Technical Note: A revised parameterisation of the Jonswap spectrum

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Equations are derived for estimating the governing parameters of the Jonswap spectrum for a sea state specified in terms of significant waveheight and average period. Two of the equations are obtained mathematically from the Jonswap equation and are of general validity. The third is derived from empirical data published by Houmb and Overvik[1] and applies only to North Sea conditions.

Key Words: sea state, wave spectrum, Jonswap spectrum, mathematical analysis, computer analysis, offshore engineering.

INTRODUCTION

The Jonswap spectrum formulation was derived by Hasselmann *et al.*[2] The governing parameters were subsequently defined by Houmb and Overvik[1] for engineering use in terms of significant waveheight and average period, the results being presented in tabular form. The purpose of the work reported here is to revise Houmb and Overvik's parameterisation in the light of certain mathematical properties of the Jonswap equation and to express the results in the form of algebraic expressions for convenience in use.

PRELIMINARY OBSERVATIONS

The Jonswap spectrum for wave amplitude η is defined by the equation:

$$S_\eta(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp(-1.25(f/fm)^{-4}) \times \gamma^{\exp\left[\frac{-(f-fm)^2}{2\sigma^2 fm^2}\right]} \tag{1}$$

$$\sigma = \sigma_a \quad \text{for } f \leqslant fm$$

$$\sigma = \sigma_b \quad \text{for } f > fm$$

where $S_\eta(f)$ = spectral density, g = acceleration due to gravity, f = frequency in Hz and the remaining parameters are constant for a given sea state, fm is the frequency corresponding to maximum $S_\eta(f)$.

Of the five parameters in equation (1), $\sigma_a = 0.07$ and $\sigma_b = 0.09$ are usually taken as absolute constants leaving α, fm, γ to be determined in such a way as to give a spectrum with the required significant waveheight, Hs, and average period, Tz; i.e. we have to obtain three unknown from two known parameters. Two of the necessary relationships can be obtained mathematically from equation (1) but the third must be based on empirical data.

In the following sections it is shown that α and fm, non-dimensionalised with respect to Hs and Tz, can be expressed as functions of γ only. Using the empirical data of Houmb and Overvik it is then shown that γ is a unique function of a single dimensionless parameter combining Hs and Tz. These expressions avoid the need for interpolation between tabulated values and are of great practical convenience in deriving Jonswap spectra with given waveheight and period.

MATHEMATICAL PROPERTIES OF THE JONSWAP SPECTRUM

Equation (1) may be rewritten in terms of the relative frequency, $r = f/fm$, noting that $S_\eta(f)\,df = S_1(r)\,dr$ to maintain constant spectral energy:

$$S_1(r) = fm \cdot S_\eta(f) = \frac{\alpha g^2}{(2\pi fm)^4} r^{-5} \exp(-1.25 r^{-4}) \times \gamma^{\exp\left[\frac{-(r-1)^2}{2\sigma^2}\right]} \tag{2}$$

This expression is the product of three terms whose significance is as follows:

(i) $\dfrac{\alpha g^2}{(2\pi fm)^4}$ is independent of frequency and is a scale factor.

(ii) $r^{-5}\exp(-1.25 r^{-4})$ is the spectrum shape term adopted by the Jonswap investigators from Pierson and Moskowitz.[3]

(iii) $\gamma^{\exp\left[\frac{-(r-1)^2}{2\sigma^2}\right]}$ is the Jonswap peak enhancement factor

We adopt the usual definition of spectral moments, viz.:

$$m_n = \int_0^\infty r^n S_1(r)\,dr \tag{3}$$

and use the standard results that significant waveheight, Hs, and average zero up crossing period, Tz, are given by

$$Hs = 4.0\sqrt{m_0} \tag{4}$$

$$r_z = \sqrt{\frac{m_2}{m_0}} \tag{5}$$

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where $r_z = fz/fm = (fm \cdot Tz)^{-1}$ and hence

$$fm \cdot Tz = \sqrt{\frac{m_0}{m_2}} \tag{6}$$

Combining equations (2) and (3) we find that

$$m_n = \frac{\alpha g^2}{(2\pi fm)^4} f_n(\gamma, \sigma) \tag{7}$$

where f_n is a function of γ and σ only. This leads immediately to the result from equation (6) that

$$fm \cdot Tz = g_1(\gamma, \sigma) \tag{8}$$

and from equation (4), with some manipulation, that

$$\frac{\alpha g^2}{(Hs \cdot fm^2)^2} = g_2(\gamma, \sigma) \tag{9}$$

where g_1, g_2 are again functions of γ and σ only. The dependence of α on the initially unknown fm can be removed by substituting for fm from equation (8). Defining an equivalent wave steepness as

$$s = \frac{2\pi Hs}{gTz^2} \tag{10}$$

by analogy with monochromatic waves in deep water we find, after further manipulation, that

$$\alpha/s^2 = g_3(\gamma, \sigma) \tag{11}$$

Assuming σ_a and σ_b are constants then g_1, g_2, g_3 reduce to functions of γ only.

Equations (8) and (11) then give fm and γ in terms of the known parameters Hs and Tz and the one remaining unknown parameter, γ.

EVALUATION OF FUNCTIONS FOR α, fm AND γ

The functions g_1, g_3 are complex, difficult and perhaps impossible to integrate analytically. They can, however, be evaluated numerically and this has been done for a range of γ values. The upper limit of integration in evaluating the moments was ∞, achieved by integrating numerically up to a predetermined truncation point and adding the contributions from the tail of the spectrum using analytically derived expressions. The following curves have been fitted by regression analysis to the results so obtained:

$$fm \cdot Tz = 0.6063 + 0.1164\gamma^{1/2} - 0.01224\gamma \tag{12}$$

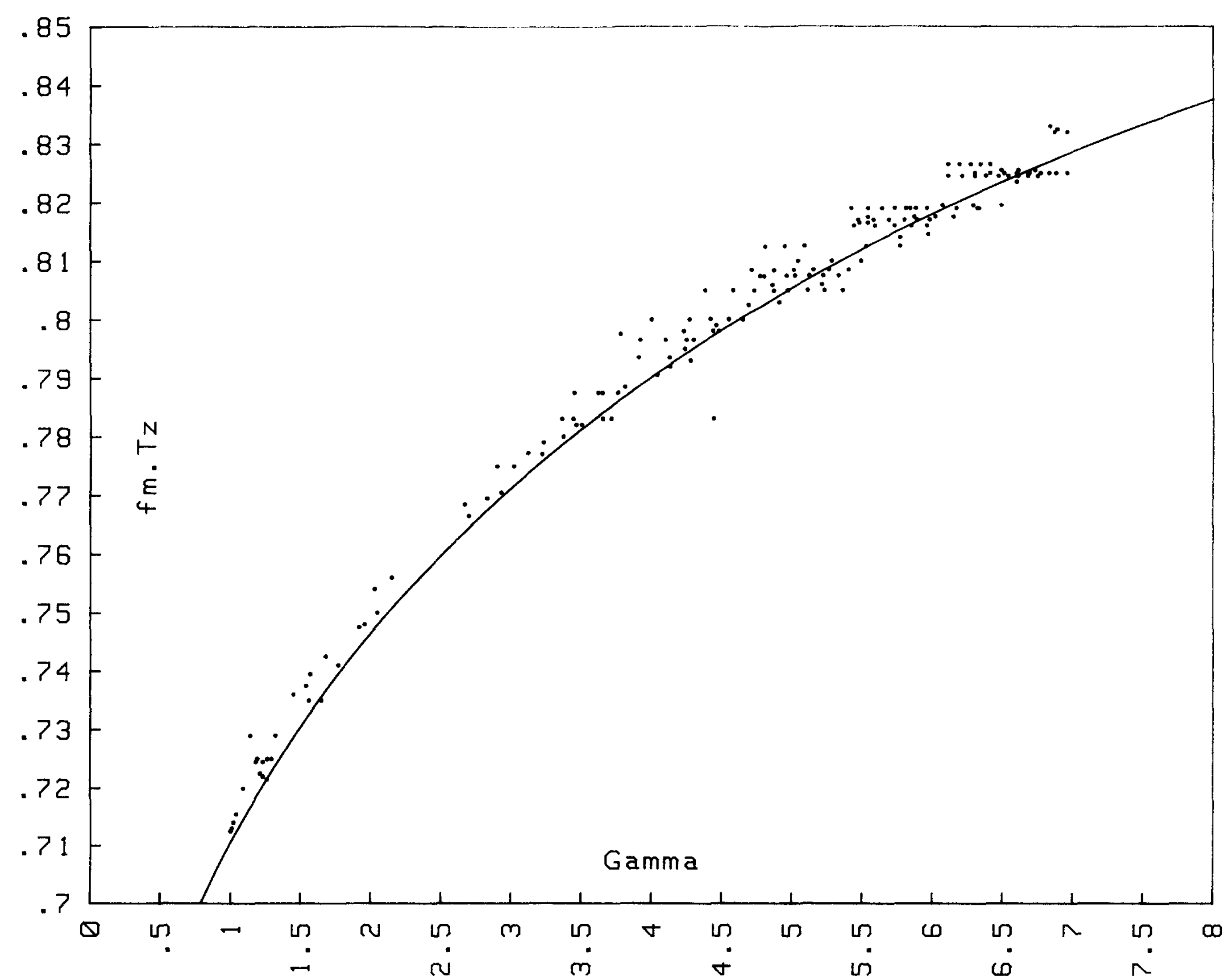

Figure 1. $fm \cdot Tz$ against γ – the continuous line represents the theoretical relationship of equation (12). The spots are from Houmb and Overvik for comparison

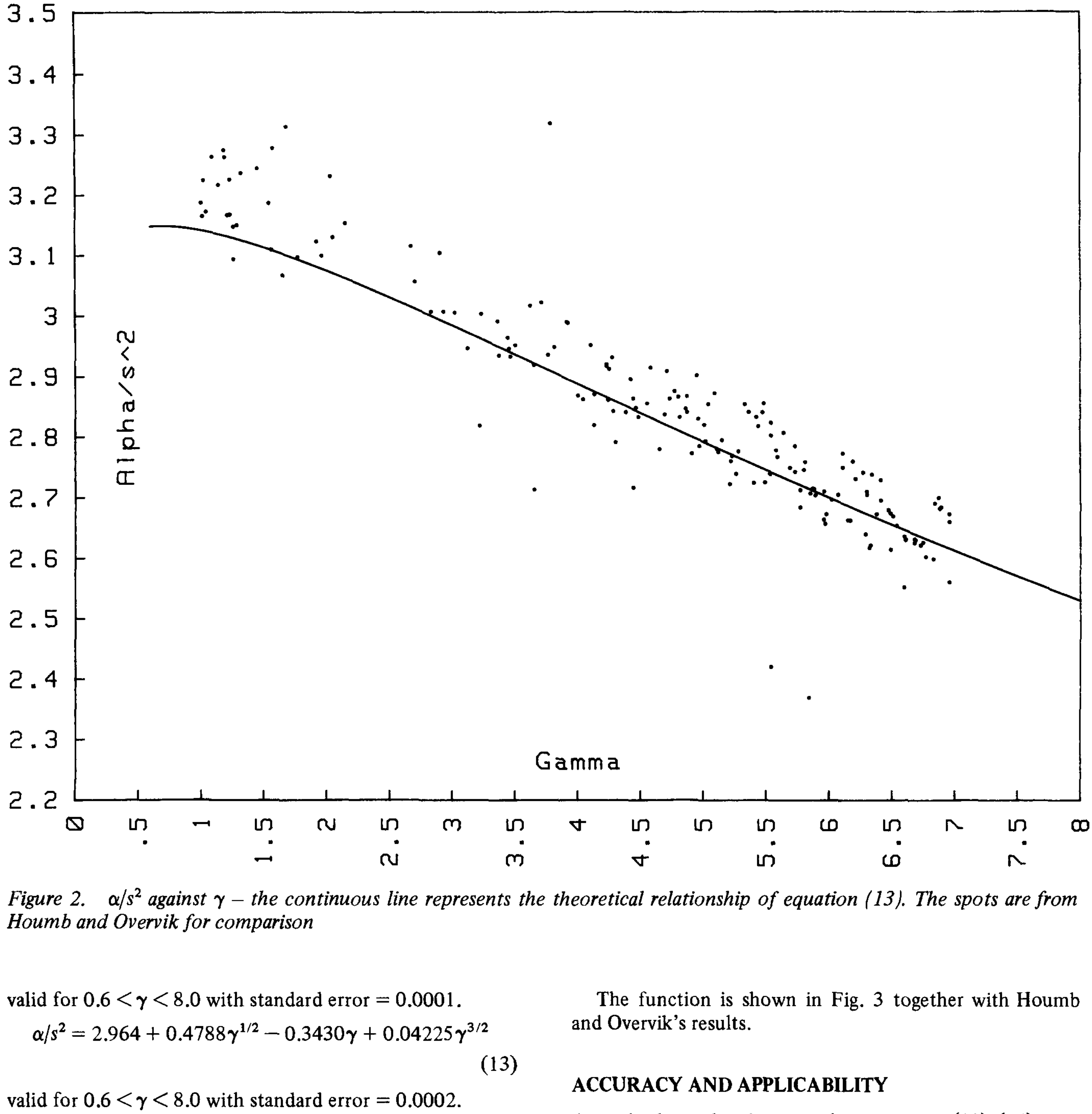

Figure 2. α/s^2 against γ – the continuous line represents the theoretical relationship of equation (13). The spots are from Houmb and Overvik for comparison

valid for $0.6 < \gamma < 8.0$ with standard error = 0.0001.

$$\alpha/s^2 = 2.964 + 0.4788\gamma^{1/2} - 0.3430\gamma + 0.04225\gamma^{3/2} \tag{13}$$

valid for $0.6 < \gamma < 8.0$ with standard error = 0.0002.

Note that equations (12) and (13) are valid for $\sigma_a = 0.07$ and $\sigma_b = 0.09$ only.

The functions are shown graphically in Figs. 1 and 2 together with the corresponding values obtained from Houmb and Overvik. In general the values derived empirically by Houmb and Overvik agree well with those obtained here from purely mathematical considerations.

Houmb and Overvik's results were then used to obtain a relationship between γ, Hs, and Tz. It was found that plotting γ on s, collapsed the data onto a unique line and the following curve fit was obtained by regression analysis:

$$\gamma = 10.54 - 1.34s^{-1/2} - \exp(-19 + 3.775s^{-1/2}) \quad \text{for } s \geqslant 0.037$$

$$\gamma = 0.9 + \exp(18.86 - 3.67s^{-1/2}) \quad \text{for } s < 0.037 \tag{14}$$

valid for $0.03 < s < 0.15$ with standard error = 0.06. This equation is also valid for $\sigma_a = 0.07$ and $\sigma_b = 0.09$ only.

The function is shown in Fig. 3 together with Houmb and Overvik's results.

ACCURACY AND APPLICABILITY

As a check on the above results, equations (12)-(14) were used to generate Jonswap spectra corresponding to each of the populated boxes in Houmb and Overvik's Table 1 and the values of Hs and Tz were then calculated from the spectral moments and compared with the input values. The accuracy of the parameterisation is indicated by the following error values:

Errors in Hs (metres)		Errors in Tz (seconds)	
Max	RMS	Max	RMS
0.005	0.002	0.002	0.001

This is clearly adequate for engineering purposes.

Equations (12) and (13) are valid for all applications of the Jonswap equation with $\sigma_a = 0.07$, $\sigma_b = 0.09$ and $0.6 < \gamma < 8.0$. Equation (14) is derived from Houmb and Overvik's results for spectra measured in the northern North Sea and may require revision for other sea areas. Equation (14) is valid for sea states up to the maximum probable steepness

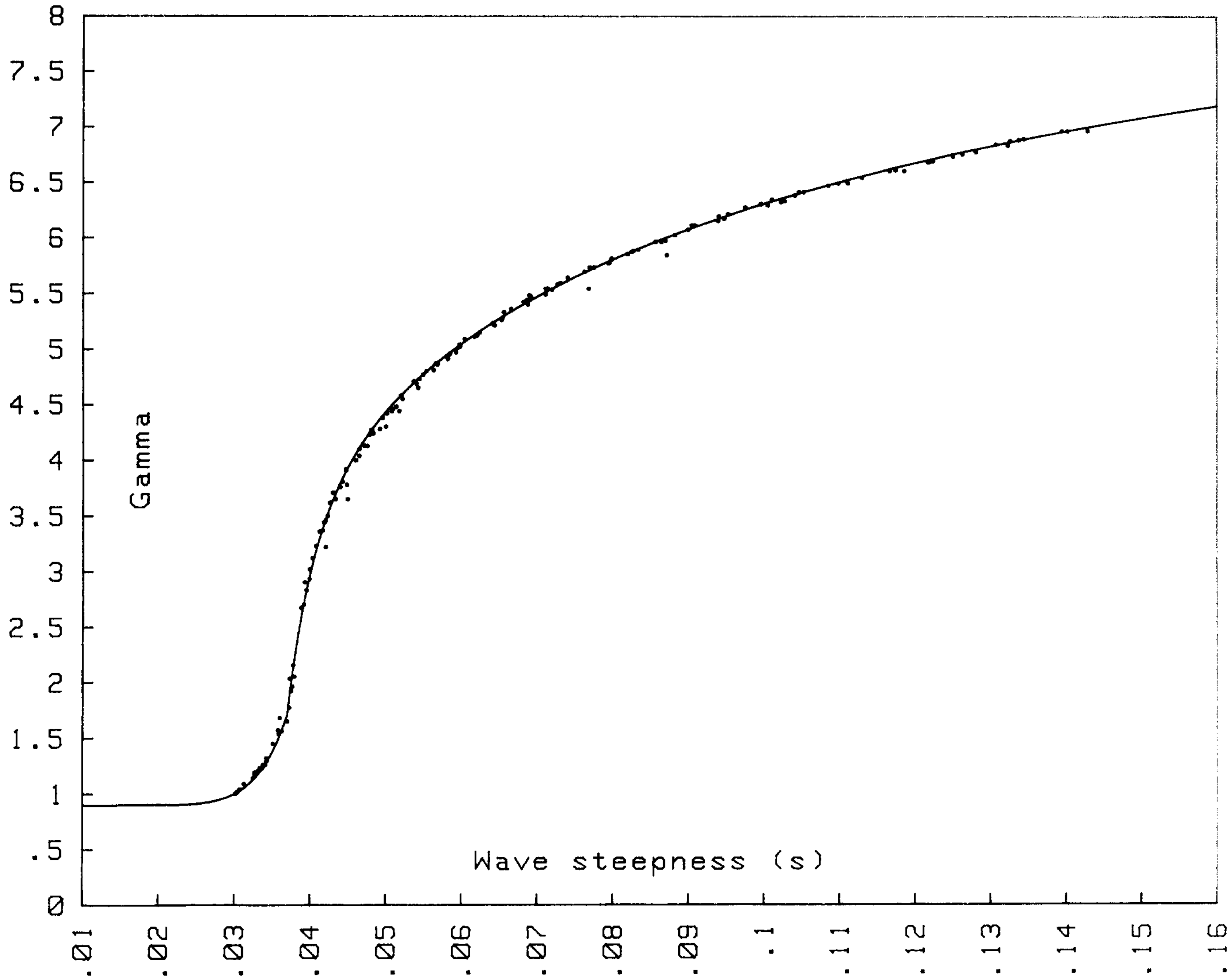

Figure 3. *γ against wave steepness – data from Houmb and Overvik with regression relationship of equation (14)*

($s = 0.14$) but Houmb and Overvik's data only extend to $s = 0.03$ at the low steepness end which only covers about half the sea states included in a typical North Sea wave scatter diagram. This is not of great importance in practical terms since the steeper sea states usually govern in engineering design work. In the absence of other data it is suggested that equation (14) be used throughout the range of wave steepness.

CONCLUSION

Given the significant waveheight and average period for a sea state, good estimates of the three parameters of the corresponding Jonswap spectrum can conveniently be obtained using equations (12)-(14). For areas other than the North Sea, equation (14) may require modification to fit empirical data for the sea area in question.

REFERENCES

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