Infinite Series Practice

Series Convergence

Use the tools we looked at in class to determine the convergence or divergence of each of the following series. If a series converges and it is possible to do so, find its sum.

1. $\sum_{n=1}^{\infty} \frac{3}{n^2+n}$ $= \sum \frac{3}{n(n+1)} = \sum \left(\frac{3}{n} - \frac{3}{n+1}\right)$

Partial Fractions

$$\left[\frac{3}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}\right] n(n+1)$$

$$3 = A(n+1) + Bn$$

$$0n + 3 = (A+B)n + A$$

$$A+B=0, \quad A=3 \quad B=-3$$

$$S_1 = \left(\frac{3}{1} - \frac{3}{2}\right)$$

$$S_2 = \left(3 - \frac{3}{2}\right) + \left(\frac{3}{2} - \frac{3}{3}\right) = 3 - \frac{3}{3}$$

$$S_3 = \left(3 - \frac{3}{2}\right) + \left(\frac{3}{2} - \frac{3}{3}\right) + \left(\frac{3}{3} - \frac{3}{4}\right) = 3 - 3/4$$

$$S_N = 3 - \frac{3}{N+1}$$

$$\lim_{N\to\infty} S_N = \lim_{N\to\infty} 3 - \frac{3}{N+1} = 3 - 0 = 3.$$

$\frac{3}{\infty} = 0$

converges with sum 3

[note! could use comparison test... but that won't get you an exact sum.]

2. $\sum_{n=1}^{\infty} \frac{3^n + \sin^2 n}{2^n - 1}$

$$2^n - 1 \leq 2^n$$

$$\frac{1}{2^n-1} \geq \frac{1}{2^n}$$

$$\frac{3^n + \sin^2 n}{2^n - 1} \geq \frac{3^n + \sin^2 n}{2^n} \geq \frac{3^n + 0}{2^n} = \frac{3^n}{2^n}$$

$\sum \frac{3^n}{2^n} = \sum \left(\frac{3}{2}\right)^n$ diverges (geo, $r = 3/2$, $|r| \geq 1$).

So $\sum \frac{3^n + \sin^2 n}{2^n - 1}$ diverges by DCT.

3. $\displaystyle\sum_{j=2}^{\infty} \frac{1}{\ln j + 2^j}$

$\ln j + 2^j \geq 2^j$

$\dfrac{1}{\ln j + 2^j} \leq \dfrac{1}{2^j}$

$\sum \frac{1}{2^j} = \sum \left(\frac{1}{2}\right)^j$ converges (geo, $r = \frac{1}{2}$, $|r| < 1$).

So $\sum \frac{1}{\ln j + 2^j}$ converges by DCT

4. $\displaystyle\sum_{n=3}^{\infty} \frac{n^2+3n-1}{(2n+1)(n-2)} = \sum \frac{n^2+3n-1}{2n^2-3n-2}$

$\displaystyle\lim_{n\to\infty} \frac{n^2+3n-1}{2n^2-3n-2} = \lim_{n\to\infty} \frac{n^2}{2n^2} = \frac{1}{2} \neq 0$

Series diverges by Test for Divergence.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{2n^2+7}$

LCT! compare to $b_n = \frac{n}{2n^2} = \frac{1}{2n}$.

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+3}{2n^2+7}}{1/2n} = \lim_{n\to\infty} \frac{n+3}{2n^2+7} \cdot \frac{2n}{1} = \lim_{n\to\infty} \frac{2n^2+6n}{2n^2+7}$

$\displaystyle= \lim_{n\to\infty} \frac{2n^2}{2n^2} = 1$, $0 < 1 < \infty$, so LCT applies.

$\sum \frac{1}{2n} = \frac{1}{2}\sum\frac{1}{n}$ diverges (harmonic).

So $\sum \frac{n+3}{2n^2+7}$ diverges as well by LCT.