2. Counting Problem

1) UNUSUAL

a. Unique subsets of 5 letters

case 1: 1 fixed u: {u, _, _, _, _} $\binom{4}{4} = 1$

{u, s, a, l}

case 2: 2 fixed u: {u, u, _, _, _} $\binom{4}{3} = 4$

case 3: 3 fixed u: {u, u, u, _, _} $\binom{4}{2} = 6$

1 + 4 + 6 = **11 unique subsets**

b. diff strings of 5 letters

1 u: 5! = 120

2 u's: $\frac{5!}{2!} \times 4$ subsets = 240

3 u's: $\frac{5!}{3!} \times 6$ subsets = 120

120 + 240 + 120 = **480 diff strings**

2)

$$\binom{13}{2} \times \binom{4}{2} \times \binom{4}{2} \times \binom{44}{1}$$

- $\binom{13}{2}$: 13 vals, choose 2
- $\binom{4}{2}$: 1st pair, choose 2 suits
- $\binom{4}{2}$: 2nd pair, choose 2 suits
- $\binom{44}{1}$: 52 − 8 (2 vals, each w/ 4 suits can't be used)

3) Stars and Bars

* | ** | * | ** | * | *

can only be 1 or 0

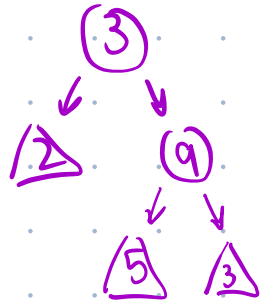
16 stars, 5 bars

case 1 - 0 songs: $\binom{21}{5} = 20349$

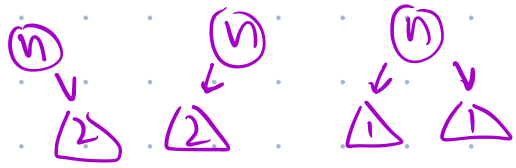
case 2 - 1 song: $\binom{20}{5} = 15504$

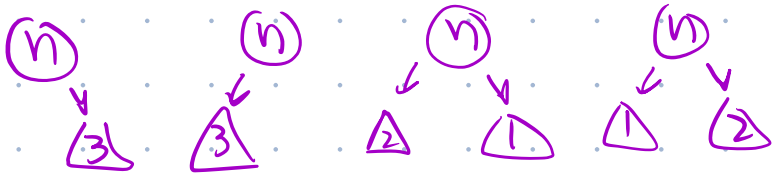
\+ = **35853**

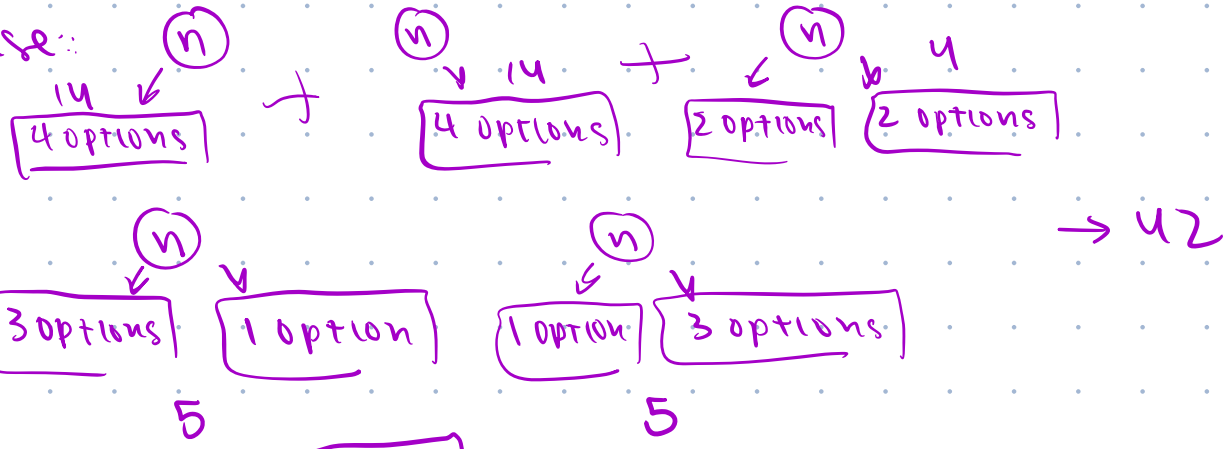
4)



2 node case:

3 node case:  → 5

4 node case:  → 14

5 no case:  → 42

$2 \cdot 42 \cdot 5 = \boxed{420}$

5) 10 friends
4 nurses, sometimes 3

case 1: 4 nurses

1	7	6	5	5	4	4	4	3	3
2	1	2	3	2	4	3	2	3	3
3	1	1	1	2	1	2	2	2	3
4	1	1	1	1	1	1	2	2	1

9

case 2: 3 nurses

1	8	7	6	6	5	5	4	4
2	1	2	3	2	4	3	3	4
3	1	1	1	2	1	2	3	2

8

$9 + 8 = \boxed{17}$