Older wing designs (the brilliant DC-3 for example) had very thick wings with a very large camber. By todays standard, the DC-3 wing is inefficient; however, using low-tech deicing boots it could travel through poor weather conditions with ease. Many modern and very efficient wings (like the Air Force's supersonic trainer, the T-38 Talon) are so efficient that even the smallest amount of ice (which changes the shape of the airfoil and can result in a wing that cannot produce lift) can become a major hazard.

2.2.4 Chord

The chord is an imaginary line from the leading edge to the trailing edge. It is used to determine the angle of attack.

2.3 Thrust

Thrust is one of the four forces acting on an aircraft. Thrust (measured in pounds or Newton) provides the velocity required for an aircraft's wings to produce lift. Thrust is the forward force and lift is the force acting in the upward direction.

The thrust force due to the propulsion system can have components that act along, each of the body axis directions. In addition, the propulsive forces also can create moments if the thrust does not act through the center of gravity. The propulsive forces and moments acting along the body axis system are denoted as follows

Figure 2.4 Plane Thrust

$$(F_x)_{propulsive} = X_T \quad (F_y)_{propulsive} = Y_T \quad (F_z)_{propulsive} = Z_T \qquad 2.4$$

$$(L)_{propulsive} = L_T \quad (M)_{propulsive} = M_T \quad (N)_{propulsive} = N_T \qquad 2.5$$

2.4 Drag

Like all objects moving through the atmosphere, aircraft create drag. For aircraft flying at high airspeeds, drag is a surprisingly complicated subject. For example, the total drag of an aircraft is the sum of three types of drag - parasite, induced, and wave.

2.4.1 Induced Drag

This is the drag that is a result of the wing producing lift. Keeping the Bernoulli Principle in view we know that how the air traveling over the top of the wing is forced to travel a greater distance than the air moving across the bottom of the wing. The higher-pressure air on the bottom will attempt to flow to the top of the wing to equalize the low pressure (the wind blowing across the earth is due to this attempt to equalize the pressure in the air mass). The high-pressure air will succeed along the wing tips and the trailing edge of the wing. This results in a "downwash" effect that produces a drag that must be overcome by the engines. This effect is most noticeable at high angles of attack and low airspeeds.

2.4.2 Parasite Drag

Parasite drag is what most people think of when considering drag. When riding a bike, lowering your body to a horizontal position reduces the amount of resistance by reducing the amount of air being disturbed by you and the bike. A simple concept, however, parasite drag actually consists of interference drag and profile drag (which can be further divided into skin friction drag and pressure drag). When the F-15E carries external fuel tanks the range of the aircraft is extended considerable. However, the increased parasitic drag results in higher fuel flows and lower top speeds (due to the increased drag).

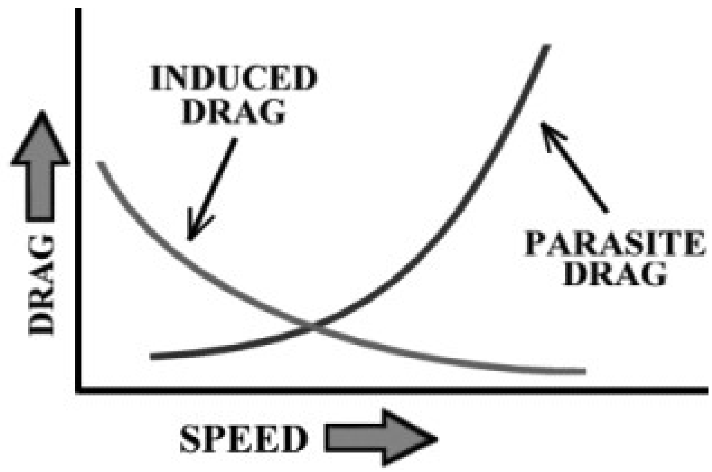


Figure 2.5 Drag effect

2.4.3 Wave Drag

Until aircraft were capable of speeds near the speed of sound, wave drag was not a concern. Aircraft in flight cause pressure disturbances to move away from the aircraft at the speed of sound. When the aircraft approaches the speed of sound the pressure disturbance (or "wave") is caught by the aircraft (which is still producing the pressure disturbance) and the disturbances stack up to produce a wall of pressure that creates the wave drag effect. The wave drag effect caused early fighters to experience violent shaking and even loss of control (thus the belief of the "sound barrier"). Once an aircraft reaches supersonic flight, wave drag is reduced significantly.

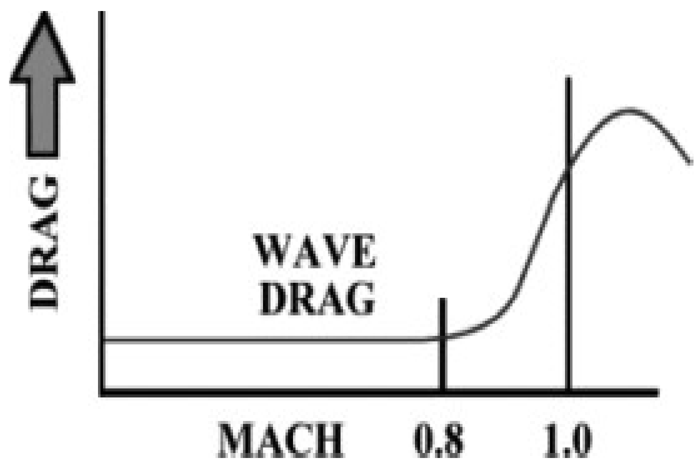


Figure 2.6 Drag and Mach number

CHAPTER 3

FLIGHT CONTROL SURFACES

Aircraft can maneuver in three axes or planes of motion. The point in which these axes intersect is the aircraft's center of gravity. The center of gravity is the point that the aircraft is perfectly balanced (think of a large object balancing on a narrow pole – the point where the pole touches the larger object is it's center of gravity). All movements of the aircraft will be based off this one point.

Control of an airplane can be achieved by providing an incremental lift force on one or more of the airplane's lifting surfaces. The incremental lift force can be produced by deflecting the entire lifting surface or by deflecting a flap incorporated in the lifting surface. See control surfaces on figure 3.1. Because the control flaps or movable lifting surfaces are located at some distance from the center of gravity, the incremental lift force creates a moment about the airplane's center of gravity. See figure 3.2

- Movement in the LATERAL AXIS refers to the pitch of the aircraft. ELEVATORS are the flight controls that control movement in this axis.

- Movement in the VERTICAL AXIS is called yaw.
  RUDDERS perform this function.

- Movement in the LONGITUDINAL AXIS is a roll motion.
  AILERONS are responsible for rolling the aircraft.

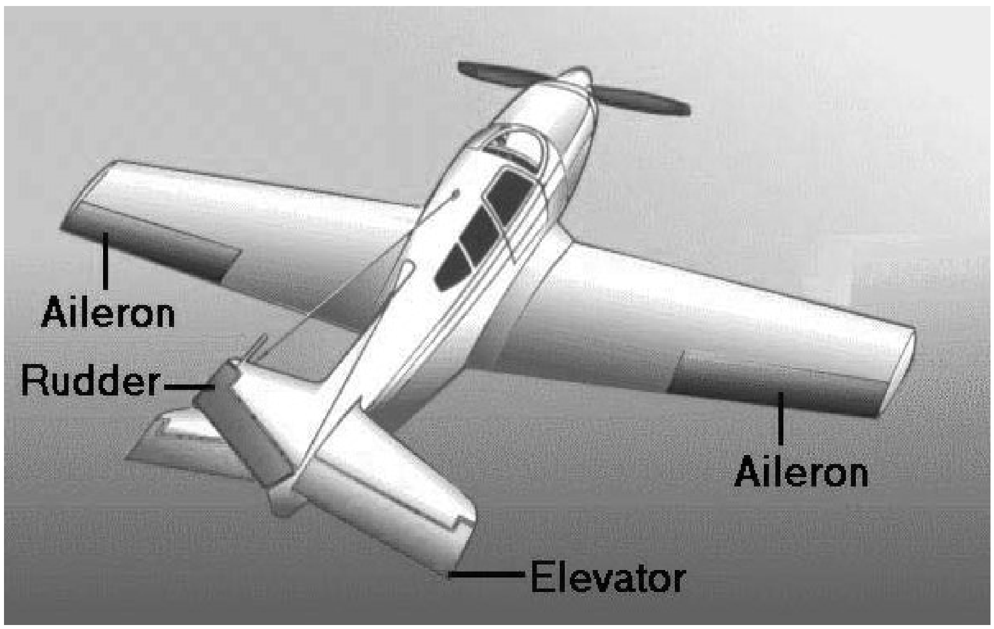


Figure 3.1 The three primary flight controls are the ailerons, elevator and rudder.

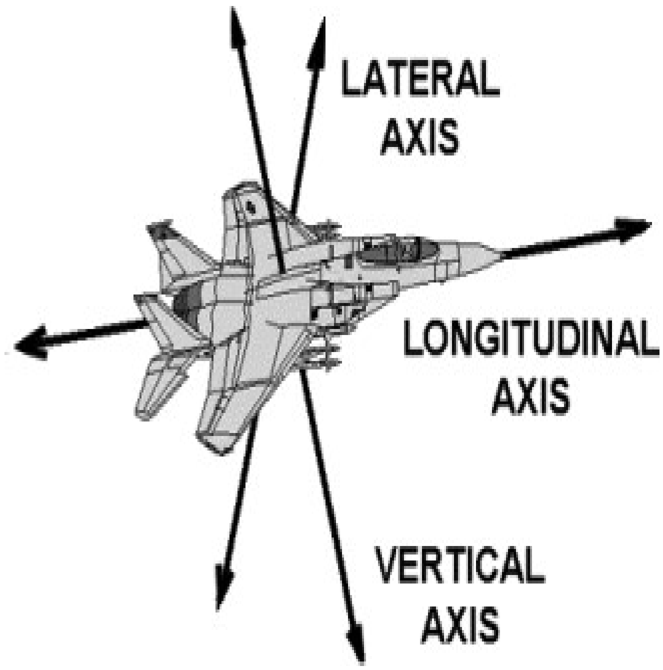


Figure 3.2 Axis of rotation

3.1 Elevators

Elevators (or Stabilators) control movement in the lateral axis (also called pitch). Elevators are connected to the horizontal stabilizer and can move both up and down. Stabilators are used in many aircraft - they differ from elevators in that the entire horizontal stabilizer moves (the F-15 like most modern fighter aircraft has stabilators). Moving the control wheel or stick in the cockpit aft will raise the trailing edge of the elevator. This modifies the camber of the airfoil (i.e. the aircraft) and changes the amount of lift. The trailing edge up in our case will force the tail of the aircraft down and the nose of the aircraft up.

The elevators control the movement of the airplane about its lateral axis. This motion is pitch. The elevators form the rear part of the horizontal tail assembly and are free to swing up and down. They are hinged to a fixed surface--the horizontal stabilizer.

Together, the horizontal stabilizer and the elevators form a single airfoil. A change in position of the elevators modifies the camber of the airfoil, which increases or decreases lift.

Like the ailerons, the elevators are connected to the control wheel (or stick) by control cables. When forward pressure is applied on the wheel, the elevators move downward. This increases the lift produced by the horizontal tail surfaces. The increased lift forces the tail upward, causing the nose to drop. Conversely, when back pressure is applied on the wheel, the elevators move upward, decreasing the lift produced by the horizontal tail surfaces, or maybe even producing a downward force. The tail is forced downward and the nose up.

The elevators control the angle of attack of the wings. When backpressure is applied on the control wheel, the tail lowers and the nose rises, increasing the angle of attack. Conversely, when forward pressure is applied, the tail raises and the nose lowers, decreasing the angle of attack.

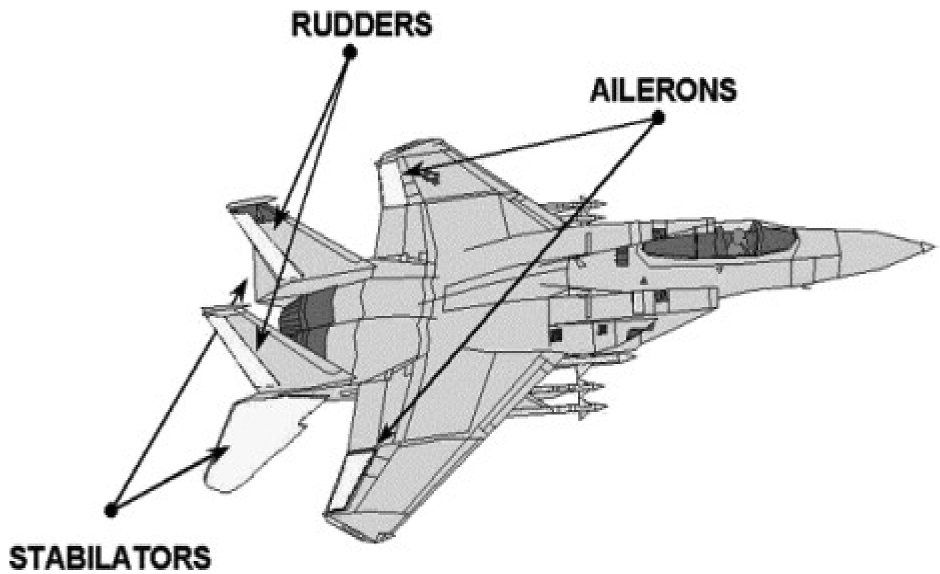


Figure 3.3 Control surfaces of Fighter aircraft

3.2 Aileron

Ailerons are mounted on the outside of the wings and control movement in the longitudinal axis (or roll). When the stick or control wheel is moved to the left, the right aileron will deflect down and the left aileron will deflect up. The right aileron (now lower than the wing) will produce increased lift and will lift the right wing. The left aileron (now above the wing - REMEMBER: lift is produced by the faster moving air across the TOP of the wing) reduces the camber of the left wing and it produces less lift. The increased lift from the right wing and the reduced lift from the left wing cause the aircraft to roll to the left.

The two ailerons, one at the outer trailing edge of each wing, are movable surfaces that control movement about the longitudinal axis. The movement is roll. Lowering the aileron on one wing raises the aileron on the other. The wing with the lowered aileron goes up because of its increased lift, and the wing with the raised aileron goes down because of its decreased lift. Thus, the effect of moving either aileron is aided by the simultaneous and opposite movement of the aileron on the other wing.

Rods or cables connect the ailerons to each other and to the control wheel (or stick) in the Cockpit. When pressure is applied to the right on the control wheel, the left aileron goes down and the right aileron goes up, rolling the airplane to the right. This happens because the down movement of the left aileron increases the wing camber (curvature) and thus increases the angle of attack. The right aileron moves upward and decreases the camber, resulting in a decreased angle of attack. Thus, decreased lift on the right wing and increased lift on the left wing cause a roll and bank to the right.

3.3 Rudders

Rudders move the aircraft along the vertical axis. This is called YAW. The rudders are connected to the vertical stabilizer can are controlled by rudder pedals as opposed to the control stick. Kicking the rudder to the right causes the rudder to move right and to produce greater lift on the right side of the vertical stabilizer. This increased lift pushes the tail to the left and the nose to the right. The rudder is used to with the ailerons to produce a coordinated turn.

The rudder controls movement of the airplane about its vertical axis. This motion is yaw. Like the other primary control surfaces, the rudder is a movable surface hinged to a fixed surface, which, in this case, is the vertical stabilizer, or fin. Its action is very much like that of the elevators, except that it swings in a different plane--from side to side instead of up and down. Control cables connect the rudder to the rudder pedals.

3.3.1 Trim Tabs: A trim tab is a small, adjustable hinged surface on the trailing edge of the aileron, rudder, or elevator control surfaces. Trim tabs are labor saving devices that enable the pilot to release manual pressure on the primary controls. Some airplanes have trim tabs on all three control surfaces that are adjustable from the cockpit; others have them only on the elevator and rudder; and some have them only on the elevator. Some trim tabs are the ground-adjustable type only. The tab is moved in the direction opposite that of the primary control surface, to relieve pressure on the control wheel or rudder control. For example, consider the situation in which we wish to adjust the elevator trim for level flight. ("Level flight" is the attitude of the airplane that will maintain a constant altitude.) Assume that back pressure is required on the control wheel to maintain level flight and that we wish to adjust the elevator trim tab to relieve this pressure. Since we are holding back pressure, the elevator will be in the "up" position. The trim tab must then be adjusted downward so that the airflow striking the tab will hold the elevators in the desired position. Conversely, if forward pressure is being held, the elevators will be in the down position, so the tab must be moved upward to relieve this pressure. In this example, we are talking about the tab itself and not the cockpit control.

Rudder and aileron trim tabs operate on the same principle as the elevator trim tab to relieve pressure on the rudder pedals and sideward pressure on the control wheel, respectively.

3.4 Hydro mechanical System

The hydro mechanical system provides inputs to the three primary flight controls – ailerons, rudders, and the stabilator. The ailerons and rudders act fairly conventional. A conventional stabilator is used only for pitch control. While simple in concept, the actual workings of the stab and ailerons are extremely complex due to the flight envelope of the Strike Eagle.

3.5 Aileron-Rudder Interconnect

In the Flight Controls, it was mentioned that rudder is required to perform a coordinated turn. In most general aviation aircraft, the control wheel and the rudders control the ailerons and elevators by pedals. The Aileron-Rudder Interconnect (ARI) mechanically links the ailerons and rudders to the control stick. This system automatically applies rudder inputs to correspond with roll inputs requested by the pilot. In simple terms, it automatically deflects the rudder for coordinated turns. Flight above the speed of sound has a different set of rules. For one, very little rudder inputs are required (as a matter of fact, at high Mach numbers rudder inputs can cause structural failure); thus, the ARI disengages above Mach 1.0. Also, when landing in a cross wind (a wind that is not directly aligned with the runway), rudder inputs can hinder techniques to counter the wind so the ARI is disabled when the wheels on the ground.

CHAPTER 4

STABILTIY OF AIRCRAFT

By stability we mean the tendency of the airplane to return to its equilibrium position after it has been disturbed.

4.1 Disturbance faced by an Aircraft

The disturbance may be generated by the pilot's actions or atmospheric phenomena. The atmospheric disturbances can be wind gusts, wind gradients, or turbulent air.

4.2 Conditions for Airplane Stability

An airplane must have sufficient stability that the pilot does not become fatigued by constantly having to control the airplane owing to external disturbances. Although airplanes with little or no inherent aerodynamic stability can be flown, they are unsafe to fly unless they are provided artificial stability by an electromechanical device called stability augmentation system.

Two conditions are necessary for an airplane to fly its mission successfully.

1. The airplane must be able to achieve equilibrium flight and it must have the capability to maneuver for a wide range of flight velocities and altitudes. To

achieve equilibrium or perform maneuvers, the airplane must be equipped with aerodynamic and propulsive controls. The design and performance of control systems is an integral part of airplane stability and control.

2. Airplanes should have good handling qualities. The stability and control characteristics of an airplane are referred to as the vehicle's handling or flying qualities. It is important to the pilot that the airplane possesses satisfactory handling qualities. Airplanes with poor handling qualities will be difficult to fly and could be dangerous. Pilots form their opinions of an airplane on the basis of its handling characteristics. An airplane will be considered of poor design if it is difficult to handle regardless of how outstanding the airplane's performance might be.

4.3 Static Stability of Aircraft

Aircraft should be statically stable. Without static stability the pilot would have to continuously control the airplane to maintain a desired flight path, which would be quite fatiguing. The degree of static stability desired by the pilot has been determined through flying quality.

The important point at this time is to recognize that the airplane must be made statically stable, either through inherent aerodynamic characteristics or by artificial means through the use of an automatic control system.

The inherent static stability tendencies of the airplane are function of its geometric and aerodynamic properties. The designer can control the degree of longitudinal and lateral directional stability by proper sizing of the horizontal and vertical tail surfaces, whereas roll stability are consequence of dihedral effect, which is controlled by the wing's placement or dihedral angle.

4.4 Dynamic Stability of Aircraft

Note that the vehicle can be statically stable but dynamically unstable. Static stability, therefore, does not guarantee dynamic stability. However, for the vehicle to be dynamically stable it must be statically stable.

Of particular interest to the pilot and designer is the degree of dynamic stability. Dynamic stability usually is specified by the time it takes a disturbance to be damped to half of its initial amplitude or, in the case of an unstable motion, the time it takes for the initial amplitude of the disturbance to double. In the case of an oscillatory motion, the frequency and period of the motion are extremely important.

4.5 Dynamic Stability and SAS

The reduction of the disturbance with time indicates that there is resistance to the motion and, therefore, energy is being dissipated. The dissipation of energy is called positive damping. If energy is being added to the system, then we have a negative damping. Positive damping for an airplane is provided by forces and moments that arise owing to the airplane's motion. If positive damping, these forces and moments will oppose the motion of the airplane and cause the disturbance to damp out with time. An airplane that has negative aerodynamic damping will be dynamically unstable. To fly such an airplane, artificial damping must be designed into the vehicle. The artificial damping is provided by a **stability augmentation system** (SAS).

Basically, a stability augmentation system is an electromechanical device that senses the undesirable motion and moves the appropriate controls to damp out the motion. This usually is accomplished with small control movements and, therefore, the pilot's control actions are not influenced by the system.

Another responsibility of the Stability Augmentation System (SAS) system is to refine the flight control inputs from the pilot provided to the hydro mechanical system. It is a fly-by-wire system that overlays the hydro mechanical system. It incorporates a sophisticated flight control computer with numerous motion sensors to refine the inputs to the flight control surfaces to respond to the pilot's stick inputs. In other words, it precisely deflects the flight control surfaces to provide the pilot with exactly the inputs he requested based on the amount of force used to move the stick. This system has several redundant systems built within it providing outstanding reliability.

The CAS system is sub-divided into 3 systems PITCH, ROLL, and YAW controller.

(Note: The CAS system does not provide inputs to the ailerons, it uses only differential stab inputs to roll the aircraft. The hydro mechanical system provides the only inputs to the ailerons).

4.6 Pilot-induced oscillation

So far, we have been discussing the response of an airplane to external disturbances while the controls are held fixed. When we add the pilot to the system, additional complications can arise. For example, an airplane that is dynamically stable to external disturbances with the controls fixed can become unstable by the pilot's control actions. If the pilot attempts to correct for a disturbance and that control input is out of phase with the oscillatory motion of the airplane, the control actions would increase the motion rather than correct it. This type of pilot-vehicle response is called pilot-induced oscillation (PIO). Many factors contribute to the PIO tendency of an airplane. A few of the major contributions are insufficient aerodynamic damping, insufficient control system damping, and pilot reaction time.

CHAPTER 5

STATIC STABILITY AND CONTROL

5.1 Definition of Longitudinal Static Stability

Two conditions are necessary for an airplane to longitudinally stable

1. To have static longitudinal stability the aircraft pitching moment curve must have a negative slope.
2. Plane should trim at positive angle of attach

5.2 Contribution of Aircraft Components

In discussing the requirements for static stability, we so far have considered only the total airplane pitching moment curve. However, it is of interest (particularly to airplane designers) to know the contribution of the wing, fuselage, tail, propulsion system, and the like, to the pitching moment and static stability characteristics of the airplane.

In the following sections, each of the components will be considered separately. We will start by breaking down the airplane into its basic components. Such as the wing, fuselage, horizontal tail, and propulsion unit.

5.2.1 Wing Contribution

For a wing-alone design to be statically stable the aerodynamic center must lie aft of the center of gravity to make $C_m < 0$.
Since we also want to be able to trim the aircraft at a positive angle of attack, the pitching moment coefficient at zero angle of attack, C_{mo} must be greater than 0. A positive pitching moment about the aerodynamic center can be achieved by using a negative cambered airfoil section or an airfoil section that has a refluxed trailing edge.

For many airplanes, the center of gravity position is located slightly aft of the aerodynamic center. Also, the wing is normally constructed of airfoil profiles having a positive camber. Therefore, the wing contribution to static longitudinal stability is destabilizing for most conventional airplanes.

5.2.2 Tail Contribution—Aft Tail

The horizontal Tail surface can be located either forward or aft of the wing. When the surface is located forward of the wing, the surface is called a canard. The flow field created by the wing influences both surfaces. The canard surface is affected by the upwash flow from the wing, whereas the aft tail is subjected to the downwash flow. The wing flow field is due primarily to the bound and trailing vortices. The magnitude of the upwash or downwash depends on the location of the tail surface with respect to the wing.

Recall that earlier we showed you that the wing contribution to C_{mo} was negative for an airfoil has positive camber. The tail contribution to C_{mo} can be used to ensure that C_{mo} for the complete airplane is positive. This can be accomplished by adjusting the tail incidence angle

The designer can also control the level of static stability by proper selection of the horizontal tail volume ratio. In practice the only parameter making up the volume ratio that can be varied by the stability and control designer is the horizontal tail surface area.

The other parameters, such as the tail moment arm, wing area, and mean wing chord, are determined by the fuselage and wing requirements, which are related to the internal volume and performance specifications of the airplane, respectively.

5.2.3 Canard—Forward Tail Surface

A canard is a tail surface located ahead of the wing. The canard surface has several attractive features. The canard, if properly positioned, can be relatively free from wing or propulsive flow interference. Canard control is more attractive for trimming the large nose-down moment produced by high-lift devices. To counteract the nose-down pitching moment, the canard must produce lift that will add to the lift being produced by the wing. An aft tail must produce a down load to counteract the pitching moment and thus reduce the airplane's overall lift force.

The major disadvantage of the canard is that it produces a destabilizing contribution to the aircraft's static, stability. However, this is not a severe limitation. By proper location of the center of gravity, one can ensure the airplane is statically stable.

5.2.4 Fuselage Contribution

The *primary function* of the fuselage is to provide room for the flight crew and payload such as passengers and cargo. The optimum shape for the internal volume at minimum drag is a body for which the length is larger than the width or height. For most fuselage shapes used in airplane designs, the width and height are on the same order of magnitude and for many designs a circular cross-section is used.

The local angle of attack along the fuselage is greatly affected by the flow field created by the wing. The portion of the fuselage ahead of the wing is in the wing upwash; the aft portion is in the wing downwash flow.

5.2.5 Power Effects

The propulsion unit can have a significant effect on both the longitudinal trim and static stability of the airplane. If the thrust line is offset from the center of gravity, the propulsive force will create a pitching moment that must be counteracted by the aerodynamic control surface.

The static stability of the airplane also is influenced by the propulsion system. For a propeller driven airplane the propeller will develop a normal force in its plane of rotation when the propeller is at an angle of attack. The propeller's normal force will create a pitching moment about the center of gravity, producing a propulsion contribution to C_{m_α}. Although one can derive a simple expression for C_{m_α}, due to the propeller, the actual contribution of the propulsion system to the static stability is much more difficult to estimate. This is due to the indirect effects that the propulsion system has on the airplanes characteristics. For example, the propeller slipstream can have an effect on the tail efficiency and the downwash field. Because of these complicated interactions the propulsive effects on airplane stability are commonly estimated from powered wind-tunnel models.

A normal force will be created on the inlet of a jet engine when it is at an angle of attack. As in the case of the propeller-powered airplane, the normal force will produce a contribution to C_{m_α}.

5.2.6 Stick fix Neutral point

The total pitching moment C_{m_α} for the airplane can now be obtained by summing the wing fuselage and tail contribution

Note that C_{m_α} depends upon the center of gravity position as well as aerodynamic characteristics of the airplane. The center of gravity of an airplane varies during the course of its operation. Therefore it is important to know if there are any limits to the center of gravity travel. To ensure that the airplane possesses static longitudinal stability we would like to know that at what point C_{m_α} =0. If airplane center of gravity reaches this point, the airplane will be neutrally stable.

5.3 Longitudinal Control

Control of an airplane can be achieved by providing an incremental lift force on one or more of the airplane's lifting surfaces. In this section we shall be concerned with longitudinal control.

Pitch control can be achieved by changing the lift on either a forward or aft control surface. If a flap is used, the flapped portion of the tail surface is called an elevator.

Factors affecting the design of a control surface are control effectiveness, hinge moments and aerodynamic and mass balancing.

Control effectiveness is a measure of how effective the control deflection is in producing the desired control moment, control effectiveness is a function of the size of the flap and tail volume ratio.

Hinge moments also are important because they are the aerodynamic moments that must be overcome to rotate the control surface. The hinge moment governs the magnitude of force required of the pilot to move the control surface. Therefore, great care must be used in designing a control surface so that the control forces are within acceptable limits for the pilots.

Finally, aerodynamic and mass balancing deals with techniques to vary the hinge moments so that the control stick forces stay within an acceptable range.

5.4 Elevator Effectiveness

We need some form of longitudinal control to fly at various trim conditions. As shown earlier, the pitch attitude can be controlled by either an aft tail or forward tail (canard). We shall examine how an elevator on an aft tail provides the required control moments. Although we restrict our discussion to an elevator on an aft tail, the same arguments could be made with regard to a canard surface. The elevator does not change the slope of the pitching moment curves but only shifts them so that different trim angles can be achieved. When the elevator is deflected, it changes the lift and pitching moment of the airplane.

5.5 Elevator Hinge Moment

It is important to know the moment acting at the hinge line of the elevator (or other type of control surface). The hinge moment, of course, is the moment the pilot must overcome by exerting a force on the control stick. Therefore to design the control system properly we must know the hinge moment characteristics. The hinge moment can be expressed as the addition of the effects of angle of attack, elevator deflection angle, and tab angle taken separately. The hinge moment parameters are very difficult to predict analytically with great precision. Wind-tunnel tests usually are required to provide the control system designer with the information needed to design the control system properly.

5.6 Definition Of Directional Stability

Directional, or weathercock, stability is concerned with the static stability of the airplane about the z axis. Just as in the case of longitudinal static stability, it is desirable that the airplane should tend to return to an equilibrium condition when subjected to some form of yawing disturbance. To have static directional stability, the airplane must develop a yawing moment that will restore the airplane to its equilibrium state. Assume that two airplanes are disturbed from their equilibrium condition, so that the airplanes are flying with a positive sideslip angle. Airplane 1 will develop a restoring moment that will tend to rotate the airplane back to its equilibrium condition; that is, a zero sideslip angle. Airplane 2 will develop a yawing moment that will tend to increase the sideslip angle. Examining these curves, we see that to have static directional stability the slope of the yawing moment curve must be positive. Note that an airplane possessing static directional stability will always point into the relative wind, hence the name weathercock stability.

5.7 Contribution of Aircraft Components to Directional Stability

The contribution of the wing to directional stability usually is quite small in comparison to the fuselage, provided the angle of attack is not large. The fuselage and engine nacelles, in general, create a destabilizing contribution to directional stability.

Wing-fuselage contribution to directional stability is destabilizing so the vertical tail must be properly sized to ensure that the airplane has directional stability. Vertical tail does effect stability for example when the airplane is flying at a positive sideslip angle the vertical tail produces a side force (lift force in the *xy* plane) that tends to rotate the airplane about its center of gravity. The moment produced is a restoring moment.

5.8 Directional Control

Directional control is achieved by a control surface, called a rudder, located on the vertical tail. The rudder is a hinged flap that forms the aft portion of the vertical tail. By rotating the flap, the lift force (side force) on the fixed vertical surface can be varied to create a yawing moment about the center of gravity. The size of the rudder is determined by the directional control requirements. The rudder control power must be sufficient to accomplish the requirements listed in Table 5.1.

The yawing moment produced by the rudder depends on the change in lift on the vertical tail due to the deflection of the rudder times its distance from the center of gravity. For a positive rudder deflection, a positive side force is created on the vertical tail. A positive side force will produce a negative yawing moment.

5.9 Definition Of Roll Stability

An airplane possesses static roll stability if a restoring moment is developed when it is disturbed from a wings-level attitude. The requirement for stability is that $C_{l_\beta} < 0$.

The roll moment created on an airplane when it starts to sideslip depends on the wing dihedral, wing sweep, position of the wing on the fuselage, and the vertical tail. Each of these contributions will be discussed qualitatively in the following paragraphs.

The dihedral angle is defined as the spanwise inclination of the wing with respect to the horizontal. If the wing tip is higher than the root section, then the dihedral angle is positive; if the wing tip is lower than the root section, then the dihedral angle is negative. A negative dihedral angle is commonly called anhedral.

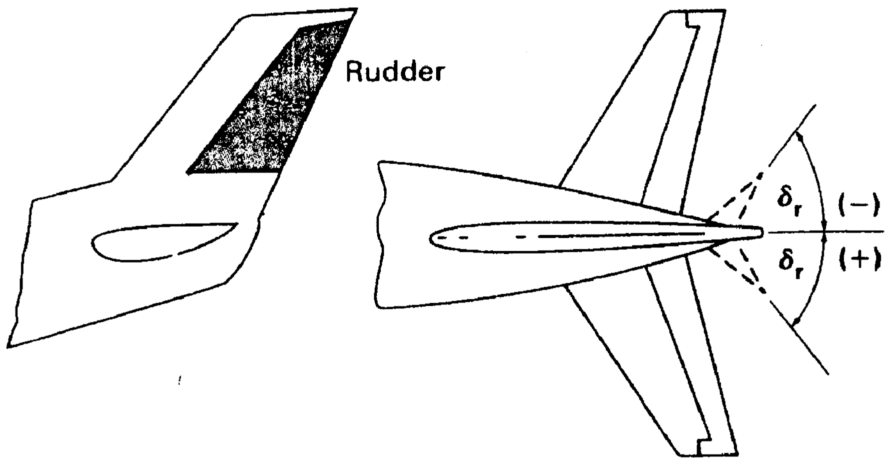


Figure 5.4 Directional Controls by mean of Rudder

Table 5.1 Requirements for directional control

Rudder requirements	Implication for rudder design
Adverse yaw	When an airplane is banked to execute a turning maneuver the ailerons may create a yawing moment that opposes the turn (i.e., adverse yaw). The rudder must be able to overcome the adverse- yaw so that a coordinated turn can be achieved. The critical condition for adverse yaw occurs when the airplane is flying slow (i.e., high C_L)
Crosswind landings	To maintain alignment with the runway during a crosswind landing requires the pilot to fly the airplane at a sideslip angle. The rudder must be powerful enough to permit the pilot to trim the airplane for the specified crosswinds. For transport airplanes, landing may be carried out for crosswinds up to 15.5 m/s or 51 ft/s.
Asymmetric power Condition	The critical asymmetric power condition occurs for a multiengine airplane condition for a multiengine airplane when one engine fails at low flight speeds. The rudder must be able to overcome the yawing moment produced by the asymmetric thrust arrangement.
Spin recovery	The primary control for spin recovery in many airplanes is a powerful rudder. The rudder must be powerful enough to oppose the spin rotation.

When an airplane is disturbed from a wings-level attitude, it will begin to sideslip. Once the airplane starts to sideslip a component of the relative wind is directed toward the side of the airplane. The leading wing experiences an increased angle of attack and consequently an increase in lift. The trailing wing experiences the opposite effect. The net result is a rolling moment that tries to bring the wing back to a wings-level attitude. This restoring moment is often referred to as the dihedral effect.

Wing sweep also contributes to the dihedral effect. In a sweptback wing, the windward wing has an effective decrease in sweep angle and the trailing wing experiences an effective increase in sweep angle. For a given angle of attack, a decrease in sweepback angle will result in a higher lift coefficient. Therefore, the windward wing (with a less effective sweep) will experience more lift than the milling wing. It can he concluded that sweepback adds to the dihedral effect. On the other hand, sweep forward will decrease the effective dihedral effect.

The fuselage contribution to dihedral effect is that the sideward flow turns in the vicinity of the fuselage and creates a local change in wing angle of attack at the inboard wing stations. For a low wing position, the fuselage contributes a negative dihedral effect; the high wing produces a positive dihedral effect. To maintain the same C_{l_β}, a low-wing aircraft will require a considerably greater wing dihedral angle than a high-wing configuration.

The horizontal tail also can contribute to the dihedral effect in a manner similar to the wing. However, owing to the size of the horizontal tail with respect to the wing, its contribution is usually small. The contribution to dihedral effect from the vertical tail is produced by the side force on the tail due to sideslip. The side force on the vertical tail produces both a yawing moment and a rolling moment. Thu rolling moment occurs because the center of pressure for the vertical tail is located above the aircraft's center of gravity. The rolling moment produced by the vertical tail tends to bring the aircraft back to a wings-level attitude.

5.10 Roll Control

Roll control is achieved by the differential deflection of small flaps called ailerons which are located outboard on .the wings, or by the use of spoilers. The basic principle behind these devices is to modify the spanwise lift distribution so that a moment is created about the x axis.

5.11 Stick Forces

To deflect a control surface the pilot must move the control stick or rudder pedals. The forces exerted by the pilot to move the control surface is called the stick force or pedal force, depending which control is being used. The stick force is proportional to the hinge moment acting on the control surface. The work of displacing the control stick is equal to the work in moving the control surface to the desired deflection angle. The magnitude of the stick force increases with the size of the airplane and the square of the airplane's speed. Similar is for the rudder pedal force and aileron stick force.

The control system is designed to convert the stick and pedal movements into control surface deflections. Although this may seem to be a relatively easy task, it in fact is quite complicated. The control system must be designed so that the control forces are within acceptable limits. On the other hand, the control forces required in normal maneuvers must not be too small; otherwise, it might be possible to overstress the airplane. Proper control system design will provide stick force magnitudes that give the pilot a feel for the commanded maneuver. The magnitude of the stick force provides the pilot with an indication of the severity of the motion that will result from the stick movement.

The convention for longitudinal control is that a pull force should always rotate. the nose upward, which causes the airplane to slow down. A push force will have the opposite effect; that is, the nose will rotate downward and the airplane will speed up. The control system designer must also be sure that the airplane does not experience control reversals due to aerodynamic or aeroelastic phenomena.

5.11.1 Trim Tabs

In addition to making sure that the stick and rudder pedal forces required to maneuver or trim the airplane are within acceptable limits, it is important that some means be provided to zero out the stick force at the trimmed flight speed. If such a provision is not made, the pilot will become fatigued by trying to maintain the necessary stick force. The stick force at trim can be made zero by incorporating a tab on either the elevator or the rudder. The tab is a small flap located at the trailing edge of the control surface. The trim tab can be used to zero out the hinge moment and thereby eliminate the stick or pedal forces. Although the trim tab has a great influence over the hinge moment, it has only a slight effect on the lift produced by the control surface.

CHAPTER 6

AIRCRAFT EQUATION OF MOTION

In chapter the nonlinear differential equations of motion of a rigid airplane is developed from Newton's second law of motion. Linearization of these equations is accomplished using the small disturbance theory. In following chapter linearized equations of motion will be developed. These equations will be valuable information on the dynamic characteristic of airplane motion.

6.1 Derivation of rigid body equations of motion

The rigid body equations of motion are obtained from Newton's second law, which states that the summation all external forces acting on a body is equal to the time rate of change of momentum of the body and the summation of the external moments acting on the body is equal to the time rate of change of the moment of momentum (angular moment). The time rates of change of linear and angular momentum are referred to an absolute or inertial reference frame. For many problems in airplane dynamic an axis system fixed to the earth can be used as an inertial reference frame. Newton's second law can be expressed in the following vector equation.

$$\sum F = \frac{d}{dt} mv \qquad 6.1$$

$$\sum M = \frac{d}{dt} H \qquad 6.2$$

The vector equations can be rewritten in scalar form and then consists of three force equations.

The forces equations can be expressed as follows

$$F_x = \frac{d}{dt} mu \qquad 6.3$$

$$F_y = \frac{d}{dt} mv \qquad 6.4$$

$$F_z = \frac{d}{dt} mw \qquad 6.5$$

Where F_x, F_y, F_z and u, v, w are the components of the force and velocity along the x , y, and z axes respectively. The forces components are composed of contributions due to the aerodynamic propulsive and gravitational forces acting on the airplane. The moment equations can be expressed in a similar manner:

$$L = \frac{d}{dt} H_x \qquad 6.6$$

$$M = \frac{d}{dt} H_y \qquad 6.7$$

$$N = \frac{d}{dt} H_z \qquad 6.8$$

Where L, M, N and H_x, H_y,H_z are the components of the moment and momentum of momentum along the x, y, and z axes respectively.

Consider the airplane shown in figure 6.1. If let δm be an element of mass of the airplane , v be the velocity of he element mass relative to be absolute or inertial frame and δF be the resulting force acting on the elemental mass then newtons second law yields

$$\delta F = \delta m \frac{dv}{dt} \qquad 6.9$$

and the total external forces acting on the airplane is found by summing all the elements of the airplane.

$$\sum \delta F = F \qquad 6.10$$

The velocity of the differential mass δm is

$$v = v_c + \frac{dr}{dt} \tag{6.11}$$

Where v_c is the velocity of the center of the mass of the airplane and dr/dt is the velocity of the element relative to the center of mass. Substituting this expression for the velocity into Newton's second law yields.

$$\sum \delta F = F = \frac{d}{dt}\sum\left(v_c + \frac{dr}{dt}\right)\delta m \tag{6.12}$$

If we assume that the mass of the vehicle is constant

$$F = m\frac{dv_c}{dt} + \frac{d}{dt}\sum\frac{dr}{dt}\delta m \tag{6.13}$$

$$F = m\frac{dv_c}{dt} + \frac{d^2}{dt^2}\sum r\delta m \tag{6.14}$$

Because r is measured from the center of the mass, the summation $\sum r\delta m$ is equal to 0. The forces equation then becomes

$$F = m\frac{dv_c}{dt} \tag{6.15}$$

which relates the external forces on the airplane to the motion of the vehicle's center of the mass.

In a similar manner we can develop the moment equation referred to a moving center of mass. For the differential element of mass, δm, the moment equation can be written as

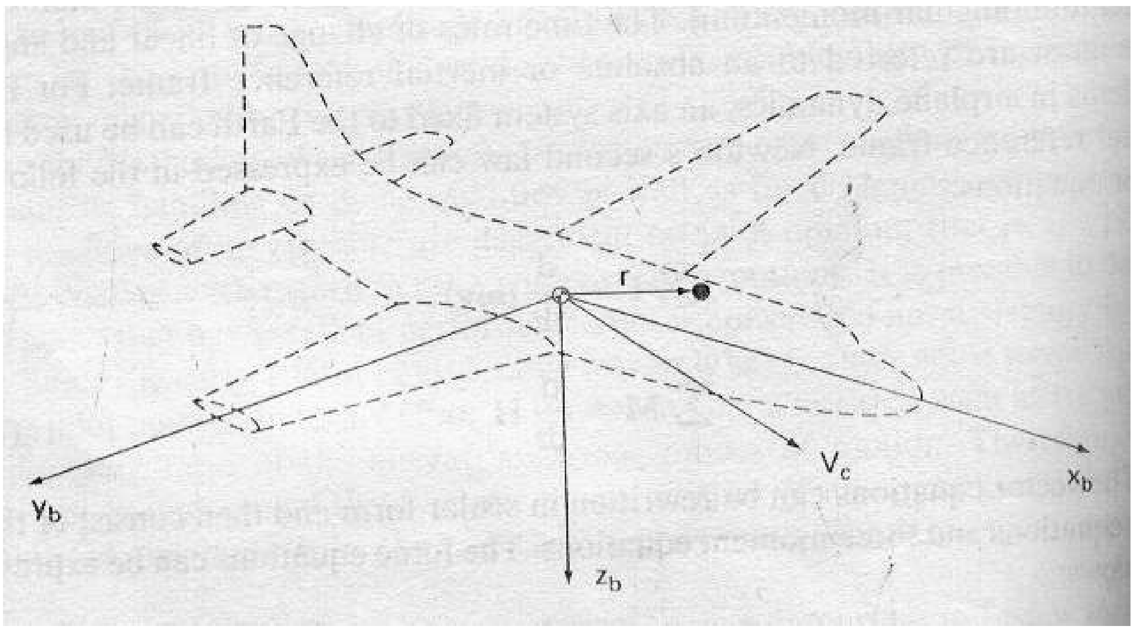


Figure 6.1 An element of mass on an airplane

$$\delta M = \frac{d}{dt}\delta H = \frac{d}{dt}(r * v)\delta m \qquad 6.16$$

The velocity of the mass element can be expressed in terms of the velocity of the center of mass and the relative velocity of the mass element to the center of mass

$$v = v_c + \frac{dr}{dt} = v_c + \omega * r \qquad 6.17$$

Where ω is the angular velocity of the vehicle and r is the position of the mass element measured from the center of mass. The total momentum can be written as

$$H = \sum \delta H = \sum (r * v_c)\delta m + \sum [r * (\omega * r)]\delta m \qquad 6.18$$

The velocity v_c is a constant with respect to the summation and can be taken outside the summation sign.

$$H = \sum r\delta m * v_c + \sum [r * (\omega * r)]\delta m \qquad 6.19$$

The first term in equation is 0 because the term $\sum r\delta m = 0$ as explained previously. If we express the angular velocity and position vector as

$$\omega = pi + qj + rk \qquad 6.20$$

And

$$r = xi + yj + zk \qquad 6.21$$

Then after expanding Equation H can written as

$$H = (pi + qi + rk)\sum (x^2 + y^2 + z^2)\delta m - \sum (xi + yi + zk)(px + qy + rz)\delta m \qquad 6.22$$

The scalar components of H are

$$H_x = p\sum\left(x^2 + z^2\right)\delta m - q\sum xy\delta m - r\sum xz\delta m \qquad 6.23$$

$$H_y = p\sum xy\delta m - q\sum\left(x^2 + z^2\right)\delta m - r\sum yz\delta m \qquad 6.24$$

$$H_z = p\sum xz\delta m - q\sum yz\delta m - r\sum\left(x^2 + z^2\right)\delta m \qquad 6.25$$

The summations in these equations are the mass moment and products of inertial of the airplane and are defined follows

$$I_x = \iiint\left(x^2 + z^2\right)\delta m \quad 6.26 \qquad , \qquad I_{xy} = \iiint xy\delta m \quad 6.27$$

$$I_y = \iiint\left(y^2 + z^2\right)\delta m \quad 6.28 \qquad , \qquad I_{xz} = \iiint xz\delta m \quad 6.29$$

$$I_z = \iiint\left(x^2 + y^2\right)\delta m \quad 6.30 \qquad , \qquad I_{yz} = \iiint yz\delta m \quad 6.31$$

The terms I_x, I_y I_z are the mass moments of inertial of the body about the x, y and z axes respectively. The term with the mixed indexes are called the products of inertia. Both the moments and products of inertia depend on the shape of the body and the manner in which its mass is distributed. The larger the moments of inertia, the greater will be the resistance to rotation. The scalar equations for the moment of momentum follow:

$$H_x = pI_x - qI_{xy} - rI_{xy} \qquad 6.32$$

$$H_y = -pI_{xy} + qI_y - rI_{yz} \qquad 6.33$$

$$H_z = pI_{xx} - qI_{yz} - rI_z \qquad 6.34$$

If the reference frame is not rotating, then as the airplane rotates the moments and products of inertia will vary with time. To avoid this difficulty we will fix the axis system to the airplane. Now we must determine the derivatives of the vector v and H refereed to the rotating body frame of reference.

It can be shown that the derivative of an arbitrary vector A referred to a rotating body frame having an angular velocity ω can be representing by the following vector identity

$$\left.\frac{dA}{dt}\right|_{I}=\left.\frac{dA}{dt}\right|_{B}+\omega * A \qquad 6.35$$

Where the subscript I and B refer to the inertial and body fixed frames of reference. Applying this identity to the equations derived earlier yields

$$F=m\left.\frac{dv_c}{dt}\right|_{B}+m(\omega * v_c) \qquad 6.36$$

$$M\Big|=\left.\frac{dH}{dt}\right|_{B}+\omega * H \qquad 6.37$$

The scalar equations are

$$F_x=m(\dot{u}+qw+rv) \qquad 6.38$$

$$F_y=m(\dot{v}+ru+pw) \qquad 6.39$$

$$F_z=m(\dot{w}+pv+qu) \qquad 6.40$$

$$L=\dot{H}_x+qH_z+rH_y \qquad 6.41$$

$$M=\dot{H}_y+rH_x+pH_z \qquad 6.42$$

$$N=\dot{H}_z+pH_y+qH_z \qquad 6.43$$

The components of the force and moment acting on the airplane are composed of aerodynamic gravitational propulsive contributions.

By the proper positioning of the body axis system one can make the products of inertia $I_{yz}=I_{xy}=0$. To do this we are assuming that the xz plane is a plane of symmetry of the airplane. With this assumption the moment equations can be written as

$$L=I_x\dot{p}-I_{xz}\dot{r}+qr(I_z-I_y)-I_{xz}pq \qquad 6.44$$

$$M=I_y\dot{q}+rp(I_x-I_z)-I_{xz}(p^2-r^2) \qquad 6.45$$

$$N=-I_{xz}\dot{p}+I_z\dot{r}+pq(I_y-I_x)+I_{xz}qr \qquad 6.46$$

6.2 Aerodynamic force and moment representation

The method of representing the aerodynamic forces and moments by stability coefficients was first introduced by Bryan over three-quarters of a century ago. The technique proposed by Bryan assumes that the aerodynamic forces and moments can be expressed as a function of the instantaneous values of the perturbation variables. The perturbation variables are the instantaneous changes from the reference conditions of the transnational velocities, angular velocities, control deflection, and their derivatives. With this assumption, we can express the aerodynamic forces and moments by means of a Taylor series expansion of the perturbation variables about the reference equilibrium condition. For example, the change in the force in the v direction can he expressed as follows:

$$\Delta X(u, w, \dot{u}, \dot{w}, \ldots\ldots\ldots\ldots, \delta_e, \dot{\delta}_e) = \frac{\partial X}{\partial u}\Delta u + \frac{\partial M}{\partial \dot{u}}\Delta \dot{u} + \ldots\ldots + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + H.O.T \qquad 6.47$$

The term $\frac{\partial \mathrm{X}}{\partial \mathrm{u}}$ called the stability derivative is evaluated at the reference flight condition.

The contribution of the change in the velocity u to change $\Delta\mathrm{X}$ in he X f**orce** is just $\frac{\partial \mathrm{X}}{\partial \mathrm{u}}\Delta \mathrm{u}$

We can also express $\frac{\partial \mathrm{X}}{\partial \mathrm{u}}$ in terms of the stability Coefficient $\mathrm{C}_{\mathrm{x_u}}$, as follows:

$$\frac{\partial X}{\partial u} = C_{x_\alpha}\frac{1}{u_o}QS \qquad 6.48$$

$$C_{x_u} = \frac{\partial C_x}{\partial u/u_o} \qquad 6.49$$

Note that the stability derivative has dimensions, whereas the stability coefficient is defined so that it is non-dimensional.

The preceding discussion may seem as though we are making the aerodynamic force and moment representation extremely complicated. However, by assuming that the perturbations are small we need to retain only the linear terms in Equation above. Even though we have retained only the linear terms, the expressions still may include numerous first-order terms. Fortunately, many of these terms also can be neglected because their contribution to a particular force or moment is negligible. For example, If we express the pitching moment in terms of the perturbation variables, as indicated next,

$$M(u, v, w, \dot{u}, \dot{v}, \dot{w}, p, q, r, \delta_a, \delta_e, \delta_r) = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial v}\Delta v + \frac{\partial M}{\partial w}\Delta w + ... + \frac{\partial M}{\partial p}\Delta p + ... \quad 6.50$$

It should be quite obvious that terms such as $\frac{\partial M}{\partial v}$ and $\frac{\partial M}{\partial p}$ are not going to be significant for an airplane. Therefore, we can neglect these terms in our analysis.

6.3 Small-disturbance theory

The equations developed in the previous section can be linear-zed using the small-disturbance theory. In applying the small-disturbance theory we assume that the motion of the airplane consists of small deviations about a steady flight condition. Obviously, this theory cannot be applied to problems in which large-amplitude motions are to be expected (e.g., spinning or stalled flight). However, in many cases the small-disturbance theory yields sufficient accuracy for practical engineering purposes.

All the variable in the equations of motion are replaced by a reference value plus a perturbation or disturbance

$u = u_o + \Delta u$ 6.51 , $v = v_o + \Delta v$ 6.52 , $w = w_o + \Delta w$ 6.53

$p = p_o + \Delta p$ 6.54 , $q = q_o + \Delta q$ 6.55 , $r = r_o + \Delta r$ 6.56

$X = X_o + \Delta X$ 6.55 , $Y = Y_o + \Delta Y$ 6.56 , $Z = Z_o + \Delta Z$ 6.57

$M = M_o + \Delta M$ 6.58 , $N = N_o + \Delta N$ 6.59 , $L = L_o + \Delta L$ 6.60

$\delta = \delta_o + \Delta\delta$ 6.61

For convenience, the reference flight condition is assumed to be symmetric and the propulsive forces are assumed to remain constant. This implies that

$$v_o = p_o = q_o = r_o = \Phi_o = \psi_o = 0 \qquad 6.62$$

Furthermore, if we initially align the *x*-axis so that it is along the direction of the airplane's velocity vector, then $w_o = 0$.

Now, if we introduce the small-disturbance notation into the equations of motion, we can simplify these equations. As an example, consider the *X* force equation:

$$X - mg\sin\theta = m(\dot{u} + qw - rv) \qquad 6.63$$

Substituting the small-disturbance variables into this equation yields

$$X_o + \Delta X - mg\sin(\theta_o + \Delta\theta) = m(\frac{d}{dt}(u_o + \Delta u) + (q_o - \Delta q)(w_o - \Delta w) - (r_o - \Delta r)(v_o - \Delta v)) \qquad 6.64$$

If we neglect products of the disturbance and assume that

$$w_o = v_o = p_o = q_o = r_o = \Phi_o = \psi_o = 0 \qquad 6.65$$

Then the *X* equation becomes

$$X_o + \Delta X - mg\sin(\theta_o + \Delta\theta) = m\Delta\dot{u} \qquad 6.66$$

Applying the following trigonometric identity can reduce this equation further

$$\sin(\theta_o + \Delta\theta) = \sin\theta_o\cos\Delta\theta + \cos\theta_o\sin\Delta\theta = \sin\theta_o + \Delta\theta\cos\theta_o \qquad 6.67$$

$$X_o + \Delta X - mg(\sin\theta_o + \Delta\theta\cos\theta_o) = m\Delta\dot{u} \qquad 6.68$$

If all the disturbance quantities are set equal to 0 in these equations, we have the reference flight condition

$$X_o - mg\sin\theta_o = 0 \qquad 6.69$$

This reduces the X-force equation to

$$X_o - mg\Delta\theta\cos\theta_o = m\Delta\dot{u} \qquad 6.70$$

The force ΔX is the change in aerodynamic and propulsive force in the *x* direction and can be expressed by means of a Taylor series in terms of the perturbation variables. If we assume that ΔX is a function only of u, w, δ_e and δ_T, then can ΔX be expressed as

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\delta_T \qquad 6.71$$

where $\frac{\partial X}{\partial u}, \frac{\partial X}{\partial w}, \frac{\partial X}{\partial \delta_e} \text{and} \frac{\partial X}{\partial \delta_T}$ are called stability derivatives, that are evaluated at the reference flight condition. The variables $\delta_e \text{and} \delta_T$ are the change in elevator angle and throttle setting, respectively. If a canard or all-moveable stabilator is used for longitudinal control, then the control term would be replaced by

$$\frac{\partial X}{\partial H}\Delta H \text{................or...........} \frac{\partial X}{\partial \delta_e}\Delta\delta_e \qquad 6.72$$

Substituting the expression for ΔX into the force equation yields:

$$\frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\theta\cos\theta_o = m\Delta\dot{u} \qquad 6.73$$

Or on rearranging

$$(m\frac{d}{dt} - \frac{\partial X}{\partial u})\Delta u + \frac{\partial X}{\partial w}\Delta w + (mg\cos\theta_o)\Delta\theta = \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\delta_T \qquad 6.74$$

The equation can be rewritten in a more convenient form by dividing through by the mass *m:*

$$(\frac{d}{dt} - X_u)\Delta u + X_w\Delta w + (g\cos\theta_o)\Delta\theta = X_{\delta_e}\Delta\delta_e + X_{\delta_T}\Delta\delta_T \qquad 6.75$$

Where $m\Delta X = \frac{\partial X}{\partial u}$ and so on are aerodynamic derivatives divided by the airplane's mass.

The change in aerodynamic forces and moments are functions of the motion variables Δu, Δw and so forth. The aerodynamic derivatives usually the most important for conventional airplane motion analysis follow:

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\Delta\delta_T \qquad 6.76$$

$$\Delta Y = \frac{\partial Y}{\partial v}\Delta v + \frac{\partial Y}{\partial p}\Delta p + \frac{\partial Y}{\partial r}\Delta r + \frac{\partial Y}{\partial \delta_r}\Delta\delta_r \qquad 6.77$$

$$\Delta Z = \frac{\partial Z}{\partial u}\Delta u + \frac{\partial Z}{\partial w}\Delta w + \frac{\partial Z}{\partial \dot{w}}\Delta \dot{w} + \frac{\partial Z}{\partial q}\Delta q + \frac{\partial Z}{\partial \delta_e}\Delta\delta_e + \frac{\partial Z}{\partial \delta_T}\Delta\delta_T \qquad 6.78$$

$$\Delta L = \frac{\partial L}{\partial v}\Delta v + \frac{\partial L}{\partial p}\Delta p + \frac{\partial L}{\partial r}\Delta r + \frac{\partial L}{\partial \delta_r}\Delta\delta_r + \frac{\partial L}{\partial \delta_a}\Delta\delta_a \qquad 6.79$$

$$\Delta M = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial w}\Delta w + \frac{\partial M}{\partial \dot{w}}\Delta \dot{w} + \frac{\partial M}{\partial q}\Delta q + \frac{\partial M}{\partial \delta_e}\Delta\delta_e + \frac{\partial M}{\partial \delta_T}\Delta\delta_T \qquad 6.80$$

$$\Delta N = \frac{\partial N}{\partial v}\Delta v + \frac{\partial N}{\partial p}\Delta p + \frac{\partial N}{\partial r}\Delta r + \frac{\partial N}{\partial \delta_r}\Delta\delta_r + \frac{\partial N}{\partial \delta_a}\Delta\delta_a \qquad 6.81$$

The aerodynamic forces and moments can be expressed as a function of all the motion variables; however, in these equations only the terms that are usually significant have been retained. Note also that the longitudinal aerodynamic control surface was assumed to be an elevator. For aircraft that use either a canard or combination of longitudinal controls, the elevator terms in the preceding equations can be replaced by the appropriate control derivatives and angular deflections. The complete set of linearized equations of motion is presented below

6.3.1 The linearized small disturbance longitudinal rigid body equation of motion

$$\left(\frac{d}{dt}-X_u\right)\Delta u - X_w\Delta w + (g\cos\theta_o)\Delta\theta = X_{\delta_e}\Delta\delta_e + X_{\delta_T}\Delta\delta_T \qquad 6.82$$

$$-Z_u\Delta u + \left((1-Z_{\dot{w}})\frac{d}{dt}-Z_w\right)\Delta w + \left((u_o+Z_q)\frac{d}{dt}-g\sin\theta_o\right)\Delta\theta = Z_{\delta_e}\Delta\delta_e + Z_{\delta_T}\Delta\delta_T \qquad 6.83$$

$$-M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt}-M_w\right)\Delta w + \left(\frac{d^2}{dt^2}-M_q\frac{d}{dt}\right)\Delta\theta = M_{\delta_e}\Delta\delta_e + N_{\delta_T}\Delta\delta_T \qquad 6.84$$

6.3.2 The linearized small disturbance lateral rigid body equation of motion

$$\left(\frac{d}{dt}-Y_v\right)\Delta\nu - Y_p\Delta p + (u_o - Y_r)\Delta r - g\cos\theta_o\Delta\Phi = Y_{\delta_r}\Delta\delta_r \qquad 6.85$$

$$-L_v\Delta\nu - \left(\frac{d}{dt}-L_p\right)\Delta p + \left(\frac{I_{xz}}{I_x}\frac{d}{dt}-L_r\right)\Delta r = L_{\delta_a}\Delta\delta_a + L_{\delta_r}\Delta\delta_r \qquad 6.85a$$

$$-N_v\Delta\nu - \left(\frac{I_{xz}}{I_x}\frac{d}{dt}-N_p\right)\Delta p + \left(\frac{d}{dt}-N_r\right)\Delta r = N_{\delta_a}\Delta\delta_a + N_{\delta_r}\Delta\delta_r \qquad 6.86$$

6.4 Aircraft Equations for Longitudinal and Lateral Directions

A brief summary of all the equations of longitudinal and lateral stability coefficients and longitudinal and lateral stability derivatives are presented in the following sections

6.4.1 Equations for Estimating Longitudinal Stability Coefficients

6.4.1.1 X-Force Derivative

U	$C_{X_U} = -[C_{D_U} + C_{D_0}] + C_{T_U}$	6.87
α	$\mathbf{C_{X_\alpha} = C_{L_0} - \frac{2C_{L_0}}{\prod e} \frac{C_{L_\alpha}}{AR}}$	6.88
$\dot{\alpha}$	0	6.89
q	0	6.90
α_e	0	6.91

6.4.1.2 Z-Force Derivative

U	$C_{Z_U} = -\frac{M^2}{1-M^2} C_{L_0} - 2C_{L_0}$	6.92
α	$\mathbf{C_{Z_a} = -(C_{L_\alpha} + C_D)}$	6.93
$\dot{\alpha}$	$C_{Za} = -2\eta C_{L\alpha} V_H \frac{d\varepsilon}{d\alpha}$	6.94
q	$C_{z_\alpha} = -2\eta\, C_{L_\alpha} V_H$	6.95
α_e	$C_{Z\delta_e} = -C_{L\delta_e} = -\frac{S_t}{S}\, \eta\, \frac{dC_{L_t}}{d\delta_e}$	6.96

6.4.1.3 Pitching moment Derivative

U	$C_{m_U} = \frac{\partial C_m}{\partial M} M_0$	6.97
α	$C_{m_\alpha} = C_{L_{\alpha w}} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) + C_{m_{\alpha fus}} - \eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\varepsilon}{d\alpha} \right)$	6.98
$\dot{\alpha}$	$C_{m_\alpha} = -2\eta C_{L\alpha} V_H \frac{l_t}{c} \frac{d\varepsilon}{d\alpha}$	6.99
q	$C_{m_\alpha} = -2\eta\, C_{L_\alpha} V_H \frac{l_t}{c}$	6.100
α_e	$C_{m_\alpha} = -\,\eta\, V_H \frac{dC_{L_t}}{d\delta_e}$	6.101

6.4.2 Equations for Estimating Lateral Stability Coefficients

6.4.2.1 Y-Force Derivative

$$C_{y\beta} = -\eta (S\upsilon / S) C_{L\alpha\upsilon}(1 + d\alpha / d\beta) \quad 6.102$$
$$C_{yp} = C_L((AR + \cos\Lambda)/(AR + 4\cos\Lambda))\tan\Lambda \quad 6.103$$
$$C_{yr} = C_L - 2(l\upsilon / b)(C_{y\beta})_{tail} \quad 6.104$$
$$C_{y\delta r} = (S\upsilon / S)\tau C_{L\alpha\upsilon} \quad 6.105$$

6.4.2.2 Yawing moment derivatives

$$C_{n\beta} = C_{n\beta wf} + \eta_\upsilon V\upsilon C_{L\alpha}(1 + d\alpha / d\beta) \quad 6.106$$
$$C_{np} = -C_L / 8 \quad 6.107$$
$$C_{nr} = -2\eta_\upsilon V_\upsilon (l_\upsilon / b) C_{L\upsilon} \quad 6.108$$
$$C_{n\delta\alpha} = 2KC_{L\upsilon} C_{l\delta\beta} \quad 6.109$$
$$C_{n\delta r} = -V\eta\tau C_{L\alpha\upsilon} \quad 6.110$$

6.4.2.3 Rolling moment derivatives

$$C_{l\beta} = (C_{l\beta} / \Gamma)\Gamma + \Delta C_{l\beta} \quad 6.111$$
$$C_{lp} = -(C_{L\alpha} / 12)((1 + 3\lambda)/(1 + \lambda)) \quad 6.112$$
$$C_{lr} = C_L / 4 - 2(l\upsilon / B)(Z\upsilon / b) C_{y\beta tail} \quad 6.113$$
$$C_{l\delta a} = 2(C_{L\alpha}\tau / Sb)\int_{y_1}^{y_2} cydy \quad 6.114$$
$$C_{l\delta r} = S\upsilon / S(z\upsilon / b)\tau C_{L\alpha w} \quad 6.115$$

6.4.3 Equations for Estimating Longitudinal Directional Derivatives

$$X_u = \frac{-(C_{D_U} + 2C_{D_0})QS}{mu_0} \; (s^{-1}) \quad 6.116$$
$$X_W = \frac{-(C_{D_\alpha} - C_{L_0})QS}{mu_0} \; (s^{-1}) \quad 6.117$$

$$Z_U = \frac{-(C_{L_u} + 2C_{L_0})QS}{mu_0}\ (s^{-1}) \qquad 6.118$$

$$Z_W = \frac{-(C_{L_\alpha} + C_{D_0})QS}{mu_0}\ (s^{-1}) \qquad 6.119$$

$$Z_{\dot{W}} = -C_{Z_{\dot{\alpha}}}\ \frac{c}{2u_0} QS/u_0 m \qquad 6.120$$

$$Z_\alpha = u_0 Z_w\ (ft/s^2)\ or\ (m/s^2) \qquad 6.121$$

$$Z_{\dot{\alpha}} = u_0 Z_{\dot{W}}\ (ft/s)\ or\ (m/s) \qquad 6.122$$

$$Z_q = -C_{Z_q}\ \frac{C}{2u_0} QS/m\ (ft/s)\ or\ (m/s) \qquad 6.123$$

$$Z_{\delta_e} = -c_{Z_{\delta_e}} QS/m\ (ft/S^2) \qquad 6.124$$

$$M_u = C_{m_u}\ \frac{(QSc)}{u_0 I_Y}\left(\frac{1}{ft\cdot s}\right) or \left(\frac{1}{m\cdot s}\right) \qquad 6.125$$

$$M_w = C_{m_\alpha}\ \frac{(QS\bar{c})}{u_0 I_Y}\left(\frac{1}{ft\cdot s}\right)\ or\ \left(\frac{1}{m\cdot s}\right) \qquad 6.126$$

$$M_{\dot{w}} = C_{m_{\dot{\alpha}}}\ \frac{\bar{c}}{2u_0}\frac{QS\bar{c}}{u_0 I_y}(ft^{-1}) \qquad 6.127$$

$$M_\alpha = u_0 M_w\ (s^{-2}) \qquad 6.128$$

$$M_{\dot{\alpha}} = u_0 M_{\dot{w}}\ (s^{-1}) \qquad 6.129$$

$$M_q = C_{m_q}\ \frac{\bar{c}}{2u_0}(QS\bar{c})/I_y\ (s^{-1}) \qquad 6.130$$

$$M_{\delta_e} = C_{m\delta_e}\ (QS\bar{c})/I_y\ (s^{-2}) \qquad 6.131$$

6.4.4 Equations for Estimating Lateral Directional Derivatives

$$Y_\beta = \frac{QSC_{y\beta}}{m}\ (ft/s^2)\ or\ (m/s^2) \qquad 6.132$$

$$Y_p = \frac{QSbC_{yp}}{2mu_0}\ (ft/s)\ (m/s) \qquad 6.133$$

$$Y_r = \frac{QSbC_{Yr}}{2mu_0}\ (ft/s)\ or\ (m/s) \qquad 6.134$$

$$Y_{\delta_\alpha} = \frac{QSC_{y\delta_\alpha}}{m}(ft/s^2) or (m/s^2) \qquad 6.135$$

$$N_{\delta_\alpha} = \frac{QSbC_{n\delta_\alpha}}{I} \ (s^{-2}) \tag{6.136}$$

$$L_{\delta_\alpha} = \frac{QSbC_{l\delta_\alpha}}{I_x} (s^{-2}) \tag{6.137}$$

$$N_\beta = \frac{QSbC_{n\beta}}{I_Z} \ (s^{-2}) \tag{6.138}$$

$$N_P = \frac{QSb^2C_{np}}{2I_x u_0} \ (S^{-1}) \tag{6.139}$$

$$N_r = \frac{QSb^2C_{nr}}{2I_x u_0} (s^{-1}) \tag{6.140}$$

$$Y_{\delta_r} = \frac{QSC_{y\delta_r}}{m} \ (ft/s^2) \ or \ (m/s^2) \tag{6.141}$$

$$N_{\delta_r} = \frac{QSbC_{n\delta_r}}{I_z} \ (s^{-2}) \tag{6.142}$$

$$L_{\delta_r} = \frac{QSbC_{l\delta_r}}{I_x} \ (s^{-2}) \tag{6.142}$$

$$L_\beta = \frac{QSbC_{l\beta}}{I_x} (s^{-2}) \tag{6.143}$$

$$L_p = \frac{QSb^2C_{lp}}{2l_x u_0} \ (s^{-1}) \tag{6.145}$$

$$L_r = \frac{QSb^2C_{lr}}{2I_x u_0} \ (s^{-1}) \tag{6.146}$$

CHAPTER 7

FLYING QUALITIES & CLASSES OF AIRPLANE

7.1 Flying Qualities

The flying qualities of an airplane are related to the stability and control characteristics and can be defined as those stability and control characteristics important in forming the pilot's impression of the airplane. The pilot forms a subjective opinion about the ease or difficulty of controlling the airplane in steady and maneuvering flight. In addition to the longitudinal dynamics, the pilot's impression of the airplane is influenced by the feel of the airplane, which is provided by the stick force and stick force gradients. Normally Department of Defense and Federal Aviation Administration has a list of specifications dealing with airplane flying qualities. These requirements are used by the procuring and regulatory agencies to determine whether an airplane is acceptable for certification. The purpose of these requirements is to ensure that the airplane has flying qualities that place no limitation in the vehicle's flight safety nor restrict the ability of the airplane to perform its intended mission.

Flying qualities research provides the designer information to assess the flying qualities of a new design early in the design process. If the flying qualities are found to be inadequate then the designer can improve the handing qualities by making design changes that influence the dynamic characteristics of the airplane. A designer that follows the flying qualities guidelines can be confident that when the airplane is finally is built it will have flying qualities acceptable to its pilots.

The flying qualities are specified in terms of three levels:

Level 1 Flying qualities clearly adequate for the mission flight phase.

Level 2 Flying qualities adequate to accomplish the mission flight phase but with some increase in pilot workload and or degradation in mission effectiveness or both.

Level 3 Flying qualities such that the airplane can be controlled safely but pilot workload is excessive and/or mission effectiveness is inadequate or both. Category A night phases can be terminated safely and Category B and C flight phases can be completed.

The levels are determined on the basis of the pilot's opinion of the flying characteristics of the airplane.

7.2 Classification of airplane

As one might guess, the flying qualities expected by the pilot depend on the type of aircraft and the flight phase. Aircraft are classified according to size and maneuverability as shown below.

7.3 Flight phase categories

The flight phase is divided into three categories as shown below. Category A deals exclusively with military aircraft. Most of the flight phases listed in categories B and C are applicable to either commercial or military aircraft.

7.4 Pilot Opinion

Handling or flying qualities of an airplane are related to the dynamic and control characteristics of the airplane. For example, the short- and long-period damping ratios and un damped natural frequencies influence the pilot's opinion of how easy or difficult the airplane is to fly. Although we can calculate these qualities, the question that needs to

Table 7.1 Classification of airplanes

Class I Small, light airplanes, such as light utility, primary trainer, and light observation craft

Class II Medium-weight, low-to-medium maneuverability airplanes, such as heavy utility/search and rescue, light or medium transport/cargo/tanker, reconnaissance, tactical bomber, heavy attack and trainer for Class II.

Class III Large, heavy, low-to-medium maneuverability airplanes, such as heavy transport/cargo/tanker, heavy bomber and trainer for Class III **.**

Class IV High-maneuverability airplanes, such as fighter/interceptor, attack, tactical reconnaissance, observation and trainer for Class IV.

Table 7.2 Flight phase categories

Non-terminal flight phase

Category A Non-terminal flight phase that require rapid maneuvering, precision tracking, or precise flight-path control, Included in the category are air-to-air combat ground attack, weapon delivery/launch, aerial recovery, reconnaissance, in-night refueling (receiver), terrain-following, antisubmarine search, and close-formation flying.

Category B Nonterminal flight phases that are normally accomplished using gradual maneuvers , precision tracking , although accurate flight path control may be required. Included in the category are climb, cruise, loiter, in-flight refueling (tanker), descent, emergency descent, emergency deceleration, and aerial delivery.

Terminal flight phases

Category C Terminal flight phases are normally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this category are takeoff, catapult takeoff, approach, wave-off/go-around and landing.

be answered is what values should ξ and ω_n takes so that the pilot finds the airplane easy to fly. Researchers have studied this problem using ground-based simulators and flight test aircraft. To establish relationships between the stability and control parameters of the airplane and the pilot's opinion of the airplane a pilot rating system was developed. A variety of rating scales have been used over the years; however, the rating system proposed by Cooper and Harper has found widespread acceptance. The Cooper-Harper scale is presented in Table below. The rating scale goes from 1 to 10 with low numbers corresponding to good flying or handling qualities. The scale is an indication of the difficulty in achieving the desired performance that the pilot expects.

Table 7.3 Cooper-Harper Scales

Pilot Rating	Aircraft characteristic	Demand of pilot	Overall Assessment
1	Excellent, highly desirable	Pilot compensation not a factor for desired performance	Good Flying Qualities
2	Good, negligible deficiencies	Pilot compensation not a factor for desired performance	Good Flying Qualities
3	Pair, some mildly unpleasant deficiencies	Minimal Pilot Compensation required for desired performance	Good Flying Qualities
4	Minor but annoying deficiencies	Desired performance requires moderate pilot compensation.	Flying Qualities warrant improvement
5	Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	Flying Qualities warrant improvement
6	Very objectionable but tolerable deficiencies	Adequate performance requires extensive pilot compensation	Flying Qualities warrant improvement
7	Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation; controllability not in question	Flying Qualities deficiencies require improvement
8	Major deficiencies	Considerable pilot compensation is required for control	Flying Qualities deficiencies require improvement
9	Major deficiencies	Intense pilot compensation is required to retain control	Flying Qualities deficiencies require

			improvement
10	Major deficiencies	Control will be lost during some portion of required operation	Improvement mandatory

CHAPTER 8

LONGITUDINAL MOTION

8.1 Introduction

In the following sections we shall examine the longitudinal motion of an airplane without control input. The longitudinal motion of an airplane (control fixed) disturbed from its equilibrium flight condition is characterized by two oscillatory modes of motion.

Long-period mode: This mode of motion is lightly damped and has a long period.
Short period mode: This mode of motion is heavily damped and has a very short period.

8.2 Longitudinal Equation Of Motion

The linear-zed longitudinal equations are simple ordinary linear differential equations with constant coefficients. The coefficients in the differential equations are made up of the aerodynamic stability derivatives, mass and inertia characteristics of the airplane.

8.2.1 The linearized small disturbance longitudinal rigid body equation of motion

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w \Delta w + (g\cos\theta_o)\Delta\theta = X_{\delta_e}\Delta\delta_e + X_{\delta_T}\Delta\delta_T \qquad 8.1$$

$$-Z_u\Delta u + \left((1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right)\Delta w + \left((u_o + Z_q)\frac{d}{dt} - g\sin\theta_o\right)\Delta\theta = Z_{\delta_e}\Delta\delta_e + Z_{\delta_T}\Delta\delta_T \qquad 8.2$$

$$-M_u\Delta u-\left(M_{\dot{w}}\frac{d}{dt}-M_w\right)\Delta w+\left(\frac{d^2}{dt^2}-M_q\frac{d}{dt}\right)\Delta\theta=M_{\delta_e}\Delta\delta_e+N_{\delta_T}\Delta\delta_T \quad 8.3$$

The longitudinal equations of motion can be rearranged into state space form in following manner. (I_{xz}=0).

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{w}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix}=\begin{bmatrix}X_u & X_w & 0 & -g\\ Z_u & Z_w & u_o & 0\\ M_u+M_{\dot{w}}Z_u & M_q+M_{\dot{w}}Z_w & M_q+M_{\dot{w}}u_o & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix}$$
$$+\begin{bmatrix}X_{\delta_e} & X_{\delta_T}\\ Z_{\delta_e} & Z_{\delta_T}\\ M_{\delta_e}+M_{\dot{w}}Z_{\delta_e} & M_{\delta_T}+M_{\dot{w}}Z_{\delta_T}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta_T\end{bmatrix} \quad 8.4$$

8.3 Longitudinal Approximations

8.3.1 Long-Period Approximation

We can think of the long-period or phugoid mode as a gradual interchange of potential and kinetic energy about the equilibrium altitude and airspeed. The long-period mode is characterized by changes in pitch attitude, altitude, and velocity at a nearly constant angle of attack. Neglecting the pitching moment equation and assuming that the change in angle of attack is 0 can obtain an approximation to the long-period mode. That is

$$\Delta\alpha=\frac{\Delta w}{u_o} \qquad \Delta\alpha=0\rightarrow\Delta w=0 \quad 8.5$$

Making these assumptions the homogeneous longitudinal state equations reduce to the following

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ \dfrac{-Z_u}{u_o} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta \theta \end{bmatrix} \qquad 8.6$$

The characteristic equation is

$$\lambda^2 - X_u \lambda - \sqrt{\frac{Z_u g}{u_o}} = 0 \qquad 8.7$$

The characteristic root for this equation is

$$\lambda_p = \left[X_u \pm \sqrt{X_u^2 + 4\frac{Z_u g}{u_o}} \right] / 2 \qquad 8.8$$

The frequency and damping ratio can be expressed as

$$\omega_{n_p} = \sqrt{\frac{-Z_u g}{u_o}} \qquad 8.9$$

$$\zeta_p = \frac{-X_u}{2\omega_{n_p}} \qquad 8.10$$

If we neglect compressibility effects, the frequency and damping ratios for the long-period motion can be approximated by the following equations:

$$\omega_{n_p} = \sqrt{2}\,\frac{g}{u_o} \qquad 8.11$$

$$\zeta_p = \frac{1}{\sqrt{2}} \frac{1}{L/D} \qquad 8.12$$

Notice that the frequency of oscillation and the damping ratio are inversely proportional to the forward speed and the lift-to-drag ratio, respectively.

We see from this approximation that the phugoid damping is degraded as the aerodynamic efficiency (L/D) is increased.

When pilots are flying an airplane under visual flight rules the phugoid damping and frequency can vary over a wide range and they will still find the airplane acceptable to fly. On the other hand, if they are flying the airplane under instrument flight rules low phugoid damping will become very object able.

To improve the damping of the phugoid motion, the designer would have to reduce the lift-to-drag ratio of the airplane. Because this would degrade the performance of the airplane, the designer would find such a choice unacceptable and would look for another alternative, such as an automatic stabilization system to provide the proper damping characteristics.

8.3.2 Short-Period Approximation

An approximation to the short-period mode of motion can be obtained by assuming $\Delta u = 0$ and dropping the *X-force* equation. The longitudinal state-space equations reduce to the following

$$\begin{bmatrix} \Delta \dot{w} \\ \Delta \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & u_o \\ M_w + M_{\dot{w}} & M_q + M_{\dot{w}} u_o \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta q \end{bmatrix} \qquad 8.13$$

This equation can be written in terms of the angle of attack by using the relationship

$$\Delta\alpha = \frac{\Delta w}{u_o} \qquad 8.14$$

In addition, one can replace the derivatives due to *w* and $\dot{w}$ with derivatives due to α and $\dot{\alpha}$ by using the following equations. The definition of the derivative M_α is

$$M_\alpha = \frac{1}{I_y}\frac{\partial M}{\partial \alpha} = \frac{1}{I_y}\frac{\partial M}{\partial(\Delta w / u_o)} = \frac{u_o}{I_y}\frac{\partial M}{\partial w} = u_o M_w \qquad 8.15$$

In a similar way we can show that

$$Z_\alpha = u_o Z_w \qquad 8.16 \qquad \text{and} \qquad M_{\dot{\alpha}} = u_o M_{\dot{w}} \qquad 8.17$$

Using these expressions, the state equations for the short period approximation can be rewritten as

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} \dfrac{Z_\alpha}{u_o} & 1 \\ M_\alpha + M_{\dot{\alpha}}\dfrac{Z_\alpha}{u_o} & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} \qquad 8.18$$

The characteristic equation is

$$\lambda^2 - \left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_o} \right)\lambda + M_q \frac{Z_\alpha}{u_o} - M_\alpha = 0 \qquad 8.19$$

The approximate short-period roots can be obtained easily from the characteristic equation

$$\lambda_{sp} = \frac{\left(M_q + M_{\dot{\alpha}} + \dfrac{Z_\alpha}{u_o} \right)}{2} \pm \frac{[(M_q + M_{\dot{\alpha}} + \dfrac{Z_\alpha}{u_o})^2 - 4(M_q \dfrac{Z_\alpha}{u_o} + M_\alpha)]^{1/2}}{2} \qquad 8.20$$

or in terms of the damping and frequency

$$\omega_{n_{SP}} = \sqrt{\left(M_q \frac{Z_\alpha}{u_o} - M_\alpha \right)} \qquad 8.21$$

$$\zeta_{sp} = -\frac{\left[M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_o}\right]}{2\omega_{n_{SP}}} \qquad 8.22$$

8.4 The Influence Of Stability Derivatives On The Longitudinal Modes Of Motion

The type of response we obtain from solving the differential equations of motion depends on the magnitude of the stability coefficients. This easily can be seen by examining the expressions for the damping ratio and frequency of the long- and short-period approximations.

Of the two characteristic modes, the short-period mode is the more important. If this mode has a high frequency and is heavily damped, then the airplane will respond rapidly to an elevator input without any undesirable overshoot. When the short-period mode is lightly damped or has a relatively low frequency, the airplane will be difficult to control and in some cases may even he dangerous to fly.

The phugoid or long-period mode occurs so slowly that the pilot can easily negate the disturbance by small control movements. Even though the pilot can correct easily for the phugoid mode it would become extremely fatiguing if the damping were too low.

If the center of gravity is moved rearward the longitudinal modes become a periodic and, eventually, unstable. From a performance standpoint, it would be desirable to move the center of gravity further aft so that trim drags during the cruise portion of the flight could be reduced. Unfortunately, this leads to a less stable airplane. By using an active control stability augmentation system, the requirement of static stability can be relaxed without degrading the airplane's flying qualities.

Recent studies by the commercial aircraft industry have shown that fuel saving of 3 or 4 percent is possible if relaxed stability requirements and active control stability augmentation are incorporated into the design. With the ever-rising costs of jet fuel, this small percentage could mean the savings of many millions of dollars for the commercial airlines.

8.5 Longitudinal Flying Qualities

In the previous sections we examined the stick fixed longitudinal characteristics of an airplane. The damping and frequency of both the short- and long-period motions were determined in terms of the aerodynamic stability derivatives. Because the stability derivatives are a function of the geometric and aerodynamic characteristics of the airplane, designers have some control over the longitudinal dynamics by their selection of the vehicle's geometric and aerodynamic characteristics. For example, increasing the tail size would increase both the static stability of the airplane and the damping of the short-period motion. However, the increased tail area also would increase the weight and drag of the airplane and thereby reduce the airplane's performance. The designer is faced with the challenge of providing an airplane with optimum performance that is both safe and easy to fly. To achieve such a goal, the designer needs to know what degree of stability and control is required for the pilot to consider the airplane safe and flyable.

Table 8.2 is summary of the longitudinal specifications for the phugoid and short-period motions that is valid for all classes of aircraft.

The information provided by table provides the designer with valuable design data. As we showed earlier, the longitudinal response characteristics of an airplane are related to its stability derivatives. Because the stability derivatives are related to the airplane's geometric and aerodynamic characteristics it is possible for the designer to consider flying qualities in the preliminary design phase.

Table 8.1 Influence of stability derivatives on long and short period motion

Stability derivative	Mode affected	How affected
$M_q + M_{\dot{\alpha}}$	Damping of short period mode of motion	Increasing $M_q + M_{\dot{\alpha}}$ increases damping
M_α	Frequency of short period mode of motion	Increasing M_α or static stability increases the frequency
X_u	Damping of long period mode of motion	Increasing X_u increases damping
Z_u	Frequency of long period mode of motion	Increasing Z_u increases the frequency

Table 8.2 Longitudinal Flying Qualities

Long period mode	
Level 1	$\zeta > 0.04$
Level 2	$\zeta > 0$
Level 3	T > 55s

Short period mode				
	Category A and C		Category B	
Level	ζ_{SP} Min	ζ_{SP} Max	ζ_{SP} Min	ζ_{SP} Max
Level 1	0.35	1.3	0.3	2.0
Level 2	0.25	2.0	0.2	2.0
Level 3	0.15	--	0.15	--

CHAPTER 9

LATERAL MOTION

9.1 Introduction

The stick fixed lateral motion of an airplane disturbed from its equilibrium state is a complicated combination of rolling, yawing and side slipping motions.

An airplane produces both yawing and rolling moments due to the sideslip angle. This interaction between the roll and the yaw produces the coupled motion.
Three potential lateral dynamic instabilities are of interest to the airplane designer.

1. Directional divergence
2. Spiral divergence,
3. Dutch roll oscillation.

Directional divergence can occur when the airplane lacks directional or weathercock stability. If disturbed from its equilibrium state such an airplane will tend to rotate to ever-increasing angles of sideslip. Owing to the side force acting on the airplane, it will fly a curved path at large sideslip angles. For an airplane that has lateral static stability (i.e., dihedral effect) the motion can occur with no significant change in bank angle. Obviously, such a motion cannot be tolerated and readily can be avoided by proper design of the vertical tail surface to ensure directional stability.

Spiral divergence is a no oscillatory divergent motion that can occur when directional stability is large and lateral stability is small. When disturbed from equilibrium, the airplane enters a gradual spiraling motion. The spiral becomes tighter and steeper as time proceeds and can result in a high-speed spiral dive if corrective action is not taken. This motion normally occurs so gradually that the pilot unconsciously corrects for it.

Dutch roll oscillation is a coupled lateral-directional oscillation that can be quite objectionable to pilots and passengers. The motion is characterized by a combination of rolling and yawing oscillations that have the same frequency but are out of phase with each other. The period can be on the order of 3 to 15 seconds, so that if the amplitude is appreciable the motion can be very annoying.

9.2 Lateral-Directional Equation Of Motion

The lateral directional equation of motion consists of the side force, rolling moment and yawing moment equation of motion.

9.2.1 The linear zed small disturbance lateral rigid body equation of motion

$$\left(\frac{d}{dt} - Y_v\right)\Delta\nu - Y_p\Delta p + (u_o - Y_r)\Delta r - g\cos\theta_o\Delta\Phi = Y_{\delta_r}\Delta\delta_r \qquad 9.1$$

$$-L_v\Delta\nu - \left(\frac{d}{dt} - L_p\right)\Delta p + \left(\frac{I_{xz}}{I_x}\frac{d}{dt} - L_r\right)\Delta r = L_{\delta_a}\Delta\delta_a + L_{\delta_r}\Delta\delta_r \qquad 9.2$$

$$-N_v\Delta\nu - \left(\frac{I_{xz}}{I_x}\frac{d}{dt} - N_p\right)\Delta p + \left(\frac{d}{dt} - N_r\right)\Delta r = N_{\delta_a}\Delta\delta_a + N_{\delta_r}\Delta\delta_r \qquad 9.3$$

The lateral equations of motion can be rearranged into state space form in following manner. (I_{xz}=0).

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\Phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & -(u_o - Y_r) & g\cos\theta_o \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\Phi \end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \qquad 9..4$$

It sometimes is convenient to use the sideslip angle $\Delta\beta$ instead of the side velocity $\Delta\nu$.

$$\Delta\beta \approx \tan^{-1}\frac{\Delta\nu}{u_o} = \frac{\Delta\nu}{u_o} \qquad 9.5$$

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\Phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_o} & \frac{Y_p}{u_o} & -\left(1 - \frac{Y_r}{u_o}\right) & \frac{g\cos\theta_o}{u_o} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\Phi \end{bmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta_r}}{u_o} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \qquad 9.6$$

In general, we will find the roots to the lateral-directional characteristic equation to be composed of two real roots and a pair of complex roots. The roots will be such that the following motions can characterize the airplane response:

1. A slowly convergent or divergent motion, called the spiral mode.
2. A highly convergent motion, called the rolling mode.
3. A lightly damped oscillatory motion having a low frequency, called the Dutch roll mode.

An unstable **spiral mode** results in a turning flight trajectory. The airplane's bank angle increases slowly and it flies in an ever-tightening spiral dive. The **rolling motion** usually is highly damped and will reach a steady state in a very short time. The combination of the yawing and rolling oscillations is called the **Dutch roll motion** because it reminded someone of the weaving motion of a Dutch ice skater.

9.3 Lateral approximation

9.3.1 Spiral Approximation

The spiral mode is characterized by changes in the bank angle Φ and the heading angle Ψ. The sideslip angle usually is quite small but cannot be neglected because the aerodynamic moments do not depend on the roll angle Φ or the heading angle Ψ but on the sideslip angle β, roll rate *p,* and yawing rate r.

The aerodynamic contributions due to β and r usually are on the same order of magnitude. Therefore, to obtain an approximation of the spiral mode we shall neglect the side force equation and $\Delta\Phi$. With these assumptions, the equation of motion becomes:

$$L_\beta \, \Delta\beta + L_r \, \Delta r = 0 \qquad 9.7$$

$$\Delta \dot{r} = N_\beta \, \Delta\beta + N_r \, \Delta r \qquad 9.8$$

$$\Delta \dot{r} + \frac{N_r L_\beta - L_r N_\beta}{L_\beta} \Delta r = 0 \qquad 9.9$$

The characteristic root for this equation is

$$\lambda_{\text{Spiral}} = \frac{L_\beta N_r - N_\beta L_r}{L_\beta} \qquad 9.10$$

The stability derivatives L_p (dihedral effect) and N_r (yaw rate damping) usually are negative quantities. On the other hand, N_p (directional stability) and L_r (roll moment due to yaw rate) generally are positive quantities. If the derivatives have the usual sign, then the condition for a stable spiral model is

$$L_\beta N_r > N_\beta L_r \qquad 9.11$$

Increasing the dihedral effect L_p or the yaw damping or both can make the spiral mode stable.

9.3.2 Roll Approximation

This motion can be approximated by the single degree of freedom (pure) rolling motion

$$\tau \Delta \dot{p} + \Delta p = 0 \qquad 9.12$$

$$\lambda_{Roll} = -\frac{1}{\tau} = L_p \qquad 9.13$$

The magnitude of the roll damping L_p is dependent on the size of the wing and tail surfaces. τ is a time constant .0

9.3.3 Dutch Roll Approximation

If we consider the Dutch roll mode to consist primarily of sideslipping and yawing motions, then we can neglect the rolling moment equation. Equation reduces to

$$\begin{bmatrix} \Delta \dot{\beta} \\ \Delta \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_o} & -\left(1 - \frac{Y_r}{u_o}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta \beta \\ \Delta r \end{bmatrix} \qquad 9.14$$

Characteristic equation

$$\lambda^2 - \left(\frac{Y_\beta + u_o N_r}{u_o}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + u_o N_\beta}{u_o} = 0 \qquad 9.15$$

From this expression we can determine the un-damped natural frequency and the damping ratio as follows:

$$\omega_{n_{DR}} = \sqrt{\frac{(Y_\beta N_r - N_\beta Y_r + u_o N_\beta)}{u_o}} \qquad 9.16$$

$$\zeta_{DR} = -\frac{1}{2\omega_{n_{DR}}}\left(\frac{Y_\beta + u_o N_r}{u_o}\right) \qquad 9.17$$

The approximations developed in this section give, at best, only a rough estimate of the spiral and Dutch roll modes. The approximate formulas should, therefore, be used with caution. The reason for the poor agreement between the approximate and exact solutions is that the Dutch roll motion is truly a three-degree-of-freedom motion with strong coupling between the equations.

9.4 Influence Of Stability Derivatives On Lateral Motion

An airplane possesses static stability in both the directional and roll modes. This implies the $C_{n_\beta} > 0$ and $C_{l_\beta} < 0$. However, if we examine the influence of these stability coefficients on the lateral roots by means of a root locus plot, we observe the following. As the dihedral effect is increased, that is, C_{l_β} becomes more negative. The Dutch roll root moves toward the right half-plane. Which means the Dutch roll root is becoming less stable and the spiral root is moving in the direction of increased stability.

Increasing directional stability of the airplane, that is, C_{n_β} becomes more positive, causes the spiral root to become less stable and the frequency of the Dutch roll root is increased. Increasing the yaw damping, that is, C_{n_r} becomes more negative, will result in better Dutch roll damping. Unfortunately, this is not easy to achieve simply by geometric design changes. Increasing the vertical tail size will cause an increase in both C_{n_β} and C_{n_r}. Many airplanes are provided with a rate damper to artificially provide adequate damping in Dutch roll.

9.5 Lateral Flying Qualities

In this section we developed the necessary equations and analysis procedures to calculate the lateral dynamics. Although these techniques tells us to determine whether an airplane design is stable or unstable, by itself the analysis does not tell us whether the pilot will judge the airplane to have acceptable flying characteristics. To determine this designer needs to know what dynamic characteristics the pilots who will fly the airplane consider favorable. This information is available through the lateral-directional flying quality specifications.

9.6 Inertial Coupling

In the analysis we treated the longitudinal and lateral equations separately. In so doing we assumed that there is no coupling between the equations. However, slender high-performance fighter aircraft can experience significant roll coupling that can result in divergence from the desired Right path, causing loss of control or structural failure.

The mechanisms that cause this undesirable behavior can be due to inertial or aerodynamic coupling of the equations of motion. To explain how inertial coupling occurs, we examine the nonlinear zed moment equations.

Moment equations

Roll moments $= I_x \dot{p} + qr(I_z - I_y) - (\dot{r} + qp) I_{xz}$ 9.19

Pitching moments $= I_x \dot{q} + pr(I_x - I_z) - (p^2 + r^2) I_{xz}$ 9.20

Yawing moments $= I_x \dot{r} + pq(I_y - I_x) - (qr + \dot{p}) I_{xz}$ 9.21

The first cases of inertial coupling started to appear when fighter aircraft designs were developed for supersonic flight. These aircraft were designed with low aspect ratio wings and long, slender fuselages. In these designs, more of the aircraft's weight was

Table 9.1 Spiral mode flying qualities

Class	Category	Level I	Level 2	Level 3
I and IV	A	12s	12s	4s
	B and C	20s	12s	4s
II and III	ALL	20s	12s	4s

Table 9.2 Roll mode flying qualities

Class	Category	Level 1	Level 2	Level 3
I , IV	A	1.0s	1.4s	10s
II , III	A	1.4s	3.0s	10s
ALL	B	1.4s	3.0s	10s
I,IV	C	1.0s	1.4s	10s
II,III	C	1.4s	3.0s	10s

Table 9.3 Dutch roll flying qualities

Level	Category	Class	Min ξ *	Min $\xi\omega_{n,*}$	Min ω_n
1	A	I, IV	0.19	0.35 rad/s	1.0 rad/s
		II , III	0.19	0.35 rad/s	0.4 rad/s
	B	All	0.08	0.15 rad/s	0.4 rad/s
	C	I, II-C	0.08	0.15 rad/s	1.0 rad/s
		IV			
		II-L, III	0.08	0.15 rad/s	0.4 rad/s
2	All	All	0.02	0.15 rad/s	0.4 rad/s
3	All	All	0.02		0.4 rad/s

Where C and *L* denote carrier- or land-based aircraft.

*The governing damping requirement is that yielding the larger value of ξ .

concentrated in the fuselage than in the earlier subsonic fighters. With the weight concentrated in the fuselage, the moments of inertia around the pitch angle yaw axis increased and the inertia around the roll axis decreased in comparison with subsonic fighter aircraft.

On examining above Equation we see that the second term in the pitch equation could be significant if the difference in the moments of inertia becomes large. For the case of a slender high-performance fighter executing a rapid rolling maneuver the term $pr(I_x - I_z)$ can become large enough to produce an uncontrollable pitching motion.

A similar argument can be made for the product of inertia terms in the equations of motion. The product of inertia I_{xy} is a measure of the uniformity of the distribution of mass about the *x*-axis. For modern fighter aircraft I_{xy} typically is not 0. Again we see that if the airplane is executing a rapid roll maneuver the term $(p^2+ r^2)I_{xz}$ may be as significant as the other terms in the equation.

Finally, aerodynamic coupling also must be considered when aircraft are maneuvering at high angular rates or at high angles of attack. As was discussed in Chapter 4 high angle of attack flow asymmetries can cause out-of-plane forces and moments even for symmetric flight conditions. Such forces and moments couple the longitudinal and lateral equations of motion.

CHAPTER 10

AIRCRAFT TRANSFER FUNCTIONS

Avery useful concept in the analysis and design of control system is the transfer function. The transfer function gives the relationship between the output of and input to the system. In the case of aircraft dynamics it specifies the relationship between the motion variable and the control input. The transfer function is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input with all the initial conditions set to 0. (I.e., the system is assumed to be initially in equilibrium). In the following sections we develop the transfer function based on the longitudinal and lateral approximations developed earlier. We develop these simpler and mathematical models so that we can examine the idea behind the various autopilots without undue mathematical complexity.

10.1 Short-Period Dynamics

In earlier chapters the equations for the short-period motions were developed for the case where the control was held fixed. The equation with control input from the elevator in state space form can be written as

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha/u_0 & 1 \\ M_a + M_a Z_\alpha/u_0 & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e}/u_0 \\ M_{\delta_e} + M_{\dot{\alpha}} Z_{\delta_e}/u_0 \end{bmatrix} [\Delta\delta_e] \qquad 10.1$$

The control due to the propulsion system is neglected here for simplicity. Taking the Laplace transform of this equation yields

$$(s - Z_\alpha / u_0)\,\Delta\alpha(s) - \Delta q(s) = Z_{\delta_e} \big/ u_0 \;\; \Delta\delta_e(s) \qquad 10.2$$

$$-(M_\alpha + M_\alpha Z_\alpha / u_0)\,\Delta\alpha(s) + [s - (M_q + M_{\dot{\alpha}})]\,\Delta q(s) = (M_{\delta_e} + M_{\dot{\alpha}} Z_{\delta_e} / u_0)\,\Delta\delta_e \qquad 10.3$$

If we divide these equations by $\Delta\delta_e(s)$ we obtain a set of algebraic equations in terms of the transfer functions $\Delta\alpha(s)/\Delta\delta_e(s)$ and $\Delta q(s)/\Delta\delta_e(s)$

$$(s - Z_\alpha \big/ u_0)\,\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} - \frac{\Delta q(s)}{\Delta\delta_e(s)} = Z_{\delta_e} \big/ u_0 \qquad 10.4$$

$$-(M_\alpha + M_{\dot{\alpha}} Z_\alpha / u_0)\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} + [s - (M_q + M_{\dot{\alpha}})]\frac{\Delta q(s)}{\Delta\delta_e(s)} = M_{\delta_e} + M_{\dot{\alpha}}\frac{Z_{\delta_e}}{u_o} \qquad 10.5$$

Solving for $\Delta\alpha(s)/\Delta\delta_e(s)$ and $\Delta q(s)/\Delta\delta_e(s)$ by Cramer's rule yields

$$\frac{\Delta\alpha}{\Delta\delta_e} = \frac{N^\alpha_{\delta_e}}{\Delta_{sp}} = \frac{\begin{vmatrix} Z_{\delta_e}/u_o & -1 \\ M_{\delta_e} + M_{\dot{\alpha}}\dfrac{Z_{\delta_e}}{u_o} & s - (M_q + M_{\dot{\alpha}}) \end{vmatrix}}{\begin{vmatrix} s - Z_\alpha / u_o & -1 \\ -(M_\alpha + M_{\dot{\alpha}} Z_\alpha / u_o) & s - (M_q + M_{\dot{\alpha}}) \end{vmatrix}} \qquad 10.6$$

$$\frac{\Delta\alpha}{\Delta\delta_e} = \frac{N^\alpha_{\delta_e}}{\Delta_{sp}} = \frac{A_\alpha + B_\alpha}{As^2 + Bs + C} \qquad 10.7$$

Where the coefficients in the numerator and denominator are given in Table 10.1

The transfer function for the change in pitch rate to the change in elevator angle can be shown to be

$$\frac{\Delta q}{\Delta \delta_e} = \frac{N_{\delta_e}^{q}}{\Delta_{sp}} = \frac{\begin{vmatrix} s - Z_\alpha / u_o & Z_{\delta_e} / u_o \\ (M_\alpha + M_{\dot{\alpha}} \dfrac{Z_\alpha}{u_o} & M_{\delta_e} + M_{\dot{\alpha}} \dfrac{Z_{\delta_e}}{u_o}) \end{vmatrix}}{\begin{vmatrix} s - Z_\alpha / u_o & -1 \\ -(M_\alpha + M_{\dot{\alpha}} Z_\alpha / u_o) & s - (M_q + M_{\dot{\alpha}}) \end{vmatrix}} \qquad 10.8$$

$$\frac{\Delta q}{\Delta \delta_e} = \frac{N_{\delta_e}^{q}}{\Delta_{sp}} = \frac{A_q + B_q}{As^2 + Bs + C} \qquad 10.9$$

Again the coefficients of the polynomials are defined in Table 11.1.

10.2 Long Period or Phugoid Dynamics

The state-space equation for the long period or phugoid approximation is its follows:

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ \dfrac{Z_u}{u_0} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ -\dfrac{Z_{\delta_e}}{u_0} & -\dfrac{Z_{\delta_T}}{u_0} \end{bmatrix} \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_T \end{bmatrix} \qquad 10.10$$

The Laplace transformation of the approximate equations for long period is

$$(s - X_u)\ \Delta u(s)\ + g\ \Delta\theta = X_{\delta_e}\ \Delta\delta_e(s)\ +\ X_{\theta_T}\ \Delta\theta_T(s) \qquad 10.11$$

$$\frac{Z_u}{u_0}\ \Delta u(s) + s\,\Delta\theta(s)\ =\ -\frac{Z_{\delta_e}}{u_0}\ \Delta\delta_e - \frac{Z_{\delta_T}}{u_0}\ \Delta\delta_T(s) \qquad 10.12$$

The transfer function $\Delta u(s) / \Delta\delta_e(s)$ and $\Delta\theta(s)\ / \Delta\delta_e(s)$ can be found by setting $\Delta\delta_T(s)$ to 0 and solving for the appropriate transfer functions as follows:

$$(s - X_u)\frac{\Delta u(s)}{\Delta\delta_e} + g\frac{\Delta\theta(s)}{\Delta\theta_e(s)}\ =\ X_{\delta_e} \qquad 10.13$$

Table 10.1 Short–Period transfer function approximation

	A, A_α or A_q	B, B_α or B_q	C
$\Delta_p(s)$	1	$-(M_q + M_{\dot{\alpha}} + Z_\alpha / u_o)$	$-Z_\alpha M_q / u_0 - M_\alpha$
$N^\alpha_{\delta_e}(s)$	Z_{δ_e} / u_o	$M_{\delta_e} - M_q Z_{\delta_e} / u_0$	
$N^q_{\delta_e}(s)$	$M_{\delta_e} + M_\alpha Z_{\delta_e} / u_0$	$M_\alpha Z_{\delta_e} / u_0 - M_{\delta_e} Z_\alpha / u_0$	

$$\frac{Z_u}{u_0}\frac{\Delta u(s)}{\Delta\theta_e(s)} + s\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = -\frac{Z_{\delta_e}}{u_0} \qquad 10.14$$

The equations of motion have been reduced to a set of algebraic equations in terms of the desired transfer function. These equations can be solved to yield the transfer functions

$$\frac{\Delta u(s)}{\Delta\theta_e(s)} = \frac{\begin{vmatrix} X_{\delta_e} & g \\ -\dfrac{Z_{\delta_e}}{u_0} & s \end{vmatrix}}{\begin{vmatrix} s - X_u & g \\ Z & s \end{vmatrix}} \qquad 10.15$$

$$\frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{X_{\delta_e} s + g Z_{\delta_e}/u_0}{s^2 + X_u s - \dfrac{Z_u g}{u_0}} \qquad 10.16$$

In a similar manner $\Delta\theta(s)/\Delta\delta(s)$ can be shown to be

$$\frac{\Delta\theta(S)}{\Delta\delta_e(S)} = \frac{-\dfrac{Z_{\delta_e}}{u_0}s + \left(\dfrac{X_u Z_{\delta_e}}{u_0} - \dfrac{Z_u X_{\delta_e}}{u_0}\right)}{s^2 - X_u s - \dfrac{Z_u g}{u_0}} \qquad 10.17$$

The transfer functions can be written in a symbolic form in the following manner:

$$\frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{N^u{}_{\delta_e}(s)}{\Delta_p(s)} = \frac{A_u + B_u}{As^2 + Bs + C} \qquad 10.18$$

$$\frac{\Delta\theta(s)}{\Delta\delta(s)} = \frac{N^{\theta}{}_{\delta_e}}{\Delta_p(s)} = \frac{A_0 + B_0}{As^2 + Bs + C} \qquad 10.19$$

Where A_u, B_u, and so forth are defined in Table 11.2. The transfer functions for the propulsive control, that is, $\Delta u / \Delta\delta_T$ and $\Delta\theta / \Delta\delta_T$ has the same form except that the derivatives with respect to δ_e are replaced by derivatives with respect to δ_T. Therefore, Table 11.2 can be used for both aerodynamic and propulsive control transfer functions provided that the appropriate control derivatives are used.

10.3 Roll Dynamics

The equation of motion for a pure rolling motion, developed in earlier is

$$\Delta\dot{p} - L_p\ \Delta p = L_{\delta_\alpha}\ \Delta\delta_a \qquad 10.20$$

The transfer function $\Delta p / \Delta\delta_a$ and $\Delta\Phi / \Delta\delta_a$ can be obtained by taking the Laplace transform of the roll equation:

$$\frac{\Delta p(s)}{\Delta\delta(s)} = \frac{L_{\delta_\alpha}}{s - L_p} \qquad 10.21$$

But the roll rate Δp is defined as $\Delta\dot{\Phi}$ therefore,

$$\Delta p(s) = s\,\Delta\phi(S) \qquad 10.22$$

$$\frac{\Delta\phi(s)}{\Delta\delta_\alpha(s)} = \frac{L_{\delta_\alpha}}{s(s - L_p)} \qquad 10.23$$

10.4 Dutch Roll Approximations

The final simplified transfer function we will develop is for the Dutch roll motion. The approximate equations can be shown to be

<u>Table 10.2 Long –Period transfer function approximation</u>

	$\boldsymbol{A, A_u}$ ***or*** $\boldsymbol{A_0}$	$\boldsymbol{B}$, $\boldsymbol{B_u}$ ***or*** $\boldsymbol{B_0}$	$\boldsymbol{C}$
$\Delta_p(s)$	1	$-X_u$	$-Z_u g/u_0$
$N^u{}_{\delta_e}(s)$	X_{δ_e}	$g Z_{\delta_e}/u_0$	
$N^\theta{}_{\delta e}(s)$	$-Z_{\delta_e}/u_0$	$X_u Z_{\delta_e}/u_0 - Z_u X_{\delta_e}/u_0$	

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{Y_\beta}{u_0} & -(1-\dfrac{Y_r}{u_0}) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \dfrac{Y_{\delta_r}}{u_0} & 0 \\ N_{\delta_r} & N_{\delta_\alpha} \end{bmatrix} \begin{bmatrix} \Delta\delta_r \\ \Delta\delta_\alpha \end{bmatrix} \qquad 10.24$$

Taking the Laplace transform and rearranging yields

$$(s - {Y_\beta}/{u_0})\,\Delta\beta(S) + (1 + Y_r/u_0)\,\Delta r(s) = Y_{\delta_r}/u_0\ \Delta\delta_r(s) \qquad 10.25$$

$$N_\beta\,\Delta\beta(s) + (s - N_r)\,\Delta r(s) = N_{\delta_a}\ \Delta\delta_a(s) + N_{\delta_r}\ \Delta\delta_r(s) \qquad 10.26$$

The transfer equations $\Delta\beta/\Delta\delta_r$, $\Delta r/\Delta\delta_r$, $\Delta\beta/\Delta\delta_a$ and $\Delta r/\Delta\delta_a$ can be obtained by setting $\delta_a(s)$ to 0 and solving for $\Delta\beta/\Delta\delta_r$ and $\Delta r/\Delta\delta_r$. Next set $\delta_r(s)$ equal to 0 and solve for $\Delta\beta/\Delta\delta_a$ and $\Delta r/\Delta\delta_a$. The transfer function $\Delta\beta/\Delta\delta_r$ and $\Delta r/\Delta\delta_r$ are obtained as follows

$$(s - Y_\beta/u_0)\,\frac{\Delta\beta(s)}{\Delta\delta_r(s)} + (1 + Y_r/u_0)\,\frac{\Delta r(s)}{\Delta\delta_r(S)} = Y_{\delta_r}/u_0 \qquad 10.27$$

$$-N_\beta\,\frac{\Delta\beta(s)}{\Delta\delta_r(s)} + (s - N_r)\,\frac{\Delta r(s)}{\Delta\delta_r(s)} = N_{\delta_r} \qquad 10.28$$

Solving for the transfer function yields

$$\frac{\Delta\beta(S)}{\Delta\delta_r(s)} = \frac{\begin{bmatrix} Y_{\delta_r}/u_0 & 1 - Y_r/u_0 \\ N_{\delta_r} & s - N_r \end{bmatrix}}{\begin{bmatrix} s - Y_\beta/u_0 & 1 - Y_r/u_0 \\ -N_\beta & s - N_r \end{bmatrix}} \qquad 10.29$$

$$\frac{\Delta\beta(s)}{\Delta\delta_r(S)} = \frac{\begin{bmatrix} s - Y_\beta/u_0 & 1 - Y_r/u_0 \\ -N_\beta & N_{\delta_r} \end{bmatrix}}{\begin{bmatrix} s - Y_\beta/u_0 & 1 - Y_r/u_0 \\ -N_\beta & s - N_r \end{bmatrix}} \qquad 10.30$$

$$\frac{\Delta\beta(s)}{\Delta\delta_r(s)} = \frac{N^{\beta}_{\delta_r}(s)}{\Delta_{DR}(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C} \qquad 10.31$$

$$\frac{\Delta r\ (s)}{\Delta\delta_r(s)} = \frac{N^{\beta}_{\delta_r}(s)}{\Delta_{DR}(s)} = \frac{A_r s + B_r}{As^2 + Bs + C} \qquad 10.32$$

In a similar manner the aileron transfer function can be shown to be

$$\frac{\Delta\beta(s)}{\Delta\delta_\alpha(s)} = \frac{N^{\beta}_{\delta_\alpha}(s)}{\Delta_{DR}(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C} \qquad 10.33$$

$$\frac{\Delta r\ (s)}{\Delta\delta_\alpha(s)} = \frac{N^{\beta}_{\delta_\alpha}(s)}{\Delta_{DR}(s)} = \frac{A_r s + B_r}{As^2 + Bs + C} \qquad 10.34$$

The coefficients, of the polynomials in the Dutch roll transfer functions are included in Table 11.3 The denominator coefficients are in the first row and the numerator coefficients are defined for each transfer function in the subsequent row

10.5 Longitudinal and Lateral control transfer function

In the previous section, transfer functions were derived for both longitudinal and lateral dynamics based on the approximations to these motions. For a preliminary autopilot design these approximations are appropriate. However, as the autopilot concept is refined and developed it is necessary to examine the autopilot performance using transfer functions based on the complete set of either longitudinal or lateral equations. This is particularly important for the late equations. As we showed earlier the lateral approximations do not general give a very accurate representation of the Dutch ' roll motion.

The longitudinal and lateral transfer functions for the complete set of equation are determined in the same manner as the approximate transfer functions derived here. The transfer functions for the complete set of rigid body equations are give in below

Table 10.3 Dutch roll transfer function approximation

	A, B_β or A_r	B, B_β or B_r	C
$\Delta_{DR}(s)$	1	$-(Y_\beta + u_0 N_r)/u_0$	$(Y_\beta N_r - N_\beta Y_r + N_\beta u_0)/u_0$
$N^{\beta}_{\delta_r}(s)$	Y_r/u_0	$(Y_r N_{\delta_r} - Y_{\delta_r} N_r - N_{\delta_r} u_0)/u_0$	
$N^{r}_{\delta_r}(s)$	N_{δ_r}	$(N_\beta Y_{\delta_r} - Y_\beta N_{\delta_r})/u_0$	
$N^{\beta}_{\delta_\alpha}(s)$	0	$(Y_r N_{\delta_\alpha} - u_0 N_{\delta_\alpha})/u_0$	
$N^{r}_{\delta_\alpha}(s)$	N_{δ_α}	$-Y_\beta N_{\delta_\alpha}/u_0$	

10.4 Longitudinal Control transfer function

	A	B	C	D	E
Δ_{long}	1	$-M - u_0 M_w - Z_w - X_u$	$Z_w M_{q0} - u_0 M_w - X_w Z_u$ $+ X_u (M_q + u_0 M_w + Z_w)$	$-X(Z_w M_\theta - u_0 M_w)$ $+ Z_\alpha (X_w M_q + gM_w)$ $-M(u_0 X_w - g)$	$g(Z_u M_u - M_u Z_w)$
N_δ^θ	$M_\delta + Z_\delta M_w$	$X_\delta (Z_u M_w + M_u)$ $+ Z_\delta (M_w - X_u M_w)$ $-l(X_u + Z_w)$	$X_\delta (Z_u M_w - Z_u M_u)$ $+ Z_\delta (M_u X_w - M_w X_u)$ $+ M_\delta (Z_w X_u - X_w Z_u)$		
N_δ^w	Z_δ	$X_\delta Z_w - Z_\delta (X_u + M_q) + M_\delta u_0$	$X_\delta (u_0 M_u - Z_u M_q)$ $+ Z_\delta X_u M_q - u_0 M_\delta X_u$	$g(Z_\delta M_w - M_\delta Z_u)$	
N_δ^u	X_δ	$-X_\delta (Z_w + M_q + u_0 M_w) + Z_\delta X_w$	$X_\delta (Z_w M_q - u_0 M_w)$ $- Z_\delta (X_w M_q + gM_w)$ $+ M_\delta (u_0 X_w - g)$	$g(M_\delta Z_w - Z_\delta M_w)$	

Table10.5 Lateral control transfer function

	A	B	C	D	E
Δ_{long}	$!-\frac{I_{XZ}^2}{I_X I_Z}$	$-Y_v(1-\frac{I_{XZ}^2}{I_x I_Z})+L_p-N_r$	$u_0 N_v - L_p(Y_v + N_\rho)$ $+N_p(\frac{I_{xz}}{I_x}Y_c - L)$	$-u_0 N_v L_p - y(N_p L_r - L_p N_r)$	$g(L_v N_r - N_v L_r)$
N_δ^θ	$Y_\delta(1-\frac{I_{xz}^2}{I_x I_z})$	$-Y_\delta(L_p + N_r$ $+\frac{I_{XZ}}{I_X}N_p + \frac{I_{XZ}}{I_Z}L_r)$	$Y_\delta(L_p N_r - N_p L_r)$ $+u_0(N_\delta L_p - L_\delta N_p)$	$g(N_\delta L_r - L_\delta N_r)$	
N_δ^ϕ	$L_\delta + \frac{I_{XZ}}{I_X}N_\delta$	$Y_\delta(L_v + \frac{I_{XZ}}{I_X}N_v)$ $-L_\delta(N_r + Y_v)$ $+N_\delta(L_r - \frac{I_{XZ}}{I_X}Y_v)$	$Y_\delta(L_r N_v - L_v N_r) + L_\delta(Y_n N_r + u_0 N_v)$ $-N_\delta(u_0 L_v + Y_v L_r)$		
N_δ^r	$N_\delta + \frac{I_{XZ}}{I_Z}$	$Y_\delta(N_v - \frac{I_{XZ}}{I_X}L_v)$ $+L_\delta(N_p - \frac{I_{XZ}}{z}Y_v)$ $-N_\delta(Y_v + L_p)$	$Y_\delta(L_v N_p - N_v L_p) - L_\delta Y_v N_p + N_\delta Y_v L_p$	$g(L_\delta N_v - N_\delta L_v)$	

CHAPTER 11

AUTOPILOT DESIGN

The rapid advancement of aircraft design from the very limited capabilities of the Wright brothers' first successful airplane to today's high performance military, commercial, and general aviation aircraft required the development of many technologies: aerodynamics, structures, materials propulsions and flight controls. To day's aircraft design relay heavily on automatic control systems to monitor and control many of the aircrafts subsystems.

The development of automobile control systems has played an important role in the growth of civil and military aviation. Modern Aircraft include a variety of the automatic control systems that aid the flight crew in navigation, flight management, and augmenting the stability characteristics of the airplane. In this section we design simple autopilots that can be used by flight crew to lessen their workload during cruising and help them land their aircraft during adverse weather conditions. In addition we also discuss how control systems can be used to provide artificial stability to improve the flying qualities of the airplane.

The development of autopilots closely followed the successful development of a powered, human carrying aircraft by the Wright brothers. In 1914 the Sperry brothers demonstrated the first successful autopilot. The autopilot was capable of maintaining pitch, roll and heading angles. To demonstrate the effectiveness of their design, Lawrence Sperry trimmed his airplane for straight and level flight and then engaged the autopilot. He then proceeded to stand in the cockpit with his hands above his mechanic walked out along the wings in an attempt to upset the airplane's equilibrium. The autopilot provide aileron, rudder and elevator commands so that the airplane remained in wing level attitude.

11.1 Automatic flight control system

Flight control system to reduce pilot workload

- Attitude control systems to maintain pitch roll or heading also called displacement autopilot
- Altitude-hold control systems to maintain a desired altitude
- Speed control system to maintain a constant speed or Mach number

Stability augmentation systems

- If an airplane is marginally stable or unstable automatic control systems can provide proper flight vehicle stability.
- Automatic control can be used to ensure an airplane has the appropriate handling qualities additional damping is incorporated by using a roll pitch or yaw damper

Landing aids

- A glide slope control system to guide the airplane down an electronic beam to the runway
- A localizer to align the aircraft in the lateral direction with the runway centerline as the airplane descends down the glide slope
- A flare control system that helps the aircraft make the transition from the glide slope to the runway

11.2 Displacement Autopilot

One of the earliest autopilots to be used for aircraft control is the so-called displacement autopilot. Displacement autopilot is used to control the angular orientation of the airplane.

Types of displacement autopilot

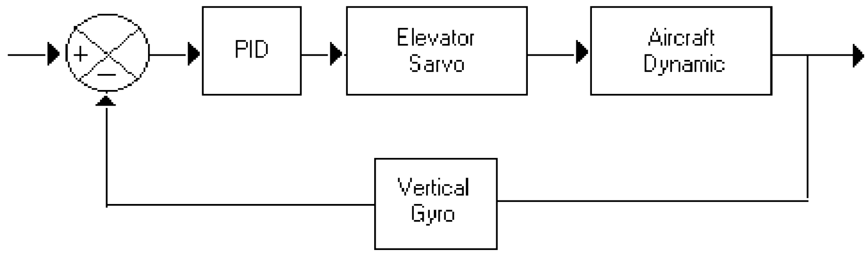
- Pitch Displacement Autopilot
- Roll Displacement Autopilot
- Heading Displacement Autopilot

11.2.1 Overview of Displacement Autopilot

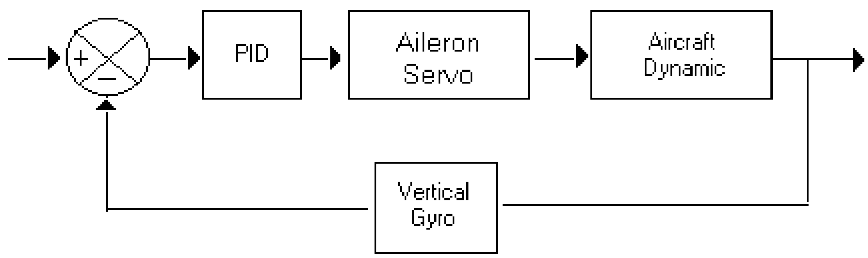
In an attitude displacement autopilot, the angle (pitch, roll, heading) is sensed by a gyro (vertical gyro in case of pitch and roll & Directional Gyro in case of heading).

The angle sensed is compared with the desired angle (pitch, roll, heading) to create an error angle. The difference or error in pitch-attitude/ roll-attitude /heading-attitude is used to produce proportional displacements of the elevator/aileron/rudder so that the error signal is reduced.

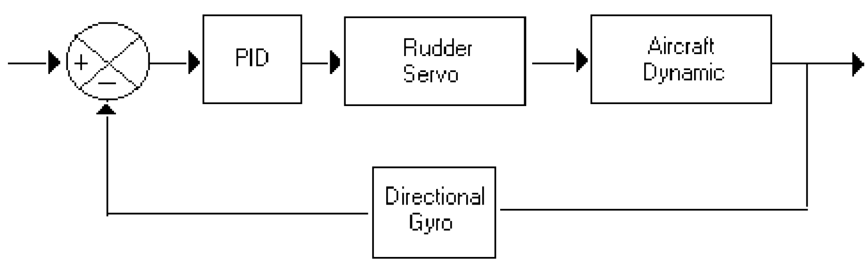
In practice, the displacement autopilot is engaged once the airplane has been trimmed in straight and level flight. To maneuver the airplane while the autopilot is engaged, the pilot must adjust the commanded signals. For example, the airplane can be made to climb or descend by changing the pitch command. Turns can be achieved by introducing the desired bank angle while simultaneously changing the heading command.



Pitch displacement Autopilot

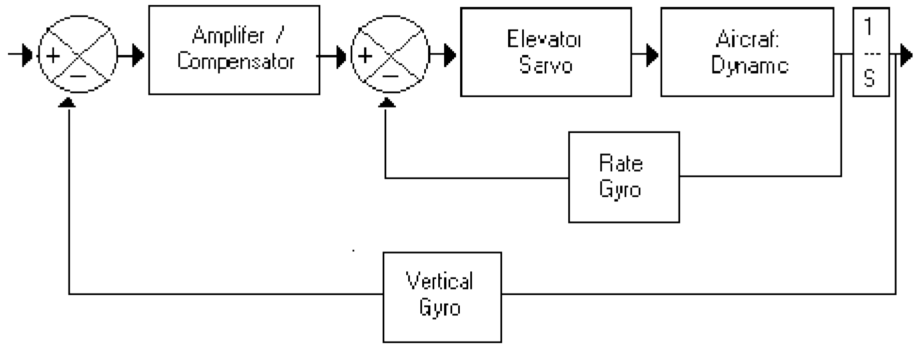


Roll Displacement Autopilot

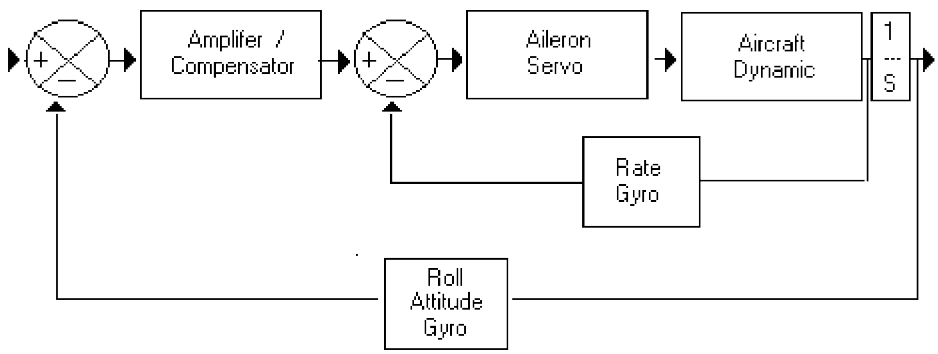


Heading Displacement Autopilot

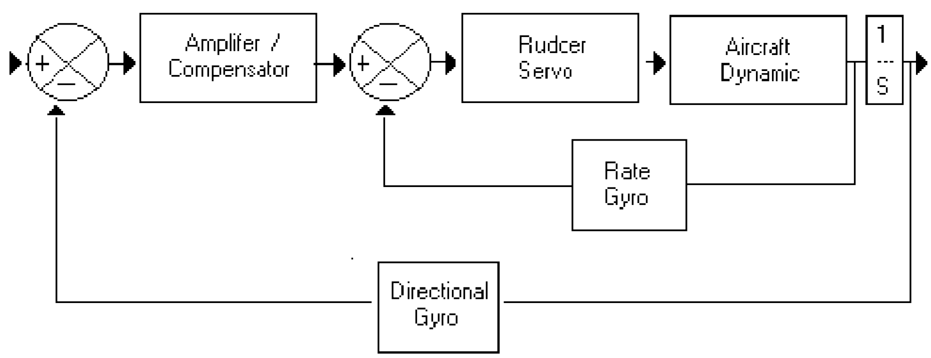
Figure 11.1 Displacement Autopilot with PID controller Block Diagram



Pitch displacement Autopilot



Roll Displacement Autopilot



Heading Displacement Autopilot

Figure 11.2 Displacement Autopilot with Rate feedback Block Diagram

11.2.2 How Displacement autopilot works

The basic components of a Displacement attitude control system are shown figure 11.1. For this design the reference pitch/roll/heading angle is compared with the actual angle measured by a gyro to produce an error signal to activate the control servo. In general the error signal is amplified and sent to the control surface actuator to deflect the control surface. Movement of the control surface causes the aircraft to achieve a new orientation, which is fed back to close the loop.

In case if aircraft has low natural damping inner rate feedback is used. Block diagram of a displacement autopilot with rate feedback for improved damping is shown in figure11.2. In the inner loop the rate is measured by rate gyro and feedback to be added with the error signal generated by the difference in pitch/roll/heading attitude. For this problem we now have two parameters to select namely the gains k_a (servo gain) and k_{rg} (rate feedback gain). The root locus method can be used to pick both parameters. The procedure is essentially is by trial and error. First the root locus is diagram is determined for the inner loop, a gyro gain is selected, and then the outer root locus plot is constructed. Several iterations may be required until the desired overall system performance is achieved.

11.3 Altitude Hold Control System

The altitude of an aircraft can be maintained by an altitude hold autopilot. A simplified altitude hold autopilot is shown in figure 11.3. Basically the autopilot is constructed to minimize the deviation between the actual altitude and the desired altitude.

To analyze how such an autopilot would function we examine an idealized case. We make the following assumptions:

- First, the airplane's speed will be controlled by, a separate control system
- Second, we neglect any lateral dynamic effects

With these restrictions we are assuming that the only motion possible is in the vertical plane. The aircraft dynamics will be represented by the short period approximate. To examine the altitude hold control system we need to find the transfer function $\Delta h / \Delta\delta_e$.

This can be obtained by examining Figure 11.4 , which shows the kinematics relationship between the airplane's rate of climb, pitch angle, and angle of attack.

From Figure 11.4 we can write the following relationship:

$$\Delta \dot{h} = u_o \sin(\Delta\theta - \Delta\alpha) \qquad 11.1$$

For small angles this can be reduced to

$$\Delta \dot{h} = u_o (\Delta\theta - \Delta\alpha) \qquad 11.2$$

Now we can find $\Delta h / \Delta\delta_e$ as follows:

$$s\Delta h(s) = u_o[\Delta\theta(s) - \Delta\alpha(s)] \qquad 11.3$$

Or

$$\Delta h(s) = \frac{u_o}{s}[\Delta\theta(s) - \Delta\alpha(s)] \qquad 11.4$$

And on dividing by $\Delta\delta_e$ we obtained the desired transfer function relationship:

$$\frac{\Delta h}{\Delta\delta_e} = \frac{u_o}{s}\left[\frac{\Delta\theta}{\Delta\delta_e} - \frac{\Delta\alpha}{\Delta\delta_e}\right] \qquad 11.5$$

The transfer function of $\frac{\Delta\alpha}{\Delta\delta_e}$ is already defined in sport period transfer function approximation.

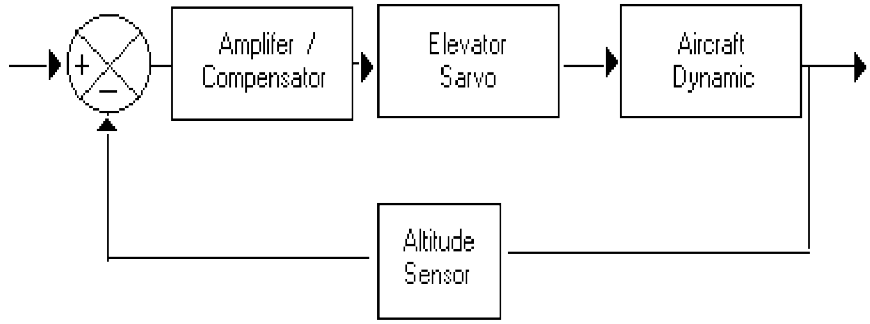


Figure 11.3 Altitude hold control system

The transfer function of $\frac{\Delta\theta}{\Delta\delta_e}$ can be obtained by following way.

$$\Delta q = \Delta\dot{\theta} \quad 11.6$$

$$\Delta q(s) = s\Delta\theta(s) \quad 11.7$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1}{s}\frac{\Delta q(s)}{\Delta\delta_e(s)} \quad 11.8$$

The transfer function of $\frac{\Delta q}{\Delta\delta_e}$ is already defined in sport period transfer function approximation.

11.4 Velocity Hold Control System

The forward speed of an airplane can be controlled by changing the thrust produced by the propulsion system. The function of the speed control system is to maintain some desired flight speed. This is accomplished by changing the engine throttle setting to increase or decrease the engine thrust. Figure 11.4 is a simplified concept for a speed control system. The components that make up the system include a compensator, engine throttle, aircraft dynamics, and a feedback path consisting of the velocity and acceleration feedback.

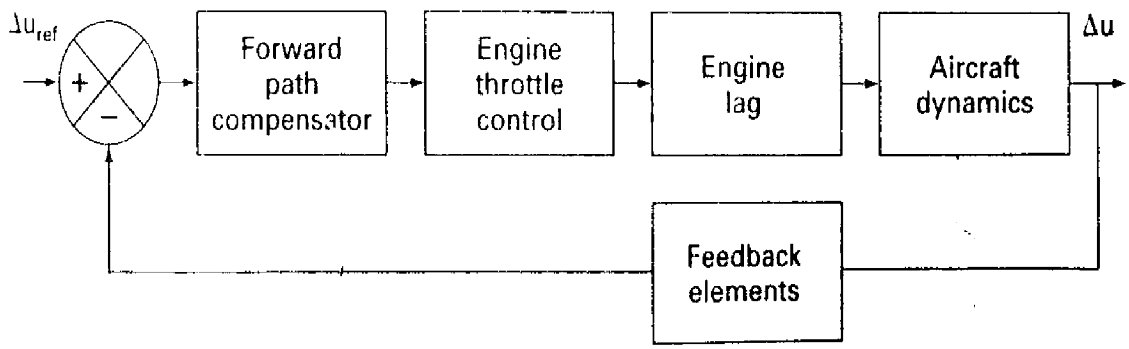


Figure 11.4 velocity hold control system

11.5 Stability Augmentation

Another application of automatic devices is to provide artificial stability for an airplane that has undesirable flying characteristics. Such control systems are commonly called stability augmentation systems (SAS).

As we showed earlier, the inherent stability of an airplane depends on the aerodynamic stability derivatives. The magnitude of the derivatives affects both the damping and frequency of the longitudinal and lateral motions of an airplane. Furthermore, it was shown that the stability derivatives were the function of the airplane's aerodynamic and geometric characteristics. For a particular flight regime it would be possible to design an airplane to possess desirable flying qualities. For example, we know that the longitudinal stability coefficients are a function of the horizontal tail volume ratio. Therefore we could select a tail size and or location so that C_{m_α} and C_{m_q}, provide the proper damping, and frequency for the short-period mode. However, for an airplane that will fly throughout an extended flight envelope, one can expect the stability to vary significantly, owing,, primarily to changes in the vehicle's configuration (lowering of flaps and landing gear) or Mach and Reynolds number effects on the stability coefficients. Because the stability derivatives vary over the flight envelope, the handling qualities also will change. Obviously, we would like to provide the flight crew with an airplane that has desirable handling qualities over its entire operational envelope. This is accomplished by employing stability, augmentation systems.

11.6 Instrument Landing

With the advent of the instrument landing system (ILS), aircraft became able to operate safely in weather conditions with restricted visibility. The instrument landing system is composed of ground-based signal transmitters and onboard receiving equipment. The ground-based equipment includes radio transmitters for the localizer, glide path, and marker beacons. The equipment on the airplane consists of receivers for detecting the signals and indicators to display the information.

The basic function of the ILS is to provide pilots with information that will permit them to guide the airplane down through the clouds to a point where the pilot re-establishes visual sighting of the runway. In a completely automatic landing, the autopilot guides the airplane all the way down to touchdown and roll out.

Before addressing the auto-land system, we briefly review the basic ideas behind the ILS equipment. To guide the airplane down toward the runway, the guidance must be lateral and vertical. The localizer beam is used to position the aircraft on a trajectory so that it will intercept the centerline of the runway. The transmitter radiates at a frequency in a band of 108-112 MHz. The purpose of this beam is to locate the airplane relative to a centerline of the runway. This is accomplished by creating azimuth guidance signals that are detected by the onboard localizer receiver. The azimuth guidance signal is created by superimposing a 90-Hz signal directed toward the left and a 150-Hz signal directed to the right on the carrier signal. Figure 11.5 shows an instrument landing localizer signal. When the aircraft is flying directly along the projected extension of the runway centerline, both superimposed signals are detected with equal strength. However, when the aircraft deviates say to the right of centerline, the 150 Hz signal is stronger. The receiver in the cockpit detects the difference and directs the pilot to fly the aircraft to the left by way of a vertical bar on the ILS indicator that shows the airplane to the right of the runway. If the airplane deviates to the left, the indicator will deflect the bar to the left of the runway marker.

The glide path or glide slope beam is located near the runway threshold and radiates at a frequency in the range 329.3 - 335.0 MHz. Its purpose is to guide the aircraft down a predetermined descent path. The glide slope is typically an angle of *2.5-3′* to the horizontal.

Figure 11.6 shows a schematic of the glide path beam. Note the glide path angle has been exaggerated in this sketch. As in the case of the localizer, two signals are superimposed on the carrier frequency to create an error signal if the aircraft is either high or low with

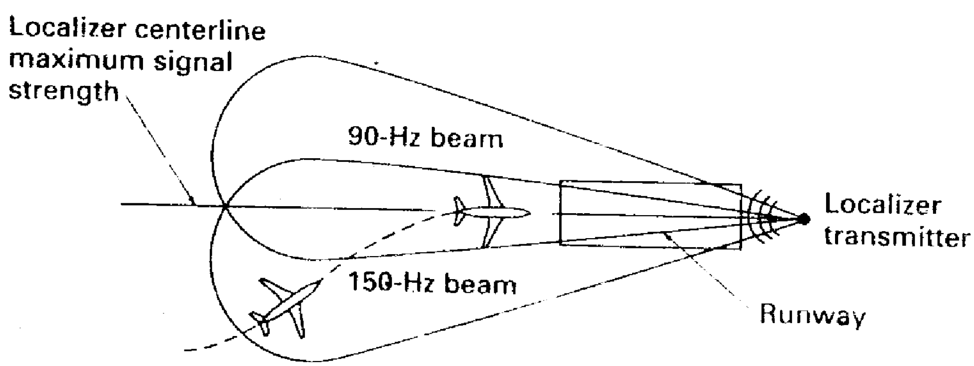


Figure 11.5: A localizer beam system

respect to the glide path. This usually is indicated by a horizontal bar on the ILS indicator that moves up or down with respect to the glide path indicator. The marker beacons are used to locate the aircraft relative to the runway. Two markers are used. One, located 4 nautical miles from the runway, is called the outer marker. The second, or inner, marker is located 3500 ft from the runway threshold. The beams are directed vertically into the descent path at a frequency of 75 MHz. The signals are coded, and when the airplane flies overhead the signals are detected by an onboard receiver. The pilot is alerted to the passage over a marker beacon by both an audio signal visual signal. The audio signal is heard over the aircraft's communication system and the visual signal is presented by way of a colored indicator light on the instrument panel.

In flying the airplane in poor visibility, the pilot uses the ILS equipment in the following manner. The pilot descends from cruise altitude under direction of, ground control to an altitude of approximately 1200 ft above the ground. The pilot then is vectored so that the aircraft intercepts the localizer at a distance of at least 6 nautical miles from the runway. The pilot positions the airplane using the localizer display so that it is on a heading toward the runway centerline. When the aircraft approaches the outer marker, the glide path signal is intercepted. The aircraft is placed in its final approach configuration and the pilot flies down the glide path slope. The pilot follows the beams by maneuvering the airplane so that the vertical and horizontal bars on the ILS indicator show no deviation from the desired flight path. The ILS system does not guide the aircraft all the way to touchdown.

At some point during the approach the pilot must look away from the instruments and outside the window to establish a visual reference for the final portion of the landing. The pilot may take 5 or 6 seconds to establish an outside visual reference. Obviously the pilot must do this at sufficient altitude and distance from the runway so that if the runway is not visible the pilot can abort the landing.

This gives rise to a "decision height," which is a predetermined height above the runway that the pilot cannot go beyond without visually sighting tile runway.

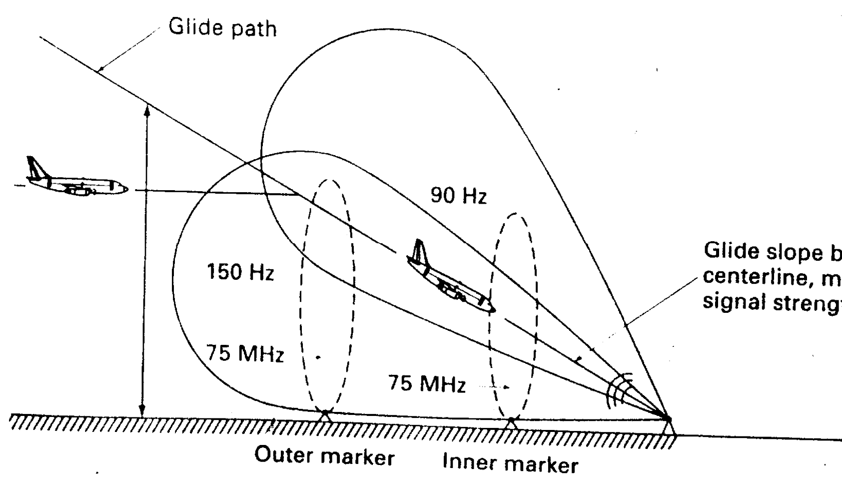


Figure 11.6:A glide slope beam system

The ILS as outlined in the previous paragraphs is an integral part of a fully automatic landing system. To be able to land an airplane with no visual reference to the runway requires an automatic landing system that can intercept the localizer and glide path signals, then guide the airplane down the glide path to some pre-selected altitude at which the aircraft's descent rate is reduced and the airplane executes a flare maneuver so that it touches down with all acceptable sink rate. The auto-land system comprises a number of automatic control systems, which include a localizer and glide path coupler, attitude and airspeed control, and an automatic flare control system.

Figure 11.9 shows an airplane descending toward the runway. The airplane shown is below the intended glide path. The deviation d of the airplane from the glide path is the normal distance of the airplane above or below the desired glide path. The angle Γ is the difference between the actual and desired glide path angle and R is the radial distance of the airplane from the glide slope transmitter. To maintain the airplane along the glide path, one must make Γ equal 0. Figure 11.8 is a conceptual design of an autopilot that will keep the airplane on the glide path. The transfer functions for d and Γ are obtained from the geometry and are noted in Figure 11.7.

As the airplane descends along the glide path, its pitch attitude and speed must be controlled. This again is accomplished by means of' a pitch displacement and speed control autopilot. The pitch displacement autopilot would be conceptually the same as the one discussed earlier in this chapter.

The difference in flight speed is used to produce a proportional displacement of' the engine throttle so that the speed difference is reduced. The component of the system labeled compensation is a device incorporated into the design so that the close loop system can meet the desired performance specifications. Finally, as the airplane gets very close to the runway threshold, the glide path control system is disengaged and a flare maneuver is executed. Figure 11.9 illustrates the flare maneuver just prior to touchdown. The flare

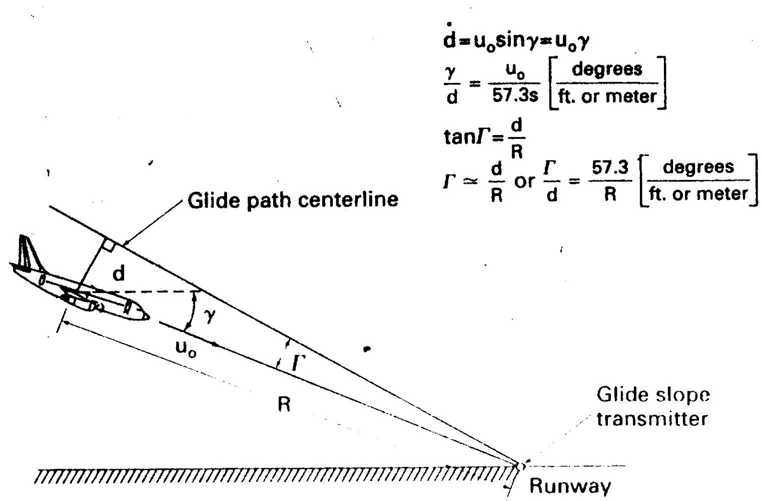


Figure 11.7: An airplane displaced from the glide path

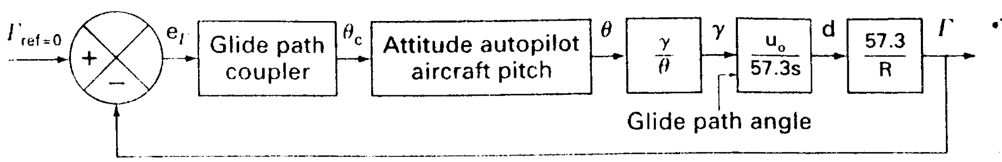


Figure 11.8 An automatic glide pathcontrol system

maneuver is needed to decrease the vertical descent rate to a level consistent with the ability of the landing gear to dissipate the energy of the impact at landing. An automatic flare control system is shown in Figure 11.10

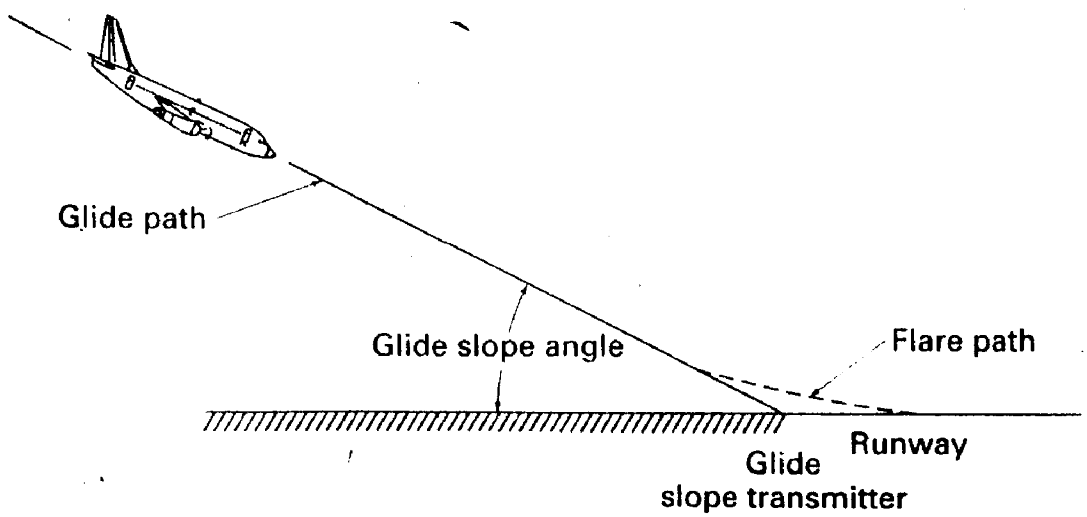


Figure 11.9: A flare maneuver

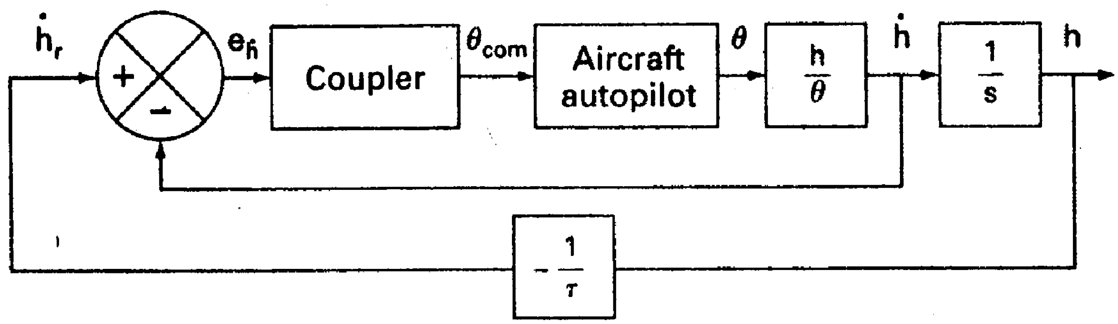


Figure 11.10 Automatic flare control system

CHAPTER 12

MODERN CONTROL THEORY AND AIRCRAFT AUTOPILOT DESIGN

In this final section we apply modern control theory to design of aircraft autopilots. Modern control theory provides the control system engineer with a valuable design tool. Unlike the classical control methods, modern control theory is ideally suited for synthesis of a control system with multiple and determining optimal control strategies.

Modern control theory is based on the state space formulation of the differential equations that govern the system. We know that higher order differential equations can be reduced to system of first order differential equations, that is the state space approach. These equations can be solved easily using a computer.

Once the system has been formulated in state space format we can use state feedback to locate the closed loop eigen values so that the system meets whatever performance requirements are desired. When some of the states are not available for feedback we can design a state observer to estimate or predict the states. The estimated states then can be used in place of the actual states in the feedback system

State feedback is used to provide stability augmentation system (SAS) to improve an aircraft's longitudinal and lateral flying qualities. In state feedback design the designer can place the close loop system poles at any location in the complex plane. In principle this technique permits the designer to completely specify the dynamic performance of the system. From mathematical standpoint the poles can be placed anywhere. However, practical considerations such as signal noise and control actuator saturation place limitations on poles placement.

To use a state feedback design the system had to be state controllable and all the state must be accessible to measurement. If any state is unavailable for feedback the design cannot be implemented. This limitation can be overcome by the use of state observer.

The classical control techniques are useful in designing many practical control systems. However, the design of control system by these techniques essentially is by trial and error. The major advantage of these design procedure is their simplicity and ease of use. This advantage disappears quickly as the complexity of the system increases.

Modern control theory permits a more systematic approach to control system design. In modern control theory system is specified as a system of first order differential equations. By formulating the problem in such manner, the control system designer can fully exploit the digital computer for solving complex control problems. Another advantage of modern control theory is optimization techniques can be applied to design optimal control system.

12.1 Modern Control Theory and SAS

State feedback control can be used to improve the stability characteristics of airplanes that lack good flying qualities. By the use of Modern Control Theory the Eigen values of a system can be changed by using state feedback. If the Eigen values do not meet the handling qualities specifications the airplane would be difficult to fly and deemed unacceptable by the pilot.

12.1.1 Longitudinal Stability Augmentation

Starting with the longitudinal state equations given in earlier section, we develop a set of linear algebraic equations in terms of the unknown feedback gains. The state equations for the longitudinal motion have been simplified by neglecting the affect of the control on the X-force equation and the stability derivative $M_{\dot{w}}$ the state equations are given below:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_o & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} + M_{\dot{w}} Z_{\delta_e} \\ 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_e \end{bmatrix} \qquad 12.1$$

$$\dot{x} = Ax + B\eta \qquad 12.2$$

Where **A** and **B** are the stability and control matrices just shown and x **and** η are the state and control vectors.

The Eigen values of the A matrix are the short- and long-period roots. If these roots are unacceptable to the pilot, a stability augmentation system will be required. State feedback design can be used to provide the stability augmentation system. In state feedback design we assume a linear control law that is proportional to the states; that is,

$$\eta = -k^T x + \eta_p \qquad 12.3$$

Where k^T is the transpose of the feedback gain vector and $-\eta_p$ is the pilot input. Substituting the control law into the state equation yields.

$$\dot{x} = (A - Bk^T)x + B\eta_p \qquad 12.4$$

Or

$$\dot{x} = A^* x + B\eta_p \qquad 12.5$$

Where **A*** is the augmented matrix, expressed as

$$A^* = A - Bk^T \tag{12.6}$$

The augmented matrix for the longitudinal system of equations is

$$A^* = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u - Z_{\delta_e}k_1 & Z_w - Z_{\delta_e}k_2 & u_o - Z_{\delta_e}k_3 & -Z_{\delta_e}k_4 \\ M_u - M_{\delta_e}k_1 & M_w - M_{\delta_e}k_2 & M_q - M_{\delta_e}k_3 & M_{\delta_e}k_4 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{12.7}$$

The characteristic equation for the augmented matrix is obtained by solving the equation

$$\left|\lambda I - A^*\right| = 0 \tag{12.8}$$

Which yields a quartic characteristic equation

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \tag{12.9}$$

Where the coefficients are defined as follows:

$$A = 1.0$$

$$B = Z_{\delta_e}k_2 + M_{\delta_e}k_3 - (X_u + Z_W + M_q)$$

$$\begin{aligned} C &= Z_{\delta_e}X_w k_1 + (u_o M_{\delta_e} - X_u Z_{\delta_e} - Z_{\delta_e}M_q)k_2 \\ &+ (Z_{\delta_e}M_w - X_u M_{\delta_e} - Z_w M_{\delta_e})k_3 + M_{\delta_e}k_4 \\ &+ X_u M_q + X_u Z_w + Z_w M_q - u_o M_w - X_w + Z_u \end{aligned} \tag{12.10}$$

$$
\begin{aligned}
D = {} & (u_0 X_W M_\delta - gM_\delta - X_w Z_\delta M_q)k_1 + (X_u Z_\delta M_q + u_0 X_u M_\delta)k_2 \\
& + (X_u Z_w M_\delta - X_u Z_\delta M - X_w Z_u M_\delta + X_w Z_\delta M_u)k_3 \\
& + (Z_\delta M_w - X_u M_\delta - Z_w M_\delta)k_4 + gM_u - X_u Z_w M_q \\
& + u_0 X_u M_w + X_w Z_u M_q - u_0 X_w M_u
\end{aligned}
\qquad 12.11
$$

$$
\begin{aligned}
E = {} & (gZ_w M_\delta - gZ_\delta M_w)k_1 + (gZ_\delta M_u - gZ_u M_\delta)k_2 \\
& + (X_u Z_w M_\delta - X_u Z_\delta M_w - X_w Z_u M_\delta + X_w Z_\delta M_u)k_4 \\
& + gZ_u M_w - gZ_w M_u
\end{aligned}
\qquad 12.12
$$

The characteristic equation of the augmented system is a function of the known stability derivatives and the unknown feedback gains. The feedback gains can be determined once the desired longitudinal characteristics are specified. For example, if the desired characteristic roots are

$$\lambda_{1,2} = -\zeta_{sp}\omega_{n_{sp}} \pm i\omega_{n_{sp}}\sqrt{1-\zeta_{sp}^2} \qquad 12.13$$

$$\lambda_{3,4} = -\zeta_p\omega_{n_p} \pm i\omega_{n_p}\sqrt{1-\zeta_p^2} \qquad 12.14$$

Then the desired character equation is

$$
\begin{aligned}
& \lambda^4 - [(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)]\lambda^3 + [\lambda_1\lambda_2 + \lambda_3\lambda_4 + (\lambda_1 + \lambda_2)(\lambda_3 + \lambda_4)]\lambda^2 \\
& - [\lambda_1\lambda_2(\lambda_3 + \lambda_4) + \lambda_3\lambda_4(\lambda_1 + \lambda_2)]\lambda + \lambda_1\lambda_2\lambda_3\lambda_4 = 0
\end{aligned}
\qquad 12.15
$$

By equating the coefficients of like powers of λ for the augmented and desired characteristic equations one obtains a set of four linear algebraic equations in terms of the unknown gains. These equations can be solved for the feedback gains.

12.1.1.1 Manual Solution

The longitudinal equations for an airplane having poor handling qualities follow. Use state feedback to provide stability augmentation so that the augmented aircraft has the following short- and long-period characteristics:

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{w} \\ \Delta \dot{q} \\ \Delta \dot{\theta} \end{bmatrix} AB = \begin{bmatrix} -0.01 & 0.1 & 0 & -32.20 \\ -0.40 & -0.8 & 180 & 0 \\ 0 & -0.003 & -0.5 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} 0 \\ -10 \\ -2.8 \\ 0 \end{bmatrix} [\Delta \delta_e]$$

$$\zeta_{sp} = 0.6 \qquad \zeta_p = 0.05 \qquad \omega_{n_{sp}} = 3.0 \text{rad/s} \qquad \omega_{n_p} = 0.1 \text{rad/s}$$

Solution. For this problem we use the Bass-Gura method, which lends itself to simple matrix manipulations. The state feedback gains can be estimated using the Bass-Gura technique. The feedback gains are found by solving the following equations:

$$k = \begin{bmatrix} V & W^T \end{bmatrix}^{-1} \begin{bmatrix} \bar{a} - a \end{bmatrix}$$

Where V is the controllability matrix, W is a transformation matrix, and $\bar{a}$ and a are vectors made up of the coefficients of the characteristic equation of the augmented or closed-loop system ($A - Bk^T$) and the characteristic equation of the open-loop plant matrix A.

The characteristic equation for the augmented or closed-loop system is determined by deciding on what closed-loop performance is desired. For this particular problem the desired eigen values are specified in terms of the short- and long-period damping ratio and un damped natural frequency. The desired characteristic equation can be written in terms of the damping and frequency as follows:

$$(\lambda^2 + 2\zeta_{sp}\omega_{n_{sp}}\lambda + \omega_{n_{sp}}^2)(\lambda^2 + 2\zeta_p\omega_{n_p}\lambda + \omega_{n_p}^2) = 0$$
$$(\lambda^2 + 3.6\lambda + 9)(\lambda^2 + 0.01\lambda + 0.01) = 0$$
$$\lambda^4 + 3.61\lambda^3 + 9.05\lambda^2 + 0.126\lambda + 0.09 = 0$$

The vector $\bar{a}$ is created from the coefficients of the desired characteristic equation

$$\lambda^4 + \bar{a}_1\lambda^3 + \bar{a}_2\lambda^2 + \bar{a}_3\lambda + \bar{a}_4 = 0$$

$$\bar{a} = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \bar{a}_3 \\ \bar{a}_4 \end{bmatrix} = \begin{bmatrix} 3.61 \\ 9.05 \\ 0.126 \\ 0.09 \end{bmatrix}$$

The characteristic equation of the open-loop system is obtained by solving the equation $|\lambda I - A| = 0$, which yields

$$\lambda^4 + 1.31\lambda^3 + 0.993\lambda^2 + 0.0294\lambda + 0.0386 = 0$$

The vector a is created from the coefficients of the open-loop characteristic equation:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1.31 \\ 0.993 \\ 0.0294 \\ 0.0386 \end{bmatrix}$$

Continuing with the solution, we need to determine the controllability matrix; V. We know that the controllability matrix is defined in terms of the plant and control matrices. For the fourth-order system under consideration here, the controllability matrix is

$$V = [B \quad AB \quad A^2B \quad A^3B]$$

The elements of the V matrix can be readily calculated by performing the appropriate matrix multiplications:

$$AB = \begin{bmatrix} -0.01 & 0.1 & 0 & -32.20 \\ -0.40 & -0.8 & 180 & 0 \\ 0 & -0.003 & -0.5 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -10 \\ -2.8 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ -496 \\ -1.43 \\ -2.8 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} -0.01 & 0.1 & 0 & -32.20 \\ -0.40 & -0.8 & 180 & 0 \\ 0 & -0.003 & -0.5 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} -1.0 \\ -496 \\ -1.43 \\ -2.8 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 40.57 \\ 654.6 \\ 0.773 \\ 1.43 \end{bmatrix}$$

$$A^3B = \begin{bmatrix} -0.01 & 0.1 & 0 & -32.20 \\ -0.40 & -0.8 & 180 & 0 \\ 0 & -0.003 & -0.5 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} 40.57 \\ 654.6 \\ 0.773 \\ 1.43 \end{bmatrix}$$

$$A^3B = \begin{bmatrix} 19.008 \\ -400.77 \\ -2.35 \\ -0.773 \end{bmatrix}$$

Substituting the column matrices into the definition of V yields

$$V = \begin{bmatrix} 0 & -1.0 & 40.57 & -19.008 \\ -10 & 496 & 654.6 & -400.77 \\ -2.8 & -1.43 & 0.773 & -2.35 \\ 0 & -2.8 & 1.43 & -773 \end{bmatrix}$$

The transformation matrix W is required if the plant matrix A is not in the companion form. For this particular problem the A matrix is not in companion form; therefore, the transformation matrix must be developed. As we know the transformation matrix is defined in terms of the coefficients of the characteristic equation of the plant matrix. For this particular example of a fourth-order, system the W matrix is defined as

$$W = \begin{bmatrix} 1 & a_1 & a_2 & a_3 \\ 0 & 1 & a_1 & a_2 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1.31 & 0.993 & 0.0294 \\ 0 & 1 & 1.31 & 0.993 \\ 0 & 0 & 1 & 1.31 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We can now calculate the state feedback gains

$$k = \left[V \quad W^T\right]^{-1} \left[\bar{a} - a\right]$$

$$VW = \begin{bmatrix} 0 & -1 & 39.23 & 71.2 \\ -10.0 & -509.1 & -5.01 & -36.1 \\ -2.8 & -2.24 & -0.134 & 0 \\ 0 & -2.8 & -2.24 & -0.13 \end{bmatrix}$$

The transpose of the matrix VW is obtained by interchanging the rows and columns of VW:

$$(VW)^T = \begin{bmatrix} 0 & -10.0 & -2.8 & 0 \\ -1 & -509.1 & -2.234 & -2.8 \\ 39.23 & -5.1 & -0.13 & -2.24 \\ 71.2 & -36.1 & 0.0 & -0.13 \end{bmatrix}$$

$$[(VW)^T]^{-1} = \begin{bmatrix} 0.0008 & -0.0010 & 0.0004 & 0.0138 \\ -0.0014 & 0.0019 & -0.0025 & -0.0014 \\ -0.3622 & -0.0068 & -0.0089 & -0.0050 \\ 0.0321 & -0.0135 & 0.4446 & -0.2451 \end{bmatrix}$$

The state feedback gains are

$$k = \left[V \quad W^T\right]^{-1}\left[\bar{a} - a\right]$$

$$k = \begin{bmatrix} 0.0008 & -0.0010 & 0.0004 & 0.0138 \\ -0.0014 & 0.0019 & -0.0025 & -0.0014 \\ -0.3622 & -0.0068 & -0.0089 & -0.0050 \\ 0.0321 & -0.0135 & 0.4446 & -0.2451 \end{bmatrix}\left(\begin{bmatrix} 3.61 \\ 9.05 \\ 0.126 \\ 0.09 \end{bmatrix} - \begin{bmatrix} 1.31 \\ 0.993 \\ 0.0294 \\ 0.0386 \end{bmatrix}\right)$$

$$k = \begin{bmatrix} -0.0055 \\ -0.0120 \\ -0.7785 \\ -0.0656 \end{bmatrix}$$

Having determined the feedback gains we can now define the control law. The stability augmentation control law is

$$\Delta\delta_e = -k^T x = -\left[-0.0055 \quad -0.0120 \quad -0.7785 \quad -0.0656\right]\begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix}$$

$$\Delta\delta_e = 0.0055\Delta u + 0.0120\Delta w + 0.7785\Delta q + 0.0656\Delta\theta$$

12.1.2 Lateral Stability Augmentation

The lateral Eigen values of an airplane also can be modified using state feedback. The lateral state equations are expressed in state-space form as follows

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\Phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & -(u_o) & g \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\Phi \end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \qquad 12.16$$

Or in shorthand mathematical form

$$\dot{x} = Ax + B\eta \qquad 12.17$$

Note that the control vector is made up of two control inputs, namely, the aileron and rudder deflector angle. The control matrix B no longer is just a column but a 4 x 2 rectangular matrix.

When we have a multiple input system the state feedback gain vector becomes a gain matrix of order n *X m* where n is the order of the system and m is the number of control input signals. Placing the eigen values at some desired location allows the designer to identify n of the gains; however, we still have n *X (m* - 1) gains that must be selected. There are techniques that can be used to handle the multiple input system but these techniques ate beyond the scope of this book.

We will use Oehman and Suddath approach to apply state feedback control for lateral stability augmentation. . Basically this technique reduces the gain matrix to a gain vector. The control law can be expressed in terms of a constant row matrix, g, the gain vector, k, and the pilot's control input, η_p

$$\eta = -gk^T x + \eta_p \qquad 12.18$$

The procedure is identical to that for the longitudinal equations. The constant vector g establishes the relationship between the aileron and rudder for augmentation. Either g1 or g2 is equal to 1, and the ratio $g_1 / g_2 = \Delta\delta_a / \Delta\delta_r$ is specified by control deflection limits.

Substituting the control vector into the state equation yields

$$\dot{x} = (A - Bgk^T)x + B\eta_p \tag{12.19}$$

$$\dot{x} = A^*x + B\eta_p \tag{12.20}$$

Where A* is the augmented matrix, expressed as

$$A^* = A - Bgk^T \tag{12.21}$$

12.1.3 Autopilot Design

The stability augmentation system discussed in the previous section is an autopilot. The function of an SAS autopilot is to provide good handling qualities for the airplane so that the pilots do not find the airplane difficult to fly. Other types of autopilot were used to lessen the flight crew's workload during cruise and help them land the airplane during adverse weather conditions. We examined autopilots to maintain the airplane's orientation, speed, and altitude. The state feedback design approach can be used to design autopilots to perform the same functions. In the following problem we demonstrate how the state feedback design approach can be used to design an altitude hold autopilot

12.1.3.1 Manual Solution

Use state feedback to design in autopilot to maintain a constant altitude. To simplify this problem we will assume that the forward speed of the airplane, Uo, is held fixed by a separate velocity control system and furthermore we neglect the control surface actuator dynamics. If the actuator dynamics were included the order of the system would be increased by 1. This assumption was made solely for the purposes of keeping the system as simple as possible. The airplane selected for this example is the STOL transport

The state equation for the airplane can be represented by the short-period approximation. The kinematics equation representing the change in vertical height in terms of the angles $\Delta\alpha$ and $\Delta\theta$ developed in earlier is:

If we add the vertical velocity equation to the short-period equations we obtain the following fourth-order system:

$$\dot{h} = u_o(\Delta\theta - \Delta\alpha)$$

If we add the vertical velocity equation to the short-period equations we obtain **the** following fourth-order system

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} z_a/u_0 \\ M_\alpha + M_\alpha Z_\alpha/u_0 \\ 0 \\ -u_0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ M_q + M_\alpha & 0 & 0 \\ 1 & 0 & 0 \\ 0 & u_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} Z_{\delta_e}/u_0 \\ M_{\delta_e} \\ 0 \\ 0 \end{bmatrix} [\Delta\delta e]$$

Substituting the numerical values of the stability derivatives for the STOL transport yields

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} -0.124 \\ -13.2 \\ 0 \\ 0 \end{bmatrix} [\Delta\delta e]$$

In state feedback design the designer can specify the desired location of the eigen values, For this example we choose to locate the eigen values at

$$\lambda_{1.2} = -1.0 \pm 3.5i \qquad \lambda_{3,4} = -2.0 \pm 1.0i$$

Solution. The state feedback gains can be again determined using the Bass-Gura method. The gains are determined by the matrix equation:

$$k = \left[V \quad W^T\right]^{-1} \left[\bar{a} - a\right]$$

Where V is the controllability matrix, W is a transformation matrix, and $\bar{a}$ and a are vectors made up of the coefficients of the characteristic equations for the closed-loop system $A^* = (A - Bk^T)$ and the characteristic equation for open-loop plant matrix A, respectively. The eigen values for the desired closed-loop system can be multiplied together to give the closed-loop characteristic equation

$$(\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)(\lambda-\lambda_4) = 0$$

Substituting the desired eigen values into the above equation and performing the indicated multiplication yields the following characteristic equation:

$$\lambda^4 + 6.0\lambda^3 + 23.25\lambda^2 + 63\lambda + 66.25 = 0$$

The vector a- is composed of the coefficients of the desired characteristic equation:

$$\lambda^4 + a_1\lambda^3 + \bar{a}_2\lambda^2 + \bar{a}_3\lambda + \bar{a}_4 = 0$$

$$\bar{a} = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \bar{a}_3 \\ \bar{a}_4 \end{bmatrix} = \begin{bmatrix} 6.0 \\ 26.25 \\ 63.0 \\ 66.25 \end{bmatrix}$$

The characteristic equation for the A matrix is found by solving for the eigenvalues the A matrix:

$$|\lambda I - A| = 0$$

$$\lambda^4 + 4.66\lambda^3 + 10.04\lambda^2 + 0\lambda + 0 = 0$$

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 4.667 \\ 10.04 \\ 0.0 \\ 0.0 \end{bmatrix}$$

The next step is to determine the controllability matrix V. The controllability matrix is defined in terms of the plant matrix A and control matrix B. For this example it is

$$V = [B \quad AB \quad A^2B \quad A^3B]$$

The elements of the V matrix can be calculated readily by simple matrix multiplication

$$AB = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} -0.124 \\ -13.20 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -13.03 \\ 43.84 \\ -13.20 \\ 49.6 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} -13.03 \\ 43.84 \\ -13.20 \\ 49.6 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 62.04 \\ -72.11 \\ 43.84 \\ -69.29 \end{bmatrix}$$

$$A^3B = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} 62.04 \\ -72.11 \\ 43.84 \\ -69.29 \end{bmatrix}$$

$$A^3B = \begin{bmatrix} -158.8 \\ -103.6 \\ -72.1 \\ -7279.4 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.12 & -13.03 & 62.04 & -158.8 \\ -13.20 & 43.84 & -72.11 & -103.6 \\ 0 & -13.20 & 43.84 & -72.1 \\ 0 & 49.60 & -69.29 & -7279.4 \end{bmatrix}$$

The rank of the V matrix is 4; therefore the system is completely state controllable. Our next step is to determine the transformation matrix W, which for this particular problem is

$$W = \begin{bmatrix} 1 & a_1 & a_2 & a_3 \\ 0 & 1 & a_1 & a_2 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4.66 & 10.04 & 0 \\ 0 & 1 & 4.66 & 10.04 \\ 0 & 0 & 1 & 4.66 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We can now calculate the state feedback gains

$$k = \begin{bmatrix} V & W^T \end{bmatrix}^{-1} \begin{bmatrix} \bar{a} - a \end{bmatrix}$$

$$VW = \begin{bmatrix} -0.12 & -13.6 & 0.1 & -0.5 \\ -13.2 & -17.7 & -0.3 & 0.6 \\ 0 & -13.2 & -17.7 & -0.3 \\ 0 & 49.6 & 161.8 & -7104 \end{bmatrix}$$

The transpose of the matrix VW is obtained by interchanging the rows and columns of VW:

$$(VW)^T = \begin{bmatrix} -0.12 & -13.2 & 0 & 0 \\ -13.6 & -17.7 & -13.2 & 49.6 \\ 0.1 & -0.3 & -17.7 & 161.8 \\ -0.5 & 0.6 & -0.3 & -7104 \end{bmatrix}$$

$$[(VW)^T]^{-1} = \begin{bmatrix} 0.0978 & -0.0741 & 0.0553 & 0.0007 \\ -0.0767 & 0.0007 & -0.0005 & 0 \\ 0.0018 & -0.0003 & -0.0563 & -0.0013 \\ 0 & 0 & 0 & -0.0001 \end{bmatrix}$$

The state feedback gains are

$$k = \left[V \quad W^T\right]^{-1}\left[\bar{a} - a\right]$$

$$k = \begin{bmatrix} 0.0978 & -0.0741 & 0.0553 & 0.0007 \\ -0.0767 & 0.0007 & -0.0005 & 0 \\ 0.0018 & -0.0003 & -0.0563 & -0.0013 \\ 0 & 0 & 0 & -0.0001 \end{bmatrix}\begin{bmatrix} 6.0 & 4.67 \\ 20.5 & 10.04 \\ 63.0 & 0 \\ 66.25 & 0 \end{bmatrix}$$

$$k = \begin{bmatrix} 2.445 \\ -0.125 \\ -3.636 \\ -0.009 \end{bmatrix}$$

Having determined the feedback gains we can now define the control law.

$$\Delta\delta_e = 2.445\Delta\alpha - 0.125\Delta q - 3.636\Delta\theta - 0.009\Delta h$$

12.1.4 Optimal Control

In the previous sections we examined the use of state feedback control for the placement of the closed-loop eigen values. By placing the eigen values in the left half portion of the complex plane we can be sure that the system is stable. However, as we move the eigen values farther to the left in the complex plane the gains may become large, resulting in excessive control- deflection. For some systems the designer may not have a good idea or feel for the best location of the closed-loop Eigen values.

Optimal control theory can be used to overcome these difficulties. Optimal control allows the designer to specify constrains on maximum allowable excursion of the states and control input . This accomplished by specifying weighting matrices for the states and control in an integral performance index. The optimal control gains are determined by solving the steady state Riccati equation.

In the following example apply optimal control theory to provide an optimal controller for maintaining a desired angle of attack, altitude excursion and control deflection. This problem is simple enough that we can solve the steady-state Riccati equations by hand. However, for higher-order systems computer methods are required. A second example of a higher-order system is examined using the software package MATLAB.

12.1.4.1 Manual Solution

Determine the optimal control law if we place constraints on the angle of attack, altitude excursion and control deflection. The weighting matrices are assumed to have the following form

$$Q = \begin{bmatrix} \left(\dfrac{1}{\alpha_{max}}\right)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\dfrac{1}{h_{max}}\right)^2 \end{bmatrix}$$

$$R = \left[\left(\dfrac{1}{\delta_{e_{max}}}\right)^2\right]$$

$$\Delta\alpha_{max} = 5^\circ = 0.087\text{rad}$$
$$\Delta h_{max} = 100\text{ft}$$
$$\Delta\delta_{e_{max}} = 10^\circ = 0.175\text{rad}$$

Solution. The equations for the STOL transport follow:

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} -0.124 \\ -13.2 \\ 0 \\ 0 \end{bmatrix} [\Delta\delta_e]$$

The MATLAB program lqr was used to determine the Riccati matrix. Listing of the MATLAB instructions used to solve this problem is given below.

A = [1.397 1 0 0 ; - 5.47 - 3.27 0 0; 0 1 0 0; - 400 0 400 0]

B = [-0.124; -13.2; 0;0]

Q = [- 1.32 1 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0.000 1]

R = [32.84]

[K, S, E] = lqr (A, B, Q, R)

Listing of MATLAB instruction.

The Ricatti matrix was found to be

$$S=\begin{bmatrix} 75.6886 & -0.9547 & -74.8768 & -0.0865 \\ -0.9547 & 0.7648 & 4.9711 & 0.0052 \\ -74.8768 & 4.9711 & 106.1849 & 0.1208 \\ -0.0865 & 0.0052 & 0.1208 & 0.0002 \end{bmatrix}$$

Once the Ricatti matrix has been determined the optimal control gains can be determined by the equation

$$k^T = R^{-1}B^TS$$

$$R^{-1} = \left[\delta^2{}_{e_{max}}\right] = 0.0306$$

$$k^T = [0.0306]\begin{bmatrix} -0.124 & -13.2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 75.6886 & -0.9547 & -74.8768 & -0.0865 \\ -0.9547 & 0.7648 & 4.9711 & 0.0052 \\ -74.8768 & 4.9711 & 106.1849 & 0.1208 \\ -0.0865 & 0.0052 & 0.1208 & 0.0002 \end{bmatrix}$$

$$k^{T} = \begin{bmatrix} 0.098 & 0.0304 & -1.715 & 0.0017 \end{bmatrix}$$

The optimal control law can now be written

$\eta = -k^{T} x$

Or

$\Delta\delta_{e} = -0.098\Delta\alpha + 0.0304\Delta q + 1.715\Delta\theta + 0.0017\Delta h$

CHAPTER 13

FLOW OF PROJECT

The file autopilot.m is the main module of our project and this file is in the directory of autopilot; a snap shot of autopilot is given in fig 13.1. There are now nine other modules which are obvious from figure 13.1, the first module is of pitch control, the second module is of roll controller, third module is of heading controller, the fourth module is about altitude controller, the fifth module is of velocity controller, the sixth module is of Glide path controller, the seventh module is of flare controller, the eight module deals with stability augmentation and the last and final module is about optimal control. Apart from this there is another module which is called change model, from this a user has an option to change the model, a user can select model from STOL a transport aircraft, NAVION a simple aviation aircraft and F104-A, a fighter aircraft, all data about these planes are included and after selection of the aircraft the data for that particular model is loaded. A user also has an option to make his own model or enter some other model whose data is available to them. First we will define flow of the project with some flow charts and then we will explain each and every module in detail, first we will start from the change model module which is on the right side of the main module autopilot and then all other modules such as pitch controller, roll controller, heading controller etc are explained which are on the left side of the main module autopilot.

13.1 Main Module (autopilot.m)

The main module of this project is autopilot.m, a snap shot of autopilot is given in figure 13.1, a flow chart is given in figure 13.2 which shows how the nine other modules are inter connected in the main module that is autopilot.m as well as the change model module.

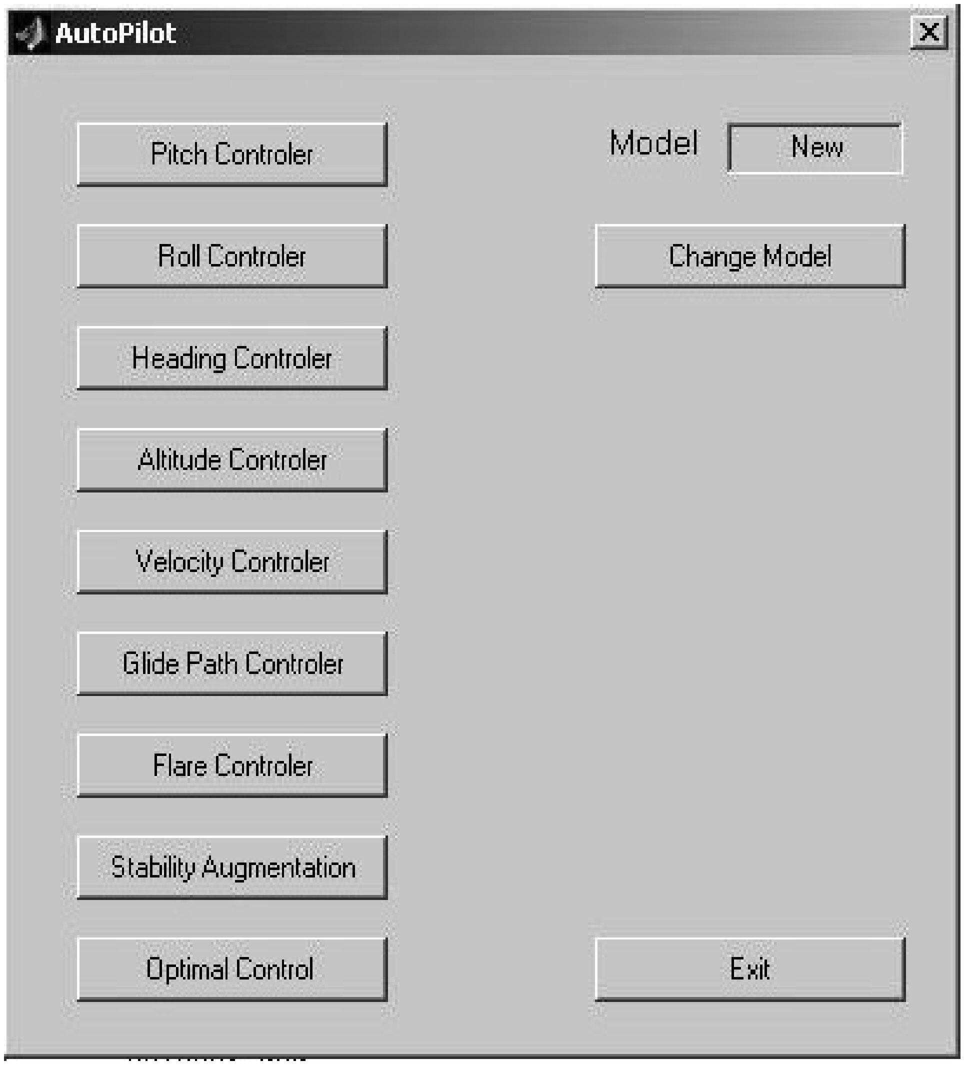


Fig 13.1 snap shot of main module autopilot

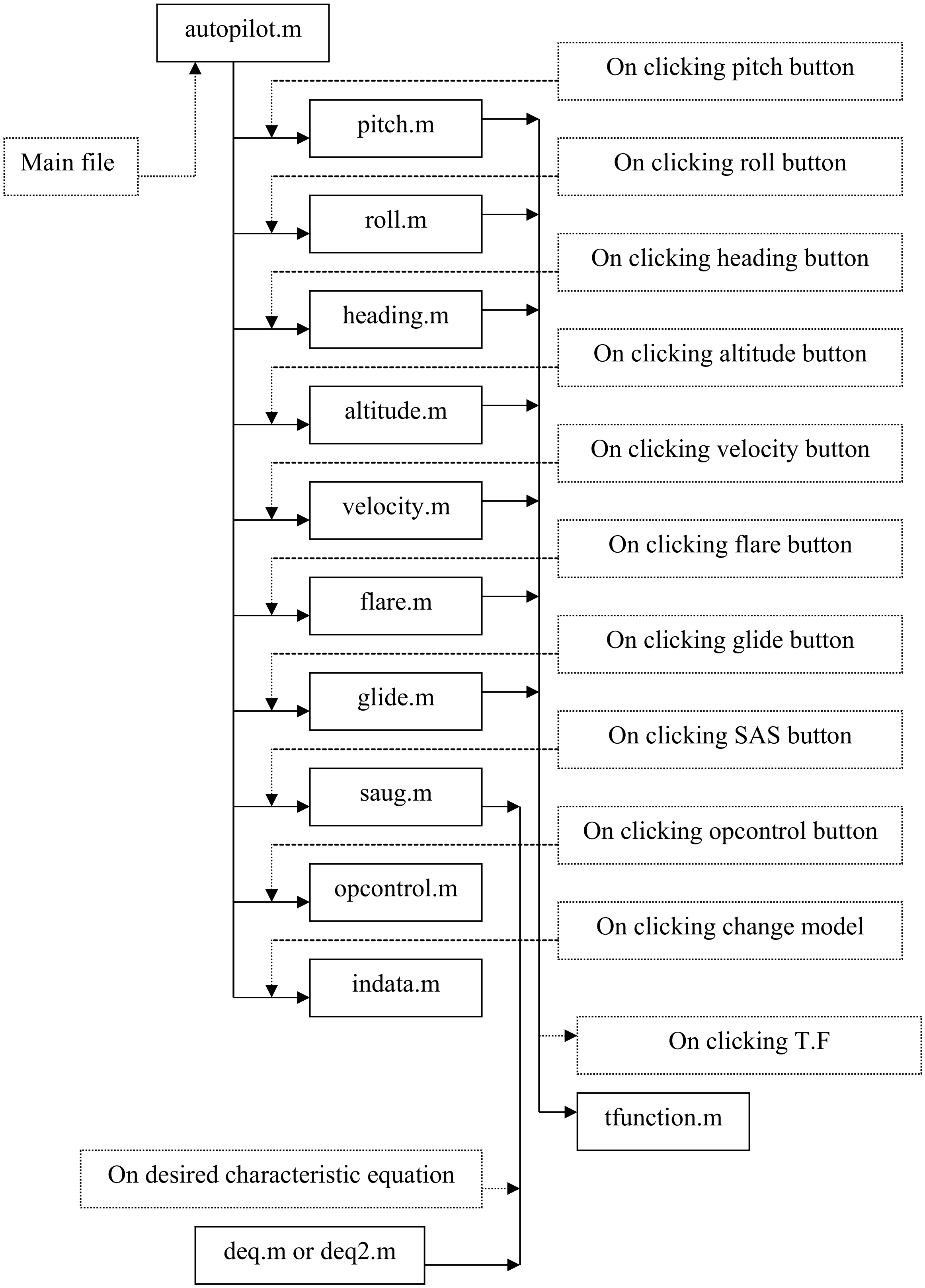


Fig 13.2 Flow chart of main module autopilot.m

13.2 Change Model Module (indata.m)

The change model module appears on the right side of the screen as is evident from the figure 13.1, In this module the user has an option to either change model from the three available models whose Geometric and aerodynamic data is already feeded in the module or the user can also make his own model or enter data for some aircraft whose data is available to the user. The three models for which we have simulated our autopilot are STOL aircraft, NAVION aircraft and F104-A aircraft. Figure 13.3 shows the change model module, In this figure there is no model selected, the user has to select a model from this pull down menu, we will show one by one about all the three aircrafts and their data. Now if we select STOL aircraft from this menu, then the model data window will appear as shown in figure 13.4. A flow chart of change model module and how all the files are interconnected is given in Figure 13.5. The different windows which appear in the Geometric and Aerodynamic mass data such as general data, aspect ratio, wing tail area etc are also given.

After selecting the model from the change model module one can see the lateral stability coefficients as shown in figure 13.20 , longitudinal stability coefficients shown in figure 13.21 and lateral stability derivatives shown in figure 13.22 and longitudinal stability derivatives shown in figure 13.23, a flow chart is given in figure 13.19 which depicts the linkage between different files in indata.m. If we select other aircraft such as NAVION or F104-A the data appears in the following way as shown in the figures 13.23 and 13.24

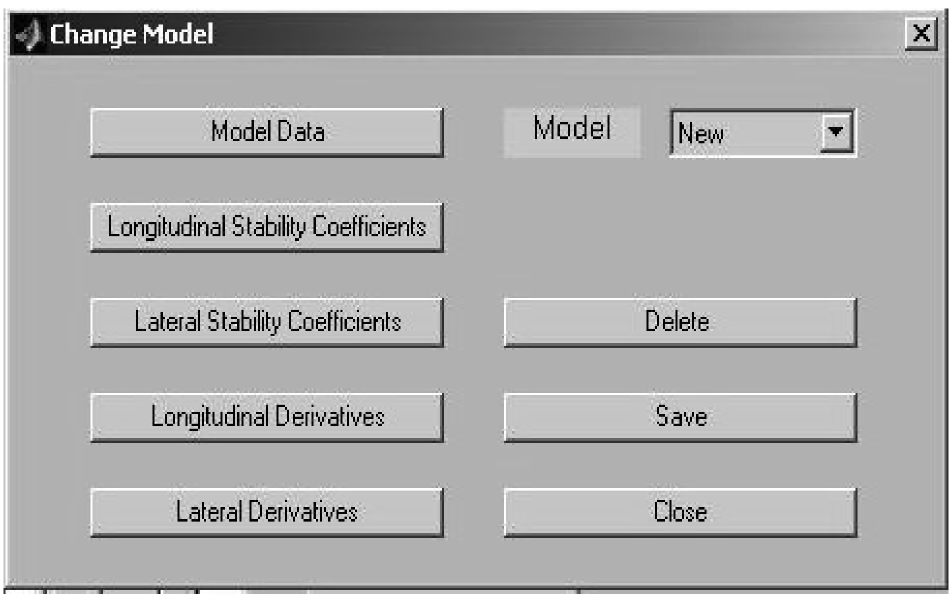


Fig 13.3 Change Model module

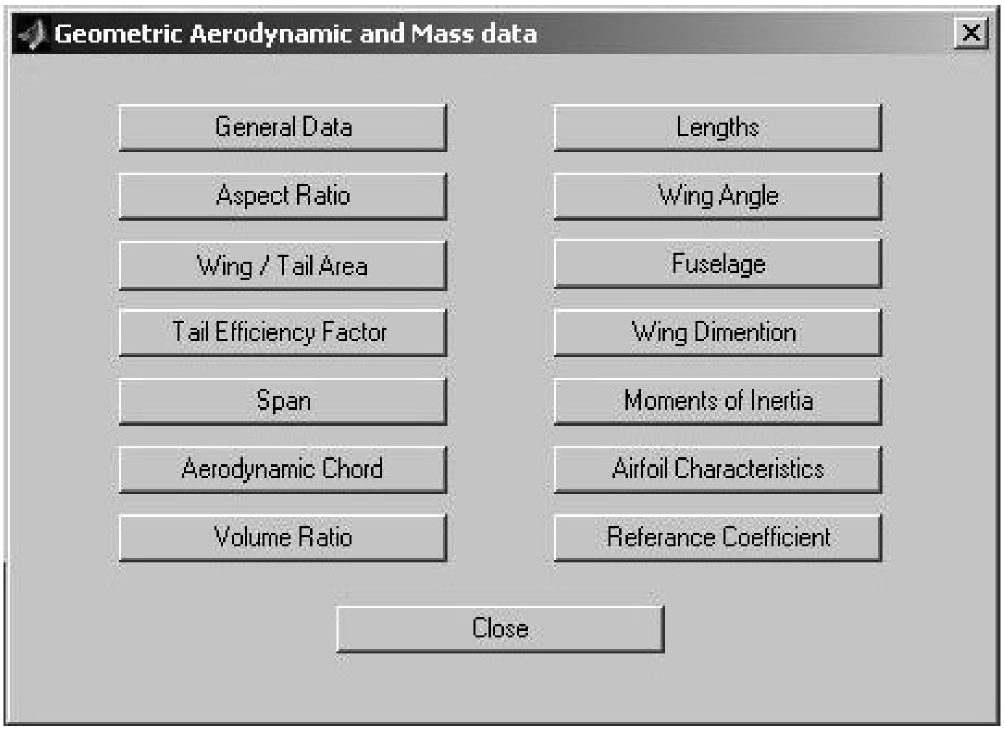


Fig 13.4 Geometric, Aerodynamic and mass data after selection of STOL aircraft

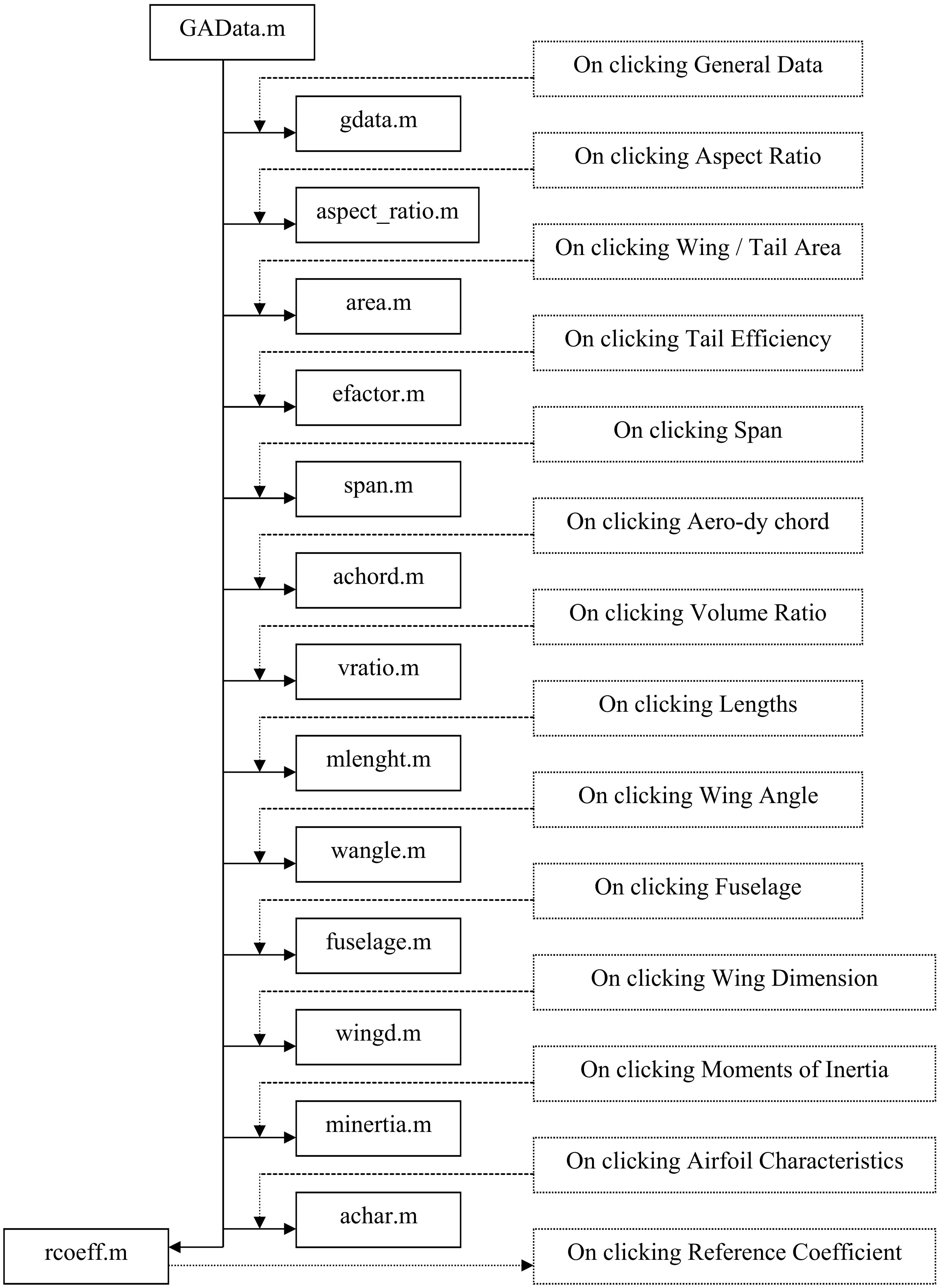


Fig 13.5 flow chart of Change Model module

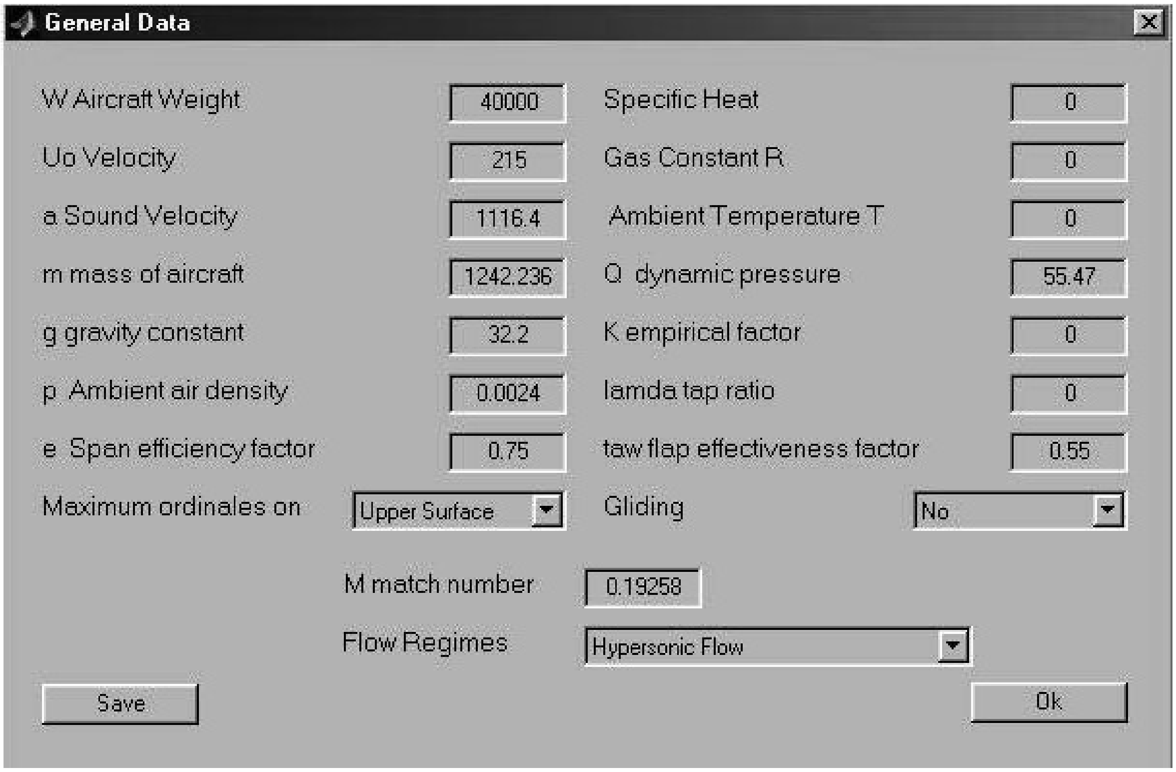


Fig 13.6 General data of STOL aircraft

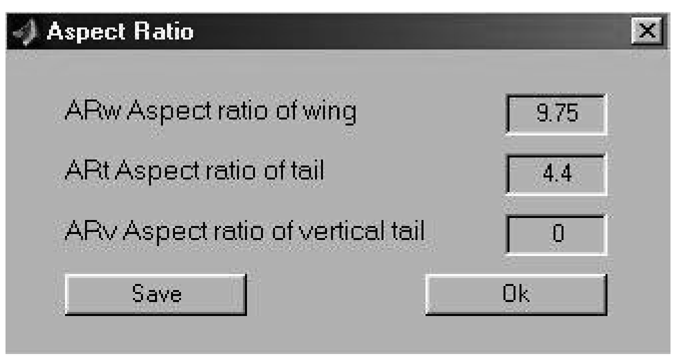


Fig 13.7 aspect ratio of STOL aircraft

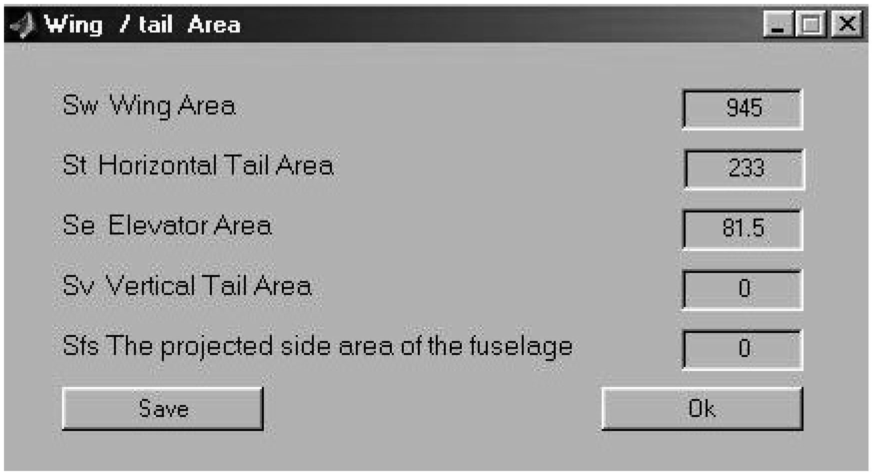


Fig 13.8 wing tail area of STOL aircraft

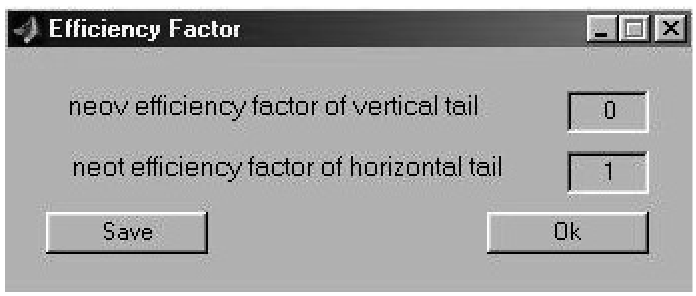


Fig 13.9 Tail Efficiency factor of STOL aircraft

Fig 13.10 span of STOL aircraft

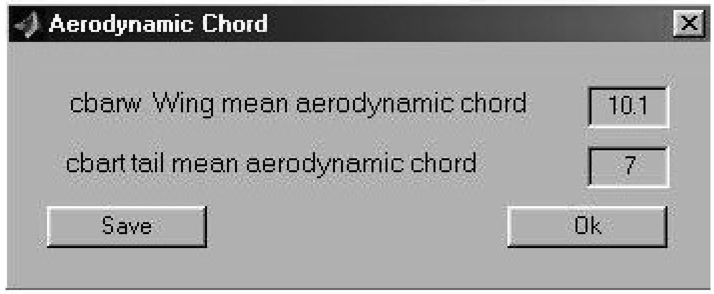


Fig 13.11 Aerodynamic chord of STOL aircraft

Fig 13.12 Volume ratio of STOL aircraft

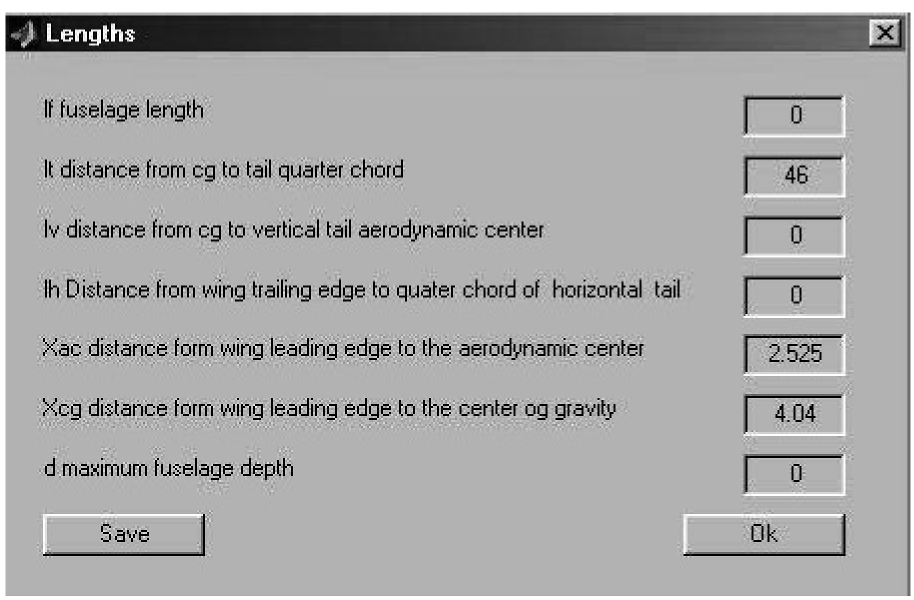


Fig 13.13 lengths of STOL aircraft

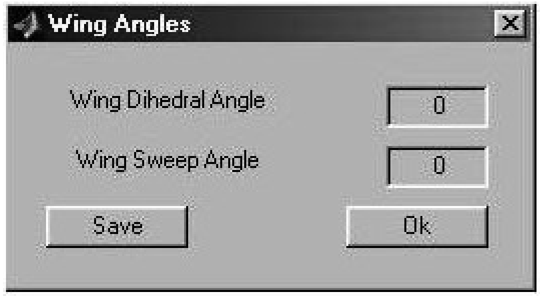
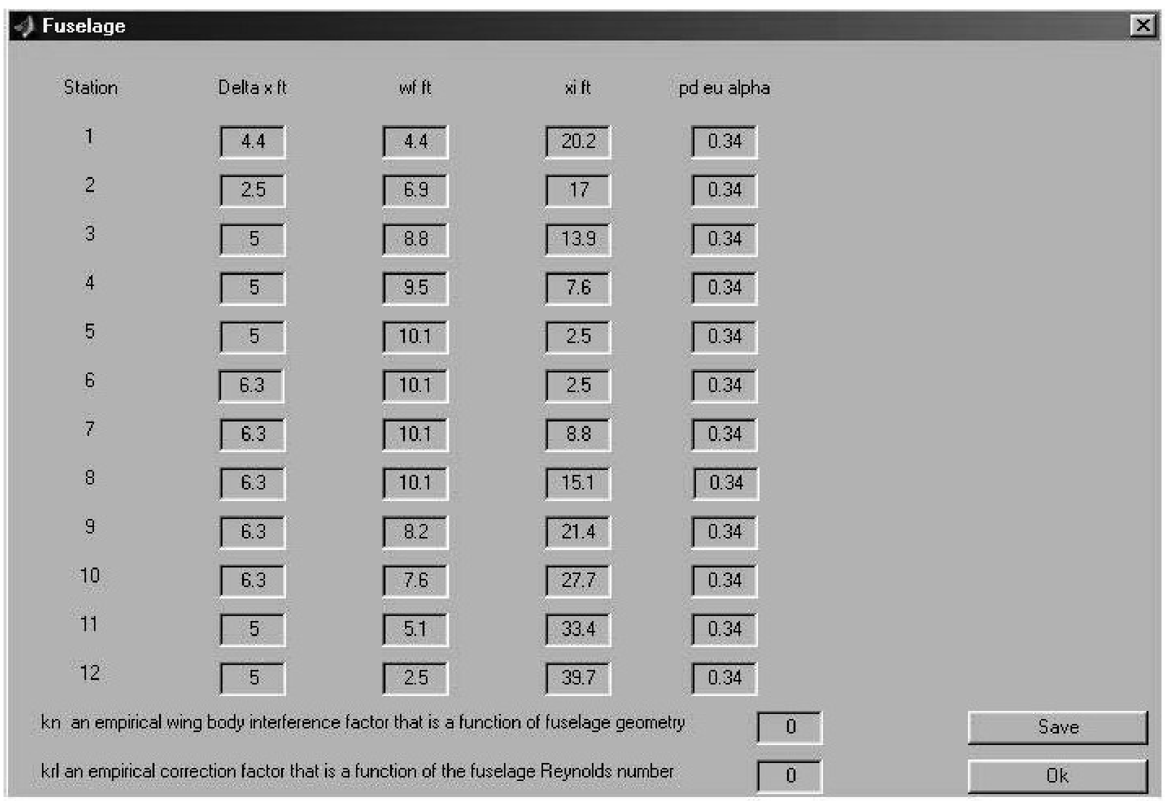


Fig 13.14 wing angles of STOL aircraft



Fuselage

Station	Delta x ft	wf ft	xi ft	pd eu alpha
1	4.4	4.4	20.2	0.34
2	2.5	6.9	17	0.34
3	5	8.8	13.9	0.34
4	5	9.5	7.6	0.34
5	5	10.1	2.5	0.34
6	6.3	10.1	2.5	0.34
7	6.3	10.1	8.8	0.34
8	6.3	10.1	15.1	0.34
9	6.3	8.2	21.4	0.34
10	6.3	7.6	27.7	0.34
11	5	5.1	33.4	0.34
12	5	2.5	39.7	0.34

kn an empirical wing body interference factor that is a function of fuselage geometry 0 Save

krl an empirical correction factor that is a function of the fuselage Reynolds number 0 Ok

Fig 13.15 lengths of STOL aircraft

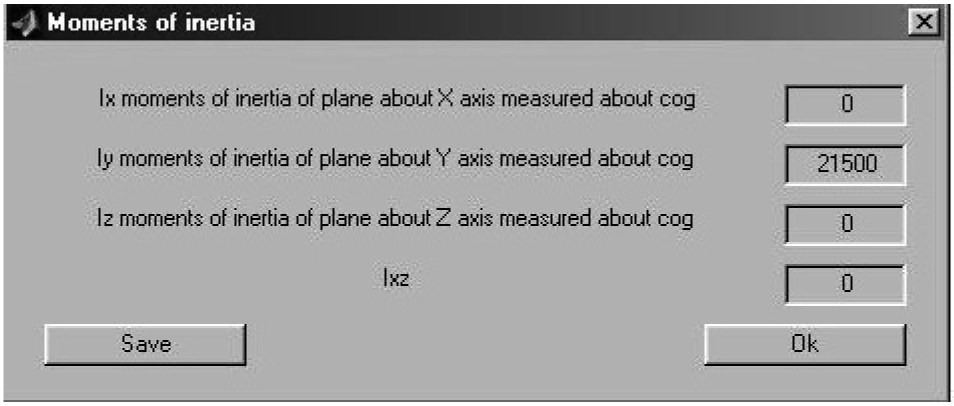


Fig 13.16 moments of inertia of STOL aircraft

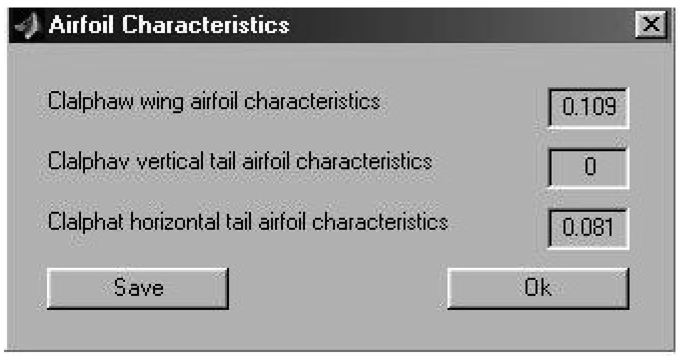


Fig 13.17 Airfoil characteristics of STOL aircraft

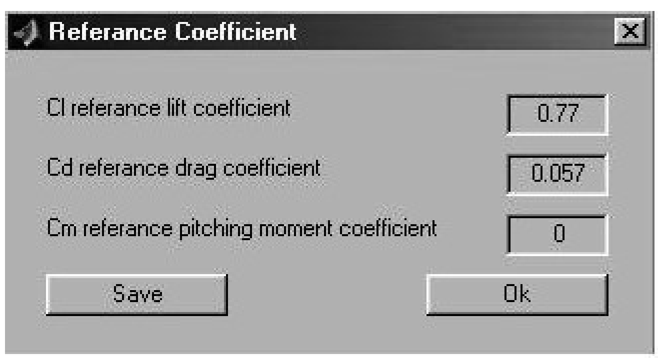


Fig 13.18 Referance coefficient of STOL aircraft

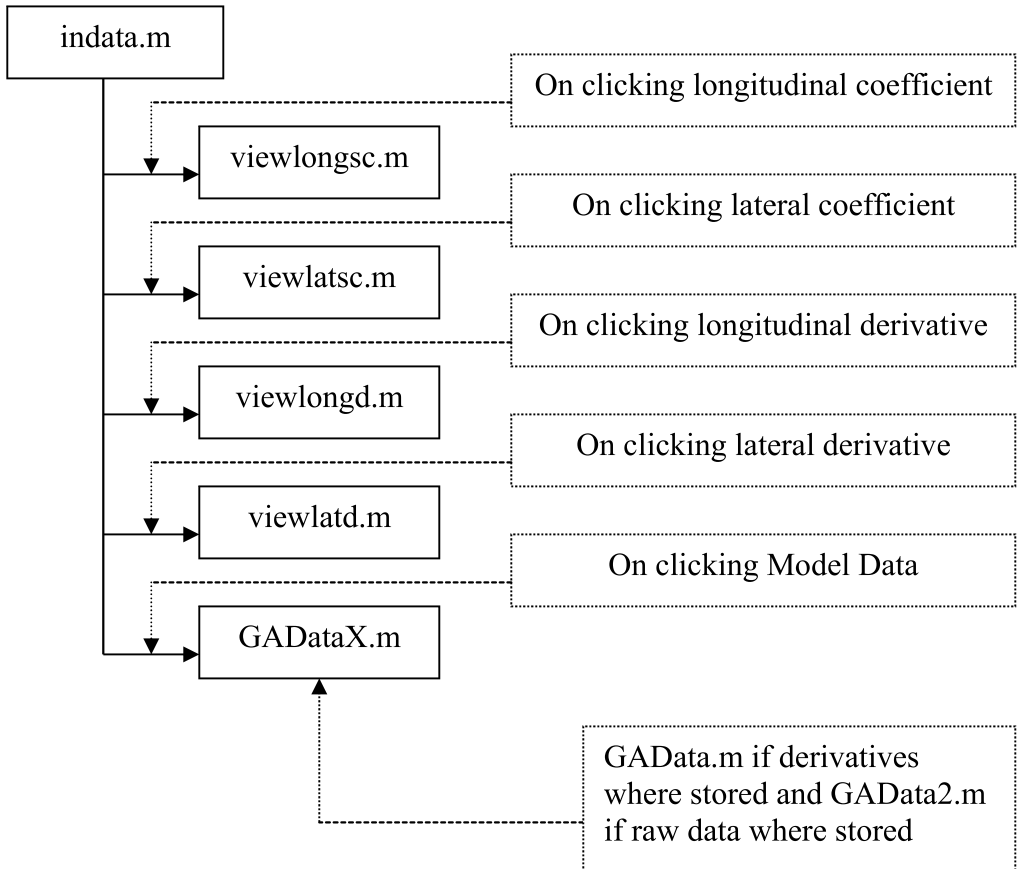


Fig 13.19 Flow chart of indata.m

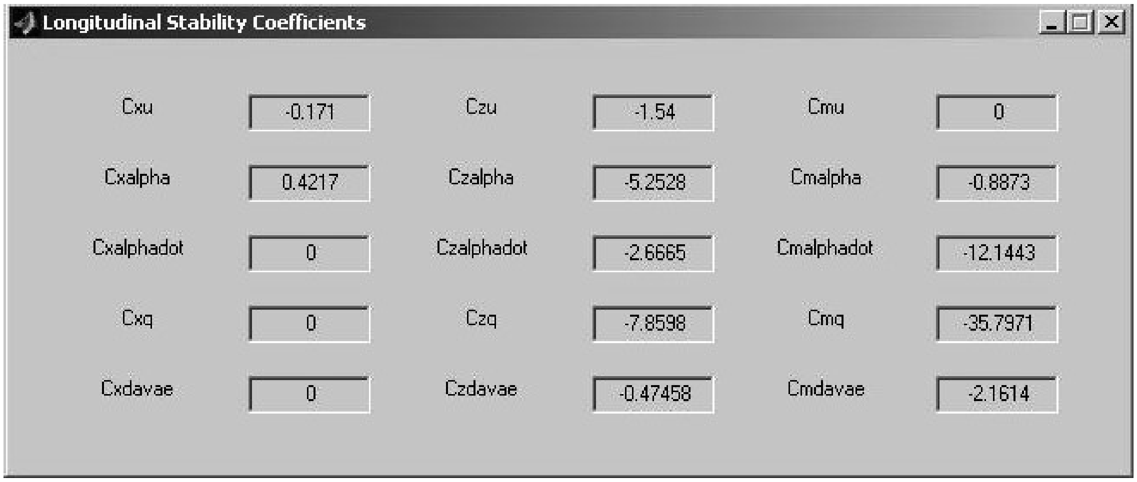


Fig 13.20 Longitudinal stability coefficients

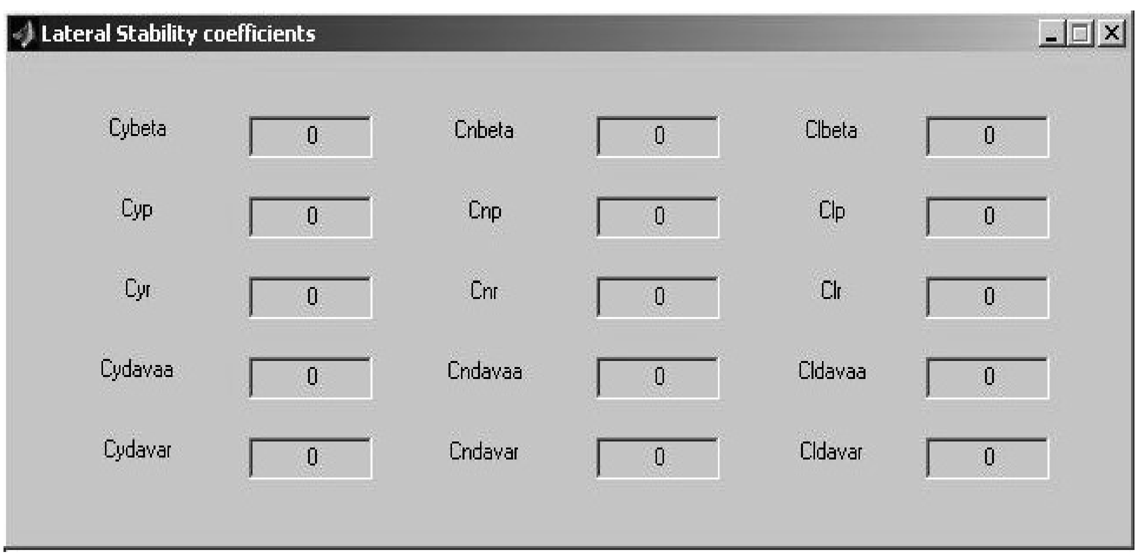


Fig 13.21 lateral stability coefficients

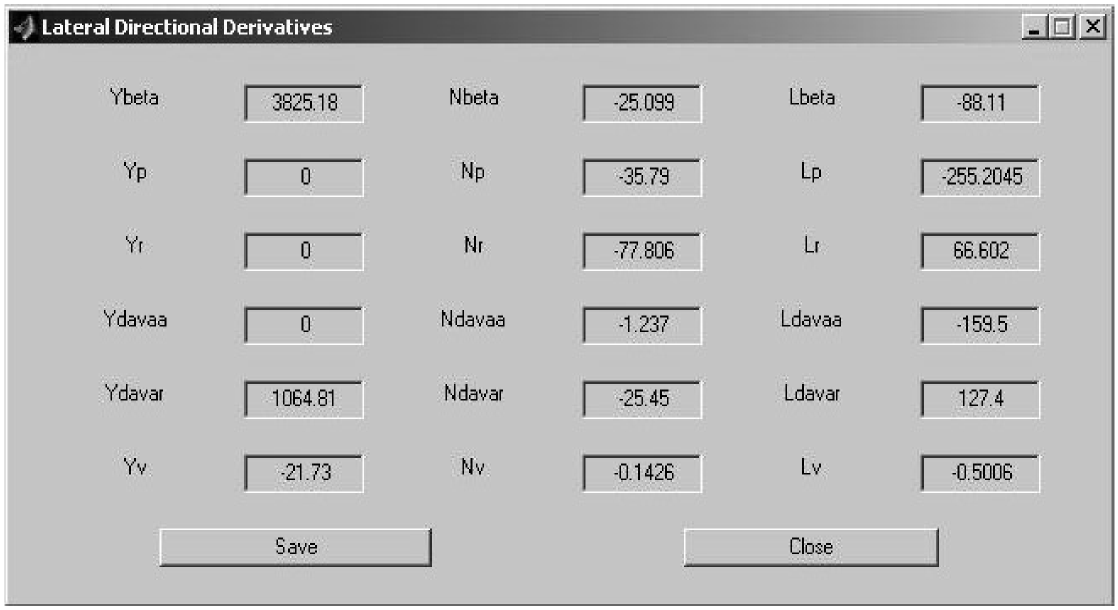


Fig 13.22 lateral stability derivatives

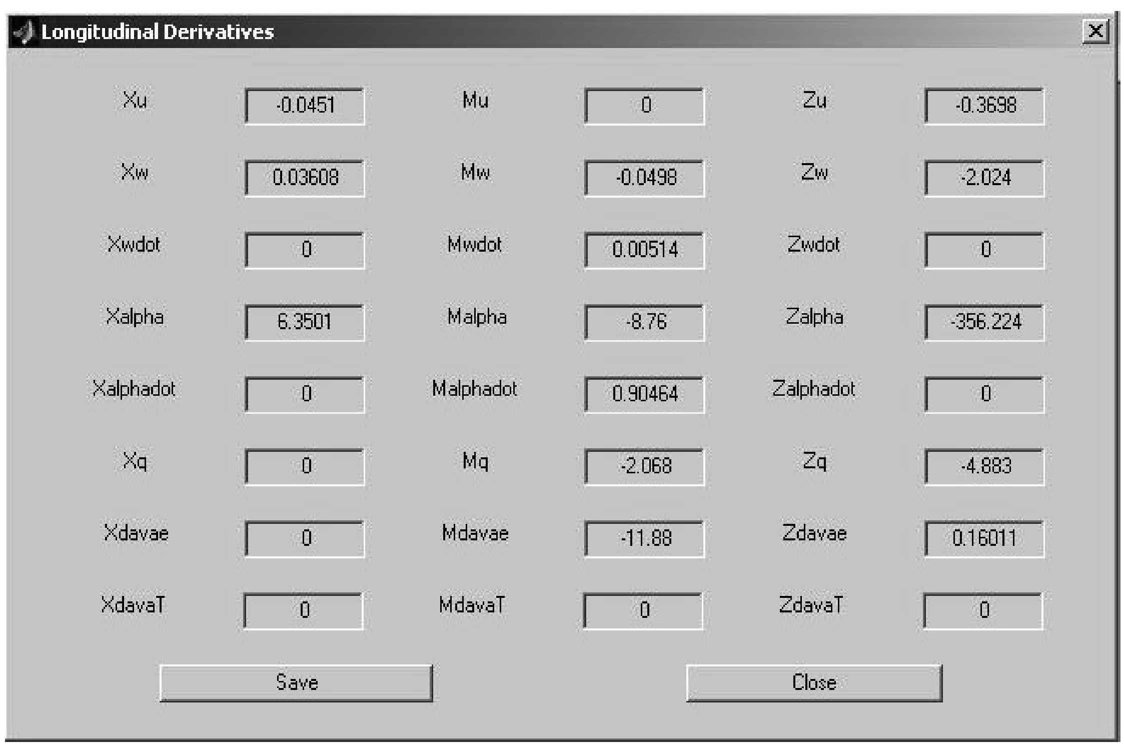


Fig 13.22 longitudinal stability derivatives

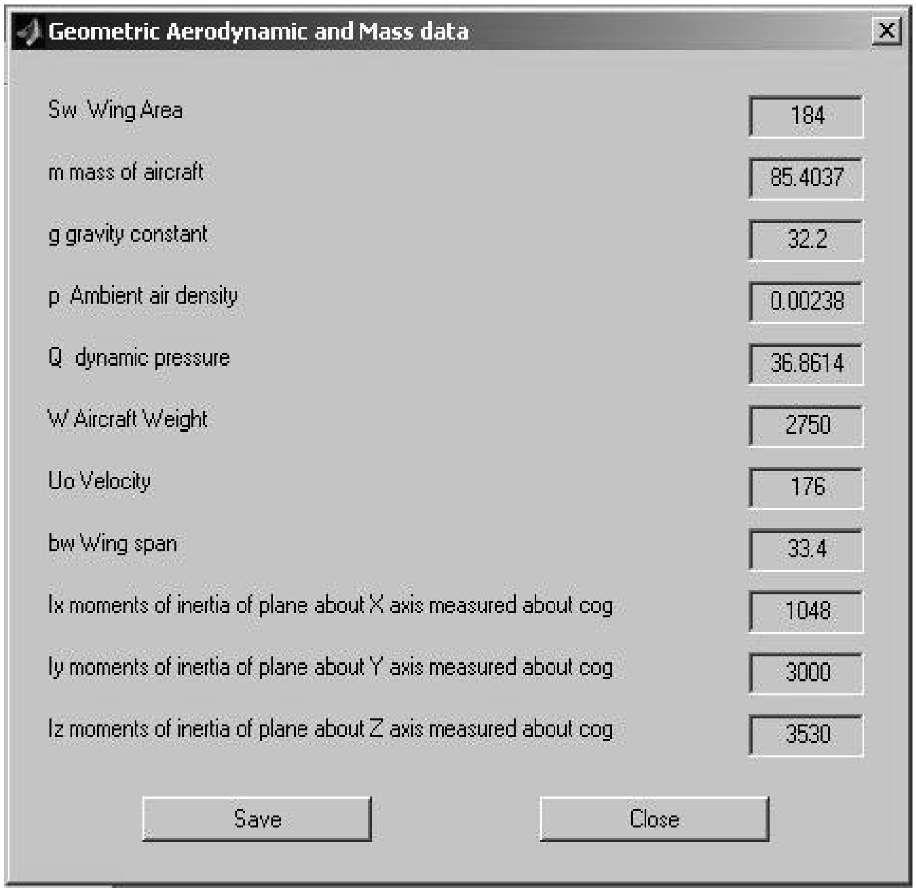
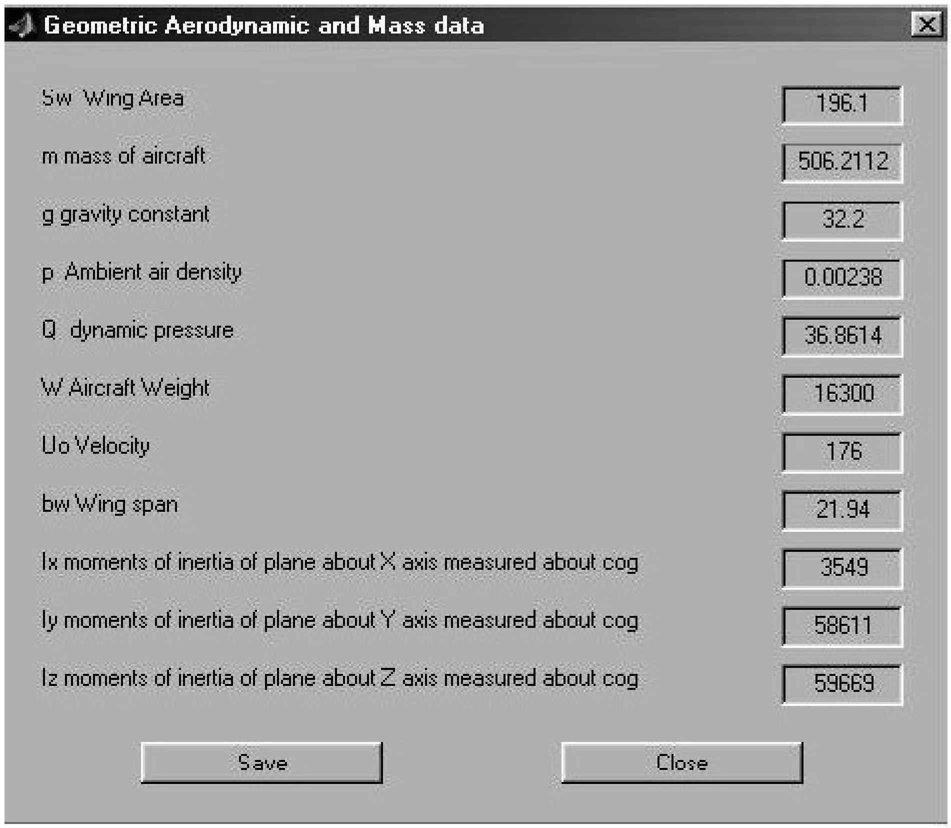


Fig 13.23 Geometric, Aerodynamic and mass data for NAVION airtcraft



'

Fig 13.24 Geometric, Aerodynamic and mass data for F104-A aircraft

13.3 Pitch Control Module (pitch m)

The first module on the left side of the main module is Pitch controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.1, a snap shot of Pitch control module is shown in figure 13.25

13.4 Roll Control Module (roll.m)

The second module on the left side of the main module is Roll controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.2, a snap shot of Roll control module is shown in figure 13.26

13.5 Heading Control Module (heading.m)

The third module on the left side of the main module is heading controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.3, a snap shot of heading control module is shown in figure 13.27

13.6 Altitude Control Module (heading.m)

The fourth module on the left side of the main module autopilot is altitude controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.4, a snap shot of heading control module is shown in figure 13.28

Table 13.1 Pitch Control Module

Caption: Pitch Controller	Control type: Button
Callback function: pitch_Callback(…..)	Action Performed: New Window Opens
File of new window: Pitch.m	Snap shot of new window: Fig 13.25

Table 13.2 Roll Control Module

Caption: Roll Controller	Control type: Button
Callback function: roll_Callback(.)	Action Performed: New Window Opens
File of new window: roll.m	Snap shot of new window: Fig 13.26

Table 13.3 Heading Control Module

Caption: Heading Controller	Control type: Button
Callback function: heading_Callback()	Action Performed: New Window opens
File of new window: Pitch.m	Snap shot of new window: Fig 13.27

Table 13.4 Altitude Control Module

Caption: Altitude Controller	Control type: Button
Callback function:altitude_Callback(.)	Action Performed: New Window Opens
File of new window: altitude.m	Snap shot of new window: Fig 13.28

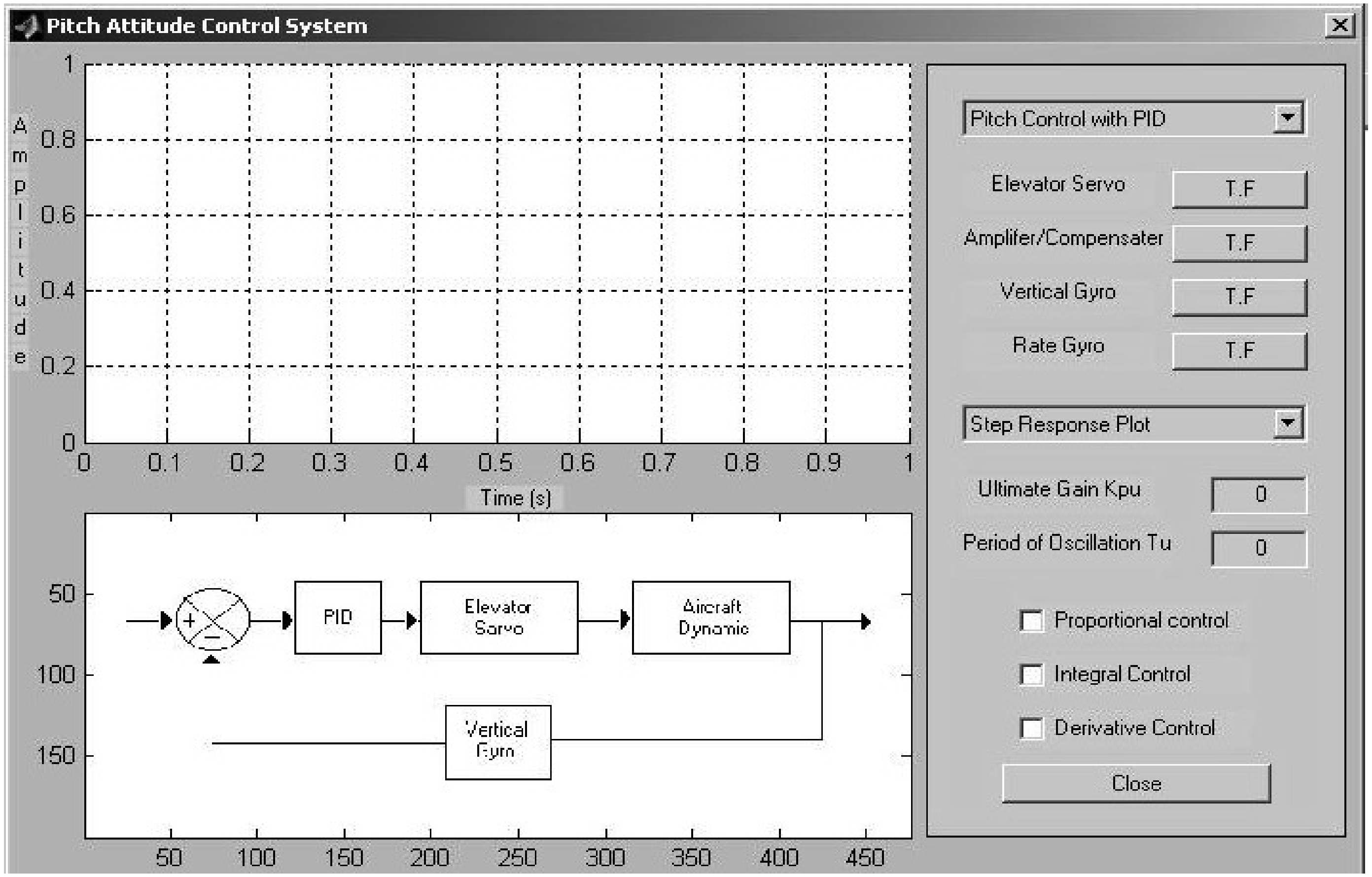


Fig 13.25 Pitch Control Module

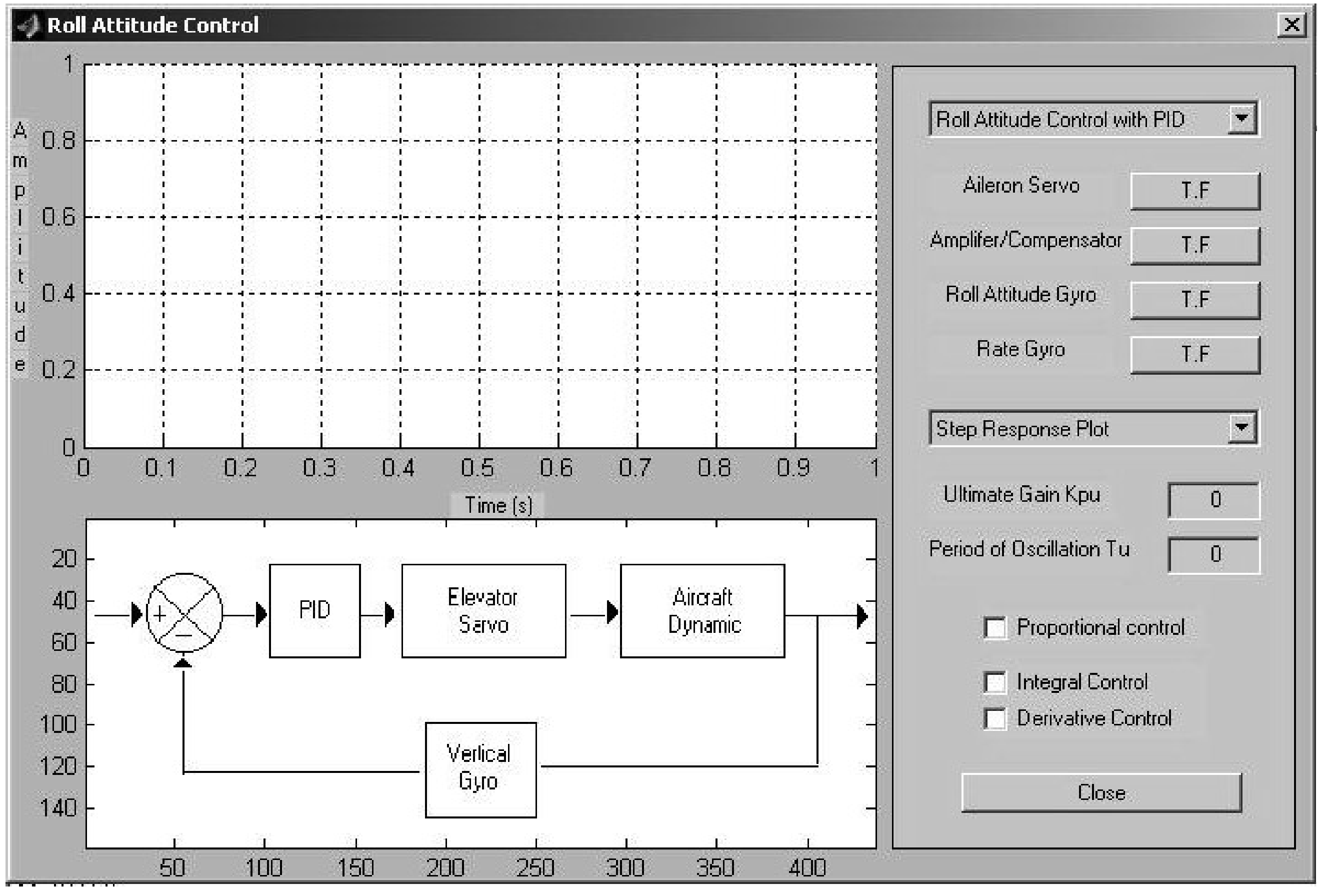


Fig 13.26 Roll Control module

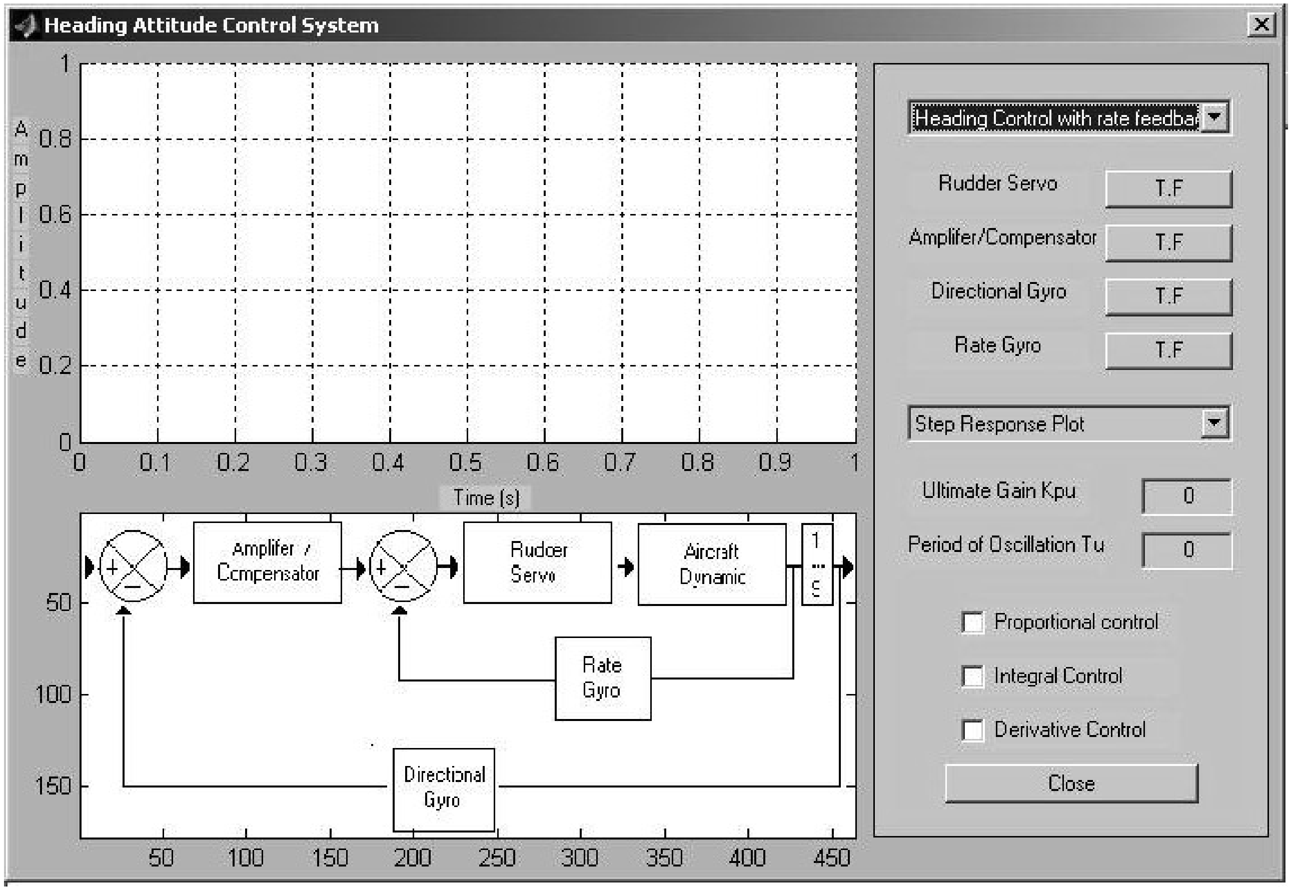


Fig 13.27 Heading Control module

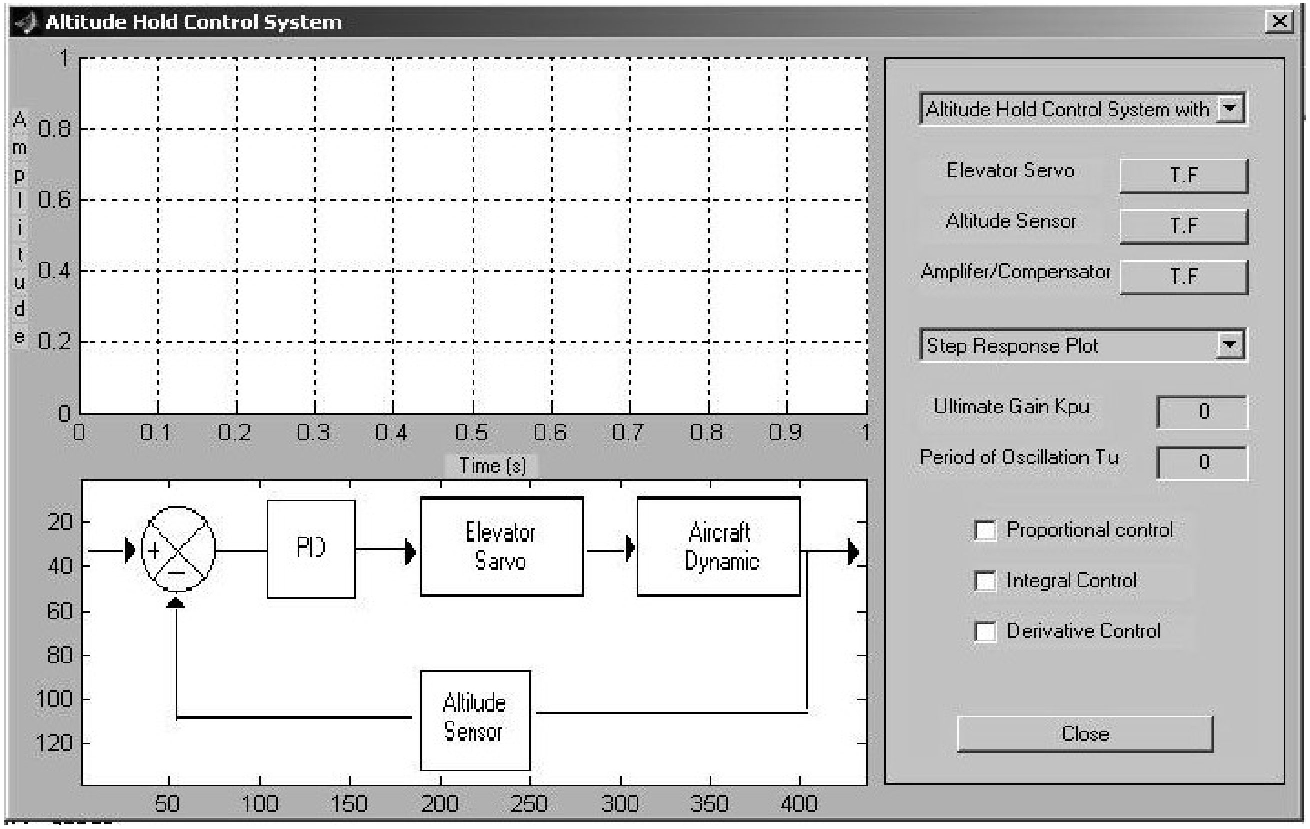


Fig 13.28 Altitude Control module

13.7 Velocity Control Module (velocity.m)

The fifth module on the main module autopilot is Velocity controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.5, a snap shot of velocity control module is shown in figure 13.29

13.8 Glide Path Control Module (glide.m)

The sixth module on the left side of the main module is Glide Path Controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.6, a snap shot of Glide path control module is shown in figure 13.30

13.9 Flare Control Module (flare.m)

The seventh module on the main module autopilot is Flare controller, a brief explanation about this file this, its callback called and action performed is shown in table 13.7, a snap shot of Flare control module is shown in figure 13.31

13.10 Stability Augmentation Module (saug.m)

The eighth module on the left side of the main module autopilot is Stability Augmentation, a brief explanation about this file this, its callback called and action performed is shown in table 13.8, a snap shot of Stability Augmentation module is shown in figure 13.32

13.11 Optimal Control Module (opcontrol.m)

The ninth module on the main module autopilot is Optimal Control module, a brief explanation about this file this, its callback called and action performed is shown in table 13.9, a snap shot of Optimal Control module is shown in figure 13.33

Table 13.5 Velocity Control Module

Caption: Velocity Controller	Control type: Button
Callback function: velocity_Callback()	Action Performed: New Window opens
File of new window: velocity.m	Snap shot of new window: Fig 13.29

Table 13.6 Glide Path Control Module

Caption: Glide path Controller	Control type: Button
Callback function: glide_Callback()	Action Performed: New Window opens
File of new window: glide.m	Snap shot of new window: Fig 13.30

Table 13.7 Flare Control Module

Caption: Flare Controller	Control type: Button
Callback function: flare_Callback()	Action Performed: New Window opens
File of new window: flare.m	Snap shot of new window: Fig 13.31

Table 13.8 Stability Augmentation Module

Caption: SAS Controller	Control type: Button
Callback function: saug_Callback()	Action Performed: New Window opens
File of new window: saug.m	Snap shot of new window: Fig 13.32

Table 13.9 Optimal Control Module

Caption: Optimal Controller	Control type: Button
Callback function:opcontrol_Callback()	Action Performed: New Window opens
File of new window: opcontrol.m	Snap shot of new window: Fig 13.33

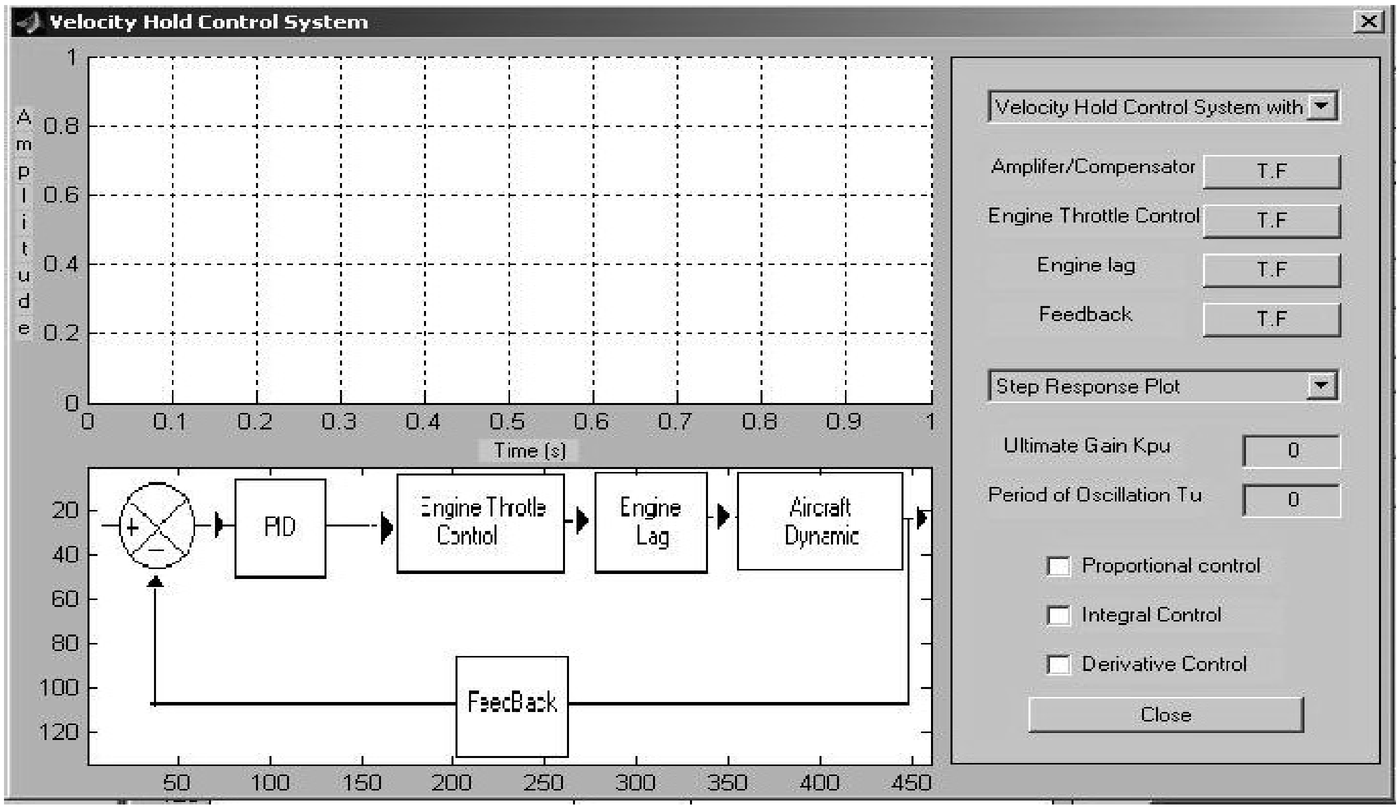


Fig 13.29 Velocity Control module

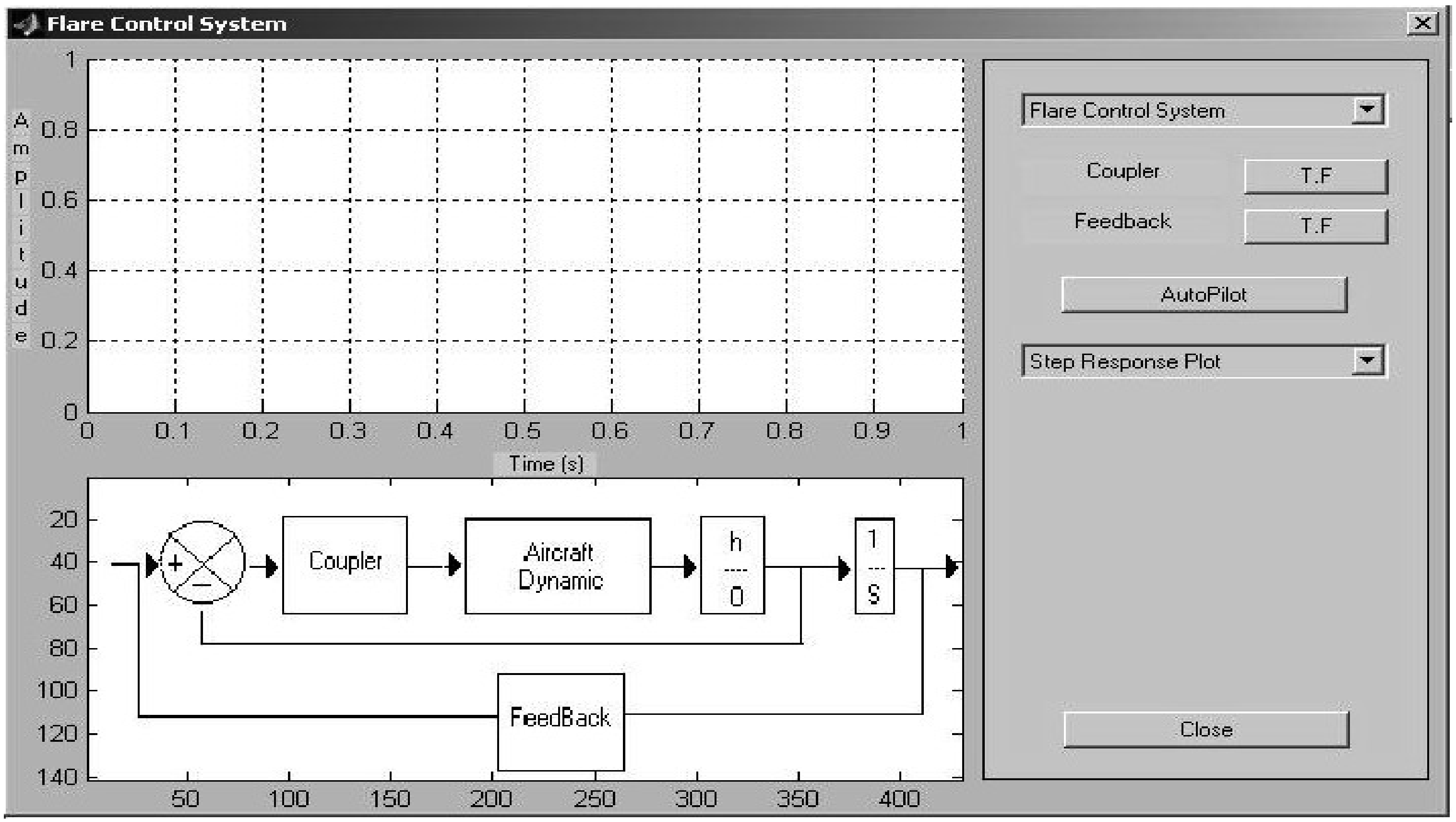


Fig 13.30 Flare Control module

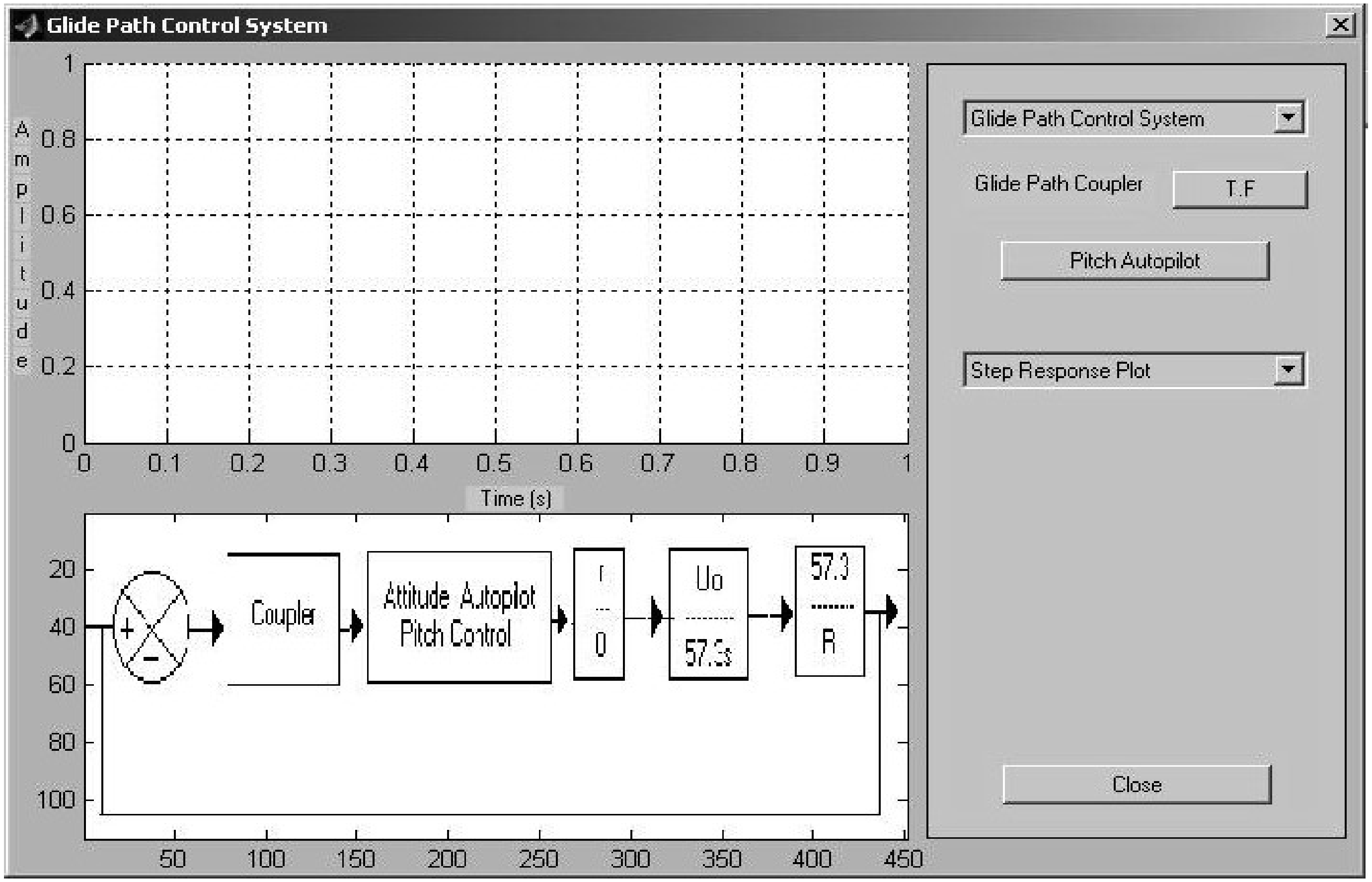


Fig 13.31 Glide Path Control module

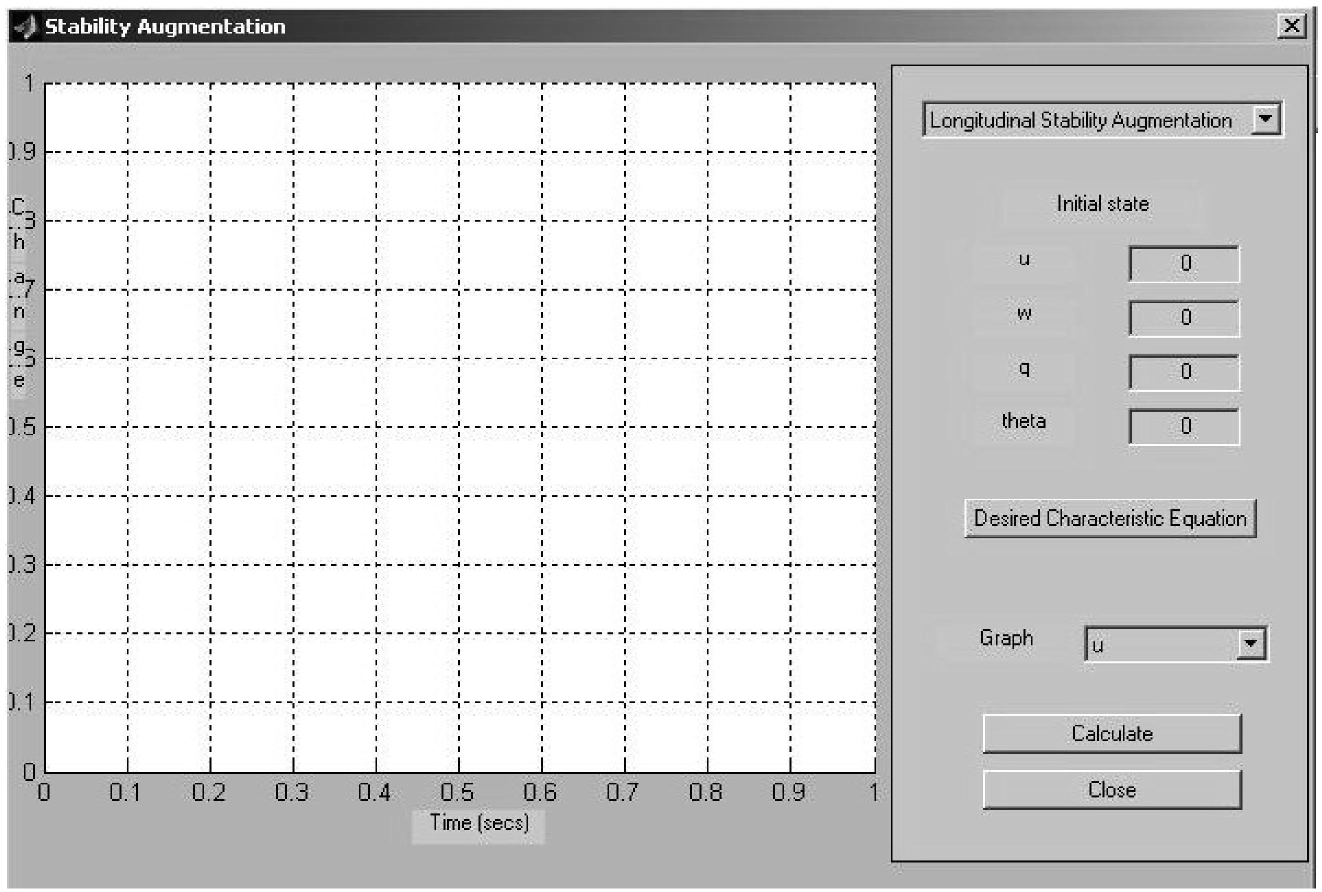


Fig 13.32 Stability Augmentation module

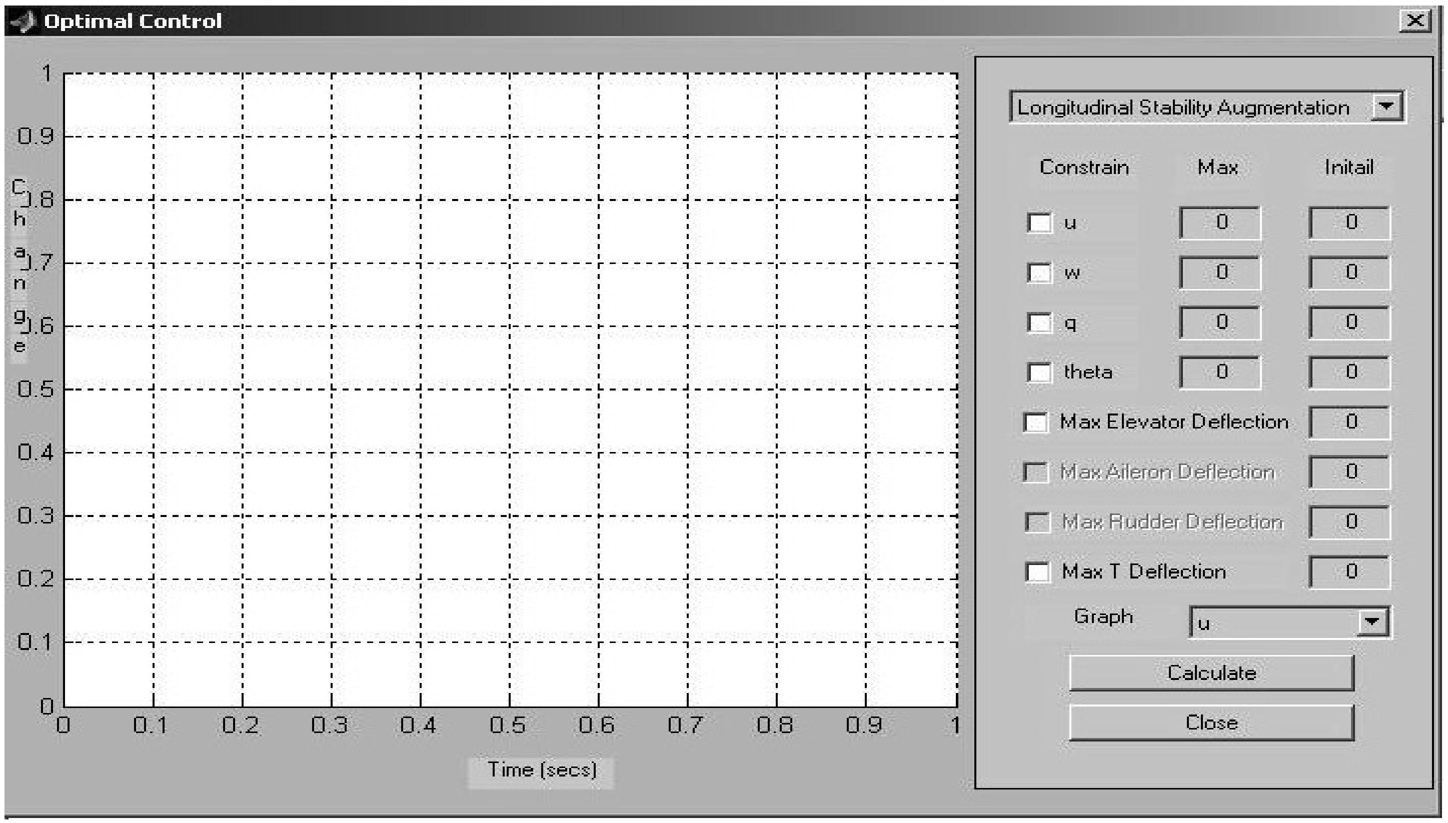


Fig 13.33 Optimal Control module

Now we will see some of the modules and how they work, we will also see them how they perform for the values we have used and how they react to different controls.

13.12 Pitch Controller

Now we will define some of the modules and we first we will see Pitch controller with Proportional control, then with Proportional and Integral control and then finally with Proportional-Integral and Differential Control.

Elevator Servo transfer function $\dfrac{-0.1}{0.1s+1}$

Aircraft Dynamic $\dfrac{-3}{s^2+2s+5}$

PID $k_p + \dfrac{k_i}{s} + k_d s$

$$k_{p_u} = 88.7$$

$$T_u = \frac{2\pi}{\omega} = 1.22$$

P Controller $k_p = 0.5k_{p_u} = 44.35$

I Controller $k_p = 0.45k_{p_u} = 39.92$

$k_i = 0.45k_{p_u} / 0.83T_u = 39.42$

D Controller $k_p = 0.6k_{p_u} = 53.22$

$k_i = 0.6k_{p_u} / 0.5T_u = 87.24$

$k_d = 0.6k_{p_u}(0.125T_u) = 8.12$

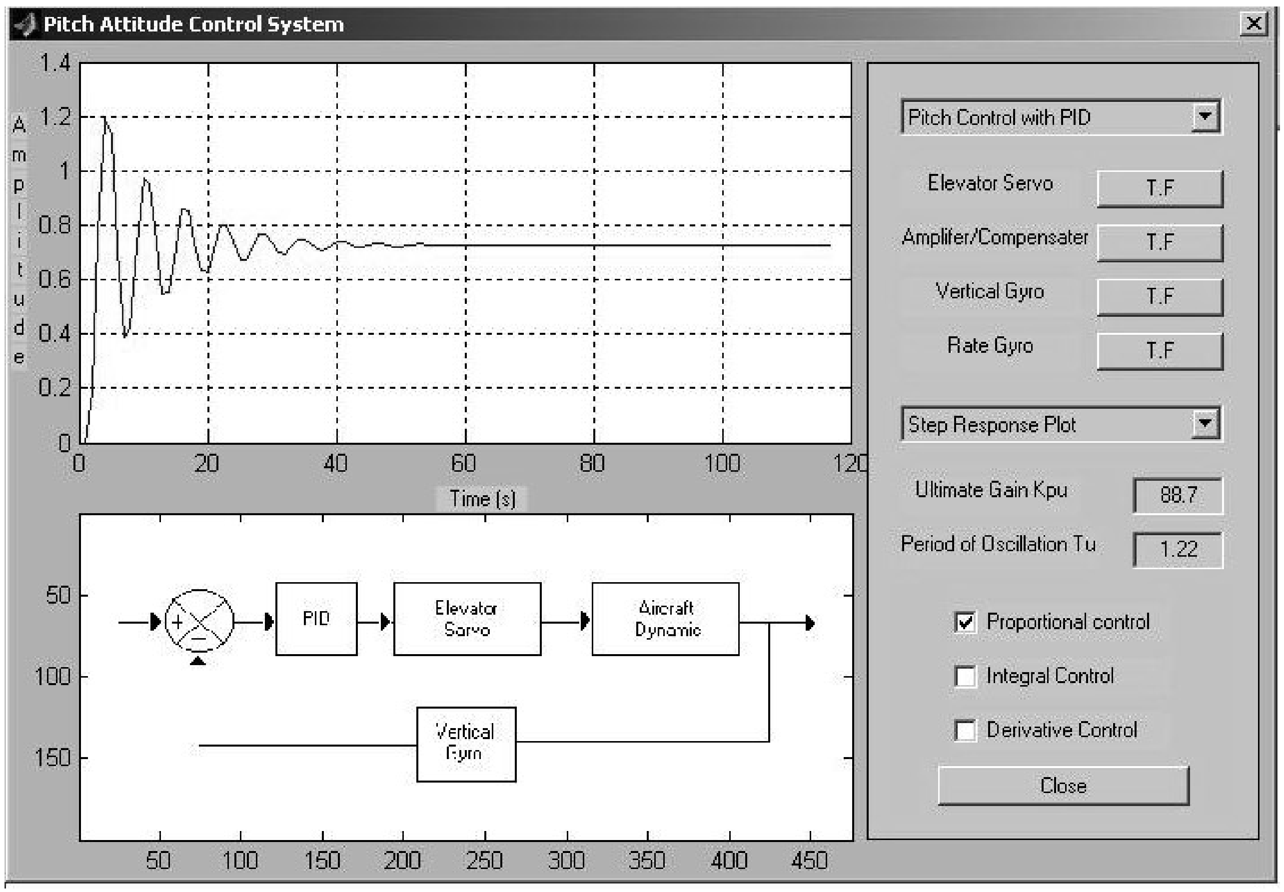


Fig13.34 Pitch controller with proportional controller

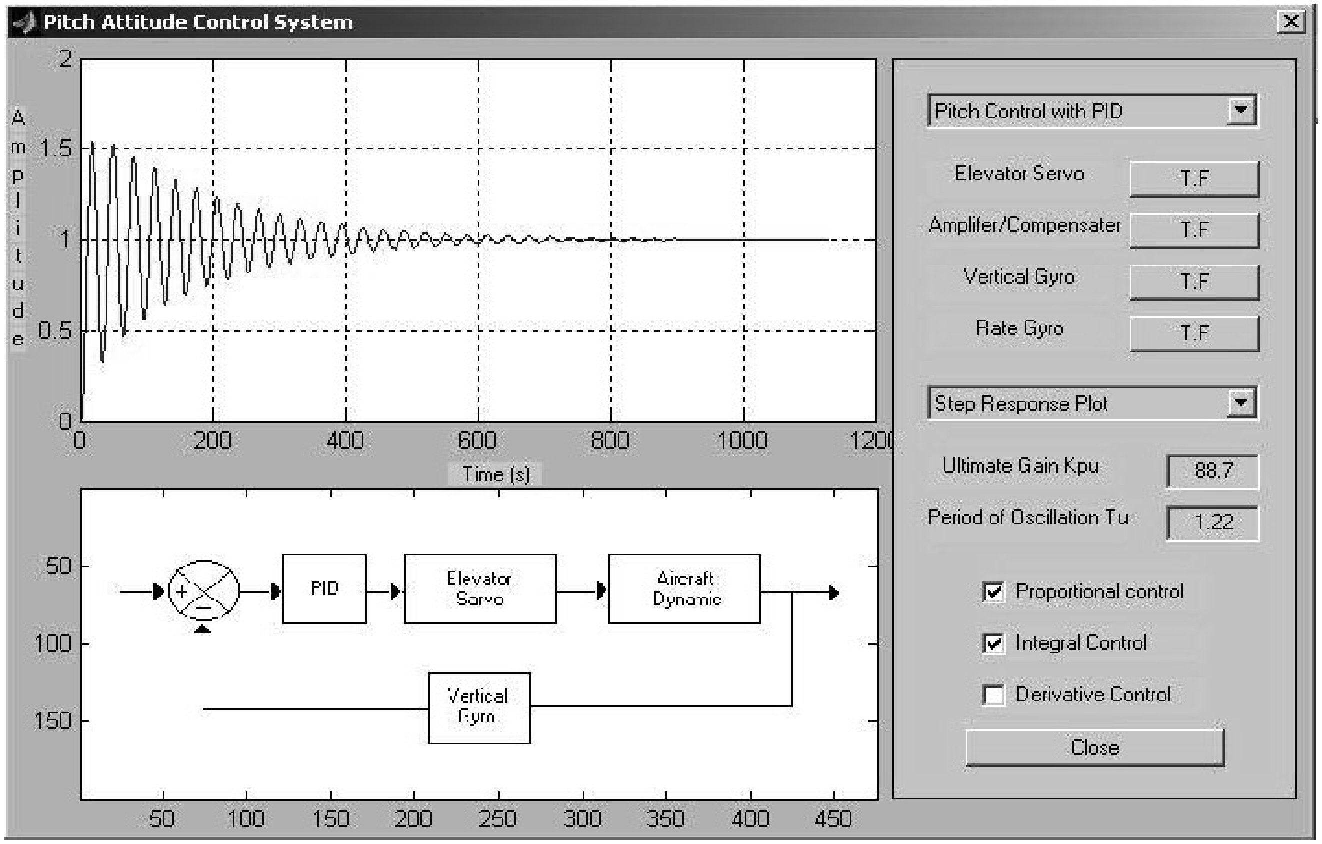


Fig 13.35 Pitch controller with proportional and integral controller

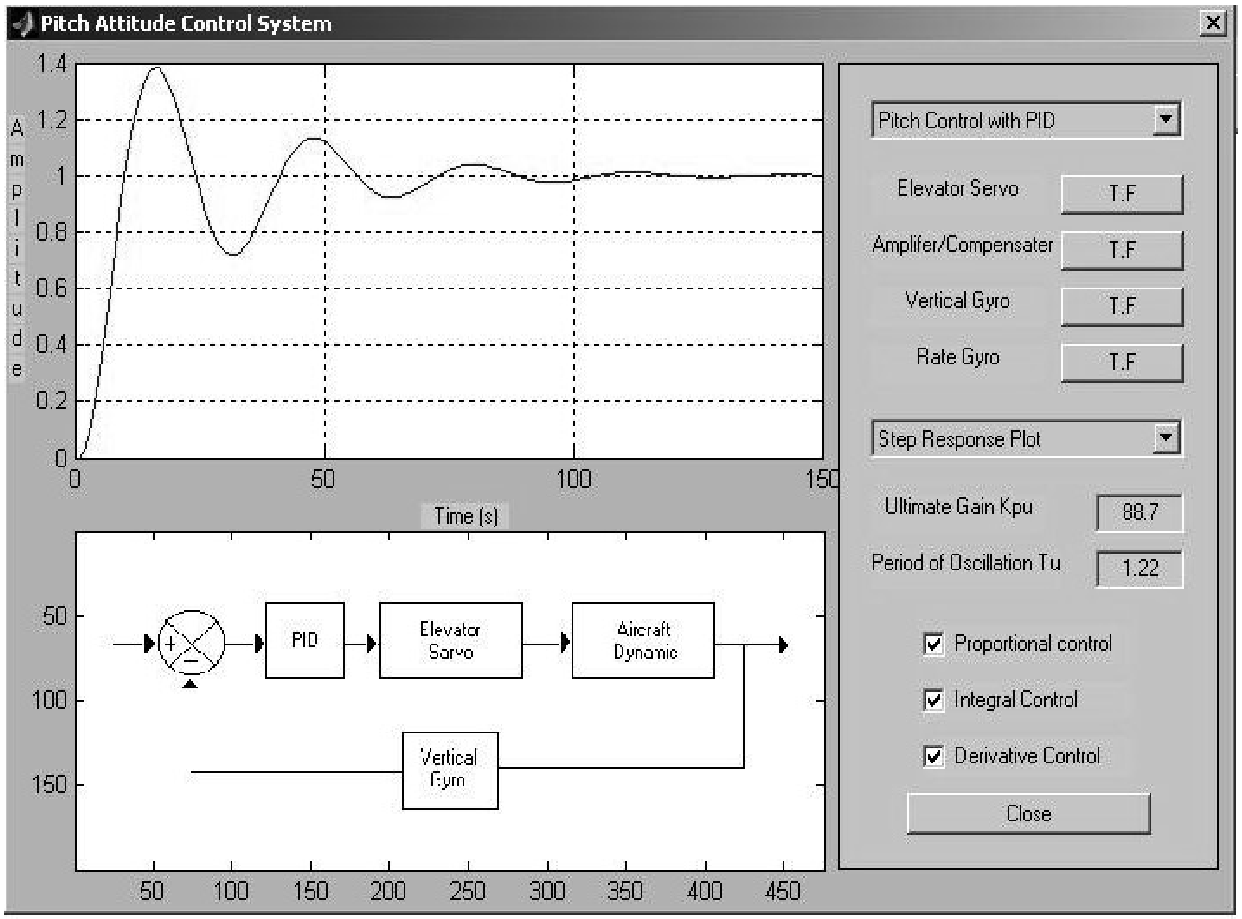
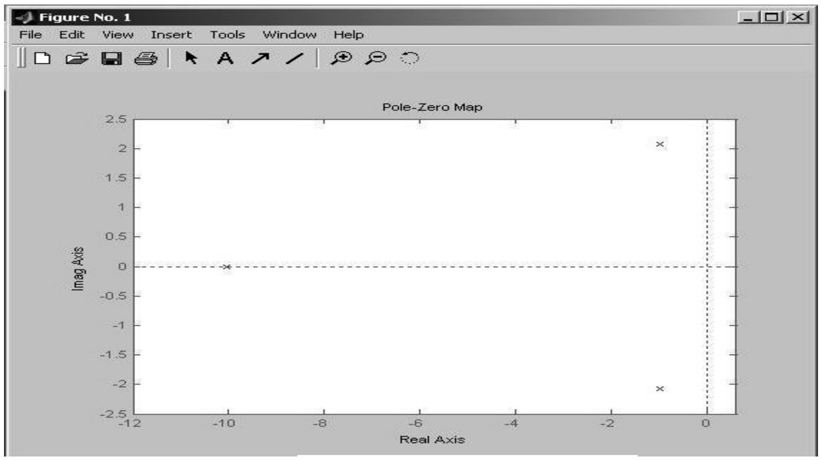
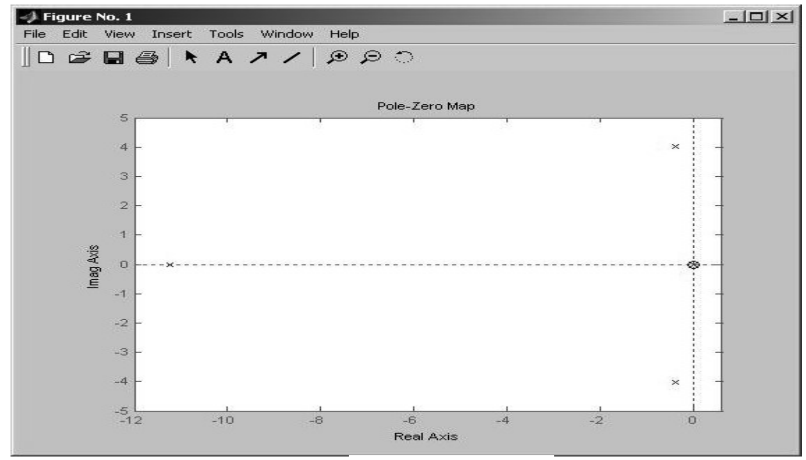


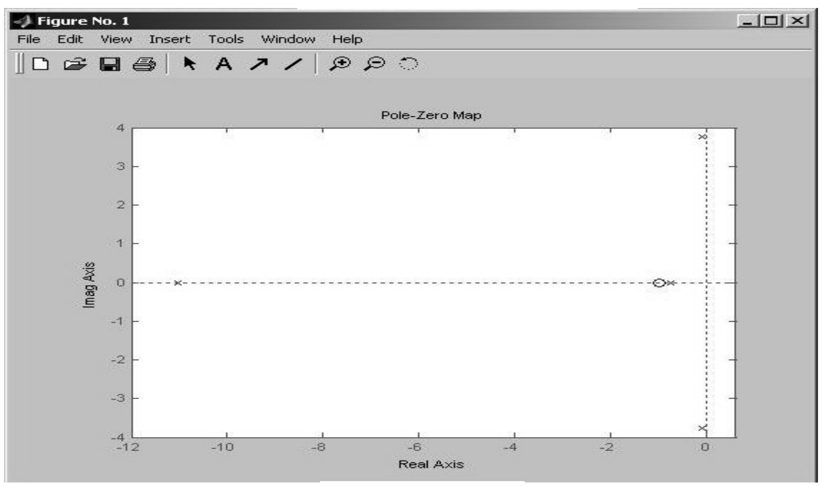
Fig 13.36 Pitch controller with proportional integral and derivative controller



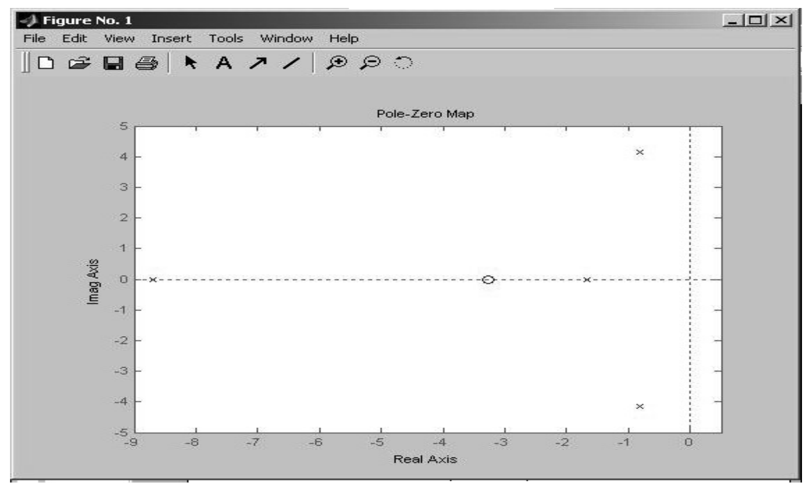
Without Controller



P Controller



PI Controller



PID Controller

Fig 13.37 Pole and Zero map

13.13 Altitude hold control system using Short period approximation and modern control theory

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} -0.124 \\ -13.2 \\ 0 \\ 0 \end{bmatrix} [\Delta\delta e]$$

Desired location of the eigen values

$$\lambda_{1.2} = -1.0 \pm 3.5i \quad , \quad \lambda_{3,4} = -2.0 \pm 1.0i$$

Characteristic equation:

$$\lambda^4 + 6.0\lambda^3 + 23.25\lambda^2 + 63\lambda + 66.25 = 0$$

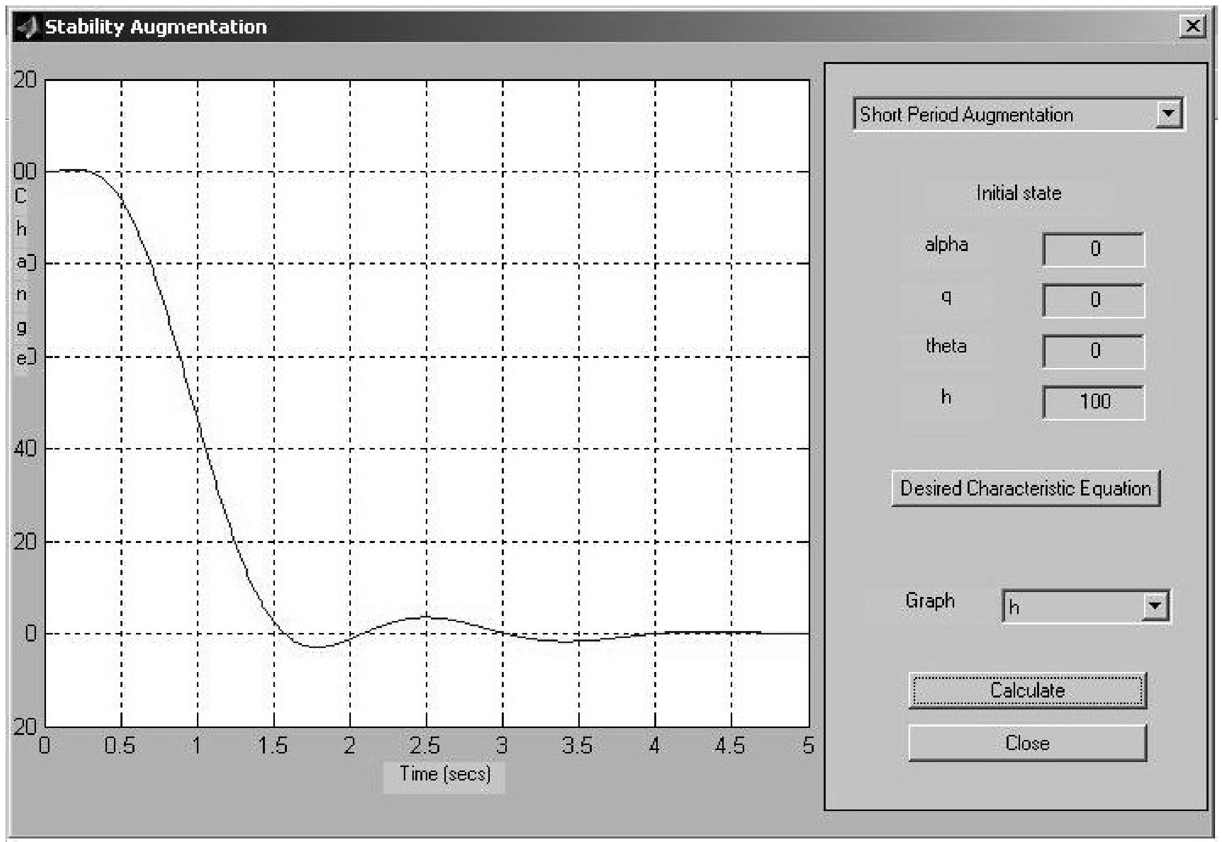


Fig 13.38 Stability augmentation system for altitude controller

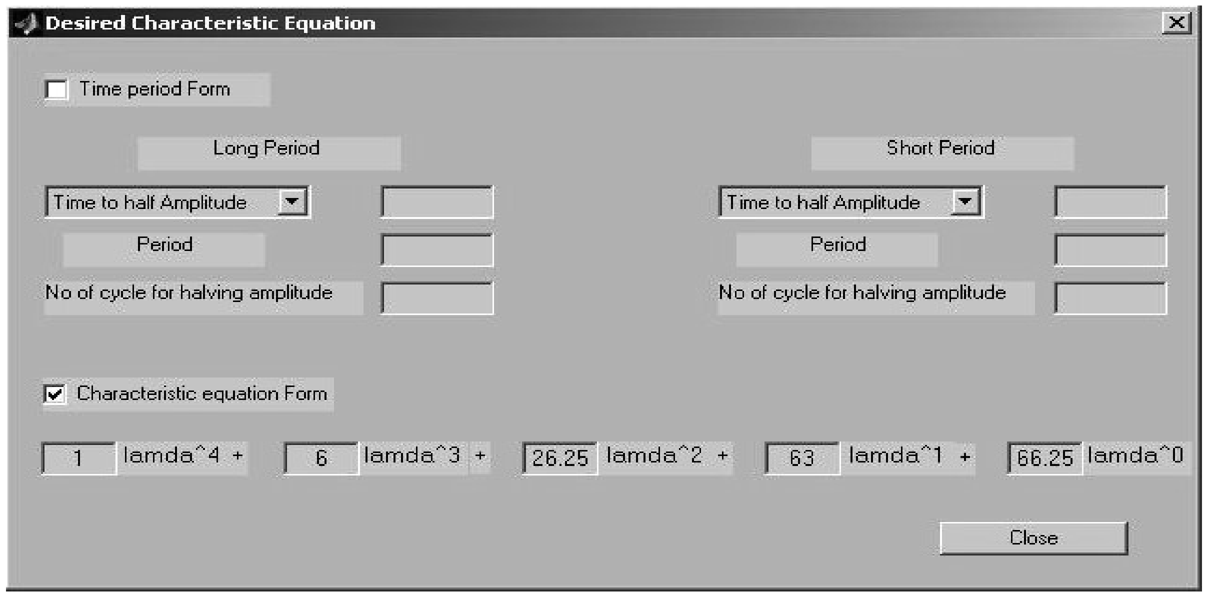


Fig 13.39 Characteristic equation for above example
This window pops up on clicking "Desired characteristic equation" on stability augmentation system windows

13.14 Optimal Control

Optimal Control constrains on angle of attack, attitude excursion and elevator deflection

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} -1.397 & 1 & 0 & 0 \\ -5.47 & -3.27 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -400 & 0 & 400 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} -0.124 \\ -13.2 \\ 0 \\ 0 \end{bmatrix} [\Delta\delta e]$$

$\Delta\alpha_{max} = 5^{o} = 0.087\text{rad}$

$\Delta h_{max} = 100\text{ft}$

$\Delta\delta_{e_{max}} = 10^{o} = 0.175\text{rad}$

$$Q = \begin{bmatrix} \left(\frac{1}{\alpha_{max}}\right)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{1}{h_{max}}\right)^2 \end{bmatrix}$$

$$R = \left[\left(\frac{1}{\delta_{e_{max}}}\right)^2\right]$$

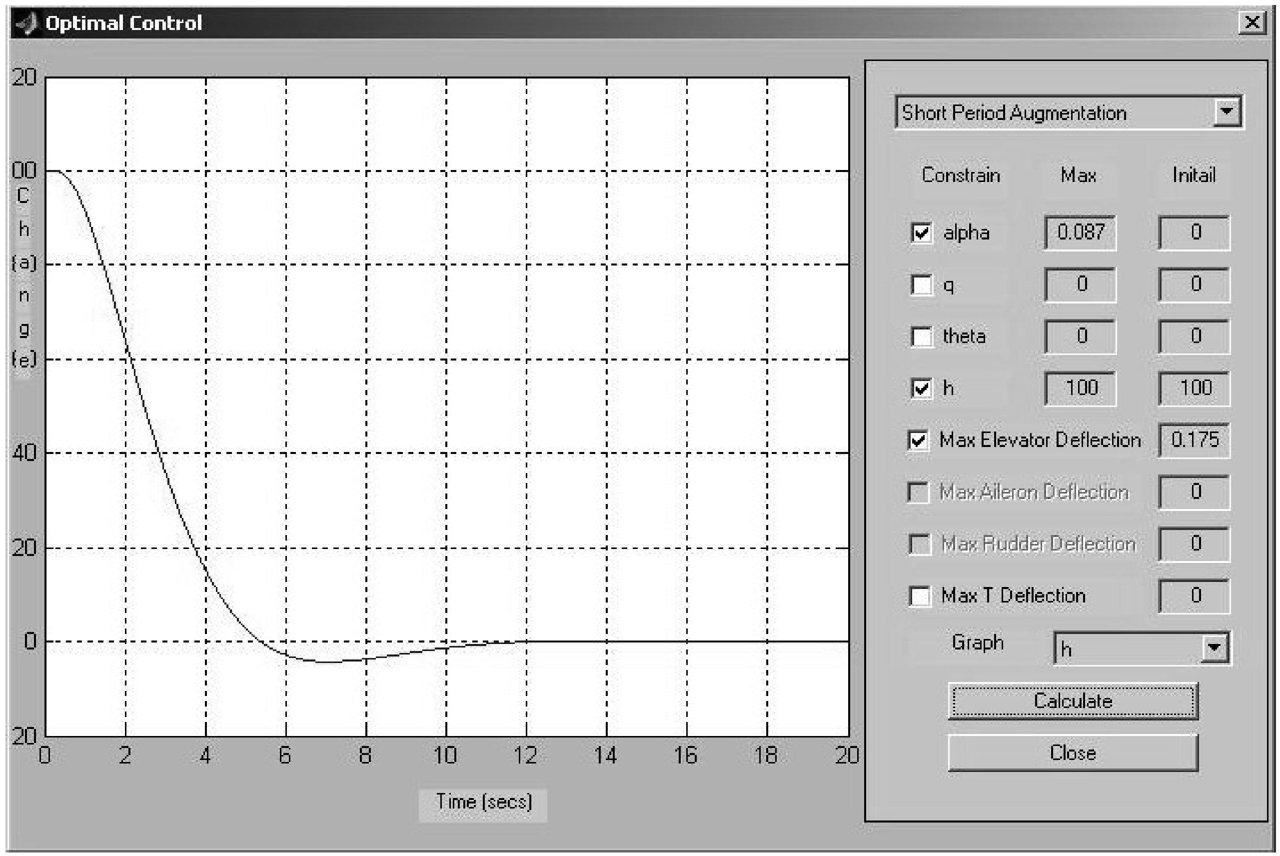


Fig 13.40 Optimal controller for with constrain on attitude angle of attack and elevator deflection

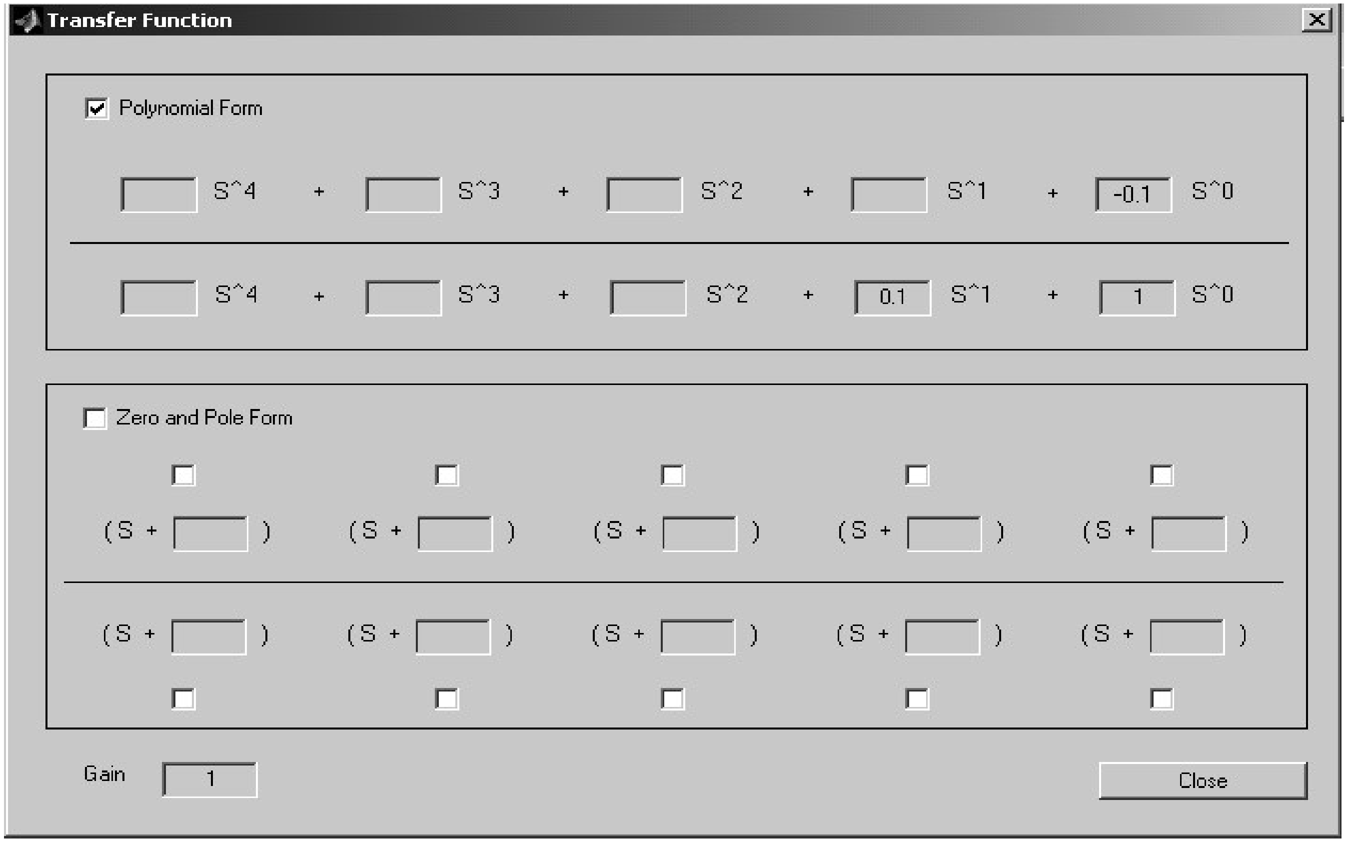


Fig 13.41 Transfer Function
This function pops up on clicking T.F button on various control windows

Appendix A: Control System

Control systems can be classified as either open loop or closed-loop systems. In the open-loop system the control action is independent of the output. In closed-loop system the control action depends on the output of the system. Closed-loop control systems are called feedback control systems. The advantage of the closed-loop system is its accuracy.

To obtain a more accurate control system some form of feedback between the output and input must be established. This can be accomplished by comparing, the controlled signal (output) with the commanded or reference input. In a feedback system one or more feedback loops are used to compare the controlled signal with the command signal to generate an error signal. The error signal then is used to drive the output signal into agreement with the desired input signal.

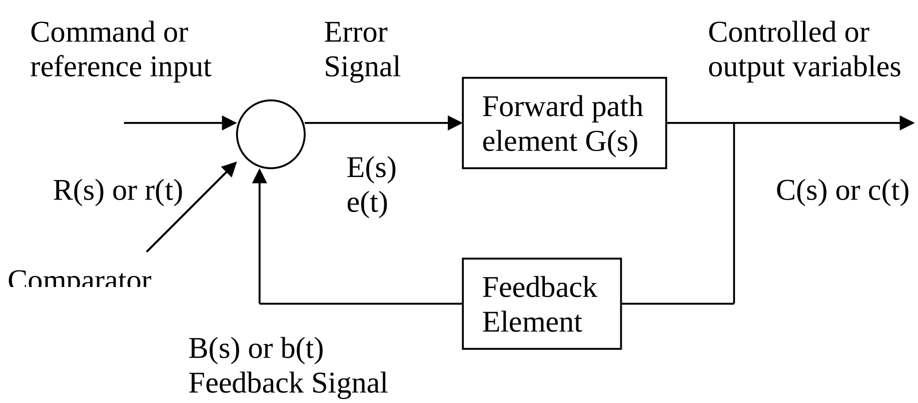


Figure A

The typical closed-loop feedback system shown in Figure A is composed of a forward path, a feedback path, and an error-detection device called a comparator. Each component of the control system is defined in terms of its transfer function

The closed-loop transfer function for the feedback control system shown in Figure A can be developed from the block diagram. The symbols used in the block diagram are defined on next page as :

R(s) reference input

C(s) output signal (variable to be controlled)

B(s) feedback signal

E(s) error or actuating signal

G(s) *C(s)IE(s)* forward path or open-loop transfer function

M(s) *C(s)/R(s)* the closed-loop transfer function

H(s) feedback transfer function

G(s)H(s) loop transfer function

The closed-loop transfer function, *C(s)/R(s)*, can be obtained by simple algebraic manipulation of the block diagram. The actuating or error signal is the difference between the input and feedback signals:

$$E(s) = R(s) - B(s)$$

The feedback signal *B(s)* can be expressed in terms of the feedback transfer function and the output signal:

$$B(s) = H(s)C(s)$$

and the output signal *C(s)* is related to the error signal and forward path transfer function in the following manner:

$$C(s) = G(s)E(s)$$

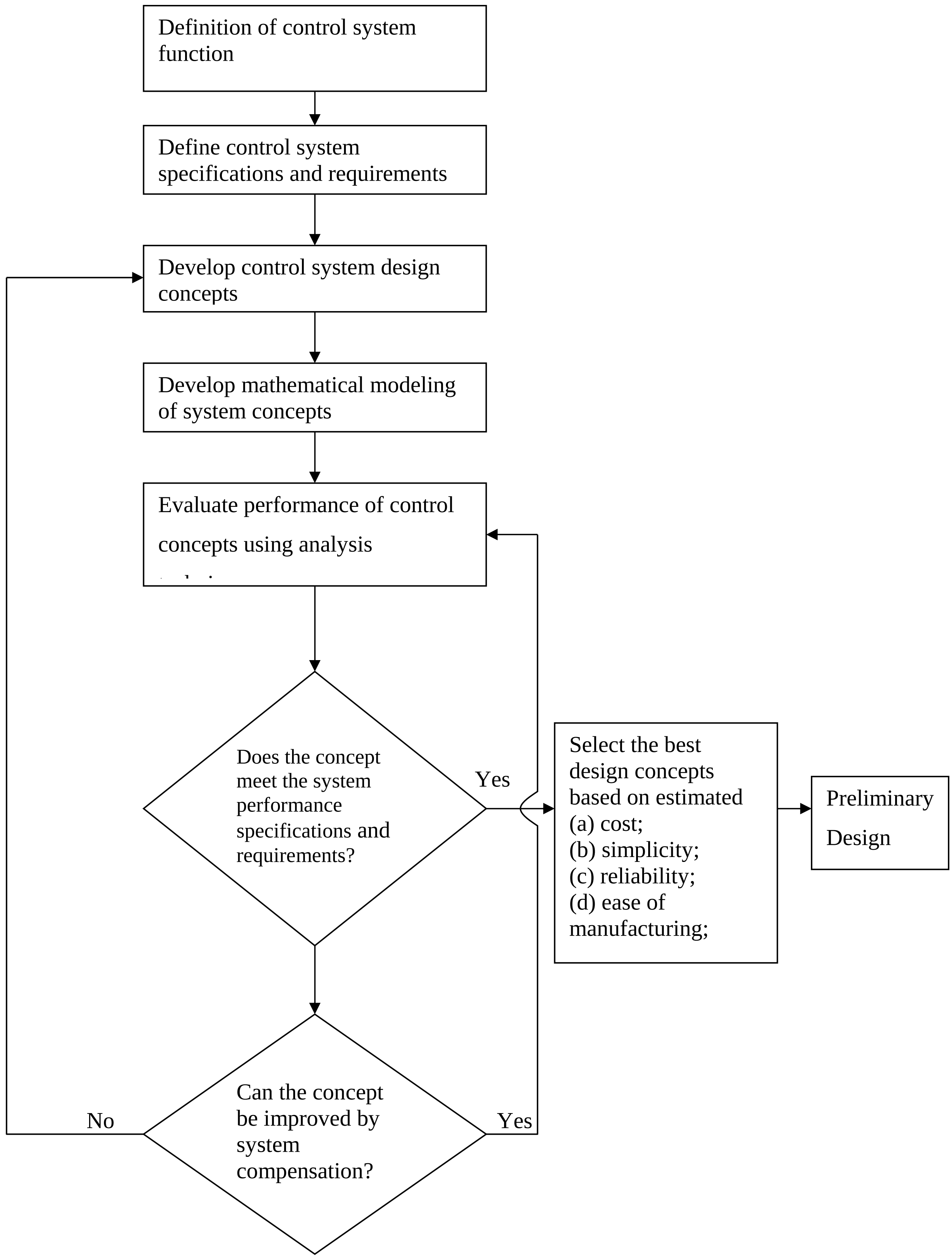
$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

Equation above can be solved for the closed-loop transfer function *C(s)/R(s)*

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Note that feedback in itself does not ensure that the system will be stable. Therefore, to design a feedback control system one needs analysis tools that allow the designer to select system parameters so that the system will be stable. In addition to determining the absolute stability, the relative stability of the control system also must be determined. A system that is stable in the absolute sense may not be a satisfactory control system. For example, if the system damping is too low the output will be characterized by large amplitude oscillations about the desired output. The large overshooting of the response may make the system unacceptable.

Appendix B: Conceptual Control System Design Process.



Appendix C: Transfer function

The transfer function of a system is a ratio of Laplace transform of output signal to the Laplace transform of input signal where the initial conditions are assumed to be 0.

$Y(s)=G(s)X(s)$ ($G(s)$ is transfer function)

Poles

Pole is any value of s that makes the denominator of the transfer function equal to zero. Equivalently it is any value of s that makes the transfer function tend to infinity

Zeros

A zero is any value of s that makes the numerator of the transfer function equal to zero. Equivalently it is any value of s that makes the transfer function equal to zero

Appendix D: Performance characteristics of the system

Overshoot

The amount by which a system overshoots its final value can be very important. Overshoot is usually expressed as a percentage of the desired final value.

Settling time

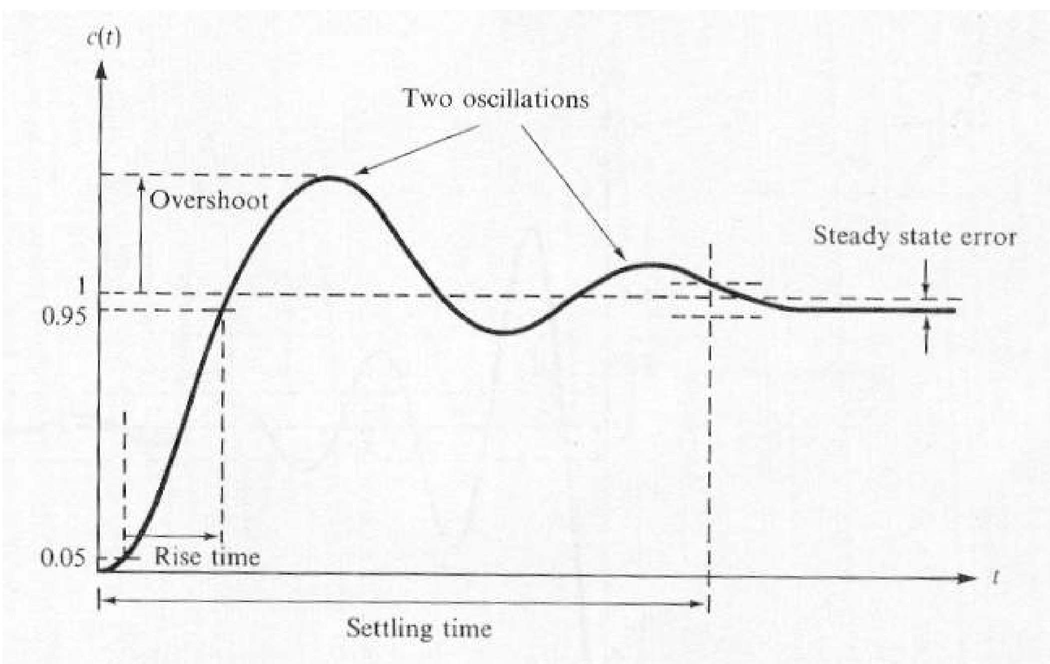
The time required to reach desired value is called settling time of the system.

Steady state error

Steady state error is the amount by which the final value differs from the desired value when the system settles to a constant value.

Rise time

Definition of rise time varies but most common one is the time to go from 5% to 95% of the desired value.



Appendix E: Compensator

The ultimate test of a design concept is whether it meets the desired performance specifications. The control system performance is specified in terms of transient behavior and the steady state error. The transient performance in the time domain can be describe in terms of damping ratio ζ the peak overshoot and the speed of the response as measured by the rise and settling time.

When the performance cannot be satisfied the designer can add an additional component to the control system called compensator. Compensator represents addition of poles and zeros to a given control system transfer system.. By Selecting the parameter in the compensator the designer can shape of the root locus plot so that the overall performance specification can be met.

When the compensator is added to the forward path it is called a cascade or series compensator and when it is placed in feedback it is called feedback or parallel compensator

Transfer function of compensator

$$G_c = \frac{(s+z_1)(s+z_2)....}{(s+p_1)(s+p_2)....}$$

Lead compensator

If $z_c / p_c < 1$ compensator is called lead compensator. It is used to improve damping ratio and settling time.

Lag compensator

If $z_c / p_c > 1$ compensator is called lag compensator. It is used to decrease steady state error.

Appendix F: PID Controller

P controller

The simplest feedback controller is one for which the controller output is proportional to error signal. Such controller is called proportional controller.

Proportional controller's main advantage is its simplicity. It has the disadvantage that there may be a steady state error.

$$\eta(t) = k_p e(t)$$

I controller

Integral controller is used for controlling steady state error.

The advantage is that the output is proportional to accumulated error. The disadvantage is that system becomes less stable.

$$\eta(t) = k_i \int_0^t e(t)dt$$

D controller

Derivative controller is used for large error corrections.

The major disadvantage is that it will not produce a control output if error is constant. Another problem is that it is susceptible to noise.

$$\eta(t) = k_d \frac{de(t)}{dt}$$

Each controller has its advantage and disadvantages. The disadvantages can be controlled by combination of all three controllers.

	Rise time	***Overshoot***	***Settling Time***	***SS Error***
k_p	Decrease	Increase	Small Change	Decrease
k_i	Decrease	Increase	Increase	Eliminate
k_d	Small Change	Decrease	Decrease	Small Change

Appendix G: Ziegler and Nichols tuning Theory

The selection of gain of PID controller can be determined by a method developed by Ziegler and Nichols. According to Ziegler and Nichols the gains k_p, k_i and k_d are determined in terms of two parameters k_{p_u} called ultimate gain and T_u the period of the oscillation that occurs at the ultimate gain. Table below gives value for gains for P, PI and PID controllers.

To apply this technique the root locus plot for control system with the k_i and k_d set to 0 must become marginally stable. That is as the k_p is increased the locus must intersect the imaginary axis. The k_p for which this occurs is called the ultimate gain k_{p_u}. The purely imaginary roots $\lambda = \pm i\omega$ determine the value of T_u

$$T_u = \frac{2\pi}{\omega}$$

One additional restriction must be must be met. All other roots of the system must have negative real parts. That is they must be in left hand portion of the complex s plane. If these restrictions are satisfied the P, PI and PID gains easily can be determined

Gain P, PI and PID Controllers

Type of Controller	k_p	k_i	k_d
P	$k_p=0.5k_{pu}$		
PI	$k_p=0.45k_{pu}$	$k_i=0.45k_{pu}/(0.83T_u)$	
PID	$k_p=0.6k_{pu}$	$k_i=0.6k_{pu}/(0.5T_u)$	$k_d=0.6k_{pu}(0.125T_u)$

Appendix H: State space method

The application of state variable to control problems is called modern control theory. The state space equations are simply first order differential equations that govern the dynamics of the system being analyzed. It should be noted that higher order system could be decomposed into set of first order differential equations.

A state space model happily deals with multiple inputs and outputs as well as taking care of nonzero initial conditions. A state space model models the internal state of a system rather then directly describing the relationship between the systems inputs and output.

At the heart of the approach is the concept of the state space variables, which are used to describe the current state of the system. These state variables are related to each other by means of a set of state equations, which are a set of coupled first order differential equations.

It is convention to write the state space in a standard form. This reduces the chances of errors being made and makes it easier to present the equations to computer for solutions

State space equations $\dot{x} = Ax + Bu \qquad y = Cx + Du$

Solution $\qquad X(s) = (sI - A)^{-1} x(0) + (sI - A)^{-1} BU(s)$

x is state vector representing the state of the nth order system.(n x 1)

$\dot{x}$ is the time derivative of the state vector

u is the input vector composed of the input signals to the system.(p x 1)

y is the output vector composed of the output signal from the system (q x 1)

A is the state matrix (n x n) B is the input matrix (n x p)

C is the output matrix (q x n) D is the direct transmission matrix (q x p)

I is identity matrix U(s) depends on number of inputs for one input it is 1/s

This is a solution to the state equations and given the initial value of the state vector x(o) and knowledge of the inputs to the system, U(s) we can calculate the value of the state vector for all t>0.

Nomenclature

global dataStruct;	A structure containing data, coefficient and derivative of model
dataStruct.name	Name of model
dataStruct.code	Determines whether raw data or derivatives are stored
dataStruct.Ix	mass moments of inertia of plane about X axis measured about cg
dataStruct.Iy	mass moments of inertia of plane about Y axis measured about cg
dataStruct.Iz	mass moments of inertia of plane about Z axis measured about cg
dataStruct.Ixz	
dataStruct.Sw	Wing area
dataStruct.St	Horizontal tail area
dataStruct.Se	Elevator area
dataStruct.Sv	Vertical tail area
dataStruct.Sfs	The projected side area of the fuselage
dataStruct.ARw	Aspect ratio of wing
dataStruct.ARt	Aspect ratio of tail
dataStruct.ARv	Aspect ratio of vertical tail
dataStruct.bw	Wing span
dataStruct.bt	Horizontal tail span
dataStruct.W	Aircraft weight
dataStruct.Uo	Velocity
dataStruct.cbarw	Wing mean aerodynamic chord
dataStruct.cbart	tail mean aerodynamic chord
dataStruct.e	Span efficiency factor
dataStruct.p	Ambient air density
dataStruct.K	empirical factor to be used in Cndavaa
dataStruct.g	gravity constant
dataStruct.m	mass of aircraft
dataStruct.M	match number
dataStruct.d	maximum fuselage depth
dataStruct.Q	Dynamic presure
dataStruct.a	Local Speed of Sound
dataStruct.s_heat	Specific heats
dataStruct.R	Gas constant
dataStruct.T	Ambient Temperature
dataStruct.flow	Various flow regimes
dataStruct.neov	efficiency factor of vertical tail
dataStruct.neot	efficiency factor of horizontal tail
dataStruct.Zv	Distance from center of pressure of vertical tail to fuselage centerline
dataStruct.Zw	Distance from wing root quarter chord point to fuselage centerline
dataStruct.kn	An empirical wing body interference factor that is a function of the fuselage geometry
dataStruct.krl	an empirical correction factor that is a function of the fuselage Reynolds number
dataStruct.lf	fuselage length
dataStruct.lv	distance from cg to vertical tail aerodynamic center
dataStruct.lt	distance from cg to tail quarter chord

dataStruct.lh Distance from the wing trailing edge to the quater chord of the horizontal tail

dataStruct.lamda tap ratio
dataStruct.taw flap effectiveness factor
dataStruct.Xac distance form wing leading edge to the aerodynamic center
dataStruct.Xc g distance form wing leading edge to the center og gravity
dataStruct.wf Fuselage width
dataStruct.Dx
dataStruct.xi
dataStruct.pd
dataStruct.surface boolean
dataStruct.gliding boolean
dataStruct.wsweep sweep angle of wing quarter chord
dataStruct.wdehidral Dehidral angle of wing
dataStruct.Clalphaw wing airfoil characteristics (unit 1/deg)
dataStruct.Clalphav vertical tail airfoil characteristics (unit 1/deg)
dataStruct.Clalphat horizontal tail airfoil characteristics (unit 1/deg)
dataStruct.Cm referance pitching moment coefficient
dataStruct.Cd referance drag coefficient
dataStruct.Cl referance lift coefficient
dataStruct.y1
dataStruct.y2 limits of region of aileron
dataStruct.cr
dataStruct.ct
dataStruct.Vh
dataStruct.Vv

global lon_sc_Struct; Longitudinal stability coefficient Structure
lon_sc_Struct.Cxu
lon_sc_Struct.Cxalpha
lon_sc_Struct.Cxalphadot
lon_sc_Struct.Cxq
lon_sc_Struct.Cxdavae
lon_sc_Struct.Czu
lon_sc_Struct.Czalpha
lon_sc_Struct.Czalphadot
lon_sc_Struct.Czq
lon_sc_Struct.Czdavae
lon_sc_Struct.Cmu
lon_sc_Struct.Cmalpha
lon_sc_Struct.Cmalphadot
lon_sc_Struct.Cmq
lon_sc_Struct.Cmdavae

global lat_sc_Struct; Lateral stability coefficient Structure
lat_sc_Struct.Cybeta
lat_sc_Struct.Cyp
lat_sc_Struct.Cyr

lat_sc_Struct.Cydavaa
lat_sc_Struct.Cydavar
lat_sc_Struct.Cnbeta
lat_sc_Struct.Cnp
lat_sc_Struct.Cnr
lat_sc_Struct.Cndavaa
lat_sc_Struct.Cndavar
lat_sc_Struct.Clbeta
lat_sc_Struct.Clp
lat_sc_Struct.Clr
lat_sc_Struct.Cldavaa
lat_sc_Struct.Cldavar

global lon_d_Struct;   Longitudinal derivative Structure
lon_d_Struct.Xu
lon_d_Struct.Xw
lon_d_Struct.Xwdot
lon_d_Struct.Xalpha
lon_d_Struct.Xalphadot
lon_d_Struct.Xq
lon_d_Struct.Xdavae
lon_d_Struct.XdavaT
lon_d_Struct.Zu
lon_d_Struct.Zw
lon_d_Struct.Zwdot
lon_d_Struct.Zalpha
lon_d_Struct.Zalphadot
lon_d_Struct.Zq
lon_d_Struct.Zdavae
lon_d_Struct.ZdavaT
lon_d_Struct.Mu
lon_d_Struct.Mw
lon_d_Struct.Mwdot
lon_d_Struct.Malpha
lon_d_Struct.Malphadot
lon_d_Struct.Mq
lon_d_Struct.Mdavae
lon_d_Struct.MdavaT

global lat_d_Struct;   Lateral derivative Structure
lat_d_Struct.Ybeta
lat_d_Struct.Yp
lat_d_Struct.Yr
lat_d_Struct.Ydavaa
lat_d_Struct.Ydavar
lat_d_Struct.Yv
lat_d_Struct.Nbeta
lat_d_Struct.Np
lat_d_Struct.Nr
lat_d_Struct.Ndavaa

lat_d_Struct.Ndavar	
lat_d_Struct.Nv	
lat_d_Struct.Lbeta	
lat_d_Struct.Lp	
lat_d_Struct.Lr	
lat_d_Struct.Ldavaa	
lat_d_Struct.Ldavar	
lat_d_Struct.Lv	
emptytfdata	For empty transfer function structure
emptytfdata.n4	For numerators of transfer function
emptytfdata.n3	
emptytfdata.n2	
emptytfdata.n1	
emptytfdata.n0	
emptytfdata.d4	For Denominator of transfer function
emptytfdata.d3	
emptytfdata.d2	
emptytfdata.d1	
emptytfdata.d0	
emptytfdata.z0	For zeros of transfer function
emptytfdata.z1	
emptytfdata.z2	
emptytfdata.z3	
emptytfdata.z4	
emptytfdata.p0	For poles of transfer function
emptytfdata.p1	
emptytfdata.p2	
emptytfdata.p3	
emptytfdata.p4	
emptytfdata.gain	For gain of transfer function
emptytfdata.type	For type or form of transfer function
global autoTF	This structure save transfer function of whole autopilot
autoTF.pitch	
autoTF.roll	
autoTF.heading	
autoTF.velocity	
autoTF.altitude	
global tfes;	elevator servo transfer function
tfes.tfdata	
tfes.tf	
global tfampc;	amplifier or compensator transfer function
tfampc.tfdata	
tfampc.tf	
global tfvg;	vertical gyro transfer function
tfvg.tfdata	

```
tfvg.tf

global tfrg;            rate gyro transfer function
tfrg.tfdata
tfrg.tf

global reqtf;           For required transfer function
reqtf.tfdata
reqtf.tf

global tfad;            Aircraft dynamic transfer function
global Kpu;
global Tu;

global tfas;            Aileron servo transfer function
tfas.tfdata
tfas.tf

global tfrag;
tfrag.tfdata
tfrag.tf

global tfrs;            Rudder servo transfer function
tfrs.tfdata
tfrs.tf

global tfdg;            Directional gyro transfer function
tfdg.tfdata
tfdg.tf

global tfc;             Compensator transfer function
tfc.tfdata
tfc.tf

global tffb;            For feedback transfer function
tffb.tfdata
tffb.tf


global tfel;            For engine lag transfer function
tfel.tfdata
tfel.tf

global tfetc;           For engine throttle transfer function
tfetc.tfdata
tfetc.tf

global dim              For dimension of matrix
global A                State space matrix
global B                State space matrix
```

global C	State space matrix
global g	For gain if multiple input
global de2	For desired transfer function of order two
global Q	For optimal control
global R	For optimal control
global de	For desired transfer function of order four

emptyceq.ts	Time for short period
emptyceq.tl	Time for long period
emptyceq.Ps	Period for short period
emptyceq.Pl	Period for long period
emptyceq.Ns	Number of cycle for short period
emptyceq.Nl	Number of cycle for long period
emptyceq.signs	For positive 2 for negative for short period
emptyceq.signl	For positive 2 for negative for long period
emptyceq.l4	Constant for desired character equation
emptyceq.l3	
emptyceq.l2	
emptyceq.l1	
emptyceq.l0	
emptyceq.eq	Final desired character equation
emptyceq.type	From of data inputed for desired character equation

emptyceq2.t	
emptyceq2.P	
emptyceq2.N	
emptyceq2.sign	1 for positive 2 for negative
emptyceq2.l2	
emptyceq2.l1	
emptyceq2.l0	
emptyceq2.eq	
emptyceq2.type	

Nomenclature of Equations

U	Velocity component along rolling axis
V	Velocity component along pitching axis
W	Velocity component along yawing axis
X	Aerodynamic force component along rolling axis
Y	Aerodynamic force component along pitching axis
Z	Aerodynamic force component along yawing axis
L	Aerodynamic moment component along rolling axis
M	Aerodynamic moment component along pitching axis
N	Aerodynamic moment component along yawing axis
P	Angular rates along rolling axis
Q	Angular rates along pitching axis
R	Angular rates along yawing axis
α	Angle of attack
β	Sideslip angle

There are two types of equations, first there are longitudinal and lateral stability coefficients and then there are longitudinal and lateral stability derivatives.

For eg some of the equations are defined here below, all other equations follow the same naming convention.

C_{lr}	Change in rolling moment with respect to angular rate along yawing axis.
C_{zq}	Change in Z-force with respect to pitching velocity q.
X_u	X-force derivative with respect to velocity component along rolling axis.
Z_q	Z-force derivative with respect to angular rate along pitching axis.
Y_r	Y-force derivative with respect to angular rate along yawing axis.

Future Extensions

We had made our project "Simulation of Autopilot", we have used matlab as the language of computing for this project and we had also used the GUI toolbox and the Control system toolbox extensively, for anybody who has some interest or want to work further on this project we are presenting some future extensions which could be very useful for someone to further work on this project.

- Real Time Simulation

  This project can be enhanced and made a real time autopilot; this can be done if know the effect of change in Mach number and Velocity and other variable on coefficients of stability.

- Considering Change in atmosphere

  It is fact that not only control input affects the stability of airplane the change in atmosphere also affects it. Therefore dynamics of variable atmosphere and its effects on stability of airplane should be incorporated in this project.

- 3D Visualization of Aircraft

  Creating a 3D visualization of the aircraft can be big step forward. In this visualization, ailerons, rudder and elevator of an airplane can be shown moving in response of control input or due to change in atmosphere. (OpenGL could be used).

- Control through Joystick

  We can use joystick through a parallel port to control airplane movement manually. The dials and gauges toolbox, which is available in MATLAB, might prove helpful in creating an inner control panel of the cockpit.

References

1. Robert C. Nelson “Flight Stability and Automatic control” , McGraw Hill, 1998

2. Richard J.Adams “Robust Multivariable Flight Control”, Springer Verlag, 1994

3. Silvia Ferrari “Tutorials in Aircraft Flight Dynamics”, Princeton University, 1999

4. Richard Drof “ Modern control system”, Addison Wesley 1998

5. Martin Hargreaves “System engineering modeling and control” Longman 1996

MATLAB manuals used

1. Getting started with matlab
2. Using matlab
3. Getting started with Control system toolbox
4. Using the Control system toolbox
5. Using the GUI toolbox

CD Directory Structure

A CD containing all the related material is attached with the project.

- Project
- Report
- Presentation