UNIVERSITY DEPARTMENT OF MATHEMATICS

Tilka Manjhi Bhagalpur University, Bhagalpur

Class Test Examination

Time: $1\frac{1}{2}$ hour | *PAPER – VI* | Session: 2017–19

1. Choose the most appropriate option. (05)

   (a) The contour integration is **independent** of path in a domain Ω if
   - A. the integrand has an anti-derivative in Ω
   - B. the integrand is analytic in Ω
   - C. both of A and B choice
   - D. None of these

   (b) For any integer n, the value of integration $\oint_{|z|=1} \frac{1}{z^n}\,dz$, along the positively oriented contour is/are

   A. 0 B. $2\pi i$ C. depends on n D. None of these

   (c) If the integration of a complex valued function f along any closed contour in domain Ω is zero, then which of the following is true
   - A. The function f is analytic in Ω
   - B. The function f has an anti-derivative in Ω
   - C. The contour integration $\int_\gamma f(z)dz$, depends on the path.
   - D. None of these

   (d) Which of the following is **true** for contour integration?
   - A. It depends on the parametrizaton of the path.
   - B. It depends on the orientation of the path.
   - C. It depends on length of the path
   - D. None of these

   (e) If f is analytic in domain Ω, then which of the following is **not** true
   - A. Maximum value lies on the boundary of the domain.
   - B. Integration along any closed contour can be non-zero.
   - C. All the derivative of f exist and coninuous in Ω.
   - D. none of these

2. Answer any two of the following (10)

   (a) Find $\int_\gamma |z|^2 dz$ along the square with vertices at $(0,0),(1,0),(1,1),(0,1)$.

   (b) Evaluate the following integral in the positively oriented contours.

   i. $\oint_{|z-1|=\frac{5}{2}} \frac{1}{(z-4)(z+1)^4}dz$ ii. $\oint_{|z|=2} \frac{e^{i\pi z/2}}{z^2-1}dz$

   (c) Let f be an entire function such that $|f(z)| \le M|z|, \forall z \in \mathbb{C}$, where M is a fixed positive constant. Using Cauchy's Inequality, show that $f(z) = \alpha z$, where α is a complex constant.
