

Rough guide to the course Quantum Field Theory

Getting started

Step 1: start by reading pages *xix–xxi* of the textbook by Peskin & Schroeder, where the relevant conventions of the textbook are listed. These conventions involve the use of so-called natural units ($\hbar = c = \mu_0 = \epsilon_0 = 1$) by absorbing these constants in the relevant fields and quantities. As a result, a single scale remains: mass. Please familiarize yourself with these conventions and treat Chapter 1 of the textbook as reading material, as recommended by the authors.

Step 2: in case you want to prepare for the course, you could refresh your knowledge about

- special relativity (mainly Lorentz transformations),
- complex contour integrations (mainly the residue theorem),
- the Klein-Gordon equation (if you have seen it before).

Optimal way to follow the course

Using the textbook by Peskin & Schroeder in combination with the weekly updated reader will be sufficient to efficiently follow the course ... *provided that you don't fall behind!* Throughout the reader you will encounter circled numbers. These numbers match the markers listed in the storyline of the course given below.

A birds-eye view of the storyline of the course

Part 1, the free Klein-Gordon field: the first four weeks cover Chapter 2 of the textbook by Peskin & Schroeder. Its contents can be described in a sketchy way as follows.

- ① Why field theory \leftrightarrow Compton wavelength
- ② Wave equations as equations of motion for the fields:
 - a) describe this by means of the Lagrangian formalism for continuous systems, which is particularly suitable for discussing symmetries
 - b) make sure that the associated action is Lorentz-invariant (relativity principle)
- ③ Noether's theorem: continuous symmetries from fundamentally unobservable quantities \leftrightarrow conserved currents and charges \Rightarrow

- a) energy, momentum and angular momentum in field theories, which are crucial for performing quantization and determining the particle interpretation
 - b) conserved charges such as particle number or electromagnetic charge, which will feature prominently in the description of fundamental interactions in nature
- ④ Quantization of the free Klein-Gordon theory (to be repeated later for the other, more complicated higher-spin theories):
- a) canonical quantization (like $[\hat{x}, \hat{p}_x] = i\hbar \hat{1}$)
 - b) demand the energy spectrum to be bounded from below
 - c) demand causality \rightarrow antiparticles
- $\xrightarrow{b)+c)}$ bosonic commutation relations
- d) the vacuum and free spin-0 particle states \leftrightarrow plane-wave solutions
 - e) inversion of the Klein-Gordon equation (Green's functions and the Feynman propagator), to be used for performing calculations in interacting scenarios

Part 2, interacting scalar fields and Feynman diagrams: the next roughly five weeks cover large parts of Chapters 4 and 7 as well as a few aspects of Chapter 10 of the textbook by Peskin & Schroeder. Its contents can be described in a sketchy way as follows.

- ⑤ Weakly coupled field theories as a starting point for perturbation theory:
- a) dimensional analysis and smallness of interactions
 - b) effective field theories \leftrightarrow integrating out physics, parametrizing ignorance
 - c) ϕ^4 -theory (which is part of the Higgs model) and the scalar Yukawa theory (which resembles the theory that describes the interactions between fermions and scalars)
- ⑥ Perturbation theory for interacting quantum field theories:
- a) scattering matrix (S-matrix)
 - b) Wick's theorem
 - c) diagrammatic notation for time-ordered vacuum expectation values of interaction-picture fields \Rightarrow Feynman diagrams and Feynman rules
- ⑦ Plane-wave amplitudes for decay processes and scattering reactions:
- a) fully connected Feynman diagrams
 - b) amputation procedure

- c) Feynman rules for incoming/outgoing particles
- d) drawing conventions for particles and antiparticles
- e) non-relativistic limit \Rightarrow forces and force carriers

⑧ From amplitudes to probabilities:

- a) decay widths
- b) cross sections for scattering reactions, including a bit of kinematics

⑨ Dealing with energy eigenstates in the interacting theory:

- a) establishing the link between Green's functions and time-ordered vacuum expectation values of interaction-picture fields
- b) dressed states (Källén–Lehmann spectral representation)
- c) perturbative loop corrections \Rightarrow tricks and analytical structure
- d) LSZ reduction formula \Rightarrow scattering amplitudes derive from Green's functions
- e) optical theorem

⑩ Dealing with infinities in loop corrections:

- a) quantifying (regularizing) infinities
- b) the concept of energy-dependent parameters (renormalization-group equations)
- c) correct starting point for setting up the perturbative series (renormalization)
- d) the mass of a scalar particle is not naturally protected against high-scale physics
- e) power counting \leftrightarrow renormalizability, i.e. when infinities have no effect on the predictive power of the theory considered

Parts 3+4 = Parts 1+2 for matter fermions instead of scalars: during the next three weeks Chapter 3 and §4.7 of the textbook by Peskin & Schroeder will be covered. Its contents can be described in a sketchy way as follows.

⑪ The free Dirac theory:

- a) representations of the Lorentz group \leftrightarrow representations of the rotation group
- b) Dirac's trick and algebra
- c) reducibility of the Dirac representation \Rightarrow chiral eigenstates (Weyl spinors)
- d) Dirac-field currents \Rightarrow building blocks for fundamental interactions
- e) Dirac equation and its symmetries

- f) plane-wave solutions and helicity $\xleftrightarrow{\text{mass} = 0}$ chirality
- g) quantization of the free Dirac theory (repeating part 1)
 \Rightarrow fermionic anticommutation relations
- h) spin-1/2 particle interpretation and Feynman propagator
- i) discrete symmetries (such as parity) \leftrightarrow fundamental interactions

⑫ Part 2 repeated for spin-1/2 Dirac fermions:

- a) Wick's theorem revisited
- b) extra Feynman rules
- c) extending the arrow convention
- d) trace technology

Part 5, Quantum Electrodynamics (QED): during the last two weeks the QED parts of Chapters 4, 5 and 7 of the textbook by Peskin & Schroeder will be covered as well as material that is not treated in this form in the textbook. Its contents can be described in a sketchy way as follows.

⑬ The electromagnetic theory:

- a) gauge freedom, gauge fixing and charge conservation
- b) charged Dirac fermions in an electromagnetic field \Rightarrow minimal substitution and the QED Lagrangian
- c) QED from local gauge invariance (gauge principle) \Rightarrow fundamental postulate for describing all non-gravitational interactions in nature as gauge interactions (incorporated in the Standard Model of electroweak and strong interactions)
- d) quantization of the free electromagnetic field and its complications \Rightarrow massless spin-1 photons: the gauge bosons that mediate the electromagnetic force

⑭ Calculating with QED:

- a) Part 2 repeated for QED \Rightarrow extra Feynman rules, more trace technology
- b) protected masses in QED \Rightarrow fermions are protected by chiral symmetry, photons remain massless as a result of gauge invariance
- c) the Ward–Takahashi identity in QED, i.e. gauge invariance as seen in Green's functions and scattering amplitudes
- d) the “running” electromagnetic coupling (charge screening)
- e) the issue of gauge-invariant regularization \Rightarrow dimensional regularization