3. Probability Problems

1) $\frac{1}{\text{prob no student has to answer more than 1 Q}} \times \frac{14}{15} \times \frac{13}{15} \times \frac{12}{15} \times \frac{11}{15} \times \frac{10}{15} \times \frac{9}{15} \times \frac{8}{15} = 0.101$ **10.124%**

2) $\underbrace{5 \times 4}_{\text{odd}} \quad 7 \quad 6 \quad \underbrace{5}_{\text{even}} = \frac{4200}{105}$ for 1 randomly generated var

5 nums meet criteria: $0.042^5 = 1.307 \times 10^{-7}$

3 nums don't meet criteria: $(1-0.042)^3 = 0.879$

$\times \quad = \boxed{1.149 \times 10^{-7}}$

3) A and B are independent if

$P(A \cap B) = P(A) \cdot P(B)$

$P(A)$: 2+ dice show 4 or above

$= P(2 \text{ dice show } 4+) + P(3 \text{ dice show } 4+)$

$\left(\frac{1}{2^3}\right) \cdot 3 \quad + \quad \left(\frac{1}{2}\right)^3 = 4/8 = 1/2$

$P(B)$: all 3 dice show the same value:

$1 \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(A \cap B)$: all 3 dice show 4 or above (same value)

$P(\text{all 4's}) + P(\text{all 5's}) + P(\text{all 6's})$

$= \frac{3}{6^3} = \frac{3}{216} = \frac{1}{72}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$ } equal

independent

4) Prob of a flush: 5-card hand of the same suit

1 suit: ${}_{15}C_5 = 1287$

4 suits: $4 \cdot 1287 = 5148$

possible 5-card hands: ${}_{52}C_5 = 2598960$

$P(\text{flush}) = \frac{5148}{2598960}$

$P(\text{play } k \text{ hands \& get flush at } k\text{th hand}) = (1-p)^{k-1} p$

$E[X] = \sum_k k \cdot P(k) = \frac{1}{p} = \frac{2598960}{5148} = \boxed{505 \text{ hands}}$

5) P(win | superstar plays) = 70%.
P(win | superstar doesn't play) = 50%.
P(superstar plays) = 75%.

P(win 4/5 games | superstar plays) = ${}_5C_4 \cdot 0.7^4 \cdot 0.3 = 0.360$
P(win 4/5 games | superstar doesn't play) = ${}_5C_4 \cdot 0.5^5 = 0.156$

$P(\text{win } 4/5) = 0.360 \cdot 0.75 + 0.156 \cdot 0.25 = 0.309$

$$P(\text{superstar plays} \mid \text{win } 4/5) = \frac{P(\text{win } 4/5 \mid \text{superstar plays}) \cdot P(\text{superstar plays})}{P(\text{win } 4/5)}$$

$$= \frac{0.360 \cdot 0.75}{0.309} = \boxed{0.874}$$