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Effect of intrinsic and sampling variability on wave parameters and wave statistics

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Abstract Significant wave height and zero-crossing wave period are used for validation of wave models, wave climate studies, and calculations of extremes for weather forecasting purposes. They represent also important parameters for design and operations of ships and offshore structures. They can be evaluated using 20 or 30-min long wave recordings directly or using the wave spectrum. Spectral methods often reduce the 20 min to about 17 min. Due to the limited duration of wave records, estimates of significant wave height and zero-crossing wave period are affected by sampling variability, the statistical uncertainty due to limited number of observations. The study provides estimates of sampling variability associated with significant wave height and zero-crossing wave period based on measurements from the Ekofisk field in central North Sea. Further, it demonstrates the impact of intrinsic and sampling variability has on short-term and long-term description of ocean waves as well as validation of wave spectral models.

Keywords Sampling variability · Intrinsic variability · Significant wave height · Zero-crossing wave period · Field data

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1 Introduction

Description of ocean waves approximating the nature in the most accurate way has always been of concern of the oceanographic community. The shipping and offshore industry, on the other hand, needs accurate wave data and models for design and operational purposes. To describe waves, not only wave data and models are required but also associated uncertainties are needed to be specified. Uncertainties of wave data and models have been a subject of research in many years but got increasing attention in the last decades. There is still limited literature addressing systematic quantification of uncertainties related to wave description.

Generally, uncertainty related to wave description may be divided into two groups: aleatory (inherent) uncertainty and epistemic (knowledge based) uncertainty. Aleatory uncertainty represents a natural randomness of a quantity, also known as intrinsic or inherent uncertainty, e.g., the variability in wave height over time. Aleatory uncertainty cannot be reduced or eliminated. Epistemic uncertainty represents errors which can be reduced by collecting more information about a considered quantity and improving the methods of measuring it. Following Bitner-Gregersen and Hagen (1990), this uncertainty may be classified into data uncertainty, statistical uncertainty (sampling variability), model uncertainty, and climatic uncertainty (regarded often as model uncertainty).

The present study is limited to discussion of intrinsic and sampling variability associated with significant wave height and zero-crossing wave period, one of the most important parameters for loads and response calculations of marine structures. Significant wave height and zero-crossing wave period are used for validation of wave models, wave climate studies, and calculations of extremes for weather forecasting purposes.

Significant wave height is defined as the average of the one-third largest waves in a wave time series ($H_{1/3}$) and is often evaluated using 20 or 30-min long recordings. It can also be calculated from a wave spectrum (H_{m0}). The average zero-crossing wave period T_z is defined as the average of zero-crossing wave periods in a 20–30-min wave records and similarly to significant wave height can also be estimated from a wave spectrum (T_{m02}).

Spectral methods often reduce the 20 min to about 17 min. This is because a sampling frequency of 2 Hz is widely used. Slow CPUs in the 1970s and 1980s required the spectral analysis to be done with Fast Fourier Transform. A record including twice 1,024 (2,048) samples at 2 Hz is 17.07 min long. It seems that the length of the record has been chosen differently in different parts of the world. In Norway, as the oil and gas industry developed in the North Sea, a record length of 20 min was determined by the needs of frequent updates by marine operators in charge of sensitive operations. Buoys that were deployed along the coasts of US and Canada and around Great Britain seem to be trigged to measure once or twice an hour and have a 30-min record length. Record length may also have been 20 (or 17) min in the early start of recording.

The limited duration of wave time series has allowed adopting an assumption of stationarity on which most of wave models are based today. Estimates of significant wave height and zero-crossing wave period derived from the limited length of wave records are affected by sampling variability, the statistical uncertainty due to limited number of observations. The shorter the time series is, the higher the variability of significant wave height and wave period are.

Sampling variability is an epistemic uncertainty which can be reduced opposite to the aleatory (intrinsic) uncertainty of sea surface elevation, which is inherent and cannot be eliminated; it is always present. The aleatory uncertainty of surface elevation and the associated wave parameters is due to randomness of nature; thus, for given meteorological conditions, sea surface will oscillate and these oscillations can be described by a distribution function (or the standard deviation). For stationary meteorological conditions, due to randomness of sea surface, wave parameters derived from a wave record will depend on which part of a wave record is used in an analysis as well as on the length of a wave record. An error introduced by the limited length of a wave record is an epistemic uncertainty and can be reduced by increasing duration of wave measurements/numerical simulations. Ideally, a wave record should be infinite to eliminate sampling variability. Numerical simulations of water surface represent a good support to field data as they allow reducing sampling variability by increasing duration of simulations when wave input is kept constant and intrinsic variability accounted for. This is more difficult in nature where stationarity of sea states is an issue.

The importance of sampling variability on calculations of extreme values was pointed out by Longuet-Higgins (1952) who showed theoretically, under the assumption that the sea surface is Gaussian and narrow banded in frequency, that extreme wave heights are affected by the duration of a wave record. This uncertainty was also addressed later by Lipa et al. (1981), Donelan and Pierson (1983), Young (1986), Monaldo (1988), Bitner-Gregersen and Hagen (1990), Tucker (1992), and Forristall et al. (1996), among others. Lipa et al. (1981), considering over 5,000 HF radar data samples, reported the standard deviation of significant wave height and the dominant wave period to be 5 % and 0.5 s, respectively. Donelan and Pierson (1983), investigating laboratory and field data, obtained 8 % variability for significant wave height. For very narrow spectra, the sampling variability was even higher. Monaldo (1988) showed for two buoys situated 100 m apart a 7 % RMS (root mean squared) error for significant wave height.

Theoretical formulas for sampling variability of H_{m0} and T_{m02} have been derived by Bitner-Gregersen and Hagen (1990) and applied to the Pierson-Moskowitz and JONSWAP spectrum; the obtained values confirm the earlier findings based on field and laboratory data. Closed form expressions for COV (coefficient of variation) of significant wave height for the Pierson-Moskowitz and JONSWAP spectrum as functions of the peak frequency and the wave record length were proposed later by Tucker (1992). For the Pierson-Moskowitz spectrum with the peak frequency of 0.1 Hz and a length of 1,024 s, the COV calculated according to Tucker (1992) (taken from Forristall et al. 1996) is 5 %, approximately the same as the one obtained by Bitner-Gregersen and Hagen (1990).

Forristall et al. (1996) proposed a distribution of significant wave height (H_s) where sampling variability of H_s could be included. The derivation presented by the authors was based on the assumption that sea surface is linear (Gaussian). Using the Tucker (1992) expression for the COV(H_s) for the JONSWAP spectrum, good agreement between the field data in 10-min samples from Auk Platform and the theoretical distribution was obtained.

Estimation of sampling variability from wave measurements is more challenging than from model tests (or numerical simulations) because of requirement of stationarity of seas states; therefore, theoretical formulas could be of support in some cases. We validate herein the formulas of Bitner-Gregersen and Hagen (1990) using measurements collected at the Ekofisk field and discuss the impact of the measuring length on validation of wave models, climate studies, and simulations of wave surface elevation, supporting it by examples. Attention is given to extreme values being a necessary input for marine structure design and operations.

The paper is organized as follows. Section 2 is dedicated to the theoretical description of sampling variability associated with significant wave height and zero-crossing wave period estimates, as formulated in Bitner-Gregersen and Hagen (1990); some comparison to the Tucker (1992) expressions is also given. Sections 3, 4, and 5 are addressing the analysis of intrinsic and sampling variability in wave measurements from a waverider in the central North Sea. Section 6 shows examples of impact of sampling variability on wave description. The paper closes with conclusions, recommendations, and references.

2 Theoretical sampling variability of wave records

Statistical uncertainty also called as sampling variability is due to a limited number of observations of a quantity. Below sampling variability associated with significant wave height and zero-crossing wave period presented by Bitner-Gregersen and Hagen (1990) is discussed.

Sea surface oscillations are commonly recorded by 20 or 30 min, and a wave record usually includes 100–300 waves. They can also be simulated by numerical wave models or generated in a model basin. Both numerical simulations and model tests generate commonly a single wave record which corresponds to a 17–30-min wave record in full scale. Sea states characteristics and distributions derived from the 17–30-min wave records will be affected by sampling variability due to limit number of observations.

The theoretical formulas for sampling variability of H_{m0} and T_z were derived by Bitner-Gregersen and Hagen (1990), assuming that sea surface is a Gaussian narrow-banded process.

The n-th spectral moment M_n of a time series X_t is defined as

$$M_n = \int_0^\infty \omega^n S(\omega) d\omega \tag{1}$$

where ω is the angular frequency.

For a time series of N observations $\{X_t\}$, $t=\Delta t, \ldots, N+\Delta t$, an estimator $\widehat{M}_n$ of the n-th moment is

$$\widehat{M}_n = 2\int_0^\infty \theta^n I_N^*(\theta) d\theta \tag{2}$$

where the periodogram $I_N^*(\theta)$

$$I_N^* = *\frac{1}{2\pi N}\left|\sum_{t=1}^{N} X_i e^{-i\theta \cdot t}\right|^2 \tag{3}$$

The periodogram (3) can be easily calculated from the observations by the fast Fourier transform (FFT). The integral (2) can be replaced by a finite sum. For a Gaussian process, the asymptotic results for the mean value, variance, and coefficient of variation of the spectral moments may be established (Priestley 1981; Krogstad 1982).

$$\lim_{N\to\infty} E\left[\widehat{M}_n\right] = M_n \tag{4}$$

$$\lim_{N\to\infty} N \mathrm{var} E\left[\widehat{M}_n\right] = 2\pi f_s \int_0^{\pi f_s} \omega^{2n} S^2(\omega) d\omega \tag{5}$$

$$\lim_{N\to\infty} N \mathrm{cov}\left[\widehat{M}_n, \widehat{M}_\mathrm{m}\right] = 2\pi f_s \int_0^{\pi f_s} \omega^{n+m} S^2(\omega) d\omega \tag{6}$$

where $f_s=1/\Delta t$ is the sampling frequency. $\widehat{M}_n$ is a consistent estimator by expressions (4) and (5).

Assuming that a given sea state (H_s, T_z) represents a narrow-banded Gaussian process (Rayleigh-distributed wave heights), the estimators of significant wave height and zero-crossing wave period may be derived from the spectral moments (see Longuet-Higgins 1952; Cartwright and Longuet-Higgins 1956; also Thorton and Guza 1983).

$$H_s = H_{m0} = 4\sqrt{M_0} \tag{7}$$

$$T_z = T_{m02} = 2\pi (M_0/M_2)^{1/2} \tag{8}$$

Note that significant wave height and zero-crossing periods calculated from spectral moments (Eqs. 7 and 8) are approximations of the time series estimators as they are valid only for the Gaussian and narrow-banded sea surface. The discrepancy between the significant wave height and zero-crossing wave period estimators derived directly from a wave record, and the ones calculated from a wave spectrum is usually rather small. However, when rogue waves are present, the bias may not be negligible, see Bitner-Gregersen and Magnusson (2004). Both estimators will be affected by sampling variability due to limited duration of wave records.

As shown by Bitner-Gregersen and Hagen (1990), the second-moment properties of the estimators $\widehat{H}_{m0}$ and $\widehat{T}_{m02}$ become

$$\mathrm{var}\left(\widehat{H}_{m0}\right) = \frac{4}{\widehat{M}_0}\mathrm{var}\left(\widehat{M}_0\right) \tag{9}$$

$$\mathrm{var}\left(\widehat{T}_{m02}\right) = \pi^2\left(\frac{\mathrm{var}\left(\widehat{M}_0\right)}{\widehat{M}_0\widehat{M}_2} - 2\frac{\mathrm{cov}\left(\widehat{M}_0, \widehat{M}_2\right.}{\widehat{M}_2^2} + \frac{\widehat{M}_0\mathrm{var}\left(\widehat{M}_2\right)}{\widehat{M}_2^3}\right) \tag{10}$$

$$\mathrm{COV}\left(\widehat{H}_{m0}, \widehat{T}_{m02}\right) = 2\pi\left(\frac{\mathrm{var}\left(\widehat{M}_0\right)}{\widehat{M}_0\sqrt{\widehat{M}_2}} - \frac{\mathrm{cov}\left(\widehat{M}_0, \widehat{M}_2\right.}{\widehat{M}_2^{3}/2}\right) \tag{11}$$

where $\mathrm{var} = \left(\widehat{M}_n\right)$, $\mathrm{var} = \left(\widehat{M}_2\right)$, and $\mathrm{COV} = \left(\widehat{M}_0, \widehat{M}_2\right)$ for a given wave spectrum derived from Eqs. (5) and (6).

The standard deviations derived from Eqs. (9) and (10) and expressing sampling variability of significant wave height and zero-crossing wave period were calculated by Bitner-Gregersen and Hagen (1990) for a North Sea scatter diagram and a JONSWAP spectrum with the gamma parameter γ=3.3. The obtained sampling variability standard deviations in (%) are shown in Tables 1 and 2 in Appendix A. As expected, sampling variability standard deviation for T_{m02} is lower than for H_{m0}.

Bitner-Gregersen and Hagen (1990) also present results for the Pierson-Moskowitz spectrum but not in a form of Tables 1 and 2. The authors have shown that sampling variability of significant wave height and zero-crossing wave period is lower for the Pierson-Moskowitz spectrum than for the JONSWAP spectrum. The dependence of sampling variability on the shape of wave spectrum was already pointed out by Tucker (1992) who proposed a formula for COV(H_s) for the Pierson-Moskowitz and JONSWAP spectrum being a function of peak frequency f_m and the length of the wave record L (in seconds). For the Pierson-Moskowitz spectrum

$$\mathrm{COV}(H_s) \approx 0.48 \Big/ \sqrt{f_m L} \tag{12}$$

and for the JONSWAP spectrum with the standard shape parameters

$$\mathrm{COV}(H_s) \approx 0.61 \Big/ \sqrt{f_m L} \tag{13}$$

For the JONSWAP spectrum with the gamma parameter equal 3.3 and a record length 1,024 s COV(H_s) according to the Eq. (13), sampling variability of H_s is higher than the one derived from the formula proposed by Bitner-Gregersen and Hagen (1990). The difference is larger for smaller f_m. e.g., for f_m=0.10, COV=0.060 according to Tucker (1992) and 0.054 following Bitner-Gregersen and Hagen (1990) while for f_m= 0.07, Eq. (13) gives COV=0.072 and Eq. (9) gives COV=0.062.

Equation (13) does not include dependency on H_s. Thus, for a given f_m and L, all classes of H_s will have the same COV. We will show that this is not confirmed by measurements; increase of COV is observed with increase of H_s as in a wave record with a given duration the number of waves is normally decreasing with increase of H_s. This finding is supporting the formula (9).

3 Wave measurements

MET Norway is forecasting marine weather for different oil and gas fields in the North, Norwegian, and Barents Sea. Special attention to high waves is given at Ekofisk (56.5° N, 3.2° E), a platform complex operated by ConocoPhillips. The platforms are more or less subject to subsidence of the sea floor (Hjorteland et al. 1999). The field has an increased interest in precise wave height forecasts to be able to start risk reducing actions at the right time. Several wave sensors have been deployed through the last 20–30 years to assure good forecasts and validation data; a waverider buoy (Datawell) has been measuring since the start (first records started to be stored in 1980). In addition, several downlooking radars measuring the height to water at 2 Hz sampling frequency have been deployed through the years at different locations. Experience has shown that this has been with more or less success with regard to avoiding lee effects. A WAMOS directional wave sensor (based on a marine radar system) was deployed in early 1990s. A LASAR, consisting of an array of four Optech lasers was mounted on one bridge to measure sea surface variations at 5 Hz sampling frequency. The configuration is used to evaluate directionality in the waves as well.

Table 1 The sampling variability standard deviation $\sigma_{H_{m0}}$ (in %) of H_{m0} for the JONSWAP spectrum (Bitner-Gregersen and Hagen 1990)

The sampling variability is recommended to be modeled as a normally distributed variable.

H_{m0}	T_{m02} (sec)												
(m)	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	Average
0–1	3.3	3.8	4.1	4.4	4.8	5.1	5.5	5.6	5.9	6.2	6.5	6.7	5.2
1–2	4.5	3.7	4.1	4.4	4.8	5.1	5.5	5.6	5.9	6.2	6.5	6.7	5.3
2–3		5.1	4.5	4.4	4.8	5.1	5.5	5.6	5.9	6.2	6.5	6.7	5.5
3–4			5.3	4.7	4.8	5.1	5.5	5.6	5.9	6.2	6.5	6.7	5.6
4–5			5.5	5.7	5.0	5.1	5.5	5.6	5.9	6.2	6.4		5.7
5–6				6.1	5.6	5.2	5.5	5.6	5.9	6.2	6.4		5.8
6–7				6.3	6.4	5.6	5.5	5.6	5.9	6.2	6.4		6.0
7–8					6.7	6.3	5.6	5.6	5.9	6.2	6.4		6.1
8–9						6.8	6.0	5.7	5.9	6.2	6.4		6.2
9–10							6.8	6.1	5.9	6.2	6.4		6.3
10–11							7.0	6.5	6.0	6.2	6.4		6.4
11–12								7.0	6.3	6.2	6.4		6.5
12–13								7.4	6.8	6.3	6.4		6.7
13–14								7.8	7.2	6.6	6.4		7.0
14–15								7.9	7.5	6.9	6.5		7.2
Average	3.9	4.2	4.7	5.1	5.4	5.5	5.8	6.2	6.2	6.2	6.4	6.7	

Table 2 The sampling variability standard deviation $\sigma_{T_{m0}}$ (in %) of T_{m02} for the JONSWAP spectrum (Bitner-Gregersen and Hagen 1990)

The sampling variability is recommended to be modeled as a normally distributed variable.

H_{m0}	T_{m02} (sec)												
(m)	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	Average
0–1	1.5	1.7	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.9	3.0	3.1	2.4
1–2	1.5	1.7	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.9	3.0	3.1	2.4
2–3		1.7	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.9	3.0	3.1	2.5
3–4			1.9	2.1	2.2	2.4	2.5	2.6	2.7	2.9	3.0	3.1	2.5
4–5			1.9	2.1	2.2	2.4	2.5	2.6	2.7	2.9	3.0		2.5
5–6				2.1	2.3	2.4	2.5	2.6	2.7	2.9	3.0		2.6
6–7				2.2	2.3	2.4	2.5	2.6	2.7	2.9	3.0		2.6
7–8					2.3	2.5	2.5	2.6	2.7	2.9	3.0		2.6
8–9						2.5	2.6	2.6	2.7	2.9	3.0		2.7
9–10							2.6	2.7	2.7	2.9	3.0		2.8
10–11							2.7	2.8	2.8	2.9	3.0		2.8
11–12								2.8	2.8	2.9	3.0		2.9
12–13								2.8	2.9	2.9	3.0		2.9
13–14								2.8	2.9	2.9	3.0		2.9
14–15								2.8	2.9	3.0	3.0		2.9
Average	1.5	1.7	1.9	2.1	2.2	2.4	2.5	2.7	2.8	2.9	3.0	3.1	

Observations are made available in real time to the forecasters. This special surveillance showed that wave parameters from the different sensors had a large variability, and a question is posed on what to validate the forecast against. The different sensors could show different values and different tendencies (one increasing while the others decrease). There are also different ways to evaluate significant wave height. This leads to the question "What is true sea state?", a question posed in Magnusson (2011). Possible lee effects were searched for to explain the differences, but lee effects alone could not explain the differences. Figure 2 (from Magnusson 2011) shows an example of the spread in significant wave height (H_s) during a storm reaching 11.5 m with the sensors at Ekofisk. The spread between the sensors may be due to a certain extent to lee effects on some of the sensors, but the figure demonstrates how variable one time series of H_s through a storm may be. Although the instrumental errors associated with the sensors are different, sampling variability will be approximately the same for all sensors for a given sea state and wave record length (see Eqs. 12 and 13; Tucker 1992) if the shape of a measured spectrum is not changed significantly due to instrumental errors. The spread shown in Fig. 2 needs to be accounted for when validating wave model and forecasting extreme waves, but it should not be mixed with sampling variability (Fig. 1).

An example of a time series in Fig. 3 shows that individual waves can be moderate for quite a while, like 5–10 min, then "explode" in a series of waves that are suddenly twice as high.

A basic assumption adopted in numerical short-term wave models and probabilistic description of waves is stationarity of sea surface (a sea state considered). A stationary process, called either strictly or strongly stationary, is a stochastic process whose joint probability distribution does not change when shifted in time or space. Consequently, parameters such as the mean and variance also do not change over time or position. As a result, the mean and the variance of the process do not follow trends. A weaker form of stationarity commonly employed in signal processing is known as weak-sense stationarity, wide-sense stationarity (WSS) or covariance stationarity. WSS random processes only require that first moment and covariance do not vary with respect to time. Any strictly

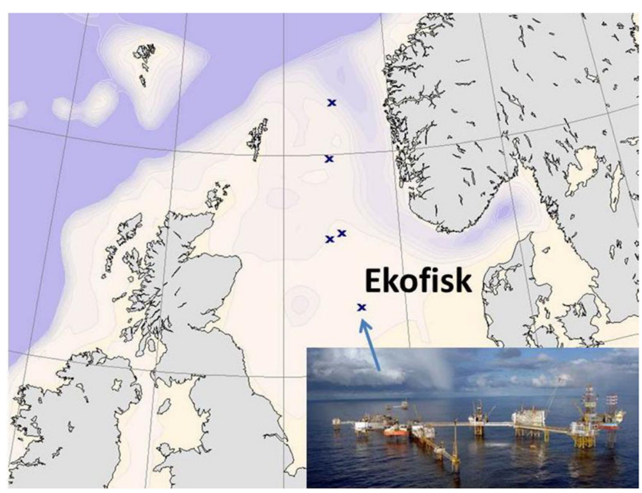

Fig. 1 Location of the Ekofisk platform complex in central North Sea (56.5° N, 3.2° E), operated by ConocoPhillips. Indent is a picture of the complex taken towards north from 2009. New platforms are being built while older ones are decommissioned

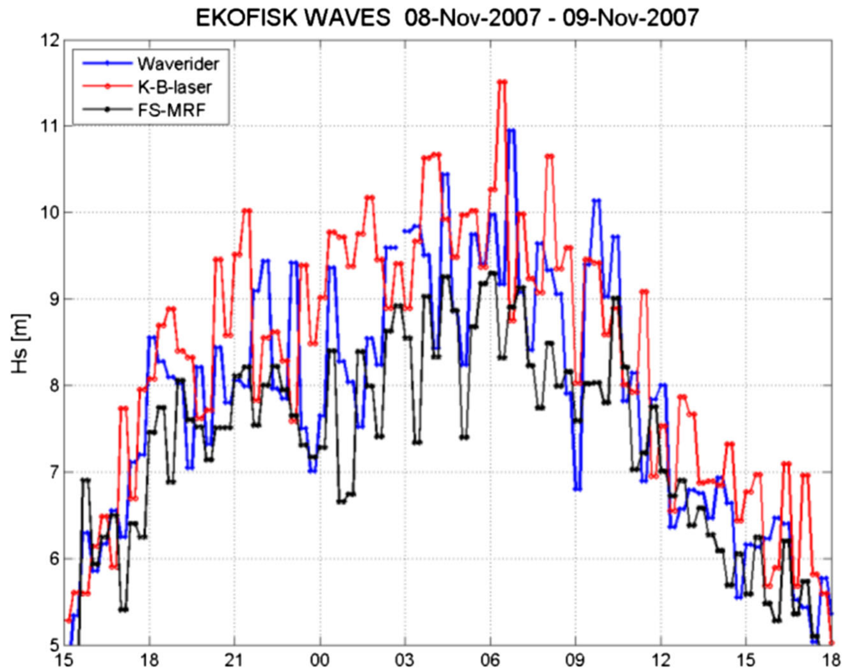

Fig. 2 Significant wave height through a storm (here ranging 27 h, from 5–11.5 m) at Ekofisk (56.5° N, 3.2° E), measured by a waverider (*blue line*), Miros Range Finder (*black*), and an Optech laser (*red*)

stationary process which has a mean and a covariance is also WSS.

During varying met-ocean conditions, even if we want to satisfy the stationarity assumption, we must use records limited in time to calculate wave parameters and derived distributions of sea surface characteristics. Therefore, uncertainty due to sampling variability will be commonly present in a sea state description. Note that to account for non-stationary character of sea surface, a time variable would need to be introduced in description of sea states.

In the next section, we present an approach in wave data analysis to demonstrate and quantify the sampling variability of H_s and T_z measurements within an hour.

4 Quantifying variability in wave height and period

The waverider measurements of the wave profile are sampled at 2 Hz and stored in 20-min files. The files have 2,400 samples each, but that is not always the case. We have chosen a period with good data return of 1 year (January–December 2007) to perform the analysis presented herein.

To analyze the effect of sampling variability within a record assumed to have stationary conditions with respect to changing weather conditions, we have gathered samples in groups of 60 min, and within each of these 60 min, consecutive sequences of 17.5 min each are picked, at 1-min interval. The method is demonstrated in Fig. 4. For clarity of the figure, the 2-min interval is presented.

The distance between the first red bar (at time=0) to the first green bar is the length of the first 17.5-min record possible to extract in this 60-min record. It is 1,050 s long or 2,100 samples. The significant wave height in this first 17.5-min record is 9.39 m. Note that we here use the formula H_s= 4*std($z(t)$), $z(t)$ being the detrended measurements of surface elevation around the mean level. The second H_s value in Fig. 4, from the recordings shifted by 2 min (starting from second red bar and ending at second green bar) is 9.58 m. The 21 possible H_s values taken this way within this hour are noted in the figure at top. The values vary between 7.47 and 9.58 m. The variability in these measurements of H_s from short records is

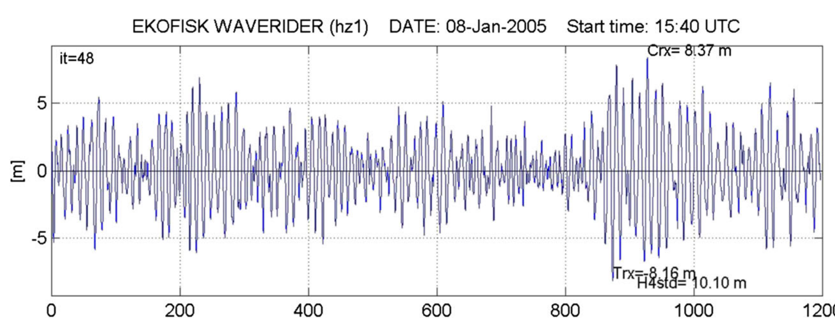

Fig. 3 Example of a 20-min time series measured with the Ekofisk waverider. H_s is 10.1 m

evaluated as the standard deviation of these values, divided by the H_s over 60 min, times 100, and this gives 6.93 % (top line indent in figure). The variability in T_z measurements is evaluated identically, T_z in each record being the record length divided by number of waves in each record. By decreasing the step from 2 to 1 min between each start of record, the number of records is increased from 21–42. The observed spread of the H_s and T_z estimators is a result of intrinsic variability, but the limited length of the wave records (limited number of observations) influences also H_s and T_z estimators significantly, as demonstrated below.

Figures 5 and 6 show waverider significant wave heights and zero-crossing wave periods calculated from 20–60-min interval through the Andrea storm on 8–9 November 2007. The 42 series of 17.5-min length defined at 1-min intervals within each hour (according to the approach described in Fig. 4) are shown with black dots, dated at midpoint times of each series. Values from NORA10 hindcast are also shown (3-hourly data). We see that the 10-km resolution WAM model used to develop the NORA10 database validates better with 60-min records than with 17.5 or 20-min records.

The figures clearly demonstrate the effect of sampling variability on the estimates of these two parameters. As expected, the shorter a wave record is, the larger the variability of H_s and T_z is observed. And we can note that the variability is higher the higher the hourly H_s is. Same tendency is seen in T_z. Thus, the variability is changing during the storm history.

5 Comparison of sampling variability derived from field data with the oretical values

The 1-hour approach described in Section 4 has been adopted to derive sampling variability of significant wave height and zero-crossing wave period from the Ekofisk waverider in 2007 (1 January–31 December 2007). Figure 7 shows standard deviation of the 17.5-min estimates of H_s ($H_{1/3}$) and T_z relative to the 60-min estimates in % as a function of the 60-min values. The average values of sampling variability of H_s and T_z as well as sampling variability of steep sea states (the left side of the H_{m0} and T_{m02} scatter diagram) from Tables 1 and 2 are also plotted in Fig. 7.

According to Bitner-Gregersen and Hagen (1990) (see Appendix A), for H_s equal 2–6 m, the sampling variability is in the range of 4–6 % while for H_s values, 6–8 m in the range of 5.5–6.8 %. Thus, sampling variability increases with increasing H_s.

The average observed values seen in Fig. 7 (left) indicate lower values than the theoretical ones for the lowest range of H_s. The spread is large, but we may say that the theoretical values otherwise are in the middle of the observations. An explanation for the discrepancy for smaller waves is that we can assume that in this range, we have much more waves for the same record length, therefore lower sampling variability.

The theoretical evaluation of sampling variability for T_{m0} compares well with observed ones (Fig. 7, right) for wave periods in the range up to about 8 s, while above there is a tendency for too low values.

It should be noted that the formula of Bitner-Gregersen and Hagen (1990) are based on wave spectral moments. The estimators of significant wave height and period derived from the wave spectral moment are equivalent to the ones obtained from the time series only under the assumption that sea surface is Gaussian (linear) and narrow banded (see Longuet-Higgins 1952; Cartwright and Longuet-Higgins 1956; also Thorton and Guza 1983), as discussed in Section 2. That is obviously questionable in nature and in particular, in the observations

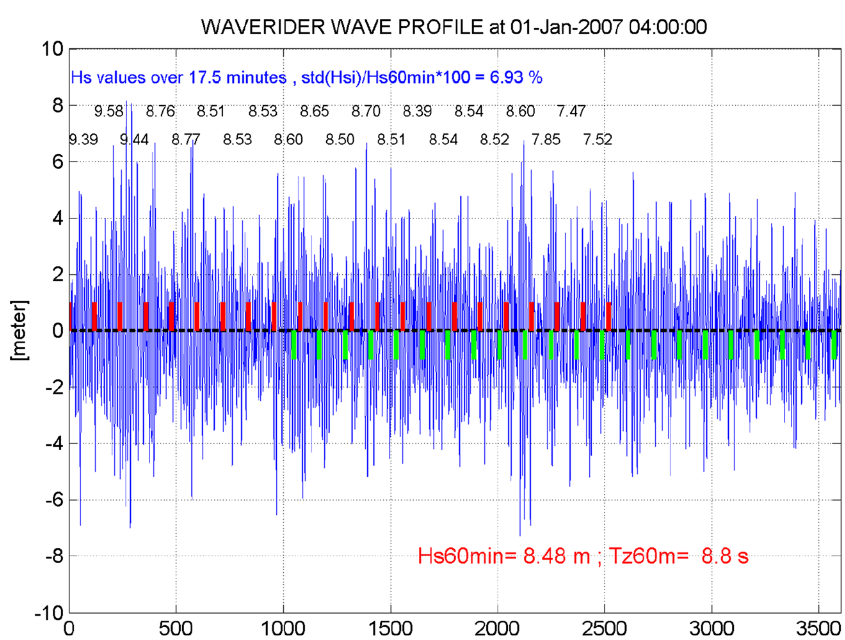

Fig. 4 Illustration of the approach used in calculating sampling variability. *Red bars* in figure indicate start points of each 17.5-min records, and *green bars* (*downwards*) indicate the end of each of them. H_s values from these records are found indent in figure, at *top* of time series. They range from 7.47–9.58 m. The hourly value H_s (60 min) is 8.48 m. The average of the 21 short records is 8.56 m

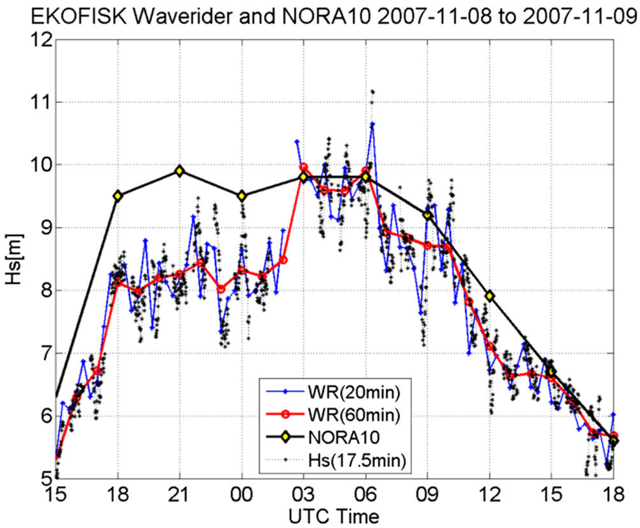

Fig. 5 Time series through one day (8–9 November 2007) of different values of H_s. *Thin blue line* H_s(20 min) for consecutive records of 20-min length, *red line* H_s(60 min), from consecutive records of 60-min length, dated with midpoint time, and *thick black line with yellow diamond* is H_s from the wave model (NORA10 database). The *black dots* are all 17.5-min values within each hour, as described in previous figure

from Ekofisk. Further, the theoretical values of the sampling variability were derived for the JONSWAP spectrum with the gamma parameter γ=3.3 while the measurements include wave spectra with the varying gamma parameter.

The spread in results presented in Fig. 7 indicates that the assumption of Gaussian sea surface and narrow-banded wave spectrum and adoption of the JONSWAP spectrum affect the estimates of H_s and T_z sampling variability standard deviations;

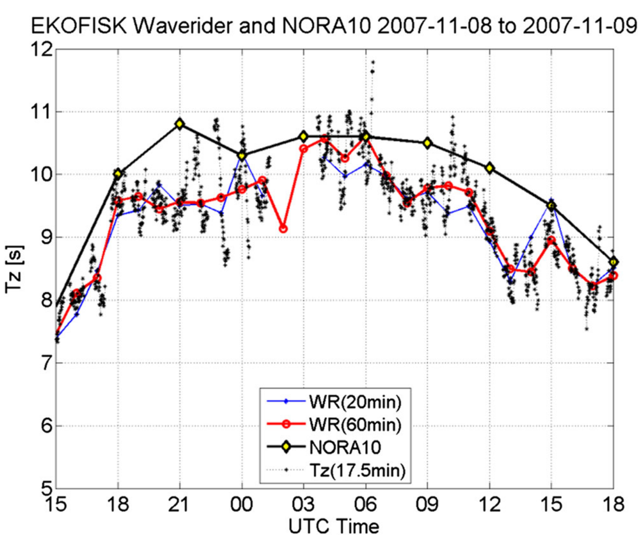

Fig. 6 Time series through one day (8–9 November 2007) of different values of average wave period T_z (record length/number of waves). *Thick cyan line with yellow diamond* is T_z from the wave model (from the NORA10 database) *thin blue line* is T_z (20 min) for consecutive records of 20-min length, *red line* T_z (60 min), from consecutive records of 60-min length evaluated using the full 60 min, dated with midpoint time. The *black dots* are all 17.5-min values within each hour, as described in previous figure

but both estimates, theoretical and observed, show the same trend of the variability increase with increasing of H_s and T_z.

A larger wave database may reduce the spread, but effect of bimodality and non-linearity will influence the spread. Both the measured and theoretical values of sampling variability will depend on the shape of a wave spectrum as shown by Bitner-Gregersen and Hagen (1990), Tucker (1992), and Forristall et al. (1996). So far, when deriving theoretical values of sampling variability, the attention has been given to a single wave spectrum, Pierson-Moskowitz and JONSWAP. The double-peak spectra (accounting for combined seas) have not been systematically investigated. Sea states with significant wave height in the range 2–8 m will typically be characterized by a two-peak spectrum in the northern North Sea (see Torsethaugen 1996). At Ekofisk, this is expected be true for less high waves due to limitations in fetch.

The analyzed 1-year data include variety of sea states: low, moderated, and high sea states. The degree of non-linearity of sea surface (deviation from the Gaussian distribution) is also widely varying. It should be mentioned that within the time period considered, sea states including rogue waves are also present, e.g., the Andrea storm (8–9 November 2007), see Magnusson and Donelan (2013). Herein, we have not grouped the analyzed sea states according to a degree of non-linearity, being associated with the wave spectrum shape. Therefore, at this stage, it is difficult to conclude on sensitivity of the H_s and T_z estimates of sampling variability to the deviation of sea surface from Gaussianity. Very weak non-linearities of sea surface are not expected to affect a measured sampling variability significantly. When the energy spectrum is concentrated on a narrow range of frequencies and directions, modulational instability is responsible for generation of rogue waves in deep and intermediate water depth. As the wave field evolves, a large fraction of the spectral energy is moved towards lower wavenumbers (or wave frequencies), generating the downshift of the spectral peak; the wave field is becoming less steep. Furthermore, a fraction of the energy is also transferred across directions (see e.g., Toffoli and Bitner-Gregersen 2011 and references therein for details). Thus, changes of the wave spectrum shape can be expected during and after occurrence of an extreme wave event, and they will impact sampling variability (see Eqs. 12 and 13); an open question is to what degree.

The model tests carried out in the MARIN basin (Forristall 2009) with a JONSWAP spectrum with the gamma parameter 3.3 showed that the empirical sampling variability does not deviate significantly from the values presented by Bitner-Gregersen and Hagen (1990). It should be noted that Forristall (2009) derived the estimators of sampling variability in a different way than proposed herein; the model tests have been repeated to increase the length of the wave record keeping wave input unchanged; this is not possible in nature.

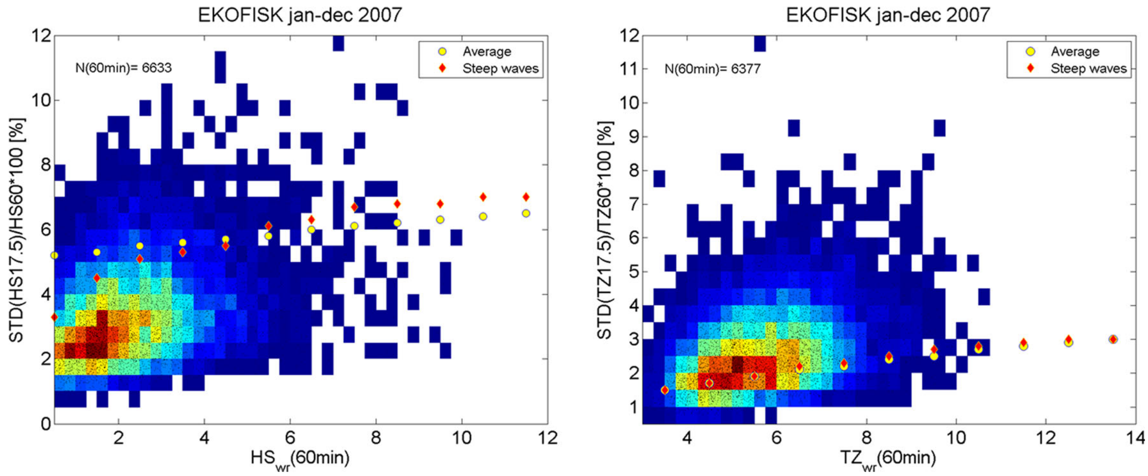

Fig. 7 Relative (with respect to the H_s(60 min)) standard deviation of the 17.5-min estimates of H_s (*left*) and T_z (*right*) in % versus the hourly estimates of H_s and T_z. Samples cover a period of 1 year

6 Implications of sampling variability on description of waves

Sampling variability, even if it is not very large, will have impact on short-term and long-term description of ocean waves as well as validation of wave spectral models and different data sources. We discuss it below supporting the discussion by selected examples.

Figure 2 shows observed significant wave heights from three different instruments installed in the Ekofisk field. These

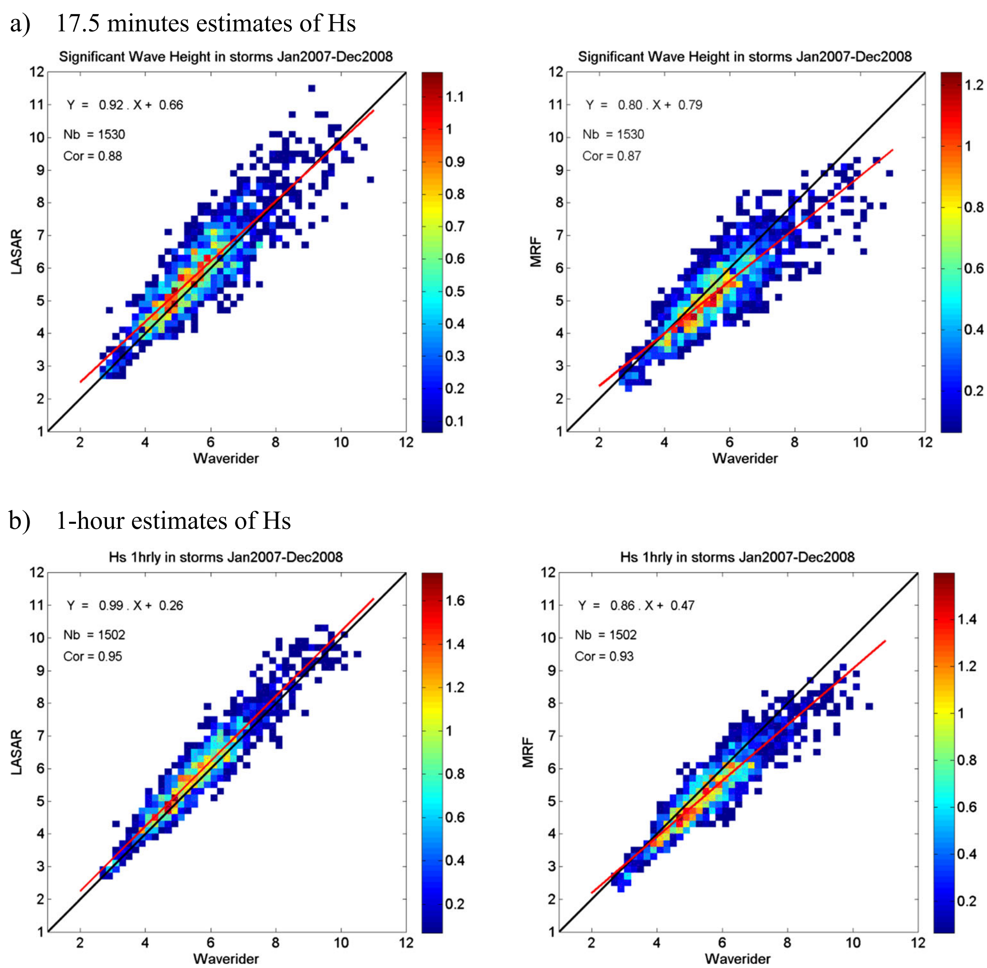

Fig. 8 Comparison of significant wave height derived from different wave sensors and time averaging of 17.5 min (*upper panel*) and 1 h (*lower panel*)

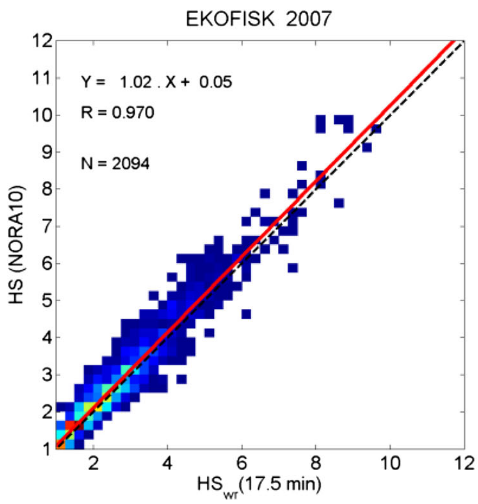

Fig. 9 Comparison of significant wave height derived from the 17.5-min (*left panel*) and 1-h (*right panel*) waverider records at the 3-hourly time interval with NORA10 (1 January–31 December 2007)

data are very important during extreme storm forecasting that is performed by MET Norway for ConocoPhillips. First, they are important during the storm development (having several sensors as backup for one another), and they are used to validate forecasts and actions to be performed for risk reduction at the field. Secondly, they are important for post-storm validation and validation of numerical wave models used for hindcast. Several hindcast databases assimilate satellite data, which are calibrated against in-situ data. Quality of the in-situ data is therefore very important; if one sensor is not performing well, another sensor can replace it. Knowledge about data uncertainty is also important when sampling variability is calculated from the measurements; we discuss it shortly below. Note that assimilation of satellite wave data is not performed in the WAM model used to develop the NORA10 database, where much effort instead was used to model the atmospheric forcing (Reistad et al. 2011).

Figure 8 shows comparison of significant wave height derived from different wave sensors (Optech laser and MRF) against the waverider buoy at Ekofisk based on 17.5 min and 1-h wave records. As expected, the longer a wave record is, the less spread there is between measurements from different sensors. Consistency of an analysis comparing predictions of different sensors is therefore of importance.

Figure 9 shows comparison of the 17.5-min and 60-min H_s derived from the waverider measurements at the 3-hourly time interval with NORA10. Increasing duration of wave records from 17.5 min to 60 min does not change much correlation (R) between the measured and NORA10 data; R increases from 0.970–0.974, but spread around 1:1 axis is slightly less for the 60-min H_s. Similar comparison for zero-crossing wave periods is shown in Fig. 10. Again, only slight increase of correlation between the measured and NORA10 data is seen for the 60 min T_z compared to 17.5-min T_z, from 0.929–0.932. Spread around 1:1 axis is also slightly reduced for the 60 min T_z. Thus, the 10-km resolution WAM model validates better with 60-min records than with 17.5-min records, but the difference is not large.

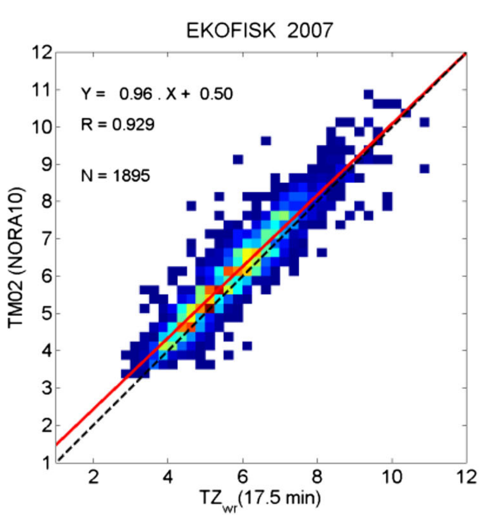

Fig. 10 Comparison of zero-crossing wave period derived from the 17.5-min (*left panel*) and 1-h (*right panel*) waverider records at the 3-hourly time interval with NORA10 (1 January–31 December 2007)

The effect of sampling variability on hindcast wave heights was demonstrated already by Forristall et al. (1996) who showed that this variability leads to positive biases in the maximum significant wave height in a storm and in design wave height estimated from extreme value distribution fitted to the data. The authors recommend correcting of the hindcast by sampling variability for design purposes.

Randomness of sea surface (intrinsic variability) is commonly accounted for in numerical wave models describing sea states by introducing random amplitude and phase, and such randomization of a numerical wave model is necessary to reflect properly natural variability of sea surface. To provide a satisfactory approximation (to reduce the effect of sampling variability) of the distribution of sea surface elevation and associated wave parameters, numerical simulations need to be repeated a sufficient number of times using different random seeds for each run. It should be noticed that higher order statistical moments will be more affected by sampling variability than significant wave height and zero-crossing wave period. As demonstrated by Bitner-Gregersen and Hagen (2003), to obtain a stable estimate of skewness, a 1,024-s wave record needed to be simulated 250 times.

Bitner-Gregersen and Hagen (2003) showed, using the second-order wave model with random amplitude and phase and the Pierson-Moskowitz spectrum, that for a 17-min time series, the coefficient of variation for significant wave height for a sample of realizations is in the range 5–7 %. Olagnon and Magnusson (2004) also showed H_{m0} calculated from simulations varied with up to 9 %, in the mean 6 %.

Due to presence of intrinsic and sampling variability, a high value of the maximum wave crest C_{max} may occur in a sea state in which also the simulated significant wave height ($H_{s,sim}$=4*standard deviation) is high. On the other side, a high crest observed in a 3-h sea state may very well occur in a part of the sea state where the 17-min averaged significant wave height is substantially higher than the 3-h averaged significant wave height $H_{s,nom}$ (input wave height to numerical simulations), making the factor $C_{max}/H_{s,nom}$ low (Bitner-Gregersen and Magnusson 2004). Figure 11 illustrates the difference between C_{max} /$H_{s,nom}$ and $C_{max}/H_{s,sim}$ simulated by the second-order wave model with random amplitude and phase and the Pierson-Moskowitz (PM) spectrum (see Bitner-Gregersen and Magnusson 2004 for details). As can be seen, there is a clear difference between the two factors, and this aspect should be kept in mind when evaluating freak wave criteria like C_{max} /H_s or H_{max}/H_s ratios (H_{max} denotes the maximum wave height within a wave record).

For the same reasons as described above, sampling variability may be responsible for underestimation or overestimation of sea state wave steepness, what may have significant consequences for design and operations of marine structures.

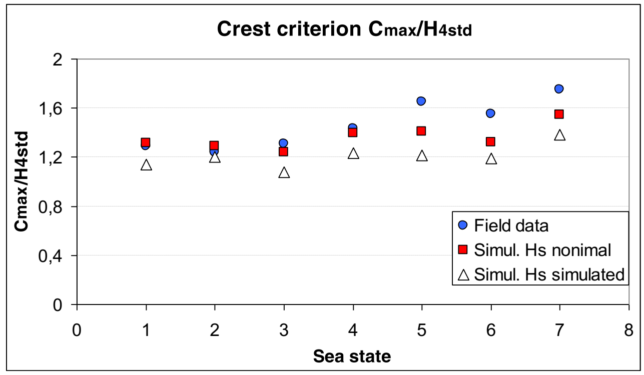

Fig. 11 Wave crest factor for nominal and simulated significant wave height, long-crested sea (2D), PM spectrum (Bitner-Gregersen and Magnusson 2004)

Hagen (2007) studied the effect of sampling variability on the predicted extreme individual wave height and crests height for long return periods, such as for the 100-year maximum wave height and 100-year maximum crest height. He showed that the effect of sampling variability is different for individual crest or wave height as compared to significant wave height. Further, he demonstrated that direct application of the Forristall (2000) crest distributions for 3-h sea state parameters give long-term extremes that are biased low, and he further showed how the short-term distributions can be modified such that consistent results for 20-min and 3-h sea states are obtained.

Sampling variability also needs to get attention when carrying out model tests. To get stable estimators of measured quantities, model tests need to be repeated a sufficient number of times.

Use of 17.5-min or 1-h H_s and T_z (or T_p) estimators will have not only significant impact on long-term description of waves (see Forristall et al. 1996; Hagen 2007) but also on the shape of parametric wave spectra used in design work. We will illustrate it for the double-peak Torsethaugen (1996) spectrum. The spectrum allows to divide the wave energy between wind sea and swell knowing the total H_s and T_p for a sea state. In the Torsethaugen model, each sea state is classified as *swell-dominated* sea or *wind-dominated* sea according to the proposed by Torsethaugen criterion. A sea state with H_{m0}=14.55 m and T_p=16.39 s will be regarded by the Torsethaugen (1996) spectrum as swell-dominated sea. If we increase significant wave height by 1 m (to H_{m0}=15.55 m), corresponding to 7 % sampling variability standard deviation, then according to the Torsethaugen (1996) spectrum, the sea state with H_{m0}= 15.55 m and T_p=16.39 s will be regarded as wind-sea dominated (see Bitner-Gregersen and Toffoli 2009 for details); thus, the physical description of the sea state provided by the Torsethaugen spectrum will be changed, and that may have consequences for design. It should be noted that the spectrum was derived for the North and Norwegian Sea and should be used with care for locations outside Norway.

7 Conclusions

The study discusses statistical uncertainty, called sampling variability (the uncertainty due to a limited number of observations of a quantity), of significant wave height and zero-crossing wave period. This uncertainty is a result of presence of intrinsic variability of sea surface. Ideally, a wave record should be infinite to eliminate sampling variability.

The intention of the paper is to put again attention to intrinsic and sampling variability and to remind practitioners that sampling variability must be taken into account for accurate use of wave measurements. Several aspects related to sampling variability need still further research and are not answered fully herein, e.g., how sampling variability is changing in different sea states and storms.

Estimation of sampling variability from wave measurements is more challenging than from model tests and numerical simulations because weather conditions are changing. Theoretical expressions for sampling variability can be of support, in some cases.

The theoretical sampling variability standard deviations of significant wave height and zero-crossing wave period proposed by Bitner-Gregersen and Hagen (1990) have been compared with the ones derived from the waverider measurements collected at the Ekofisk field. The theoretical values and the field data show the same trend. The sampling variability is higher in H_s than in T_z. Both increase with increasing H_s and T_z. The observed sampling variability ranges over a wider scale than the theoretical results. We anticipate this spread to depend on the shape of a wave spectrum; the JONSWAP spectrum gives higher variability than the Pierson-Moskowitz spectrum (Bitner-Gregersen and Hagen 1990; Tucker 1992). The effect of degree of peakedness and double-peak spectra (describing combined wave systems) on the sampling variability has not been investigated and needs further research.

Non-linearities of sea surface (deviations from Gaussianity) may have impact on the estimators of H_s and T_z sampling variability due to change of the number of observations for a given wave record length and change of the shape of the wave spectrum when steep waves are present.

We conclude that the study shows that the length of a wave record is critical for evaluating stable parameters as far as possible in varying sea states. Using some selected examples, it is demonstrated that sampling variability of H_s may have significant impact on short-term and long-term description of ocean waves as well as validation of different data sources and wave spectral models. This aspect should be kept in mind when providing short and long-term description of waves as well as specifying design and operational criteria of ship and offshore structures

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Appendix A—sampling variability

Tables and include sampling variability standard deviations of the significant wave height and zero-crossing wave period, respectively, for the Northern North Sea scatter diagram calculated by Bitner-Gregersen and Hagen (1990). Assuming Gaussian sea surface and the JONSWAP spectrum with γ= 3.3. The results are valid for any worldwide location under the adopted assumptions.

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