Exercise 20) Use the trace technology of exercise 16.

(a) $(\bar{v}(k')\gamma^\nu u(k))^* = (v^\dagger(k')\gamma^0\gamma^\nu u(k))^* = u^\dagger(k)\gamma^{\nu\dagger}\gamma^{0\dagger}v(k') \overset{16}{=\!=} u^\dagger(k)\gamma^0\gamma^0\gamma^\nu\gamma^0\gamma^0 v(k')$

$= \bar{u}(k)\gamma^\nu v(k')$,

$(\bar{v}(k')\gamma^\nu\gamma^5 u(k))^* = (v^\dagger(k')\gamma^0\gamma^\nu\gamma^5 u(k))^* = u^\dagger(k)\gamma^{5\dagger}\gamma^{\nu\dagger}\gamma^{0\dagger}v(k') \overset{16}{=\!=} u^\dagger(k)\gamma^5\gamma^0\gamma^0\gamma^\nu\gamma^0\gamma^0 v(k')$

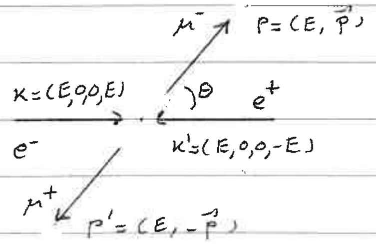
$= -u^\dagger(k)\gamma^0\gamma^5\gamma^\nu v(k') = -\bar{u}(k)\gamma^5\gamma^\nu v(k')$.

(b) Use that traces with an odd number of γ-matrices vanish:

$$\frac{1}{4}\sum_{pol.}|\mathcal{M}|^2 = \frac{e^4}{4q^4}\underbrace{Tr([\not k' - m_e]\gamma^\rho[\not k + m_e]\gamma^\nu)}_{4(k'^\rho k^\nu + k^\rho k'^\nu - g^{\rho\nu}[k\cdot k' + m_e^2])}\ \underbrace{Tr([\not p + m_\mu]\gamma_\rho[\not p' - m_\mu]\gamma_\nu)}_{4(p_\rho p'_\nu + p'_\rho p_\nu - g_{\rho\nu}[p\cdot p' + m_\mu^2])}$$

$$= \frac{8e^4}{q^4}\left[(p'\cdot k)(p\cdot k) + (p'\cdot k')(p\cdot k) + m_e^2\, p\cdot p' + m_\mu^2\, k\cdot k' + 2m_e^2 m_\mu^2\right].$$

(c) Neglecting me the following holds in the CM frame:



$E = \frac{1}{2}E_{CM}$

$p \equiv |\vec{p}| = \sqrt{E^2 - m_\mu^2} \geq 0 \implies \boxed{E \geq m_\mu}$!

$\vec{p}\cdot\vec{e}_z = p\cos\theta$

$q^2 = (p+p')^2 = (k+k')^2 = 4E^2$

$k\cdot k' = 2E^2$

$p\cdot k = p'\cdot k' = E^2 - Ep\cos\theta = E(E - p\cos\theta) > 0$

$p\cdot k' = p'\cdot k = E^2 + Ep\cos\theta = E(E + p\cos\theta) > 0$

Hence, $\frac{1}{4}\sum_{pol.}|\mathcal{M}|^2 \overset{(b)}{=\!=} \frac{e^4}{2E^4}\left[E^2(E+p\cos\theta)^2 + E^2(E-p\cos\theta)^2 + 2m_\mu^2E^2\right]$

$$\overset{p=\sqrt{E^2-m_\mu^2}}{=\!=} e^4\left[1 + \frac{m_\mu^2}{E^2} + \left(1 - \frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]$$

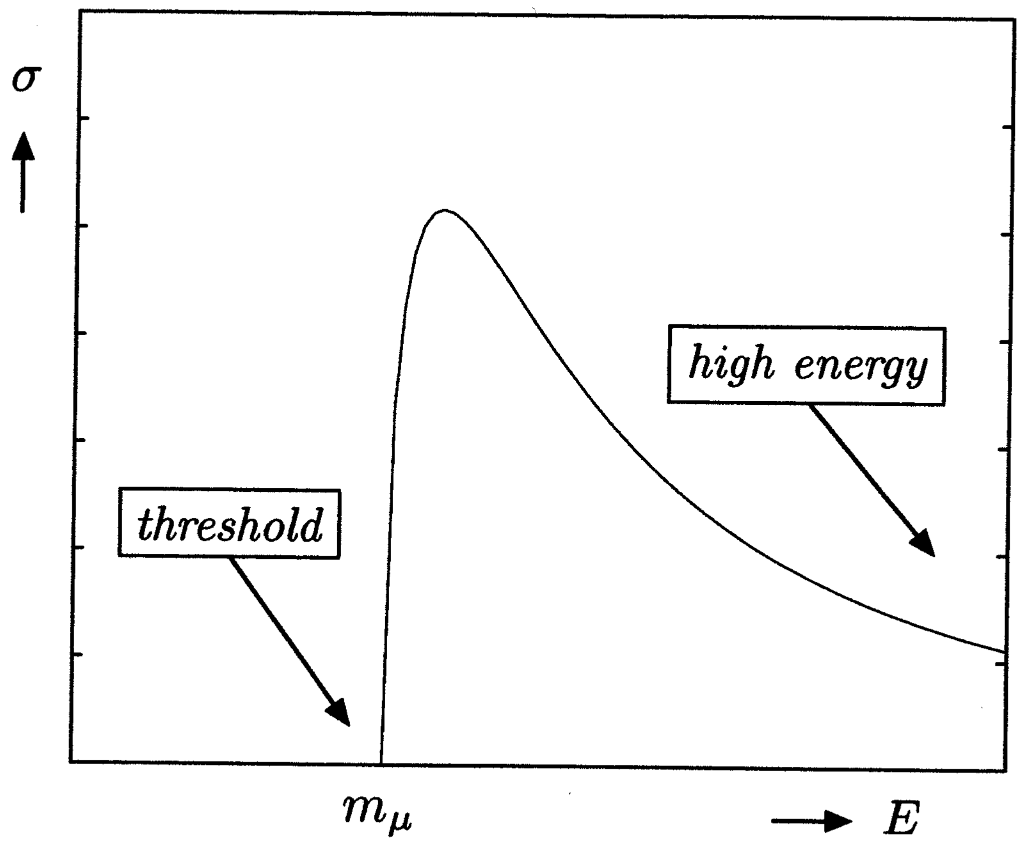
(d) $$\left(\frac{d\sigma}{d\Omega}\right)_{unpol} = \frac{\Theta(E-m_\mu)}{64\pi^2E_{CM}^2}\frac{|\vec{p}|}{|\vec{k}|}\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 \overset{e^2=4\pi\alpha}{=} \Theta(E-m_\mu)\frac{\alpha^2}{16E^2}\sqrt{1-m_\mu^2/E^2}\left[1+\frac{m_\mu^2}{E^2}+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]$$

$$\implies \sigma_{tot}^{unpol} = \Theta(E-m_\mu)\frac{\alpha^2}{16E^2}\sqrt{1-m_\mu^2/E^2}\left[2\pi * \frac{8}{3} + \frac{m_\mu^2}{E^2}2\pi * \frac{4}{3}\right]$$

$$= \boxed{\Theta(E-m_\mu)\frac{\pi\alpha^2}{3E^2}\sqrt{1-m_\mu^2/E^2}\left[1+\frac{m_\mu^2}{2E^2}\right] = \sigma_{tot}^{unpol}}$$

At high energies $(E \gg m_\mu)$: $\sigma_{tot}^{unpol} \approx \frac{\pi\alpha^2}{3E^2}$.

Near threshold $(E \approx m_\mu)$: $\sigma_{tot}^{unpol} \approx \frac{\pi\alpha^2}{2m_\mu^2}\Theta(E-m_\mu)\sqrt{1-m_\mu^2/E^2}$.



σ
threshold
high energy
m_μ
E