

PREDICTION OF WAVE HEIGHT AND PERIOD FOR A CONSTANT WIND VELOCITY USING THE JONSWAP RESULTS

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Abstract—The results of the Joint North Sea Wave Project (JONSWAP), carried out in the North Sea in 1969, are used to derive formulae for significant wave height and wave period in terms of the wind speed (assumed constant) and fetch or duration. Values from the wave height formula are compared with those from the formulae of Bretschneider (1973) and Darbyshire (1963). It is proposed that the JONSWAP results be used for the prediction of fetch and duration limited waves formed under the action of the local wind field.

INTRODUCTION

DURING the Joint North Sea Wave Project (JONSWAP) of 1969, measurements of wave energy spectra were obtained in the North Sea, west of Denmark out to 160 km from the coast. Hasselmann *et al.* (1973) give the results from an analysis of the data, including a parametric form of the wave energy spectrum for a growing sea state (based upon the Pierson–Moskowitz spectrum) which they derive from data for offshore winds (i.e. known, limited fetch). The maximum wind speed was 15 m sec^{-1} —but according to J. A. Ewing (personal communication) some measurements were subsequently made with wind speeds up to 20 msec^{-1} . The waves created by these off-shore winds were not sufficiently large to be affected by the sea floor.

Hasselmann *et al.* (1973) discuss the relationships between the peak frequency of the spectrum (f_m), the fetch, and wind speed, and conclude that, given a growing sea state, the wave energy spectrum can be described by f_m and a parameter α corresponding to Phillips' constant.

Hasselmann *et al.* (1976) derive equations, from slightly modified versions of the relationships given in the 1973 paper, to determine f_m and α given the wind field distribution in time and space. They then solve these equations for some cases when the wind distribution is a function of space or time. The solution for the specific case of constant wind velocity can be used to obtain formulae for predicting significant wave height and period for a given wind speed and fetch or duration and to determine a connection between fetch and duration. These formulae are initially cast in non-dimensional form, but are translated into numerical form of more immediate practical value.

Formulae for wave height arising from a constant wind blowing over a specified fetch or for a specified duration are often used to predict sea conditions. Formulae in common use are those derived by Bretschneider (1973) and by Darbyshire (1963); these are compared with the JONSWAP results.

WAVE ENERGY SPECTRA

(a) Pierson and Moskowitz (1964), using data from Shipborne Wave Recorders in the

northeast Atlantic Ocean Weather Ships, propose a spectrum for a fully developed sea to "serve the purpose of wave forecasting until more data . . . and better wind speed observations" were available. In terms of wind speed measured at 19.5 m, $u_{19.5}$, the spectrum, E , of wave energy distribution with frequency f is

$$p_M E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp \left[-\frac{5}{4} \left(\frac{f}{f_m} \right)^{-4} \right] \quad (1)$$

where

$$\alpha = 8.1 \times 10^{-3}$$

and

$$f_m = 0.8772 (g/2\pi u_{19.5})$$

where f_m is the frequency at the maximum of the spectrum. (Strictly, E is the distribution of the variance of surface elevation with frequency, and the energy spectrum is $\rho g E$, where ρ is water density, but conventionally E is referred to as the "energy spectrum".)

(b) The JONSWAP spectrum for a developing sea was derived by Hasselmann *et al.* (1973) by multiplying the Pierson-Moskowitz spectrum by a "peak enhancement" factor. This factor and the values of α and f_m were determined, in terms of fetch and wind speed at 10 m, from wave measurements off Denmark. The spectrum is

$$J E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp \left[-\frac{5}{4} \left(\frac{f}{f_m} \right)^{-4} \right] \cdot \gamma^q \quad (2)$$

where

$$q = \exp [-(f - f_m)^2 / 2\sigma^2 f_m^2]$$

and

$$\sigma = \begin{cases} 0.07 & f < f_m \\ 0.09 & f \geq f_m \end{cases}.$$

For the mean JONSWAP spectrum

$$\gamma = 3.3.$$

Values for α and f_m are included in Table 1.

SIGNIFICANT WAVE HEIGHT

This is defined from the energy spectrum as

$$h_s = 4 \sqrt{m_0} \quad (3)$$

where

$$m_n = \int_0^\infty f^n E(f) df.$$

TABLE 1. NON-DIMENSIONAL PARAMETERS

Dimensionless parameter	Symbol	Definition	Values			
Column 1	2	3	4	5	6	7
			Pierson-Moskowitz spectrum	JONSWAP spectrum	JONSWAP RESULTS (a) Function of fetch ξ	(b) Function of duration δ
Fetch	ξ	$gx/10^2$	—	—	—	—
Duration	δ	$gd/10$	—	—	—	—
Peak frequency	γ	uf_m/g	0.13	$3.5\xi^{-0.33}$	$2.84\xi^{-0.3}$	$16.8\delta^{-2/7}$
Surface variance	ϵ	$m_0 g^2/10^4$	3.66×10^{-3}	$1.957 \times 10^{-4} \alpha \nu^{-4}$ $9.91 \times 10^{-6} \xi^{1.1}$	$1.63 \times 10^{-7} \xi$	$4.08 \times 10^{-10} \delta^{10/7}$
Spectral shape	λ	$\frac{\epsilon \nu^4}{\alpha}$	1.28×10^{-4}	1.96×10^{-4}	1.6×10^{-4}	1.6×10^{-4}
	α	$\frac{m_0/m^4}{\alpha g^2}$	8.1×10^{-3}	$0.076\xi^{-0.22}$	$0.0662\xi^{-0.2}$	$0.203\delta^{-2/7}$

Where x is fetch, f_m is frequency of spectral peak, d is duration, u is wind speed at 10 m, m_n is the area under the energy spectrum.

The Pierson-Moskowitz spectrum gives a directly integrable value for m_0 (and for m_n if $n < 4$) which leads to a significant wave height given by

$${}_{\text{PM}}H_s = \frac{g}{\pi^2 f_m^2} \sqrt{\frac{\alpha}{5}} \simeq \frac{0.447g}{\pi^2 f_m^2} \cdot \sqrt{\alpha}.$$

Substituting for α and f_m from equation (1), taking $g = 9.81 \text{ m sec}^{-2}$ and using (from Pierson, 1977, para. 4) for wind speed at 10 m : $u_{10} = 0.93 u_{19.5}$ gives:

$${}_{\text{PM}}H_s = 0.02466 U^2 \quad (4)$$

where ${}_{\text{PM}}H_s$ is the significant wave height in m and U is the wind speed at 10 m in m sec^{-1} .

(Note the capital letters are used for numerical values, lower-case letters for physical quantities, except for the spectrum E which is conventionally given a capital letter.)

Numerical integration of the JONSWAP spectrum leads similarly to an expression for significant wave height:

$${}_J H_s = \frac{0.552 g}{\pi^2 f_m^2} \sqrt{\alpha}.$$

Substituting values for α and f_m from Table 1, leads to a significant wave height in m given by

$${}_J H_s = 0.02013 X^{0.55} U^{0.90} \quad (5)$$

where X is the fetch in km.

WAVE PERIOD

The period t_m corresponding to the peak of the spectrum is $1/f_m$. So for the Pierson-Moskowitz spectrum from equation (1)—substituting $u_{10} = 0.93 u_{19.5}$ —is given by:

$${}_{\text{PM}}t_m = 1.226 (2\pi/g) u_{10}.$$

For wind speed U in msec^{-1} and period T_m in sec, this reduces to:

$${}_{\text{PM}}T_m = 0.785 U. \quad (6)$$

Similarly for the JONSWAP spectrum, the peak period is given from equation (2) by

$${}_J T_m = 0.605 X^{0.33} U^{0.34} \quad (7)$$

where X is the fetch in km.

The zero-up-crossing wave period is given by:

$$t_z = (m_0/m_2)^{0.5}.$$

For the Pierson-Moskowitz spectrum this leads to

$${}_{\text{PM}}t_z = 0.7104 {}_{\text{PM}}t_m.$$

So for T_z in sec:

$${}_{\text{PM}}T_z = 0.558 U. \quad (8)$$

Similarly for the JONSWAP spectrum, but by numerical integration:

$${}_Jt_z = 0.777 {}_Jt_m. \quad (9)$$

Therefore

$${}_JT_z = 0.470 X^{0.33} U^{0.34}. \quad (10)$$

SOLUTIONS TO THE PARAMETRIC EQUATIONS

Hasselmann *et al.* (1973, 1976) use non-dimensional parameters for fetch, peak frequency, surface variance, and spectral shape— ξ , ν , ε and λ respectively. The definitions of these parameters and that for non-dimensional duration, δ , are given in Table 1, column 3.

Hasselmann *et al.* (1976) derive equations for the parameters α and ν , defining the spectrum, using slightly modified versions of the relationships between fetch and the other non-dimensional parameters which were deduced for the JONSWAP spectrum by Hasselmann *et al.* (1973). The original, 1973, spectral relationships and the 1976 results are given in Table 1, columns 5 and 6, respectively. The latter are based upon a linear relationship between surface variance ε and fetch ξ , whilst the former were chosen in 1973 to give the best fit to the observed spectral shape, and include putting surface variance proportional to $\xi^{1.1}$.

The JONSWAP spectrum was determined from measurements when the sea state was growing. Hasselmann *et al.* (1976) ascribe this condition to values of $\nu \geq 0.14$, describing waves with $\nu < 0.14$ as swell. They give solutions for the equations, which define the spectrum, when the wind direction is constant and the wind speed is proportional to some power, p , of either fetch or duration. If the wind velocity remains constant, independent of fetch or duration (i.e. $p = 0$), the solutions reduce to the values given in Table 1, columns 6 and 7.

EQUIVALENCE OF FETCH AND DURATION

A duration d_x equivalent to a fetch x , may be obtained, assuming constant wind velocity u , by equating any one of the corresponding expressions in columns 6 and 7 of Table 1. The results differ according to which parameter is used, indicating that no exact equivalence exists. The results are:

$$\text{From } \nu: \quad d_x = 63.3 \tau \quad (11)$$

$$\alpha: \quad d_x = 50.5 \tau \quad (12)$$

$$\varepsilon: \quad d_x = 66.2\tau \quad (13)$$

where

$$\tau = x^{0.7}/g^{0.3}u^{0.4} = (gx/u^2)^{0.7}u/g.$$

Usually estimates of significant wave height (i.e. ϵ) are required, so equation (13) is most appropriate. Using this equation, the peak frequency of the fetch-limited spectrum would be 2% higher than the peak frequency of the corresponding duration-limited spectrum; the value of α would be 8% greater. These small percentage differences indicate that for practical purposes spectra produced with limited fetch or duration according to equation (13) may be regarded as equivalent.

Parameter α describes the high frequency portion of the spectrum, which contains relatively little energy; so in general equation (12) seems the least appropriate of the three equations.

Figure 1 shows a fetch-limited JONSWAP spectrum (for a fetch of 100 km and a wind speed of 20 m sec⁻¹) with α and $f_m = g\sqrt{u}$ from Table 1, column 6, together with three equivalent duration-limited spectra, where equivalence is defined by equations (11)–(13).

An approximation to the relationship between fetch x and duration d_x may be derived as follows. Suppose a constant wind has been blowing over a fetch x for a time t which is just sufficient to produce the fetch-limited wave at x . At a distance dx downwind, the sea will not become fetch-limited before the wave energy at x has had time to travel the distance dx , until then the waves at $x + dx$ will grow as a duration-limited wave. So if the wave energy travels at a mean speed v_e , the time dt for the waves to become fetch-limited at $x + dx$ is given approximately by:

$$dx = v_e dt,$$

$$\text{i.e.} \quad dt = \frac{dx}{v_e} \quad (14)$$

The speed at which the energy travels downwind depends upon frequency and directional spread of the waves; the waves of low frequency moving ahead of the slower, high frequency waves. Assuming that the group velocity, v_g , of the waves may be taken as that value corresponding to the peak frequency $f_m (= g\sqrt{u})$ then

$$v_g = g/4\pi f_m = u/4\pi\gamma.$$

So the mean speed of energy downwind is given by:

$$v_e = \int_{-\pi/2}^{+\pi/2} v_g \cos\theta D(\theta) d\theta$$

where $D(\theta)$ is the normalized spreading function. Assuming a $\cos^2\theta$ spreading function, and integrating, gives:

$$v_e = (8/3\pi) v_g \simeq 0.85 u/4\pi\gamma.$$

Substituting in equation (14), with γ replaced by a function of x from Table 1, column 6, and integrating leads to

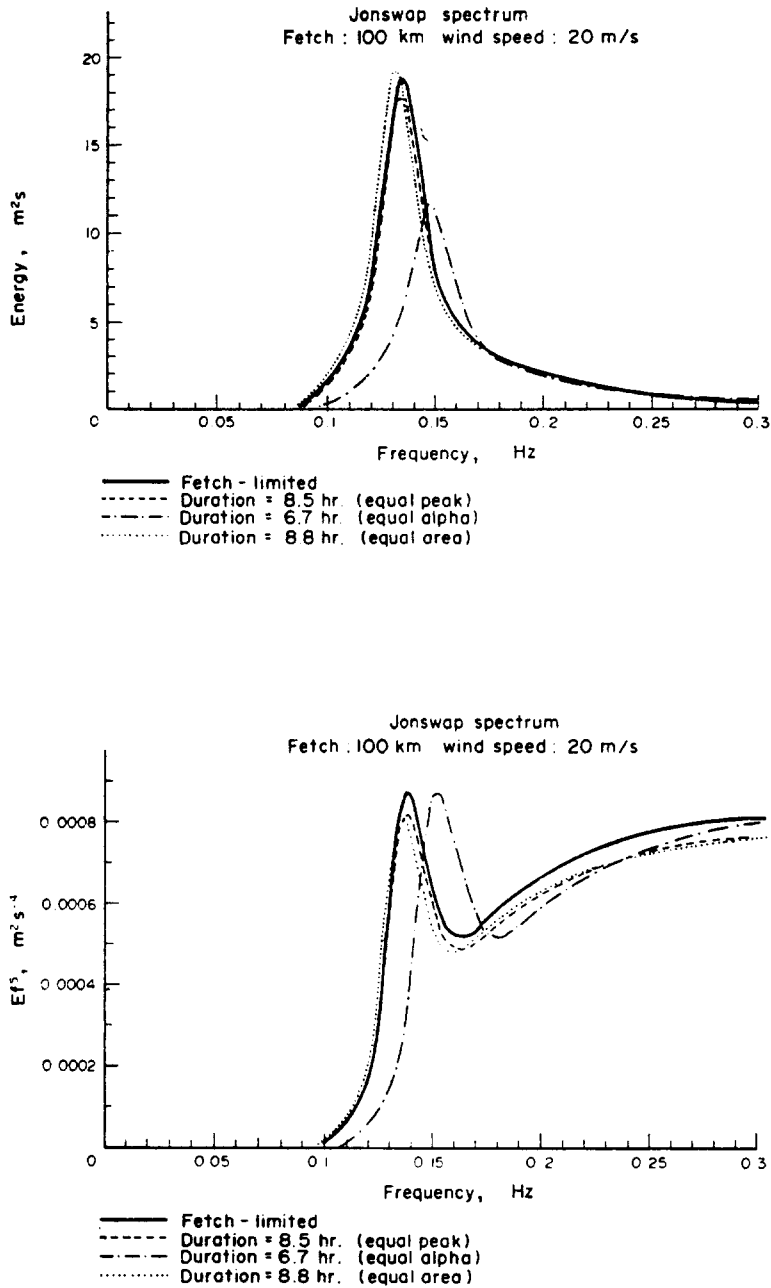


FIG. 1. Example of fetch-limited JONSWAP spectrum and equivalent duration-limited spectra. Energy E : frequency f and $E f^3$: f (equivalence defined by equations (11)–(13)).

$$d_s = 60.0\tau.$$

(This spreading factor of 0.85 is used by Hasselmann *et al.*, 1976, in their parametric transport equations.)

SIGNIFICANT WAVE HEIGHT AND PERIOD FROM THE JONSWAP RESULTS

From the formulae for ϵ in Table 1, columns 6 and 7, with significant wave height H_s in m, fetch X in km, wind speed U in m sec^{-1} and duration D in hr:

$$\begin{aligned} \text{(a)} \quad & {}_R H_s = 0.0163 X^{1/2} U \\ \text{or} \quad & \\ \text{(b)} \quad & {}_R H_s = 0.0146 D^{3/7} U^{9/7} \end{aligned} \quad (15)$$

where (a) and (b) are the fetch-limited and duration-limited cases respectively.

Similarly, from the formulae for ν in Table 1, columns 6 and 7

$$\begin{aligned} \text{(a)} \quad & {}_R T_m = 0.566 X^{3/10} U^{2/5} \\ \text{(b)} \quad & {}_R T_m = 0.540 D^{3/7} U^{4/7} \end{aligned} \quad (16)$$

so, from equation (9)

$$\begin{aligned} \text{(a)} \quad & {}_R T_z = 0.439 X^{3/10} U^{2/5} \\ \text{or} \quad & \\ \text{(b)} \quad & {}_R T_z = 0.419 D^{3/7} U^{4/7}. \end{aligned} \quad (17)$$

Solution (a) is appropriate if the wave is fetch-limited, i.e. from equation (23) if the duration D hr is such that

$$D > 1.167 X^{0.7} / U^{0.4}. \quad (18)$$

Otherwise, solution (b) applies.

The condition that $\nu \geq 0.14$ (implying sea not swell) is equivalent to

$$\begin{aligned} \text{(a)} \quad & X \leq 2.32 U^2 \\ \text{or} \quad & \\ \text{(b)} \quad & D \leq 2.01 U. \end{aligned} \quad (19)$$

Diagrams to determine wave height and period from equations (15) and (17)–(19) are given in Figs 2 and 3. In the area of Fig. 2 where the conditions in equation (19) do not hold, H_s has been obtained from equation (15a) with $X = 2.32 U^2$, i.e. $H_s = 0.0248 U^2$, which is very close to the Pierson–Moskowitz value for a fully-developed sea of $0.02466 U^2$, equation (4). (The duration-limited wave height has an upper limit from equations (15b) and $D = 2.01 U$ of $0.0240 U^2$; this slight difference is not shown in Fig. 2.) Similarly in Fig. 3 where equation (19) does not apply, a value of $T_z = 0.566 U$ has been used—derived from equa-

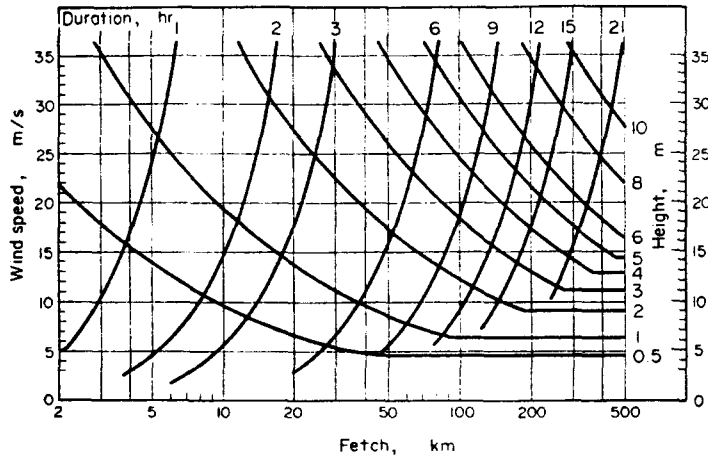


FIG. 2. Significant wave height prediction graph from JONSWAP results. (Enter with wind speed at left-hand side, move across till the limiting fetch or duration is reached, then move down the curve to the height scale.)

tion (17a) and $X = 2.32U^2$ or from equation (17b) and $D = 2.01U$ —compared with Pierson–Moskowitz value for T_z of $0.558U$, equation (8).

COMPARISON WITH WAVE HEIGHT ESTIMATED FROM THE JONSWAP SPECTRUM

The JONSWAP spectrum was derived to show the shape of the wind-generated, fetch-

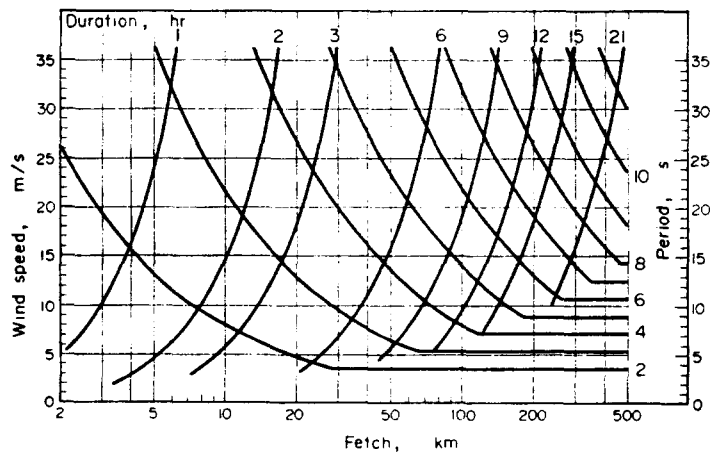


FIG. 3. Zero-up-crossing wave period prediction graph from JONSWAP results. (Enter with wind speed at left-hand side, move across till the limiting fetch or duration is reached, then move down the curve to the period scale.)

limited wave energy spectrum; it was not determined to give the best fit between wave height and fetch. So it would be a mistake to predict wave height from the spectrum, i.e. from equation (5). Indeed the ratio of fetch-limited significant wave height determined from the JONSWAP results to that from the spectrum is given—from Table 1, columns 6 and 5, respectively by:

$$R = (0.0163 X^{0.5} U) / (0.02013 X^{0.55} U^{0.9})$$

$$\text{i.e.} \quad R = 0.81 (U^2/X)^{0.05} \quad (20)$$

where U is the wind speed at 10 m in m sec^{-1} and X is the fetch in km. The value of R varies from about 0.7 at large fetches and low wind speeds (about 500 km and 5 m sec^{-1}) to about 1.0 at short fetches and high wind speeds (10 km and 30 m sec^{-1}).

COMPARISON WITH DARBYSHIRE AND BRETSCHNEIDER FETCH-LIMITED WAVE HEIGHT PREDICTIONS

Darbyshire (1963) gives two formulae for estimating root-mean square wave height (\bar{h}) from wind speed in knots and fetch in nautical miles, one formula is applicable to coastal waters and the other to oceanic waters. (The former might be expected to show closer agreement with the JONSWAP results.) Assuming that significant wave height is $\sqrt{2} \bar{h}$, converting the units to m sec^{-1} and km leads to

$$\text{oceanic waters:} \quad {}_{\text{Do}}H_s = 0.0132 Y^{3/2} U^2 \quad (21)$$

$$\text{coastal waters:} \quad {}_{\text{Dc}}H_s = 0.0630 Y^{3/2} U^{3/2}$$

$$\text{where} \quad Y = \frac{X^3 + 5.56X^2 + 223.2X}{X^3 + 22.24X^2 + 893.0X + 509}$$

where ${}_{\text{Do}}H_s$ and ${}_{\text{Dc}}H_s$ are the oceanic and coastal values of significant wave height in m with $U \text{ m sec}^{-1}$ the wind speed at 10 m and X km the fetch.

(The reference height for wind speed is not given, either in Darbyshire (1963) or in Darbyshire and Draper, 1963.) A height of 10 m has been assumed. This is correct, according to L. Draper (personal communication), for the coastal waters formula. The oceanic formula was determined from Ocean Weather Ship wind speeds estimated on the Beaufort scale (J. Darbyshire, personal communication). The conversion from this scale to knots given in the Marine Observer's Handbook (HMSO, 1950) is the same as used today and is annotated "measured at a height of 33 feet above sea level".

Bretschneider (1973) gives the following formula for significant wave height h_s .

$$gh_s/u^2 = 0.283 \tanh\{0.0125 (gx/u^2)^{0.42}\}$$

which reduces to

$${}_B H_s = 0.0288 U^2 \tanh\{0.5935 (X/U^2)^{0.42}\} \quad (22)$$

where ${}_B H_s$ is in m, U in m sec^{-1} and X in km.

Figure 4 shows the ratio of significant wave height estimated from the JONSWAP results and that estimated by Darbyshire and Bretschneider; Fig. 5 shows the difference (in m) between these estimates.

Figures 4 and 5 shows good agreement between Bretschneider's fetch-limited estimates and the JONSWAP results—except that Bretschneider gives values of significant wave height about 1 m higher at short fetches and very high wind speeds; agreement is particularly good at long fetches and very high wind speeds. Darbyshire's formulae (coastal and oceanic) give even higher estimates at rather short fetches (around 50–100 km) and very strong winds than does Bretschneider's, and generally do not agree well with the JONSWAP results; percentage differences at low wind speeds are particularly large—although absolute differences are less than 0.5 m.

COMPARISON WITH DARBYSHIRE AND BRETSCHNEIDER DURATION-LIMITED WAVE HEIGHT PREDICTIONS

Darbyshire and Draper (1963) give curves derived from the formulae in Darbyshire (1963) for predicting wave height including duration-limited values. Correspondence between fetch and duration was calculated by assuming that wave energy travelled with the group velocity, v_g , of a wave with the final peak frequency (Draper, personal communication). This peak frequency is given by Darbyshire (1963) for oceanic and coastal conditions, ${}_{D0}f_m$ and ${}_{Dc}f_m$, as converted to wind speed U in m sec^{-1} :

$${}_{D0}f_m = 1/\{Y^{0.75} (2.704 U^{0.5} + 3.6 \cdot 10^{-7} U^4)\} \text{ sec}^{-1} \quad (23)$$

and

$${}_{Dc}f_m = 1/(2.16 Y^{0.75} U^{0.5}) \text{ sec}^{-1}$$

where Y is as in equation (21), so

$$X = v_g d_x$$

where x and d_x are corresponding fetch and duration.

i.e.

$$X = g d_x / 4\pi f_m$$

and from equation (23)

$${}_{D0}D = 0.356 X / \{Y^{0.75} (2.704 U^{0.5} + 3.6 \cdot 10^{-7} U^4)\} \quad (24)$$

and

$${}_{Dc}D = 0.165 X / (Y^{0.75} U^{0.5})$$

where D is duration in hr and X is fetch in km.

Bretschneider (1973) computes duration assuming that wave energy is transmitted with the group velocity v'_g of a wave with significant wave period t_s ,

i.e.

$$v'_g = g t_s / 4\pi.$$

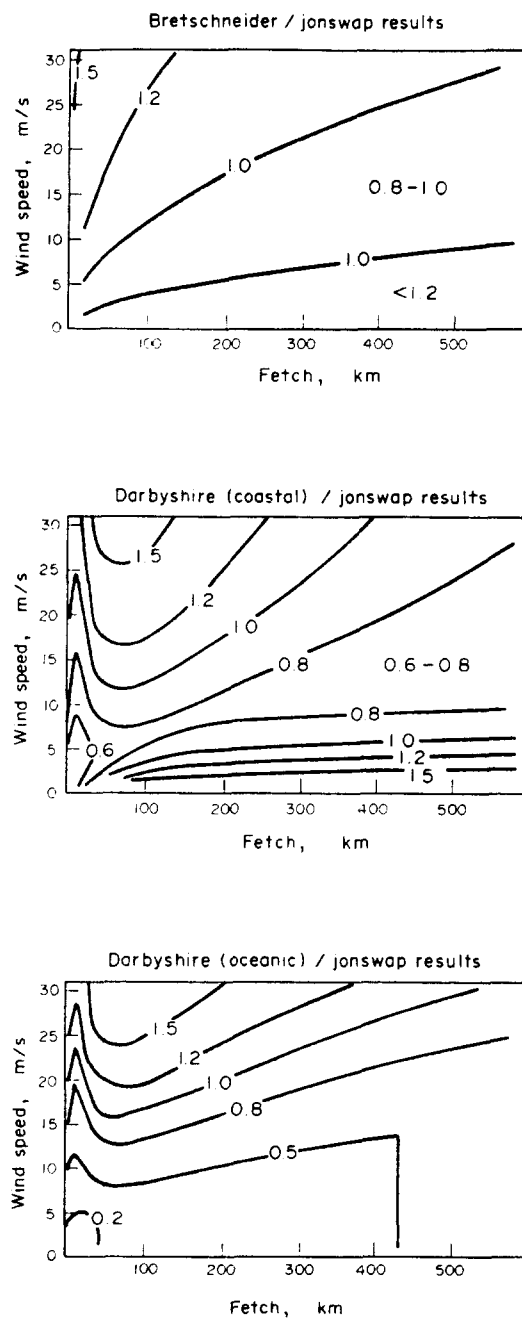


FIG. 4. Ratios of fetch-limited significant wave heights predicted by Bretschneider and Darbyshire and that predicted from the JONSWAP results.

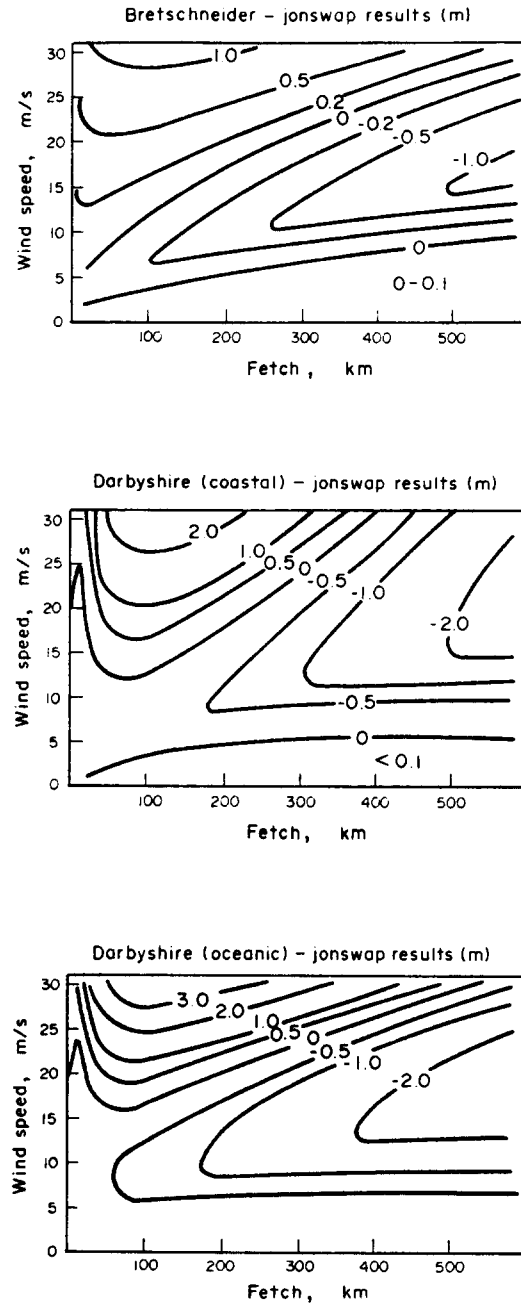


FIG. 5. Differences in fetch-limited significant wave heights predicted by Bretschneider and Darbyshire and that predicted from the JONSWAP results.

Bretschneider defines t_s by:

$$t_s = f_m^{-1}/(5/4)^{0.25}$$

and gives

$$t_s = A_2 \cdot \frac{2\pi u}{g} \cdot \tanh\{B_2(gx/u^2)^M\}$$

where $A_2 = 1.2$, $B_2 = 0.077$ and $M = 0.25$. So the duration d_x corresponding to a fetch x is given by

$$d_x = \int_0^x (4\pi/gt_s) dx.$$

Putting $z = B_2 (gx/u^2)^M$ —following Lalande (1975)—and approximating $\coth z$ leads to

$${}_B D \simeq 1.79 U Z^3 \left(1 + \frac{1}{5} Z^2 - \frac{1}{105} Z^4 + \frac{2}{2835} Z^6\right) \quad (25)$$

where

$$Z = 0.77 X^{0.25}/U^{0.5}.$$

Figures 6 and 7 show the ratios and differences in duration-limited wave heights corresponding to the fetch-limited case in Figs 4 and 5. These figures show that Bretschneiders and Darbyshire estimates of duration-limited significant wave heights do not agree so well with the JONSWAP results as the corresponding fetch-limited heights; with markedly higher values for short duration and very high wind speeds. Generally Bretschneider's estimates differ less from the JONSWAP results than the Darbyshire estimates especially for long duration and very high wind speeds.

CONCLUSIONS

Formulae have been derived which give significant wave height, zero-up-crossing period, and period corresponding to the peak frequency of the energy spectrum from the results of the JONSWAP experiment for the case of constant wind velocity with either limited fetch or limited duration. Numerical formulae are summarized in Appendix A.

Estimates of significant wave height from these formulae have been compared with those from formulae due to Bretschneider (1973) and Darbyshire (1963)—the latter for both oceanic and coastal waters. All three formulae give higher waves than the JONSWAP results for very high wind speeds and short fetch or duration. Closest agreement with JONSWAP is in general Bretschneider's formula for fetch-limited wave height. Greatest differences are generally with Darbyshire's oceanic formulae; this might be expected because it is not applicable to the area where the JONSWAP data were obtained—but Darbyshire's coastal formulae also give some large differences from the JONSWAP results.

The considerable variations between these different formulae suggest that they give only approximate estimates of wave height—particularly at very high wind speeds.

The JONSWAP results are based upon theoretical considerations of wave growth and

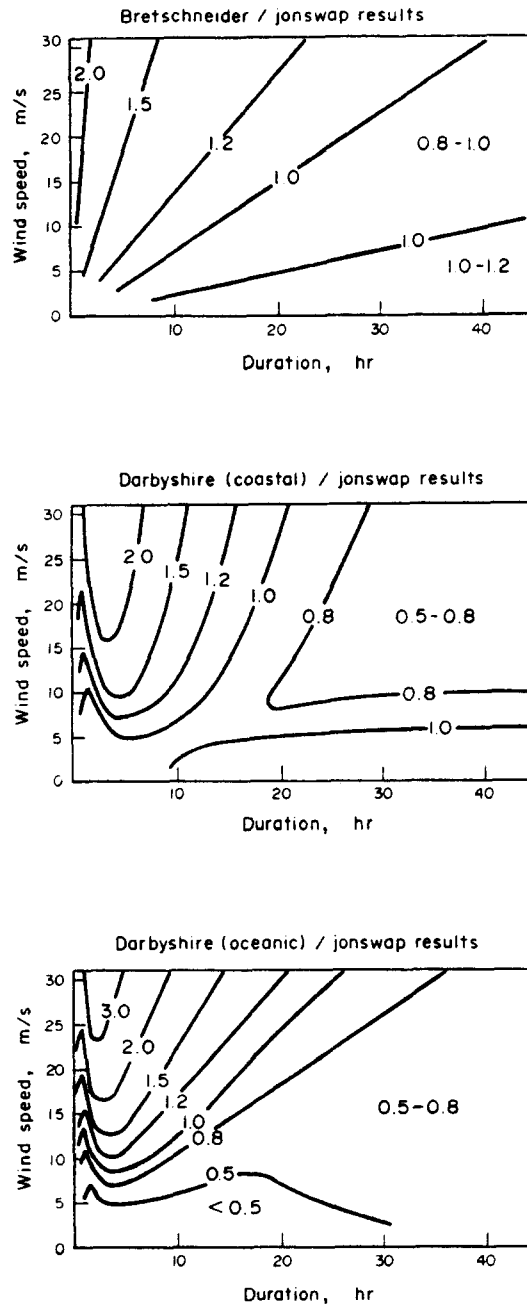


FIG. 6. Ratios of duration-limited significant wave heights predicted by Bretschneider and Darbyshire and that predicted from the JONSWAP results.

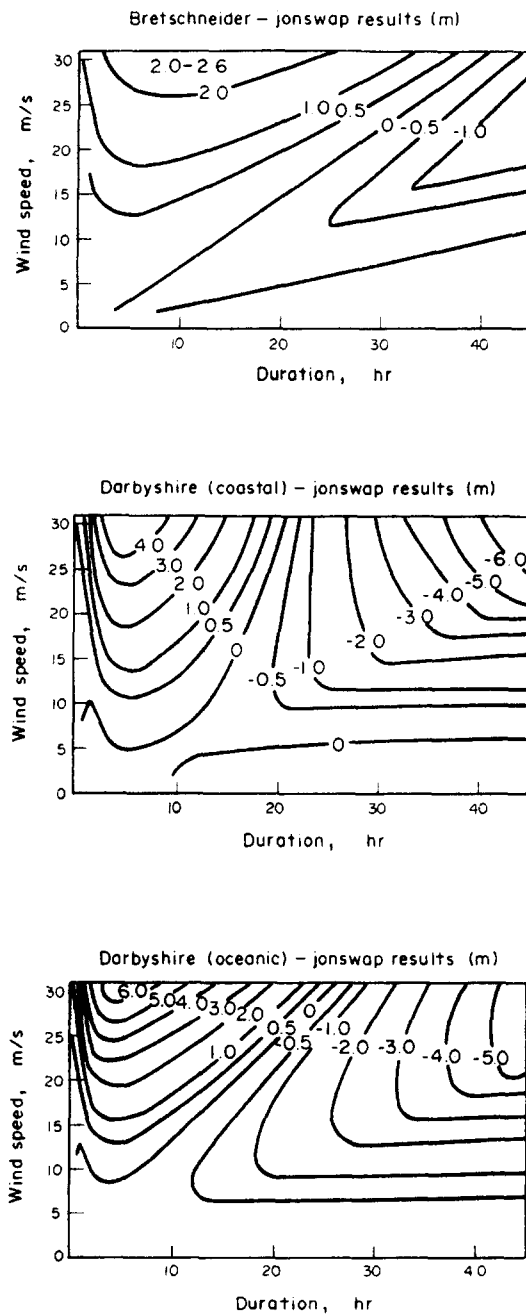


FIG. 7. Differences in duration-limited significant wave heights predicted by Bretschneider and Darbyshire and that predicted from the JONSWAP results.

upon carefully controlled measurements off Denmark. So it would seem reasonable to use these—or Figs 2 and 3 derived from them—rather than the other formulae, for estimating locally generated wave heights in the North Sea, provided that the water is sufficiently deep and that the waves are not significantly affected by the sea floor. (The criterion used by Hasselmann *et al.* (1973) for this is that water depth is greater than a quarter of the peak frequency wavelength, i.e. that depth is greater than $g/8\pi f_m^2$.)

The JONSWAP measurements were limited to a maximum fetch of 160 km and a maximum wind speed of about $15\text{--}20\text{ m sec}^{-1}$, but extrapolation of the results to greater fetches or duration and higher wind speeds give wave heights not dissimilar from those estimated by Bretschneider, and close to the value given by the Pierson–Moskowitz spectrum for a fully-developed sea.

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APPENDIX A

Numerical formulae derived from the JONSWAP results

The following symbols are used:

- D duration (hr)
 H_s significant wave height (m)
 T_m period corresponding to the peak frequency of the spectrum (sec)
 T_z zero-up-crossing period (sec)
 U wind speed at 10 m above the sea surface (m sec^{-1})
 X fetch (km)

Parameter	<div style="text-align: center;"> $D > 1.167 X^{0.7} / U^{0.4}$ Yes (fetch limited) No (duration limited) $X < 2.32 U^2$ $D < 2.01 U$ </div>			
	Yes (growing sea)	No (fully developed)	Yes (growing sea)	No (fully developed)
H_s	$0.0163 X^{0.5} U$	$0.0248 U^2$	$0.0146 D^{5/7} U^{9/7}$	$0.0240 U^2$
T_m	$0.566 X^{0.3} U^{0.4}$	$0.728 U$	$0.540 D^{3/7} U^{4/7}$	$0.728 U$
T_z	$0.439 X^{0.3} U^{0.4}$	$0.566 U$	$0.419 D^{3/7} U^{4/7}$	$0.566 U$