

Infinite Series Practice

Definitions and Theory

1. What does it mean for a series $\sum_{n=1}^{\infty} a_n$ to converge?
2. Give an example of an infinite series that converges to 7.
3. If possible, give an example of a series $\sum_{n=1}^{\infty} a_n$ that diverges and $\lim_{n \rightarrow \infty} a_n = 0$. If it's not possible, explain why.
4. If possible, give an example of a series $\sum_{n=1}^{\infty} a_n$ that converges and $\lim_{n \rightarrow \infty} a_n = 0$. If it's not possible, explain why.
5. If possible, give an example of a series $\sum_{n=1}^{\infty} a_n$ that diverges and $\lim_{n \rightarrow \infty} a_n = 0$. If it's not possible, explain why.
6. If possible, give an example of a series $\sum_{n=1}^{\infty} a_n$ that converges and $\lim_{n \rightarrow \infty} a_n \neq 0$. If it's not possible, explain why.

7. Give the argument for why $\sum_{n=1}^{\infty} ar^{n-1}$ converges when $|r| < 1$.

Series Convergence

8. Use the tools we looked at in class to determine the convergence or divergence of each of the following series. If a series converges, find its sum.

(a) $\sum_{n=1}^{\infty} 2 \left(-\frac{1}{3}\right)^{n-1}$

(b) $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}}$

(c) $\sum_{n=1}^{\infty} 3(-2)^{n-1}$

(d) $\sum_{n=2}^{\infty} 7 \left(-\frac{1}{2}\right)^{n+3}$

(e) $\sum_{n=1}^{\infty} \frac{2}{5^n}$

(f) $\sum_{n=1}^{\infty} \frac{3^{2n}}{9^{n-1}}$