QMM_Assignment_1

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a longterm contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

1. Clearly Define the Decision Variables

Let us consider Collegiate and Mini as Back Savers company's decision variables

C= Collegiate

M= Mini

C = No. of collegiate's produced per week.

M = No. of Mini's produced per week.

2. What is the Objective Function?

Objective : Management wishes to know what quantity of each type of backpack to produce per week to get the maximum profit

Z = 32C + 24M

3. What are the Constraints?

C ≤ 1000 most collegiates sold per week

M ≤ 1200 most Mini sold per week

45C + 40M ≤ 84000 Labor minutes per week (35 Labourers * 40 Hrs * 60 Mins)

$3C + 2M \le 5000$ Sqft of material per week

Non-Negativity:

$C \ge 0$

$M \ge 0$

4. Write down the full mathematical formulation for this LP Problem

Let us formulate Logical problem now

C = No. of Collegiate's to be Produced per week.

M = No. of Mini Backpacks to be Produced per week.

Max $Z = 32C + 24M$

$C \le 1000$ max Collegiate Sold per week

$M \le 1200$ max Mini Sold per week

$45C + 40M \le 84000$ Minutes per week (35 ppl * 40 Hrs * 60 Mins)

$3C + 2M \le 5000$ Sq-ft of material per week

$C \ge 0$

$M \ge 0$

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

b. Formulate a linear programming model for this problem.

1. Define the decision variables

Decision Variables are the number of units of the new product, regardless its size that should be produced on each plant to maximize the weigelt corporation’s profit.

Note:

Y_i = number of unites produced on each plant,

Where i = 1 (Plant 1), 2 (Plant 2), 3 (Plant 3).

L, M and S = Product’s Size, Where L = large, M = medium, S = small.

Decision Variables:

Y_iL = Number of Large sized items produced on plant i

Y_iM = Number of Medium sized items produced on plant i

Y_iS = Number of Small sized items produced on plant i,

Where i = 1 (Plant 1), 2 (Plant 2), 3 (Plant 3).

2. Formulate a Linear Programming for this Problem:

Y_iL = Number of Large sized items produced on plant i

Y_iM = Number of Medium sized items produced on plant i

Y_iS = Number of Small sized items produced on plant i,

Where i = 1 (Plant 1), 2 (Plant 2), 3 (Plant 3).

Maximize Profit

$Z = 420 (Y_1L + Y_2L + Y_3L) + 360 (Y_1M + Y_2M + Y_3M) +$

$300 (Y_1S + Y_2S + Y_3S)$

Constraints:

Total number of size's units produced regardless the plant:

$L = Y_1L + Y_2L + Y_3L$

$M = Y_1M + Y_2M + Y_3M$

$S = Y_1S + Y_2S + Y_3S$

Production Capacity per unit by plant each day:

Plant 1 = $Y_1L + Y_1M + Y_1S \leq 750$

Plant 2 = $Y_2L + Y_2M + Y_2S \leq 900$

Plant 3 = $Y_3L + Y_3M + Y_3S \leq 450$

Storage capacity per unit by plant each day:

Plant 1 = $20Y_1L + 15Y_1M + 12Y_1S \leq 13000$

Plant 2 = $20Y_2L + 15Y_2M + 12Y_2S \leq 12000$

Plant 3 = $20Y_3L + 15Y_3M + 12Y_3S \leq 5000$

Sales forecast per day:

$L = Y_1L + Y_2L + Y_3L \leq 900$

$M = Y_1M + Y_2M + Y_3M \leq 1200$

$S = Y_1S + Y_2S + Y_3S \leq 750$

The Plants should use the same percentage of their excess capacity to produce the new product.

$$\frac{Y_1L + Y_1M + Y_1S}{750} = \frac{Y_2L + Y_2M + Y_2S}{900} = \frac{Y_3L + Y_3M + Y_3S}{450}$$

It can be simplified as:

a) $900\,(Y_1L + Y_1M + Y_1S) - 750\,(Y_2L + Y_2M + Y_2S) = 0$

b) $450\,(Y_2L + Y_2M + Y_2S) - 900\,(Y_3L + Y_3M + Y_3S) = 0$

c) $450\,(Y_1L + Y_1M + Y_1S) - 750\,(Y_3L + Y_3M + Y_3S) = 0$

All Values must be greater or equal to zero

L, M and $S \geq 0$

Y_iL, Y_iM and $Y_iS \geq 0$