a)
```
void f1 (int n) {
    int i = 2;              ← θ(1)
    while (i < n) {
        [ O(1) ]            } θ(1)
        i = i·i;
    }
}
```

how many times does it loop?

$n = 4 \rightarrow 1$ loop

$n = 16 \rightarrow 2$ loops

$n = 256 \rightarrow 3$ loops

$n = 65536 \rightarrow 5$ loops

$2^{2^k} \geq n$

solve for k ← num of loops

$k = \log(\log(n))$

$$\theta(f1) = \theta(\log(\log(n))) \cdot \theta(1)$$
$$= \boxed{\theta(\log(\log(n)))}$$

b.
```
void f2 (int n) {
    for (int i = 1; i ≤ n; i++) {
        if ((i % (int) √n ) == 0) {
            for (int k = 0; k < i³; k++) {
                O(1)
    } } } }
```

me constant

$$\sum_{i=1}^{n} \Theta(1) + \overset{\underset{\text{sum}}{\downarrow}}{k} \sqrt{n} \times \sum_{k=0}^{i^3} \Theta(1)$$

$$= \Theta(n) + k\sqrt{n} \cdot \Theta(1^3)$$

$$= \Theta(n) + \sum_{k=1}^{\sqrt{n}} \Theta((k\sqrt{n}^3))$$

$$= \Theta(n) + \sqrt{n}^3 \sum_{k=1}^{\sqrt{n}} \Theta(k^3)$$

$$= \Theta(n) + \sqrt{n}^3 \Theta(\sqrt{n}^4)$$
$$= \Theta(n) + \Theta(\sqrt{n}^7) \longrightarrow \boxed{\Theta(n^{7/2})}$$

C.
```
for (int i=1; i≤n; i++){
  for (int k=1; k≤n; k++){
   if (A[k] == i){          condition happens ≤ n times
     for (int m=1; m≤n; m=n+m){
       O(1)
   } } } }
```

$$\sum_{i=1}^{n} \sum_{k=1}^{n} \left(\underset{\downarrow}{\Theta(1)} + \underset{?}{\sum} \sum_{m=1}^{\log n} \Theta(1) \right)$$

$$\underset{\downarrow}{\Theta(n^2)} + \underset{\downarrow}{n \cdot \log(n)}$$

$$\leqslant \boxed{\underline{\Theta(n^2)}}$$

d. size: $10 \cdot \left(\frac{3}{2}\right)^k < n$

$$k < \log_{3/2}(n/10)$$

$$T(n) = \sum_{i=0}^{n} \Theta(1) + \underbrace{\sum_{k=0}^{size} \sum_{j=0}^{size} \Theta(1)}_{merge}$$

$$= \Theta(n) + \sum_{k=0}^{\log_{3/2}(n/10)} \Theta(1)$$

$$= \Theta(n) + 10 \cdot \sum_{k=0}^{\log_{3/2}(n/10)} \left(3/2\right)^k$$

$$= \Theta(n) + 10 \cdot \Theta\left(\left(3/2\right)^{\log_{3/2}(n/10)}\right)$$

$$= \Theta(n) + \Theta(n) = \boxed{\Theta(n)}$$