13.42 Design Principles for Ocean Vehicles

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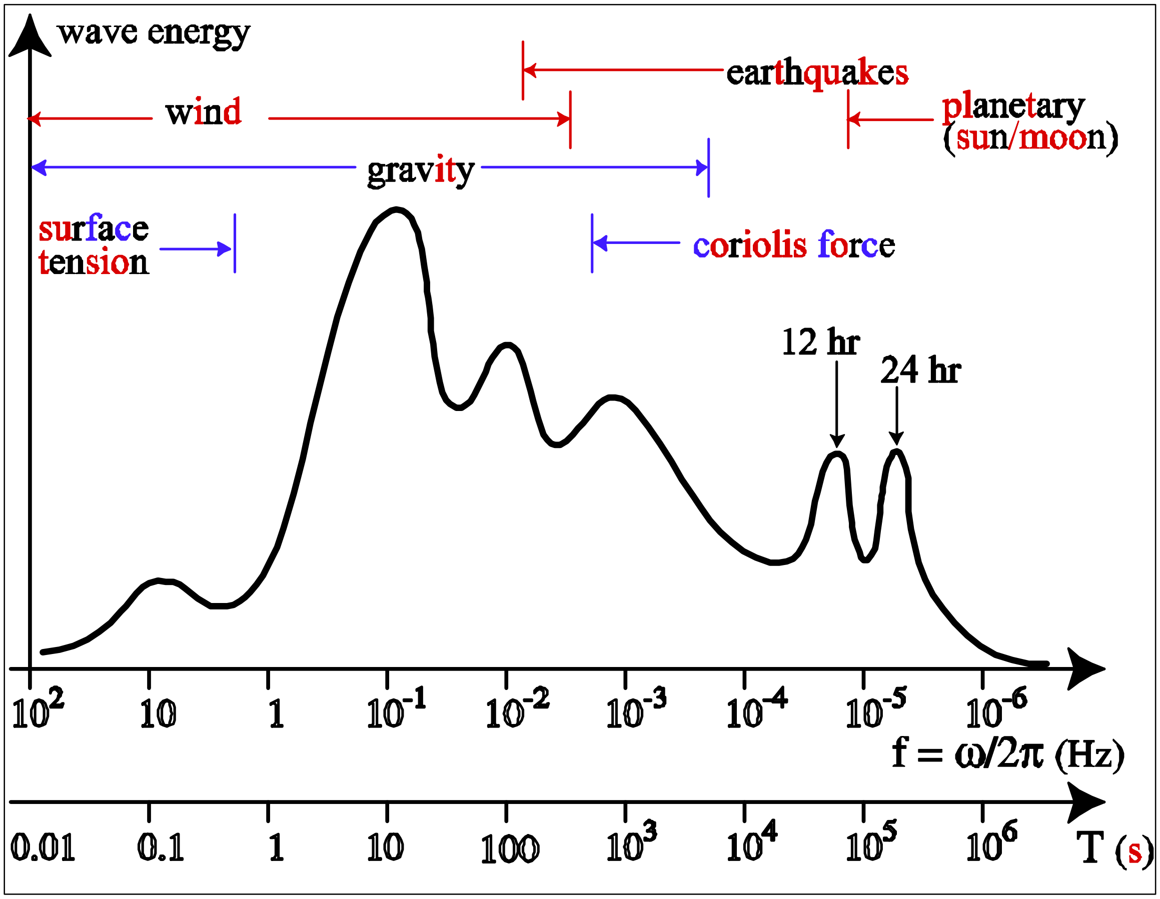
1. Ocean Wave Spectra



1. Wave energy spectra. Red text indicates wave generation mechanisms and blue text indicates damping/restoring forces.

The majority of ocean waves are wind generated. Other wave generating mechanisms include earthquakes and planetary forces. Planetary forces drive tides and cause long

period waves on the order of 12 to 24 hours. Earthquakes are the major cause of tsunamis which, while rare, can be catastrophic if the earthquake occurs near or on the coast.
Waves also encounter forces that tend to restore them to a flat surface. For small wavelength (high frequency) waves surface tension plays a large role in damping out these waves. The majority of waves are restored by gravity and longer period waves are damped by the Coriolis force.

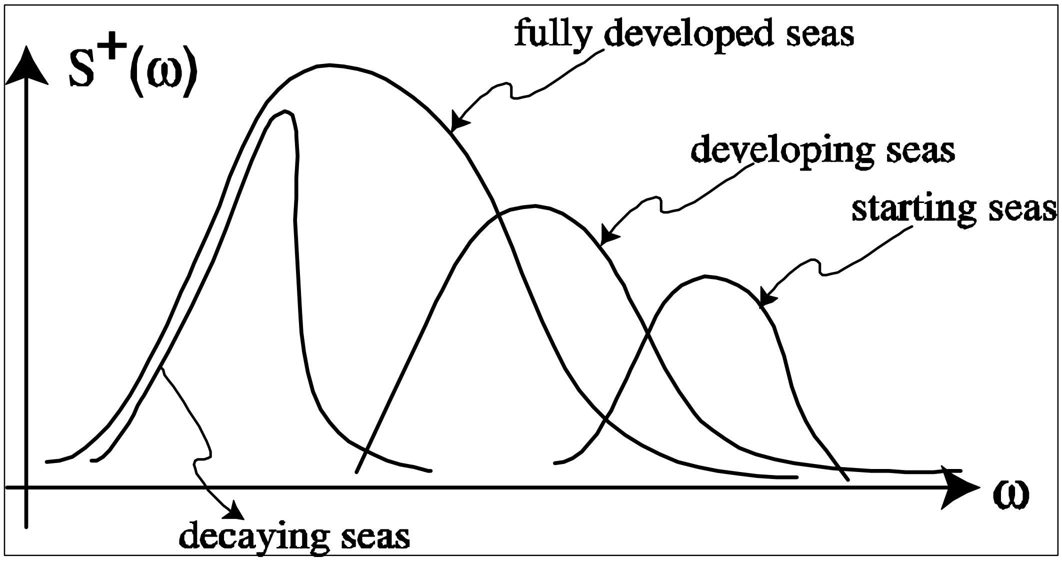
As wind begins to blow (between 0.5 - 2 knots) on a calm surface small ripples, capillary waves or “cat-paws”, tend to form. These small waves are on the order of less than 2 cm. As the wind becomes stronger wave amplitude increases and the waves become longer in order to satisfy the dispersion relationship. This growth is driven by the Bernoulli effect, frictional drag, and separation drag on the wave crests.

Wind must blow over long periods of time and large distances to reach a fully developed sea state. When the phase speed of the wave crest matches the wind speed non-linear interactions stop (except friction) and the phase speed is maximized. The limiting frequency of the waves can be determined by the equation for phase speed and the dispersion relationship:

$$C_p \approx U_w = w/k = g/w \tag{1}$$

$$w_c \approx \frac{g}{U_w} \tag{2}$$

where U_w is the wind speed and w_c is the limting frequency. Once wind stops viscosity erodes the waves slowly. The smallest wavelengths decay the fastest. Sample spectrum shapes are shown in figure 2.



For a storm with wind speed, U_w, the effects of the storm can be felt at a distance from the storm, R. The number of wave cycles between the storm and the observation location is $N = R/l$. The amplitude of the waves decays as e^{-gt} where $g = 2n k^2 = 2nw^4/g^2$ (from Landau and Lifshitz).

The development of storms can be tabulated. Fetch is the length over which the wind must blow to have fully developed seas (given in standard miles), and the storm duration, given in hours, is the time the storm must last to result in a fully developed sea.

Wind warnings	Beaufort scale	Wind speed (mph)	Fetch (miles)	Storm duration (hr)
	3-4	12	15	3
small craft	5-6	25	100	12
	7	35	400	28
gale	9	50	1050	50
hurricane		70+		

2. Typical Wave Spectra

Researchers have studying ocean waves have proposed several formulation for wave spectra dependent on a a number of parameters (such as wind speed, fetch, or modal frequency). These formulations are very useful in the absence of measured data, but they can be subject to geographical and seasonal limitations.
Most ocean wave spectra take a standard form following the mathematical formulation:

$$S^{+}(w) = \frac{A}{w^{5}} e^{-B/w^{4}} \tag{3}$$

The frequency peak is called the modal frequency. The area under the spectrum is the zeroth moment, M_o, which may be defined in terms of the significant wave height. For a narrow-banded spectrum the significant wave height is approximately four times the square root of the zeroth moment. Since the significant wave height depends on the wind speed, the spectrum could be formulated in terms of the wind speed instead of the significant wave height. While certain spectra can have more than one peak, it is assumed that a single storm produces a single-peaked spectrum and any second peak is due to a distant storm that sends waves to the considered location.

Several more specific theoretical representations of wave spectra have been developed using data collected by observation platforms and satellite data in various regions. These spectra are discussed below. When considering which spectrum formulation it is important to take into account the specific criteria that were used in developing the spectrum.

Typically parameters that influence the spectrum are:

- Fetch limitations, i.e. whether the location we are considering has some physical boundaries that do not permit the waves to *fully develop*.
- Whether the seas are developing or decaying
- Seafloor topography: Deep water wave spectra are invalid in shallow waters, and vice versa. It may also be necessary to account for wave diffraction.
- Local currents: Strong currents may significantly impact the wave spectrum

- Presence of swells: Swells are waves that result from distant storms that travel a significant distance and arrive often at an angle that differs from the wind direction. If we use a spreading function to correct a unidirectional spectrum it will not account for the presence of swell. It is also important when measuring waves that the component that results from swell be accounted for separately.

The Pierson-Moskowitz spectrum (equation 11) was developed for fully developed seas in the Northern Atlantic Ocean generated by local winds.

$$S^{+}(w) = \frac{8.1}{10^{3}} \frac{g^{2}}{w^{5}} e^{-0.032(g/zw^{2})^{2}} \tag{4}$$

where z is the significant wave height,

$$z \equiv H^{1/3} = 4\sqrt{M_{o}}, \tag{5}$$

and w_m is the modal frequency,

$$w_{m} = 0.4\sqrt{g/z}\ . \tag{6}$$

This spectrum is developed under the following conditions: unidirectional seas, North Atlantic Ocean, fully developed local wind generation with unlimited fetch. The most critical of these assumptions is the fully developed assumption. For it is possible to achieve a larger heave response for a platform from a developing sea, even though the significant wave height may be smaller that that of a fully developed sea, since the modal frequency is higher and heave motions tend to have higher natural frequencies. In the case of a rolling ship the decaying sea might excite a larger roll motion since the natural frequency of roll tends to be relatively low.

In order to overcome the limitation of fully developed seas, a *two parameter* spectrum was developed. This spectrum is the Bretschneider spectrum (equation 14). The B-S spectrum replaced the Pierson-Moskowitz spectrum as the ITTC standard.

$$S^{+}(w) = \frac{1.25}{4}\frac{w_m^4}{w^5} z \; e^{-125(w_m/w)^4} \tag{7}$$

where again, z is the significant wave height,

$$z \equiv H^{1/3} = 4\sqrt{M_o}, \tag{8}$$

If w_m satisfies equation 13 then equation 14 reduces to equation 11. By allowing the user to specify the modal frequency and significant wave height, this spectrum can be used for sea states of varying severity from developing to decaying.

The Ochi Spectrum (equation 16) is a *three parameter* spectrum that allows the user to specify the significant wave height, the modal frequency, and the steepness of the spectrum peak.

$$S^{+}(w) = \frac{1}{4}\frac{\left(\frac{4l+1}{4}w_m^4\right)^l}{\Gamma(l)}\frac{z^2}{w^{4l+1}} exp\left\{-\left(\frac{4l+1}{4}\right)\left(\frac{w_m}{w}\right)^4\right\} \tag{9}$$

where $\Gamma(l)$ is the gamma function, and l is the parameter that controls the spectrum steepness. For $l = 1$, equation 16 reduces to equation 14. The Ochi spectrum is limited in that it also considers only unidirectional seas and unlimited fetch, but the designed can now specify the spectrums severity (z), the state of development (peak frequency w_m) and isolate the important frequency range by dictating the spectrum width (l). The ability to dictate l allows the designer to account for swell from a distant storm.

The JONSWAP spectrum (equation 17) was developed by the Joint North Sea Wave Project for the *limited fetch* North Sea and is used extensively by the offshore industry. This spectrum is significant because it was developed taking into consideration the growth of waves over a *limited fetch* and wave attenuation in shallow water. Over 2,000 spectra were measured and a least squares method was used to obtain the spectral formulation assuming conditions like near uniform winds.

$$S^{+}(w) = \frac{ag^{2}}{w^{5}} e^{-\frac{5}{4}\left(\frac{w_m}{w}\right)^{4}} g^{d} \qquad (10)$$

where

$$d = -\frac{(w - w_m)^2}{2s^{2} w_m^2} \qquad (11)$$

$$a = 0.076\,\bar{x}^{(-022)} \qquad (12)$$

$$\bar{x} = \frac{gx}{U^2} \qquad (13)$$

$$s = \begin{Bmatrix} 0.07; & w \le w_m (21) \\ 0.09; & w > w_m \end{Bmatrix} \qquad (14)$$

The wind speed in knots is U , x is the fetch in nautical miles, and the modal frequency can be found as

$$w_m = 2p * 3.5 * (g/U)\,\bar{x}^{-033}. \qquad (15)$$

To recap, in general for a narrow banded spectrum:

$$\int_{-\infty}^{\infty} S^{+}(w)\, dw = M_o = \left(\frac{z}{4}\right)^2 \qquad (16)$$

We can account for the effects of two separate storms by adding the respective spectrums:

$$S^{+}(w) = S_1^{+}(w) + S_2^{+}(w) \qquad (17)$$

We can also correct for directionality multiplying the spectrum by a spreading function,

$M(m)$,

$$S^{+}(w,m) = S_{BS}^{+}(w)\, M(m) \tag{18}$$

where $M(m)$ spreads the energy over a certain angle contained within the interval $[-p, p]$ from the wind direction. The integral of $M(m)$ over this interval is one.

$$\int_{-p}^{p} M(m)\, dm = 1 \tag{19}$$

For example we can choose a spreading function such that

$$M(m) = \frac{2}{p}\cos^2 m \tag{20}$$

on the interval

$$-\frac{m}{2} < m < \frac{m}{2} \tag{21}$$

3. Bretschneider Spectrum

To recap, the 15th International Towing Tank Conference (ITTC) in 1978 recommended using a form of the Bretschneider spectrum for average sea conditions when a more specific appropriate form of the wave spectrum is well defined. The general form of this spectrum is equation refeq:specgen.

$$S^{+}(w) = \frac{A}{w^5} e^{-B/w^4} \tag{22}$$

The two parameters A and B are dependent on the modal frequency, w_m, and the variance of the spectrum, $M_o = (rms)^2 = s^2$.

$$w_m^4 = \frac{4}{5}B \;\; ; \;\; B = 5w_m^4/4 \tag{23}$$

$$Variance = s^2 = A/(4B) \;\; ; \;\; A = 4s^2 B \tag{24}$$

If we normalize the frequency, w, by the modal frequency equation 30 becomes equation 33.

$$S(w) = 5\frac{w_m^4}{w^5} s^2 \, e^{-\frac{5}{4}\left(\frac{w_m}{w}\right)^4} \tag{25}$$

For a narrow banded spectrum, $e < 0.6$, the significant wave height, $z = H^{1/3} = 4\sqrt{M_o}$, where M_o is the variance of the spectrum. For a wide banded spectrum, $e = 1$, St. Denis (1980) showed that the significant wave height was approximately, $z = 3\sqrt{M_o}$. This leaves us with the final form of the Bretschneider Spectrum.

$$S(w) = \frac{1.25}{4}\frac{w_m^4}{w^5} z^2 \, e^{-125\left(\frac{w_m}{w}\right)^4} \tag{26}$$

The moments of the spectrum can be calculated numerically. For simplification the following relationships have been given (see Principles of Naval Architecture vol. III for further discussion). The fourth moment diverges slowly as $w \to \infty$ thus the approximation helps analysis when calculation of this moment is necessary.

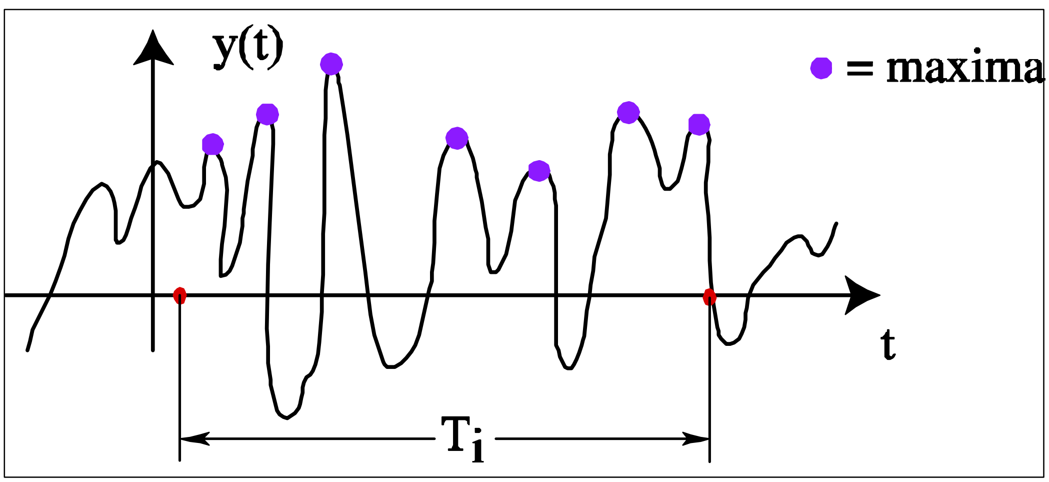
$$M_o = VARIANCE = (RMS)^2$$

$$M_2 = 1.982\, M_o\, w_m^2 = 1.982\left(\tfrac{z w_m}{4}\right)^2$$

$$M_4 \approx 7.049\, M_o\, w_m^4 \quad \text{for } 5w < 5w_m$$

4. $1/Nth$ Highest Maxima

For design purposes it is useful to determine the occurrences of wave amplitude maxima above a certain level. Given a time-trace of wave height data from a buoy deployed in the ocean we can analyze it to determine such information. If we look at a given wave train, $y(t)$, over an interval in time, T_o (T_o is not necessarily a wave period but more like a duration of time which can consist of multiple periods), then within this interval of time there are a number of maxima, $a_1, a_2, a_3, a_4, \ldots, a_n$, where $a^{1/N}$ is the value exceeded by $1/N$ of the maxima. (Note: $a^{1/N}$ is not a to the $1/N$ power)



3.

EXAMPLE: Take $N=3$; From a sequence of twelve measured wave heights find the $1/3$rd ($1/N$) highest wave height, $a^{1/3}$:

Measured wave heights:

6 5 3 4 7 11 8 9 5 4 2 5

There are 12 recorded wave maxima so there are four wave maxima above the 1/3rd highest wave height. For this sequence $a^{1/3} = 6$ since it is the next highest wave maxima below 7. Had this been a infinitely long series of observed wave heights and 7 was the lowest value of the 1/3rd highest wave maxima, then 6.99 might been a better estimation for this value. However in this short sequence the answer is 6.

Had the sequence been:

$$[1 \quad 8 \quad 3 \quad 4 \quad 11 \quad 11 \quad 11 \quad 11 \quad 5 \quad 4 \quad 2 \quad 5]$$

then $a^{1/3} = 8$, since the four highest wave heights are all equal to 11.

The probability of wave heights occurring above the 1/Nth highest wave is given by

$$P(h \geq h^{1/N}) = \frac{1}{N} \approx \frac{2\sqrt{1-e^2}}{1+\sqrt{1-e^2}} e^{-(h^{1/N})^2/2} \tag{27}$$

where

$$h^{1/N} = \frac{a^{1/N}}{\sqrt{M_o}} = \sqrt{2\ln\left(\frac{2\sqrt{1-e^2}}{1+\sqrt{1-e^2}} N\right)} \tag{28}$$

The average value of ALL maxima above $a^{1/N}$ is called the 1/N (Nth) highest average amplitude. This can be found using the formula for expected value of a variable:

$$\overline{a^{1/N}} = E\left\{a_m \,|\, (a_m > a^{1/N})\right\} \tag{29}$$

This is the expected value given a_m is greater than $a^{1/N}$ as can be represented as

$$\overline{a^{1/N}} = \int_{a^{1/N}}^{\infty} a \quad p\Big((a_m = a) \,|\, (a_m > a^{1/N})\Big) da \tag{30}$$

where the probability, $p\left(a_m = a \,|\, a_m > a^{1/N}\right)$, is simply

$$p\left(a_m = a \,|\, a_m > a^{1/N}\right) = \frac{P\Big((a_m = a) \cap (a_m > a^{1/N})\Big)}{P(a_m > a^{1/N})} \tag{31}$$

Keeping the amplitude in non-dimensional form we can calculate the Nth highest average wave height using the approximate pdf given in the last reading.

$$\overline{h^{1/N}} \approx \frac{2N\sqrt{1-e^2}}{1+\sqrt{1-e^2}} \int_{\overline{h^{1/N}}}^{\infty} h_o^2 e^{-h_o^2/2} dh_o \tag{32}$$

For a value of $N = 3$, $\overline{a^{1/N}}$ is considered the **significant wave amplitude** where $\overline{a^{1/N}} \approx 2s = 2\sqrt{M_o}$ for $e < 0.5$. The **significant wave height** is defined as twice the significant wave amplitude,

$$H^{1/3} = 2\overline{a^{1/3}}. \tag{33}$$

This value is very close to that which a casual observer would estimate as the wave height when watching the sea. This makes the significant wave height a very useful statistic. Maps of the significant wave height over the entire earth can be seen on satellite images. There are several links on the course webpage that illustrate this quantity over the earth surface. From such a satellite composite image we can see that the southern ocean is the most tumultuous ocean and has the highest significant wave height.

5. Long Term Statistics

Structural design analysis of offshore structures over a long time, specifically the total life span of the structure or T, requires knowledge of the short term statistics of waves at the system installation location. Over the life of a platform or structure there are a total of i storms, one of these can also include a "storm" which is described completely calm conditions, similar to the null set in probability. The probability of each storm event is, P_i, which can also be looked at as the fraction of the total life of the structure over which a certain storm, i, exists. Thus the life of each storm, T_i is dictated by equation 1.

$$T_i = T\ P_i \tag{34}$$

From short term statistics we have the frequency of waves, upcrossings, exceeding a certain amplitude, a_o.

$$n(a_o) = \frac{1}{2p}\sqrt{\frac{M_2}{M_o}}\, e^{-a_o^2/(2M_o)} = \frac{1}{\overline{T}}\, e^{-a_o^2/(2M_o)} = \frac{1}{\overline{T}}\, m \tag{35}$$

Where $\overline{T}$ is the average period of waves in a storm. So it is easy to find the **total number of times** the level a_o is exceeded in a storm i, given by N_i as

$$N_i = n(a_o)\, T_i = \frac{T_i}{\overline{T}_i}\, m_i \tag{36}$$

Where T_i is the life of the storm i, $\overline{T}_i$ is average period of the waves within that storm.

The total number of times level a_o is exceeded during the life of the structure then becomes:

$$N_{a_o} = \sum_{all\, i} N_i = \sum_i \frac{T}{\overline{T}}\, P_i\, e^{-a_o^2/(2M_{o\,i})} \tag{37}$$

where $M_{o\,i}$ is the zeroth moment of the spectrum of storm i. It is necessary to make the assumption that the spectrum is narrow banded so that we can write the zeroth moment in terms of the significant wave height. This simplifies the problem since the significant wave height is the data most often available that describe sea conditions over a certain time period.

Next we can adjust the equation for total number of upcrossings to reflect a number of upcrossings exceeding the height, h_o which is simply defined as twice the amplitude, a_o.

$$N_{h_o} = \sum_i \frac{T}{\overline{T}} P_i \, e^{-\left(\frac{2h_o^2}{z_i^2}\right)} \tag{38}$$

Equation 5 represents the **total number of upcrossings past a level h_o over a time interval T**. The total number of upcrossings past the mean water level, $h_o = 0$, is given by equation 6.

$$N_o = \sum_i \frac{T}{\overline{T}} P_i \tag{39}$$

Having determined the equation for the total number of waves exceeding a level h_o and also the total number of upcrossings during the life of the structure, we can find the probability of a wave height exceeding a design height h_o in equation (40)

$$P(h > h_o) = \frac{N_{h_o}}{N_o} = \frac{\sum_i P_i \frac{T}{\overline{T}_i} e^{-2h_o^2/z_i^2}}{\sum_i P_i \frac{T}{\overline{T}_i}} \tag{40}$$

This equation represents the fraction of total waves above level h_o. Realizing the life span of the structure, T, is constant we can cancel it out of equation (40) and rewrite it in the form

$$P(h > h_o) = \frac{N_{h_o}}{N_o} = \frac{\sum_i \left(\frac{1}{\overline{T}_i} e^{-2h_o^2/z_i^2} \right) P_i}{\sum_i \left(\frac{1}{\overline{T}_i} \right) P_i} \tag{41}$$

The numerator and denominator of equation (41) each have the form of the expected value formula for a random process with random variables $\overline{T}$ and z. So we can rewrite the probability as

$$P(h > h_o) = \frac{E\left\{\frac{1}{\overline{T}} e^{-2h_o^2/z^2}\right\}}{E\left\{\frac{1}{\overline{T}}\right\}} \tag{42}$$

Note the subscript i is dropped since $\overline{T}$ is the random variable and T_i is a single random event. Thus the expected value of the random variable is the sum over all the ith random events. This can also be extended to a random process.

We now have and equation for the probability of waves above a level that is not implicitly dependent on the life span of the structure. In practical application the average wave period can be neglected (cancelled) and the probability approximated by

$$P(h > h_o) \approx E\left\{e^{-2h_o^2/z^2}\right\}. \tag{43}$$

One valuable calculation is related to the *hundred year wave*. This is the wave that exceeds some value, h_100, on the average only once in every one hundred years.

$$P(h > h_100) = \frac{N(\#\text{ of waves over } h_{100})}{N_o(\#\text{ of total upcrossings})} = \frac{1}{100\ years/\overline{T}} = \frac{\overline{T}}{3.16 * 10^9\ seconds} \tag{44}$$

Note: This equation takes into consideration 100 years with the appropriate number of leap years. So now we have an equation for the probability of the hundred year wave.

$$E\left\{e^{-2h_{100}^2/z^2}\right\} = \frac{\overline{T}}{3.16e+9} \tag{45}$$

Where $\overline{T}$ is the average period of waves over all storms. In order to solve for h_{100} we must have statistics for very rough storms with high significant wave heights. Smaller, more frequent, values of wave height do not play a role in this calculation and it is only the most ferocious storms that contribute to this value. Thus you can imagine in order to collect this data observations during major storms, hurricanes, typhoons, etc., must be performed over time. Under these conditions it is too risky for human observation and often equipment is lost at sea making obtaining this challenging.

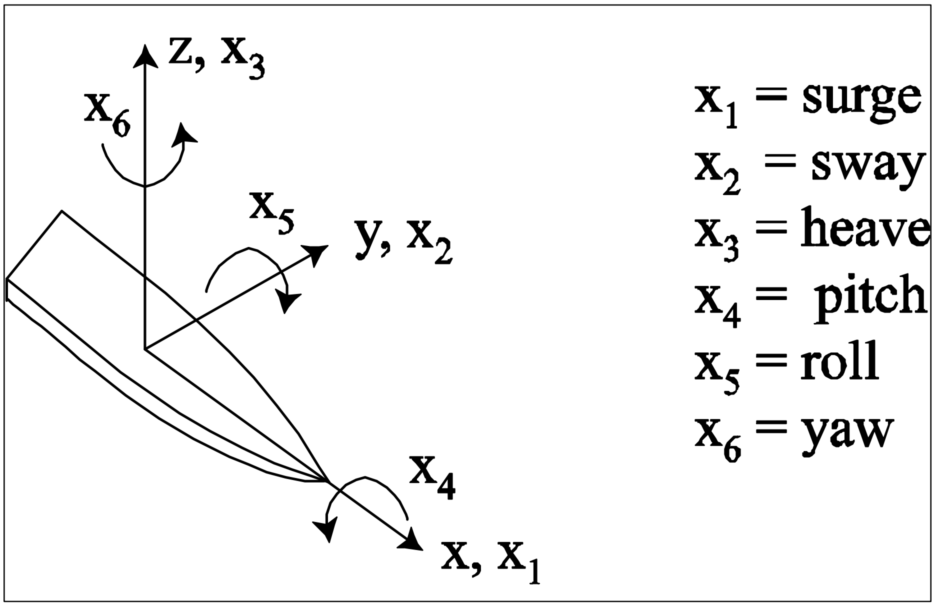
From probability theory it is possible to approximate the probability of extreme events by using a Weibull distribution in the form

$$P(x > x_o) = e^{-\left(\frac{x_o - x_1}{x_2 - x_1}\right)^g} \tag{46}$$

where x_1, x_2, and g are estimated from historical data and x_1 is a threshold level below which the statistics will not effect the extremes.

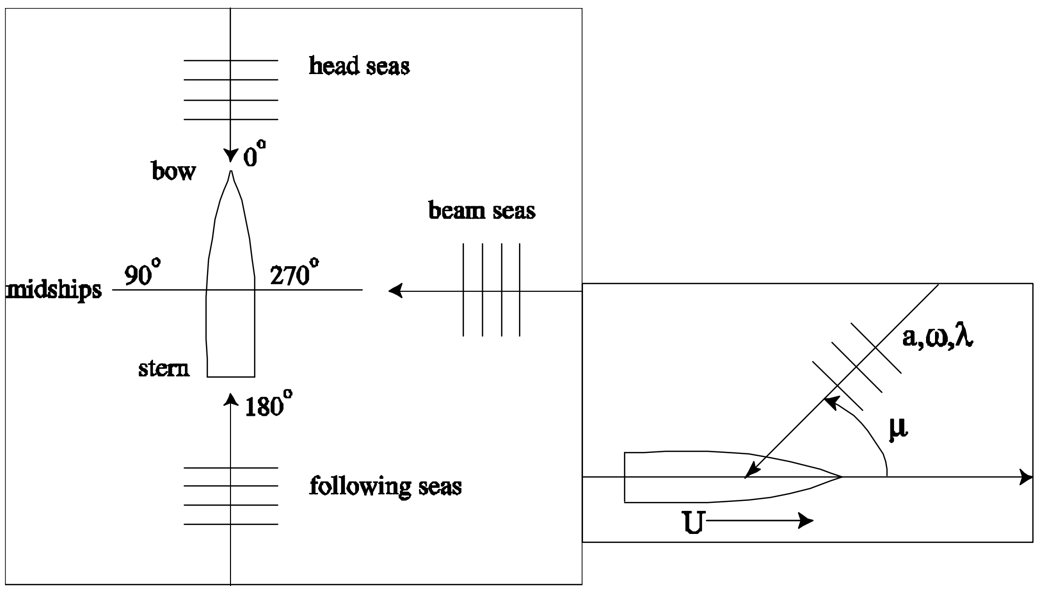
6. Encounter Frequency

The principle motions of a ship or structure in seas are defined as surge, sway, heave, pitch, roll and yaw. A diagram of the coordinate system is shown in figure 1.



4. Motion of a ship defined with a right handed coordinate system with forward motion as the positive x-direction and positive heave upwards.

Waves incident on the structure or ship can be described as head seas, following seas, beam seas, or quartering seas depending on the incident direction. Figure 2 illustrates the first three cases and quartering seas are defined as those that approach the ship from the either the port (left) or starboard (right) stern quarter of the ship (between 90^o and 180^o or 180^o and 270^o).



5. Incident sea description. Figure at left shows head, beam and following seas. The incident angle, m , is measured from the bow (x-axis is $m = 0^o$) counterclockwise.

The motion of a ship, forward or otherwise, affects the way incident waves are viewed by someone aboard the vessel. For example if the ship is making way in head seas with a constant velocity, U , then the waves will appear to meet the ship at a faster rate than the actual frequency of the waves. This new, or observed, frequency is termed the *encounter frequency*, w_e . If the waves are incident on the ship at some angle, m , then the component of the speed of the ship in the direction of wave propagation is $U_a = U \cos m$. The wave crests move at the phase speed, $C_p = w/k$ and the relative speed between the ship and the waves is

$$U_r = U_a + C_p = U\cos m + w/k \tag{47}$$

Thus it appears that the waves have a phase speed U_r such that

$$U_r = \frac{w_e}{k} = U\cos m + \frac{w}{k} \tag{48}$$

Using the dispersion relationship for waves in deep water we can rewrite the equation for encounter frequency as

$$w_e = \frac{Uw^2}{g}\cos m + w \tag{49}$$

for $w > 0$. In practice we usually have the encounter frequency since it is observed and we would like to calculate the actual wave frequency, so taking equation (49) we can solve for frequency, w.

$$w = \frac{g}{2U\cos m}\left\{-1 \pm \sqrt{1 + 4w_e\frac{U\cos m}{g}}\right\} \tag{50}$$

Looking at the equation 17 we see that there are several possible solutions for w. These are dependent on the incident angle m. For $4w_e\frac{U\cos m}{g} > -1$ (real values of w) we can look at different incident angles:

(1) HEAD SEAS: incident angle between $-p/2 < m < p/2$ and the cosine of the angle is positive ($\cos m > 0$). The encounter frequency is always positive for head seas.

(2) FOLLOWING SEAS: incident angle between $p/2 < m < 3p/2$ and $\cos m < 0$. Here there are three possible scenarios:

$$|U \cos m| > C_p, \qquad w_e < 0, \qquad \text{ship overtakes waves}$$

$$|U \cos m| = C_p, \qquad w_e = 0, \qquad \text{ship “surfs” waves}$$

$$|U \cos m| < C_p, \qquad w_e > 0, \qquad \text{waves overtake ship}$$

The final case, where the waves move faster than the ship, can tend to cause problems in controlling a vessel, especially when the seas are not directly from behind the ship. This effect is greatest in roll and yaw. When a ship is moving with the waves but overtaking them it can also appear as if the seas are approaching from the bow, thus the actual incident angle is ambiguous and could be m or $m+p$.

The effect of encounter frequency also changes the observed spectrum of the seas. Energy must be preserved under the spectrum thus we can look at the following

$$S(w_e)|dw_e| = S(w)|dw| \tag{51}$$

$$S(w_e) = \frac{S(w)}{\left|\frac{dw_e}{dw}\right|} \tag{52}$$

We can find the derivative of encounter frequency by the actual frequency by using equation (49).

$$\frac{dw_e}{dw} = 1 + \frac{2w}{g} U \cos m \tag{53}$$

So the spectrum of the encounter frequency becomes

$$S(w_e) = \frac{S(w)}{\left|1 + \frac{2w}{g} U \cos m\right|} \tag{54}$$

This spectrum has an integrable singularity at

$$w = \frac{-g}{2U \cos m}. \tag{55}$$

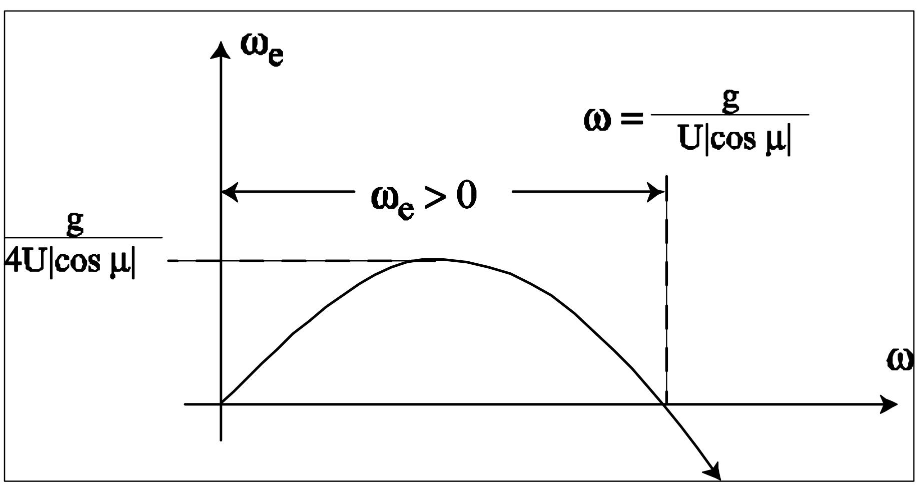
At this value of w the encounter frequency is

$$w_e = \frac{g}{4U\,|\cos m\,|}. \tag{56}$$

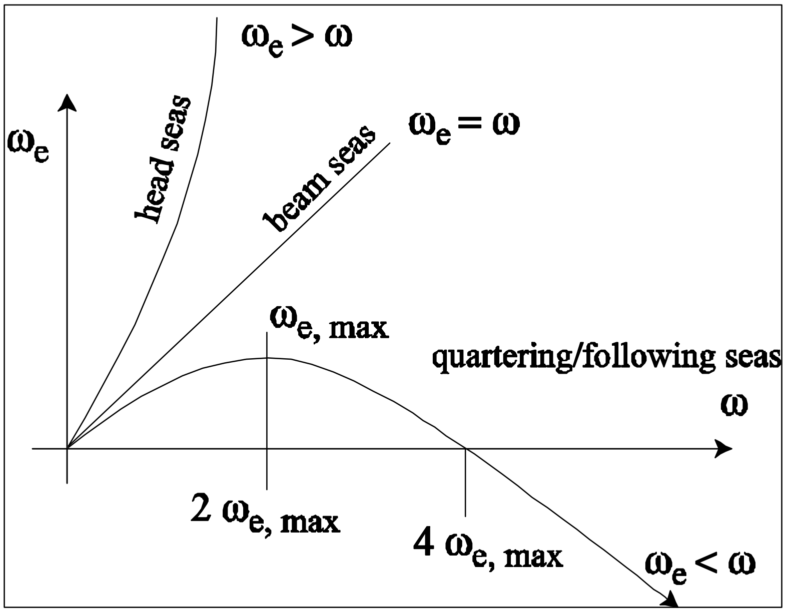
Given U, m, we can look the two cases with encounter frequency

$$w_e = w + \frac{w\,|w\,|}{g} U \cos m \tag{57}$$

1. if m is in $(-p/2, p/2)$ then $w_e > 0$ for $w > 0$ *and* $w_e > w$ for $w > 0$.
2. if m is in $(p/2, 3p/2)$ then $w_e < 0$ for $w < 0$ and for $w > \frac{g}{U|\cos m|}$.



6. Encounter frequency versus actual frequency for following seas ($\cos m < 0$) and $w > 0$.



7. Encounter frequency versus actual frequency for the three conditions: head, beam and following seas.

7. Useful References

Read section four of the supplemental notes: Triantafyllou and Chryssostomidis, (1980) "Environment Description, Force Prediction and Statistics for Design Applications in Ocean Engineering"