

1    **Privacy-Preserving and Verifiable Product Rating Aggregation in E-Commerce**  
2    **Using Threshold Homomorphic Encryption with Distributed Key Generation**

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3    **Received:** .202    •    **Accepted/Published Online:** .202    •    **Final Version:** ..202

4    **1. Background**

5    We first introduce the cryptographic primitives that form the basis of our privacy-preserving and verifiable  
6    rating-aggregation protocol. In contrast to conventional centralized architectures, our design relies on threshold  
7    homomorphic encryption, distributed key generation, and verifiable secure computation to simultaneously  
8    guarantee confidentiality, correctness, and transparency.

9    **1.1. Threshold Homomorphic Encryption**

10   Let  $N = pq$ . denote an RSA modulus where p and q are large safe primes. We adopt an additive homomorphic  
11   encryption scheme such as Paillier or CKKS [16], which enables integer addition to be performed directly over  
12   ciphertexts. A homomorphic public key  $pk_{HE}$  is published to all clients in the system. The corresponding  
13   private decryption key  $sk_{HE}$  is never constructed in full. Instead, the secret key is shared among n aggregation  
14   servers by means of a t-out-of-n threshold cryptosystem, derived from Shamir's secret sharing. Each server  $S_i$   
15   holds a private share  $sk_i$ , and decryption is only possible when a quorum of at least t servers jointly participate  
16   in the threshold decryption algorithm. This approach eliminates any single point of failure and prevents a  
17   malicious server from decrypting user ratings individually.

18   **1.2. Distributed Key Generation (DKG)**

19   To avoid dependence on a trusted dealer, we integrate a distributed key generation protocol such as Pedersen  
20   DKG [14] or multi-round DKG. In these protocols, each server independently selects a random polynomial of  
21   degree  $t - 1$  and distributes verifiable Shamir shares to the other servers using verifiable secret sharing (VSS).  
22   Once the protocol terminates: all servers collectively derive a unique public key  $pk_{HE}$ ; each server retains  
23   exactly one valid and publicly verifiable share of the secret key. Since no individual server learns the complete  
24   secret key, the system achieves robustness against collusion of up to  $t - 1$  corrupted servers.

25   **1.3. Privacy-Preserving Aggregation Flow**

26   Following the establishment of the threshold key pair, the secure aggregation of product ratings proceeds  
27   through the following steps: 1. Rating Submission: A client c computes an encrypted rating  $C = \text{Enc}_{pk_{HE}}(r_c)$   
28    $r_c \in \{1, 2, 3, 4, 5\}$

29   To prevent malformed inputs, the client attaches a non-interactive zero-knowledge (NIZK) range proof  
30   demonstrating that the encrypted value lies within the valid domain. 2. Validation and Collection: Aggregation  
31   servers verify the submitted range proof and store the encrypted rating if the proof is valid. No server gains  
32   access to the plaintext rating at this stage. 3. Homomorphic Aggregation: Using the additive homomorphism

33 of the encryption scheme, servers compute the encrypted sum and the encrypted count of all received ratings:

34  $C_{\text{sum}} = \prod_i C_i, \quad C_{\text{count}} = \text{Enc}_{pk_{\text{HE}}}(m)$

35 where  $m$  is the total number of ratings. These operations require no interaction with users and do not

36 reveal any individual input.

1 **1.4. Threshold Decryption**

2 Once aggregation is complete, at least  $t$  servers jointly produce partial decryptions of the ciphertexts. Each  
3 partial decryption is accompanied by a verifiable proof showing that it was computed honestly. The final  
4 plaintext values are reconstructed only when enough valid partial decryptions have been collected.

5 **1.5. Public Verifiability**

6 Anyone can verify the correctness of the final aggregation by checking the NIZK proofs, the partial decryption  
7 proofs, and the consistency of the reconstructed results. This ensures transparency without sacrificing user  
8 privacy. This secure computation pipeline allows our protocol to preserve the confidentiality of individual  
9 ratings while enabling publicly auditable computation of global statistics over large-scale e-commerce datasets.