

## 2. Counting Problem

### 1) UNUSUAL

a. Unique subsets of 5 letters

case 1: 1 fixed u:  $\{u, \_, \_, \_, \_ \}$   $\binom{4}{u} = 1$

$\{n, s, a, i\}$

case 2: 2 fixed u:  $\{u, u, \_, \_, \_ \}$   $\binom{4}{3} = 4$

case 3: 3 fixed u:  $\{u, u, u, \_, \_ \}$   $\binom{4}{2} = 6$

$$1 + 4 + 6 = \boxed{11 \text{ unique subsets}}$$

b. diff strings of 5 letters

1 u:  $5! = 120$

2 u's:  $\frac{5!}{2!} \times 4 \text{ subsets} = 240$

3 u's:  $\frac{5!}{3!} \times 6 \text{ subsets} = 120$

$$120 + 240 + 120 = \boxed{480 \text{ diff strings}}$$

$$2) \binom{13}{2} \times \binom{4}{2} \times \binom{4}{2} \times \binom{44}{1}$$

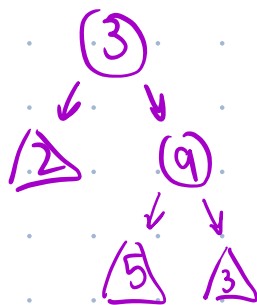
$\uparrow$  13 vals, choose 2     
  $\uparrow$  1st pair, choose 2 suits     
  $\uparrow$  2nd pair, choose 2 suits     
  $\uparrow$  52 - 8 (2 vals, each w/ 4 suits can't be used)

### 3) Stars and Bars

can only be 1 or 0  
 $\begin{array}{ccccccc} * & | & * & | & * & | & * & | & * & | & * \\ * & | & * & | & * & | & * & | & * & | & * \end{array}$   
 16 stars, 5 bars

case 1 - 0 songs:  $\binom{21}{5} = 20349$   
 case 2 - 1 song:  $\binom{20}{5} = 15504$  +  $\boxed{35853}$

4)



2 node case:



3 node case:  $\begin{array}{c} (n) \\ \swarrow \searrow \\ (2) \end{array} \quad \begin{array}{c} (n) \\ \swarrow \\ (2) \end{array} \quad \begin{array}{c} (n) \\ \swarrow \searrow \\ (1) \quad (1) \end{array} \rightarrow 5$

4 node case:  $\begin{array}{c} (n) \\ \swarrow \\ (3) \end{array} \quad \begin{array}{c} (n) \\ \swarrow \\ (3) \end{array} \quad \begin{array}{c} (n) \\ \swarrow \searrow \\ (2) \quad (1) \end{array} \quad \begin{array}{c} (n) \\ \swarrow \searrow \\ (1) \quad (2) \end{array} \rightarrow 14$

5 no case:  $\begin{array}{c} (n) \\ \swarrow \\ 14 \\ \boxed{4 \text{ options}} \end{array} + \begin{array}{c} (n) \\ \swarrow \\ 14 \\ \boxed{4 \text{ options}} \end{array} + \begin{array}{c} (n) \\ \swarrow \\ 2 \\ \boxed{2 \text{ options}} \end{array} + \begin{array}{c} (n) \\ \swarrow \\ 4 \\ \boxed{2 \text{ options}} \end{array}$

$\begin{array}{c} (n) \\ \swarrow \\ \boxed{3 \text{ options}} \end{array} + \begin{array}{c} (n) \\ \swarrow \\ \boxed{1 \text{ option}} \end{array} + \begin{array}{c} (n) \\ \swarrow \\ \boxed{1 \text{ option}} \end{array} + \begin{array}{c} (n) \\ \swarrow \\ \boxed{3 \text{ options}} \end{array}$

5                      5

$$2 \cdot 42 \cdot 5 = \boxed{420}$$

5) 10 friends  
4 nurses, sometimes 3

case 1: 4 nurses

1	7	6	5	5	4	4	4	3	3
2	1	2	3	2	4	3	2	3	3
3	1	1	1	2	1	2	2	2	3
4	1	1	1	1	1	1	2	2	1

9

case 2: 3 nurses

1	8	7	6	6	5	5	4	4
2	1	2	3	2	4	3	3	4
3	1	1	1	2	1	2	3	2

8

$$9 + 8 = \boxed{17}$$