Onderwerp: WC 4

CNS

Datum:

1a) $\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_i-\theta_j) = \omega_i - \frac{K}{N}\mathrm{Im}\left[\sum_{j=1}^{N} e^{i(\theta_i-\theta_j)}\right]$

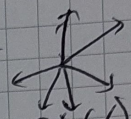
$= \omega_i - \frac{K}{N}\mathrm{Im}\left[e^{i\theta_i}\sum_{j=1}^{N} e^{-i\theta_j}\right]$

$= \omega_i - K\cdot\mathrm{Im}\left[e^{i\theta_i}\cdot\frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}\right] = \omega_i - K\cdot\mathrm{Im}\left[e^{i\theta_i}\cdot re^{-i\Theta}\right]$

$= \omega_i - Kr\,\mathrm{Im}\left[e^{i(\theta_i-\Theta)}\right] = \omega_i - Kr\sin(\theta_i-\Theta)$

where $re^{i\Theta} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$

1b) This can be interpreted as neurons as vectors



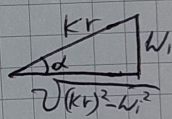
This sums (and $\frac{1}{N}$) results in a "mean vector" (mean field) with a radius (intensity) r and a phase Θ.

1c) $\dot{\theta}_i = \omega_i - Kr\sin(\theta_i-\Theta) = 0$

$\sin(\theta_i-\Theta) = \frac{\omega_i}{Kr} \Rightarrow \omega_i \le Kr$

$\ddot{\theta}_i = f(\theta_i) = -Kr\cos(\theta_i-\Theta) = -Kr\cos\left[\Theta+\sin^{-1}\left(\frac{\omega_i}{Kr}\right)-\Theta\right]$

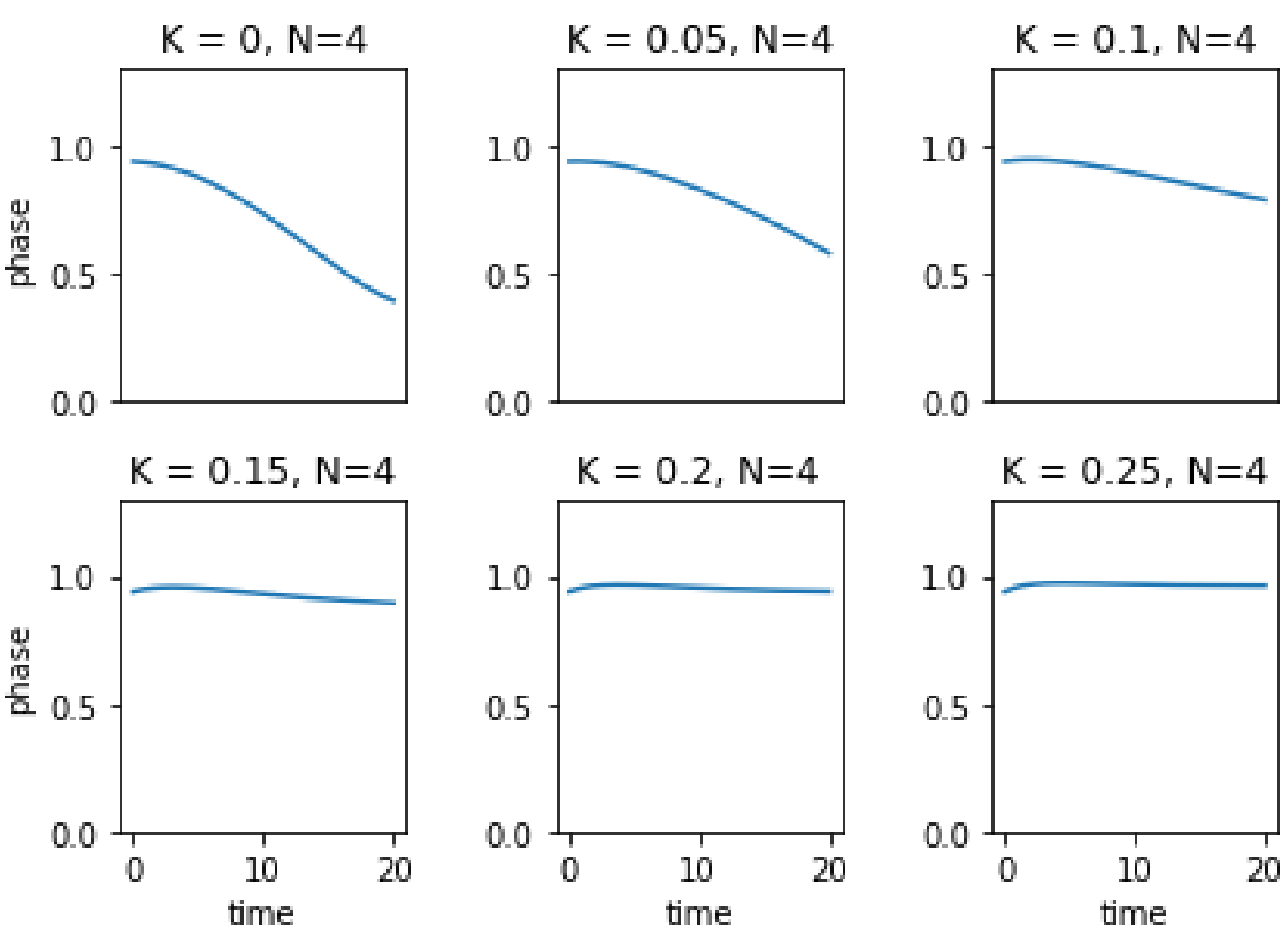
$= -Kr\cos\left(\sin^{-1}\left(\frac{\omega_i}{Kr}\right)\right) = -Kr\cdot\frac{\sqrt{(Kr)^2-\omega_i^2}}{Kr}$

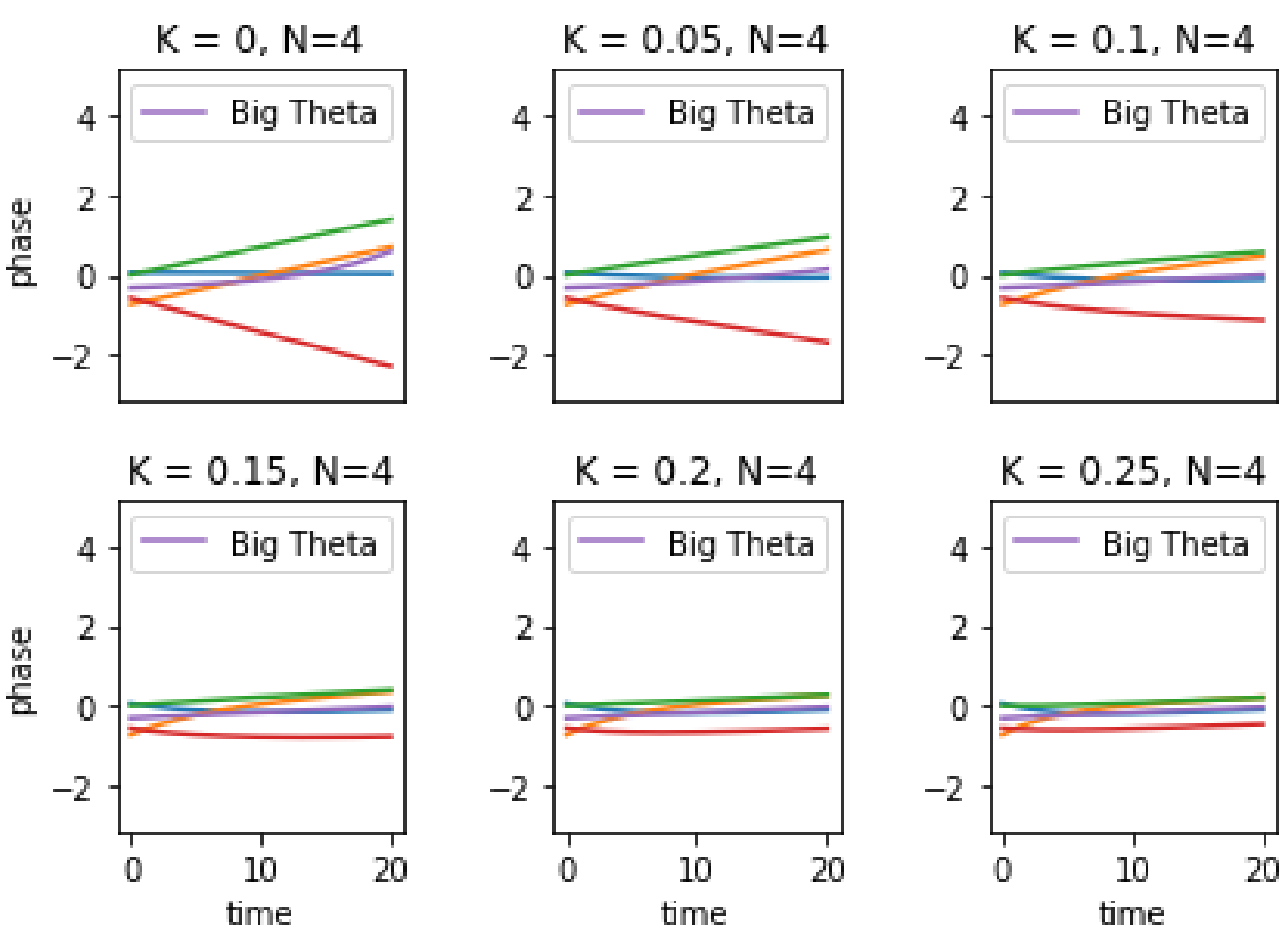


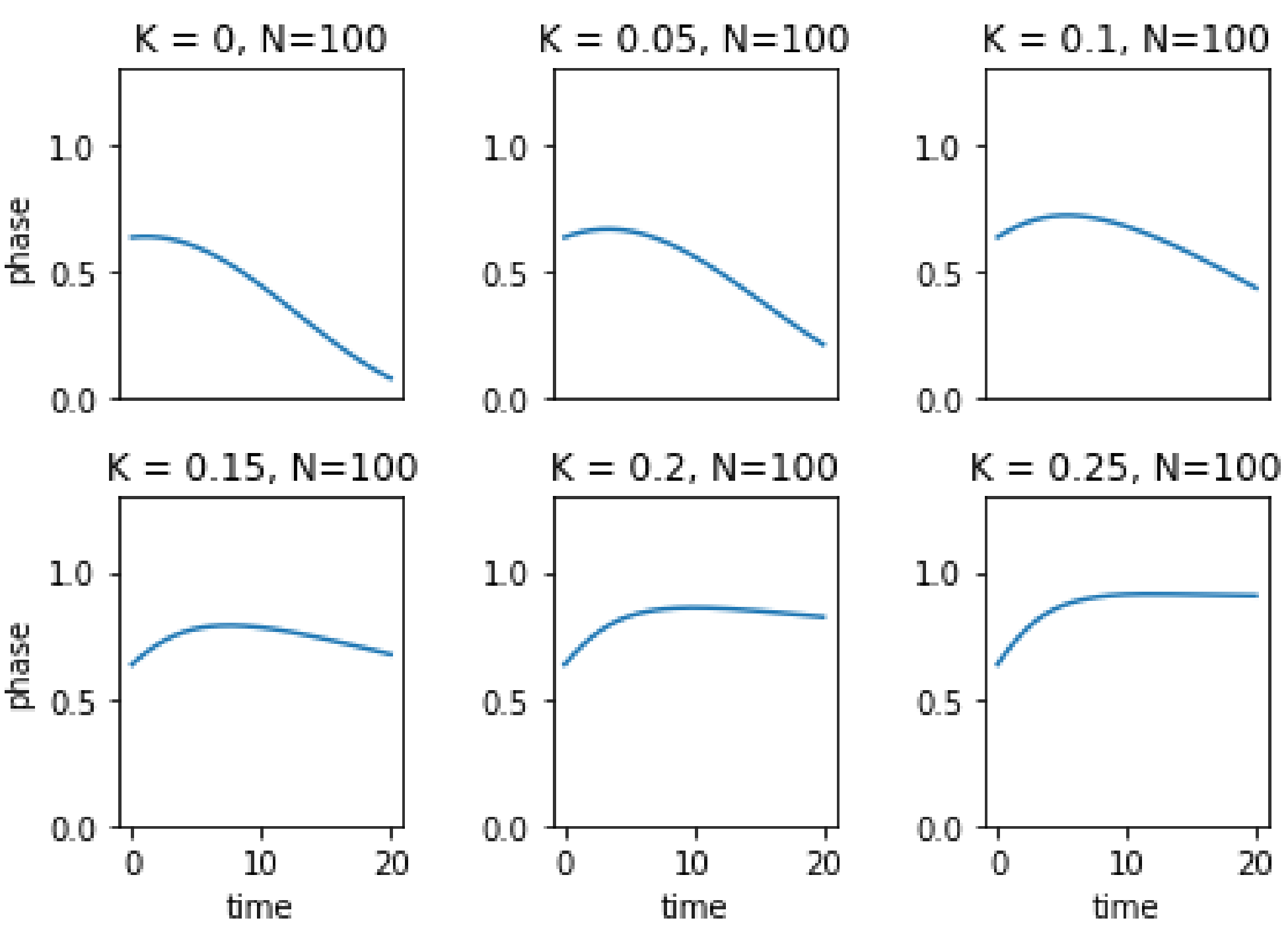
$= -\sqrt{(Kr)^2-\omega_i^2} \le 0$ for $\omega_i \le Kr$

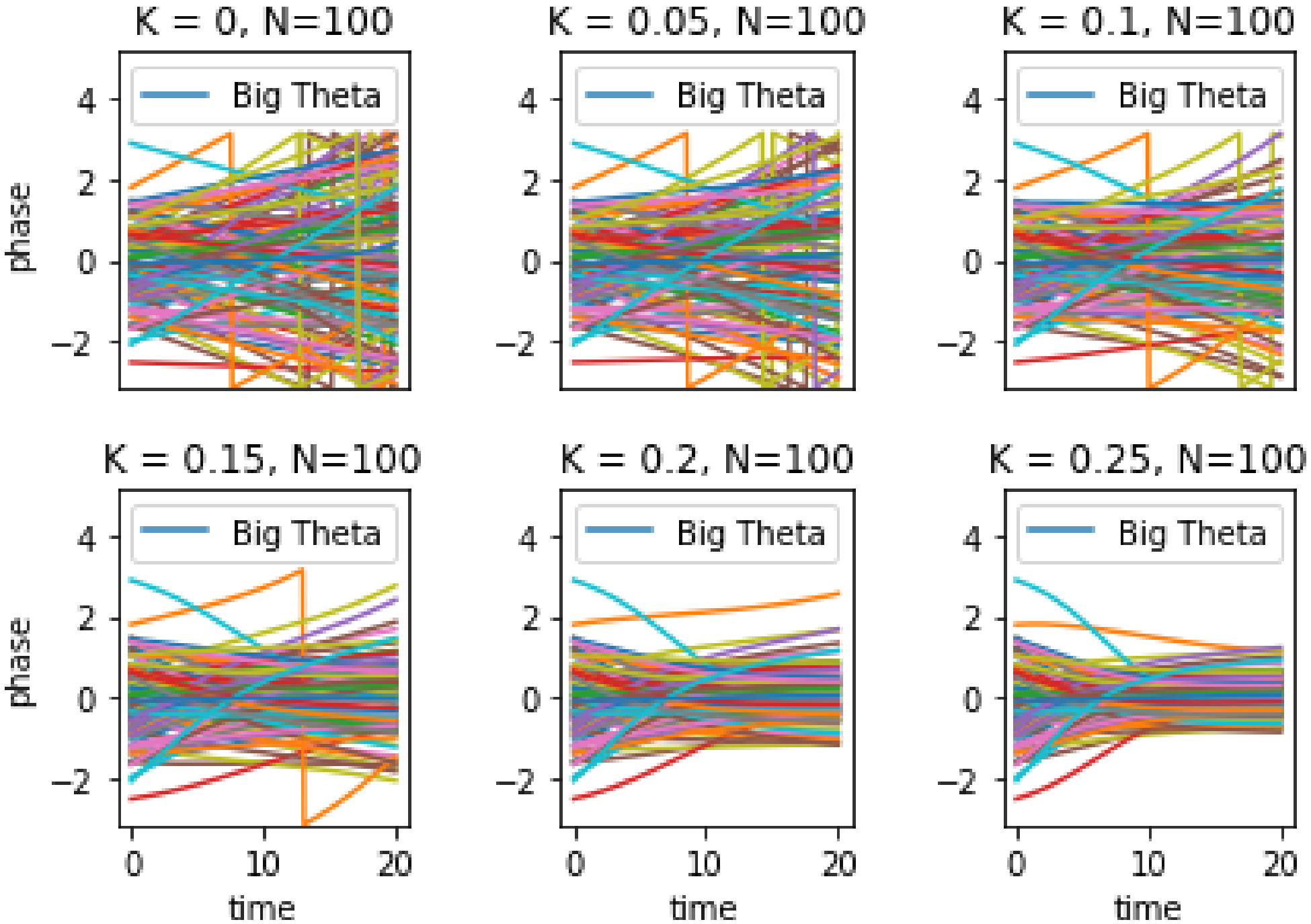
So all stable solutions are:

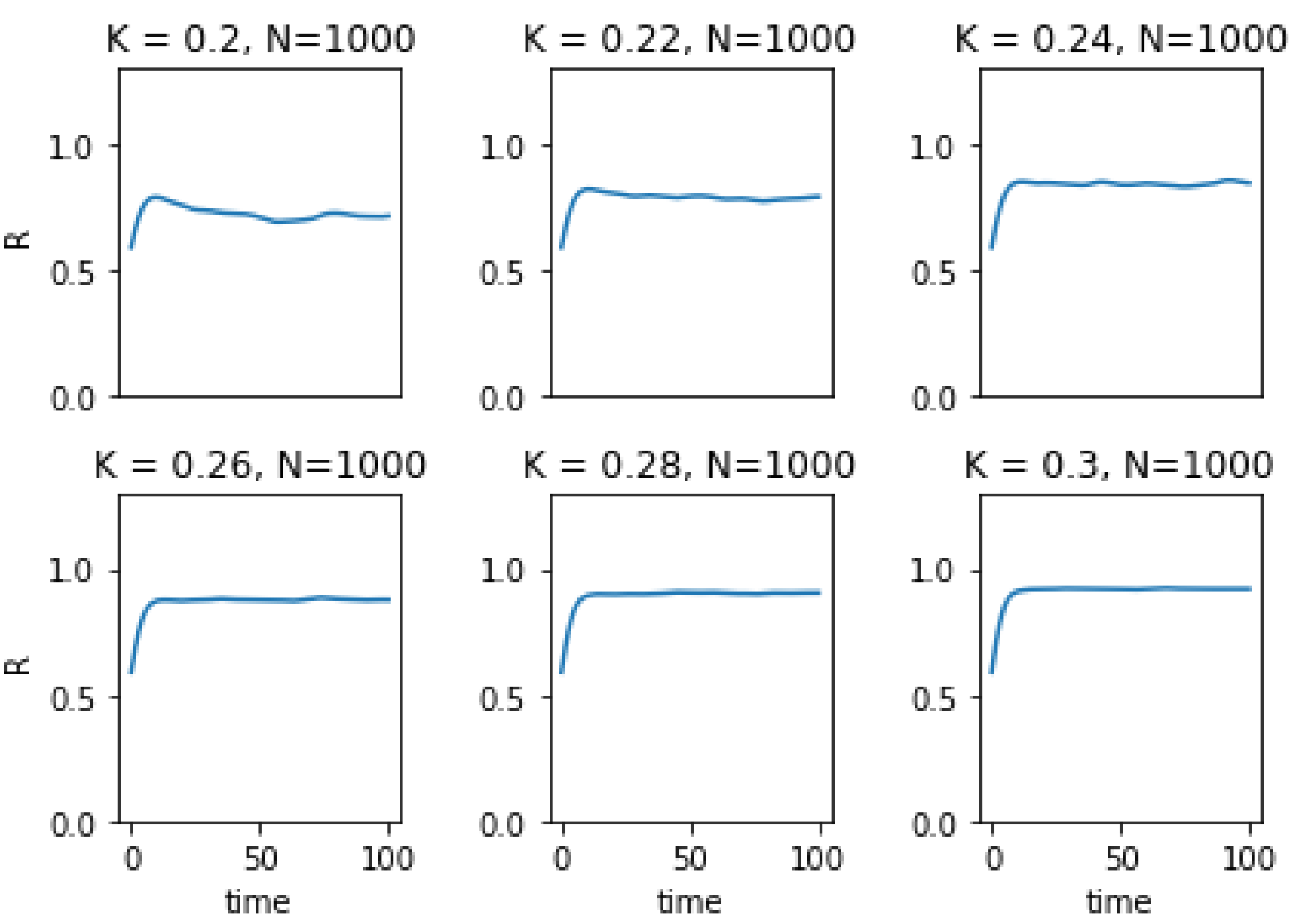
$\theta_i^* = \Theta + \sin^{-1}\left(\frac{\omega_i}{Kr}\right) \quad (\omega_i \le Kr)$

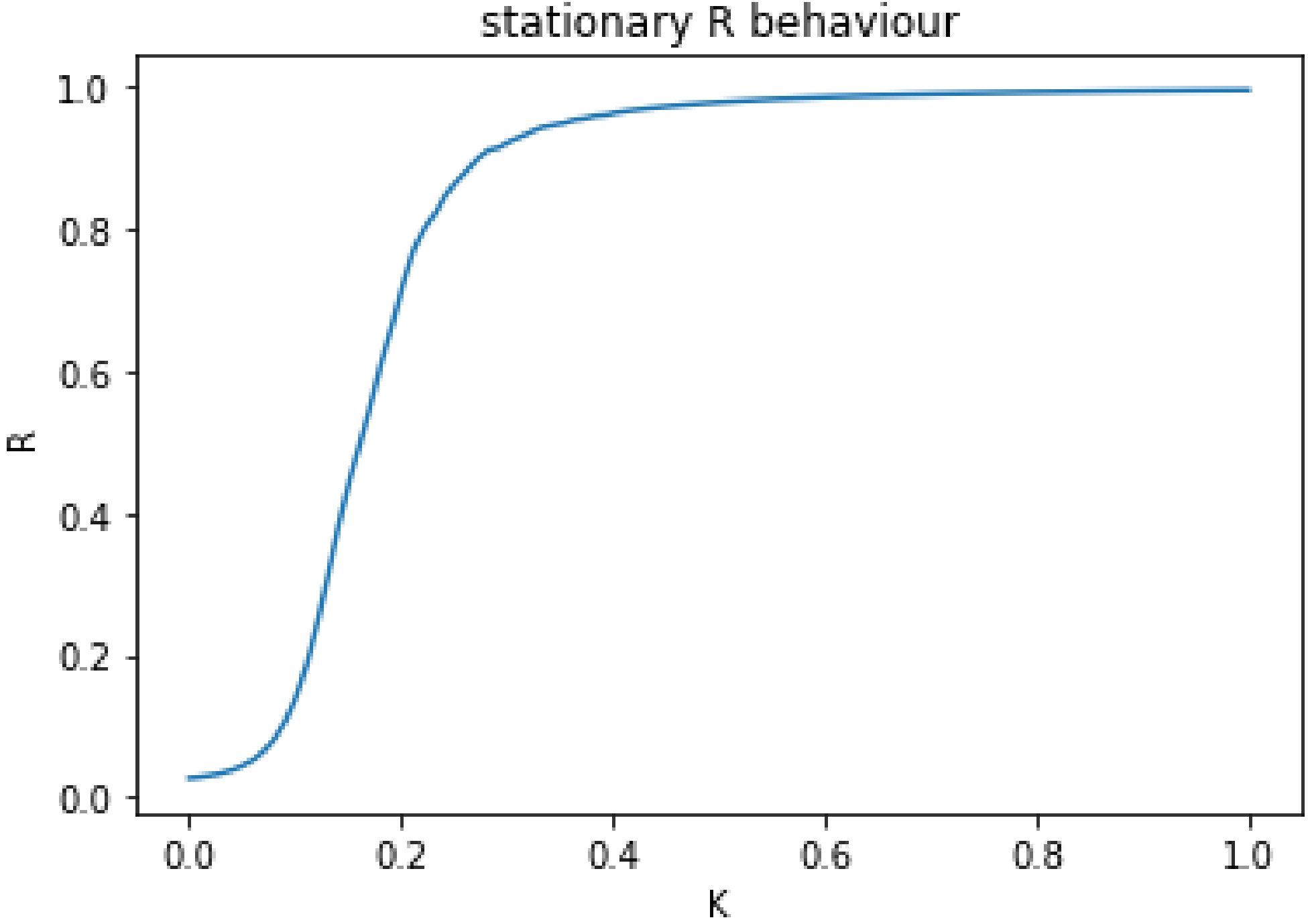

K = 0, N=4
K = 0.05, N=4
K = 0.1, N=4
K = 0.15, N=4
K = 0.2, N=4
K = 0.25, N=4
phase
phase
1.0
0.5
0.0
0
10
20
time
time

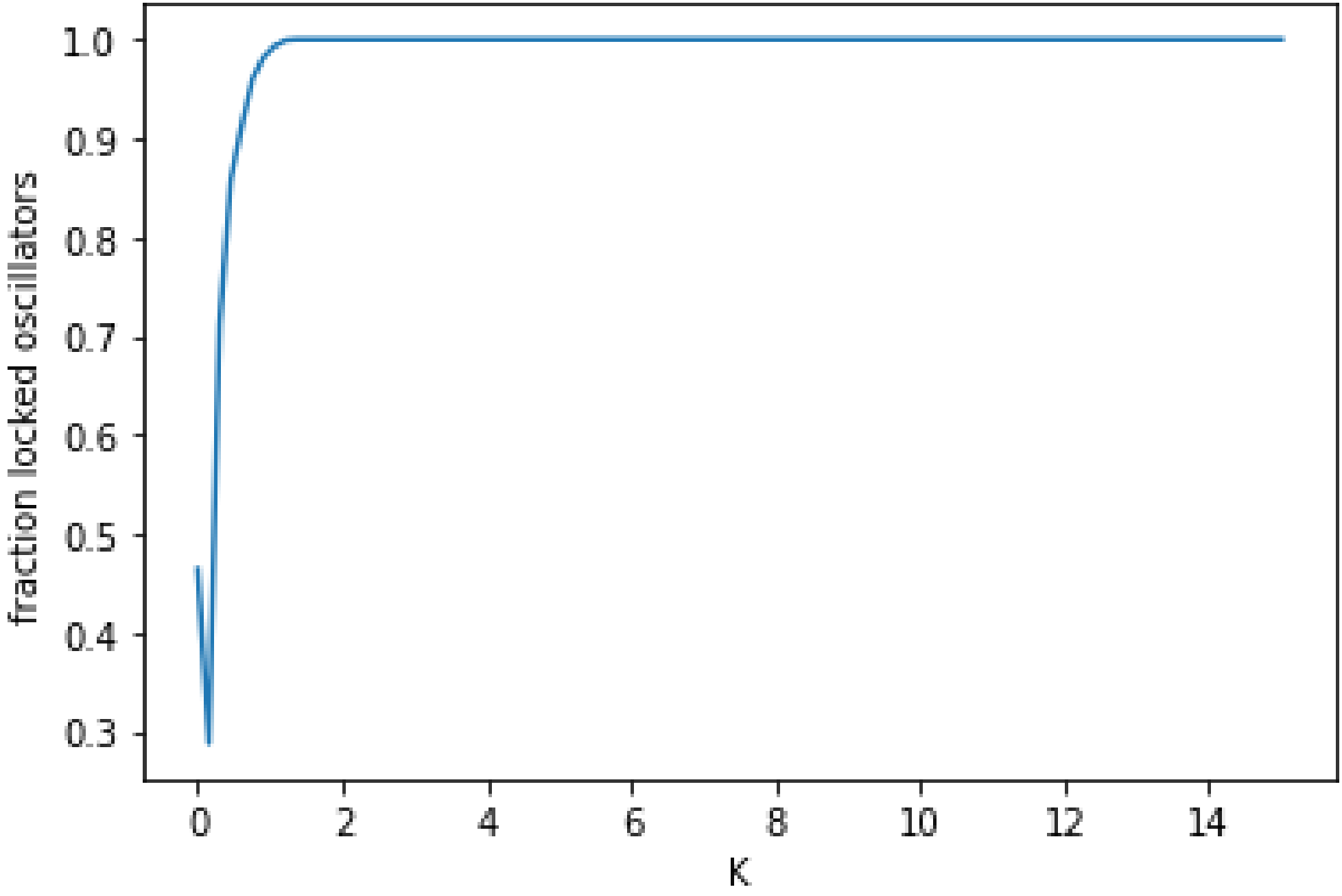

K = 0, N=4
K = 0.05, N=4
K = 0.1, N=4
K = 0.15, N=4
K = 0.2, N=4
K = 0.25, N=4
Big Theta
phase
time
4
2
0
−2
0
10
20


K = 0, N=100
K = 0.05, N=100
K = 0.1, N=100
K = 0.15, N=100
K = 0.2, N=100
K = 0.25, N=100
phase
phase
1.0
0.5
0.0
0
10
20
time
time
time


K = 0, N=100
K = 0.05, N=100
K = 0.1, N=100
K = 0.15, N=100
K = 0.2, N=100
K = 0.25, N=100
Big Theta
phase
time
4
2
0
−2
0
10
20


K = 0.2, N=1000
K = 0.22, N=1000
K = 0.24, N=1000
K = 0.26, N=1000
K = 0.28, N=1000
K = 0.3, N=1000
R
1.0
0.5
0.0
0
50
100
time


stationary R behaviour
1.0
0.8
0.6
0.4
0.2
0.0
R
0.0
0.2
0.4
0.6
0.8
1.0
K


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
fraction locked oscillators
0
2
4
6
8
10
12
14
K