UNIVERSITY DEPARTMENT OF MATHEMATICS

Tilka Manjhi Bhagalpur University, Bhagalpur

Assignment – III

Due Date: 10–08-19 | PAPER – VI | Session: 2017–19

1. Problems on singulariy of complex functions
   (a) Find the singularity and it's type of the following functions

   i. $f(z) = \dfrac{z^2}{1-\cos z}$ ii. $f(z) = \dfrac{z^2}{\sin(z)}$ iii. $f(z) = \dfrac{1}{e^{\frac{1}{z}}-1}$

   iv. $f(z) = \dfrac{\sin z^2}{z^2(z-2)}$ v. $f(z) = \sin\left(\dfrac{1}{z}\right)$ vi. $f(z) = \dfrac{1}{1-\cos\left(\frac{1}{z}\right)}$

   (b) Show that the singularity of $\sin(1/z)$ and $e^{1/z}$ is non-isolated and essential.

2. Find the radius convergence of following power series

   i. $\sum \dfrac{n!}{n^n} z^n$ ii. $\sum n^n z^n$ iii. $\sum \dfrac{(n!)^3}{3n!} z^n$

   iv. $\sum (\log n)^2 z^n$ v. $\sum \left[2+(-1)^n\right]^n z^n$ vi. $\sum z^{n!}$

3. Question on Taylor and Laurent series
   (a) Find the Maclaurin series expansion of the function

   i. $f(z) = \dfrac{z}{z^4+9}$ ii. $f(z) = z^2 e^{3z}$ iii. $f(z) = \sin z^2$

   (b) Find the Taylor's series expansion of $\dfrac{1}{1-z}$ around the point i.

   (c) Find the Laurent series for the function $\dfrac{z}{(z+1)(z-2)}$ in each of the following domains.

   i. $|z| < 1$ ii. $1 < |z| < 2$ iii. $|z| > 2$

   (d) Find the Laurent series expansion of $\dfrac{(z+1)}{z(z-4)^3}$ in the domain $0 < |z-4| < 4$.

   (e) Find the Laurent series for the function $z^2 \cos\left(\dfrac{1}{3z}\right)$ in the domain $|z| > 0$.

4. Find the zeros and poles of the following functions with their order

   i. $f(z) = \tan z$ ii. $f(z) = \dfrac{1}{z(e^z-1)}$

   iii. $f(z) = \dfrac{z+3}{z^2(z^2+4)}$ iv. $f(z) = \dfrac{z-1-i}{z^2-(4+3i)z+(1+5i)}$

5. Problem of Argument Principle
   (a) Solve the integration $\displaystyle\oint_{|z|=1} \frac{f'}{f}dz$, where f is the following

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i. $f(z) = z^2$ ii. $f(z) = \dfrac{z^3+2}{z}$ iii. $f(z) = \dfrac{(2z-1)^7}{z^3}$

(b) Solve the integration $\displaystyle\oint_{|z|=10} \frac{f'}{f} dz$, where f is the following

i. $f(z) = \cot z$ ii. $f(z) = \dfrac{e^z}{1+e^z}$ iii. $f(z) = \dfrac{z^4}{1-\cos z}$

6. Problems on Rouches Theorem
   (a) Determine the roots of $z^7 - 4z^3 + z - 1 = 0$ inside the circle $|z| = 1$.
   (b) Determine the number of zeros, counting multiplicities, of the polynomial $z^4 - 2z^3 + 9z^2 + z - 1 = 0$ inside the circle $|z| = 2$.
   (c) Determine the number of zeros, counting multiplicities, of the polynomial $z^4 - 2z^3 + 9z^2 + z - 1 = 0$ inside the circle $|z| = 2$.
   (d) If $a > e$, then show that the equation $az^n = e^z$ has n roots inside $|z| = 1$.
7. Application of contour integral in improper integral
   (a) Solve the integration of type $\displaystyle\int_{-\infty}^{\infty} f(x)dx$, where f is the following

   i. $f(x) = \dfrac{1}{(x^2+1)^2}$ ii. $f(x) = \dfrac{1}{x^4+1}$ iii. $f(x) = \dfrac{x}{(x^2+1)(x^2+2x+2)}$

   (b) Solve the following integration

   i. $\displaystyle\int_{-\infty}^{\infty} \frac{\sin 3x}{(x^2+1)^2}\, dx$ ii. $\displaystyle\int_{-\infty}^{\infty} \frac{x\sin x}{(x^2+1)(x^2+4)}\, dx$ iii. $\displaystyle\int_{-\infty}^{\infty} \frac{x\sin ax}{x^4+4}\, dx \qquad (a > 0)$