ECEN 689 **Assignment 2** Spring 2019
Each problem is worth 10 points Due: Feb 25

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1. (Owen) Find a saddle point and the value of the following zero-sum game:

$$\begin{pmatrix} 4 & 3 & 1 & 4 \\ 2 & 5 & 6 & 3 \\ 1 & 0 & 7 & 0 \end{pmatrix}$$

Clearly show all the steps you used in obtaining the saddle point. If you used a computer program, please write down the relevant LPs and attach a copy of the program.

2. (O) Repeat problem 1 for the following matrix:

$$\begin{pmatrix} 0 & 5 & -2 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{pmatrix}$$

3. (O) Repeat problem 1 for the following matrix:

$$\begin{pmatrix} 5 & 8 & 3 & 1 & 6 \\ 4 & 2 & 6 & 3 & 5 \\ 2 & 4 & 6 & 4 & 1 \\ 1 & 3 & 2 & 5 & 3 \end{pmatrix}$$

4. Consider the zero-sum game in Problem 1. Use the following procedure to compute the strategies of the two players:

- Start with some arbitrary pure strategy for each Player 1.
- At each step, compute the empirical mixed strategy for each player, based on each player's actions up until the previous step. For example, if Player 1 has played strategy $i$ $n_i$ times after the first $n$ steps, then $x_i$ is estimated to be $n_i/n$.
- At each step, find the best pure strategy response for each player based on the other player's empirically observed mixed strategy. For example, Player 2 choose $j$ to minimize $\sum_i a_{ij}x_i$.

Does the sequence (or a subsequence) of the empirical strategies converge to a saddle point? Attach a copy of the computer program that you use and show some points from the sample path of empirical mixed strategies to justify your answer.

5. Let $X$ and $Y$ be subsets of some vector space. Let $f(x,y)$ be a function from $X \times Y$ to $\Re$. Show that

$$\sup_{x\in X} \inf_{y\in Y} f(x,y) \leq \inf_{y\in Y} \sup_{x\in X} f(x,y).$$

*Note: This is a simple problem. The only purpose of the problem is to remind you that the minimax inequality holds more generally than for the case of matrix games.*

6. Again consider $f : X \times Y \to \Re$. Interpret $X$ to be the strategy space of Player 1 (the maximizer) and $Y$ to be the strategy space of Player 2 (the minimizer) in a zero-sum game. Let $P_x$ be a probability distribution over $X$ and $P_y$ be a probability distribution over $y$. The expected payoff to Player 1 is given by

$$\int_X \int_Y f(x,y) dP_y dP_x.$$

Suppose there exists two saddle points $(P_x^*, P_y^*)$ and $(P_x, P_y)$ to this game. (The definition of a saddle point extends naturally to situations more general than the case of matrix games discussed in class so far.) Show that $(P_x^*, \hat{P}_y)$ and $(\hat{P}_x, P_y^*)$ are also saddle points.

*Note: This is also a simple problem. The only purpose of the problem is to show that saddle-point interchangeability holds more generally than for the case of matrix games. However, Nash's theorem doesn't apply here, so saddle points may or may not exist. We are only stating that if they exist and there is more than one saddle point, then they are interchangeable.*