Alternating Series Practice

1. Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}b_n$, where b_n is always positive and decreasing and $\lim_{n\to\infty} b_n = 0$. Explain why $s_2 < s_4 < s_6 < \ldots$. Also explain why the even partial sums must converge.

$s_4 = s_2 + b_4 - b_5 > s_2$ since $b_4 > b_5$.
The thing I add is bigger than the thing I subtract so $s_4 > s_2$. Similarly for each s_{2N}. This is a bounded monotonic sequence, so it must converge.

Show that $\lim_{N\to\infty} |s_{N+1} - s_N| = 0$.

$$\lim_{N\to\infty} |s_{N+1} - s_N| = \lim_{N\to\infty} |(-1)^{N+2} b_{N+1}|$$
$$= \lim_{N\to\infty} b_{N+1} = 0.$$

Explain how this shows that the series $\sum_{n=1}^{\infty}(-1)^{n+1}b_n$ converges.

The evens & odds must approach the same number since $s_{N+1} - s_N \to 0$. That is, the difference between subsequent terms goes to 0.