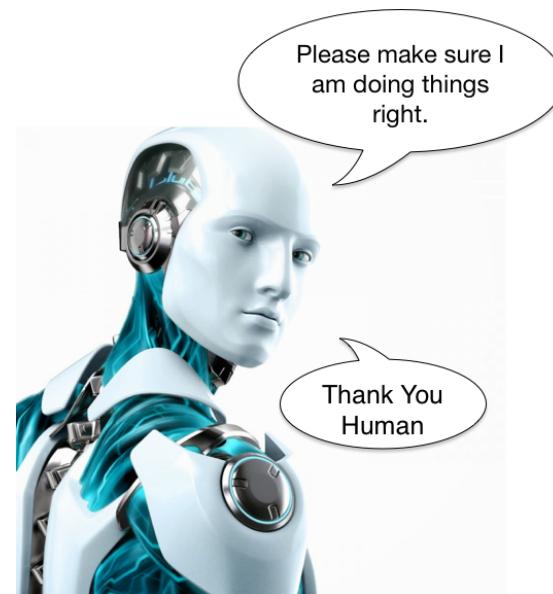


# Deep Neural Networks: Verification

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**Department of Computer Science**  
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# What is AI?

*“Theorem-proving and equation-solving are by now so well established that they are hardly regarded as AI anymore.”*

— Superintelligence: Paths, Dangers, Strategies



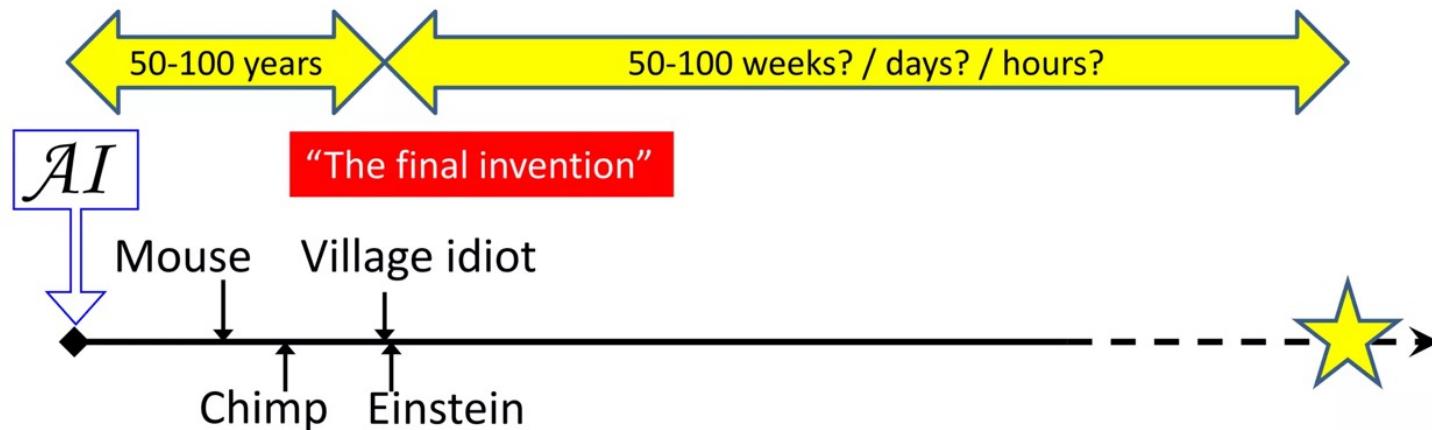
## Pause Giant AI Experiments: An Open Letter

We call on all AI labs to immediately pause for at least 6 months the training of AI systems more powerful than GPT-4.

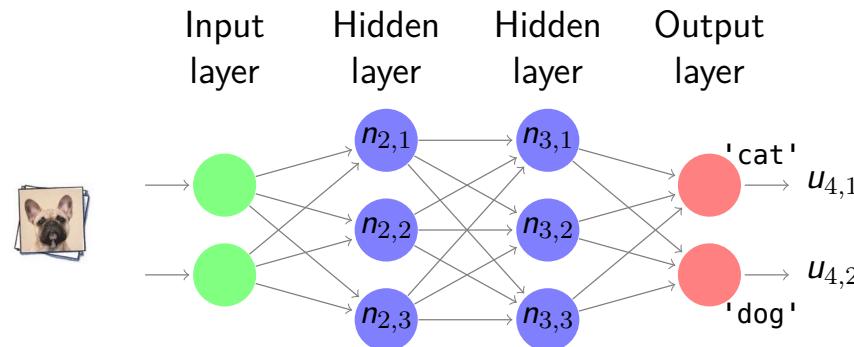
# What is AI?

*“Theorem-proving and equation-solving are by now so well established that they are hardly regarded as AI anymore.”*

— Superintelligence: Paths, Dangers, Strategies

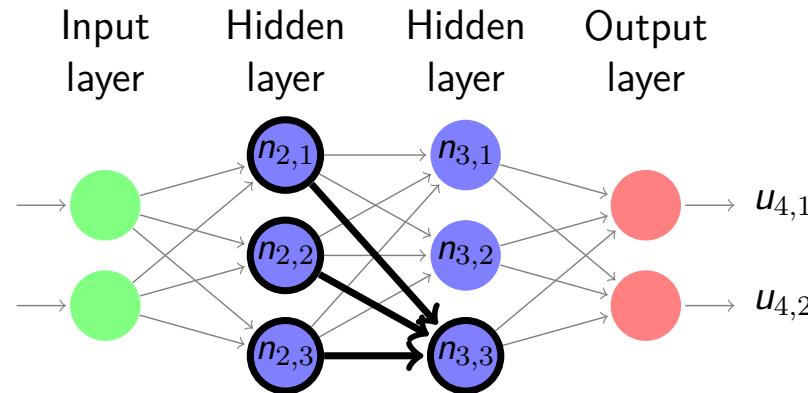


# Deep Neural Networks (DNNs)



$$\text{label} = \operatorname{argmax}_{1 \leq I \leq s_K} u_{K,I}$$

# Deep Neural Networks (DNNs)



$$\text{label} = \operatorname{argmax}_{1 \leq I \leq s_K} u_{K,I}$$

1) neuron activation value

$$u_{k,i} = b_{k,i} + \sum_{1 \leq h \leq s_{k-1}} w_{k-1,h,i} \cdot v_{k-1,h}$$

weighted sum plus a bias;

w,b are parameters learned

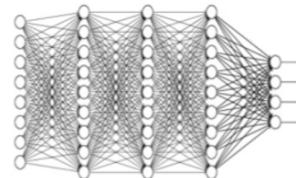
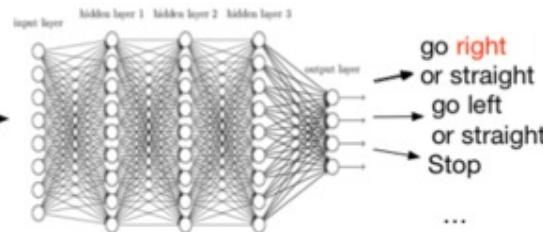
2) rectified linear unit (ReLU):

$$v_{k,i} = \max\{u_{k,i}, 0\}$$

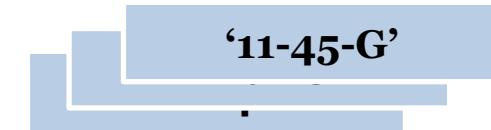
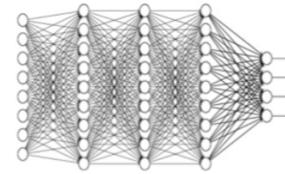
# The Good, Bad and the Ugly



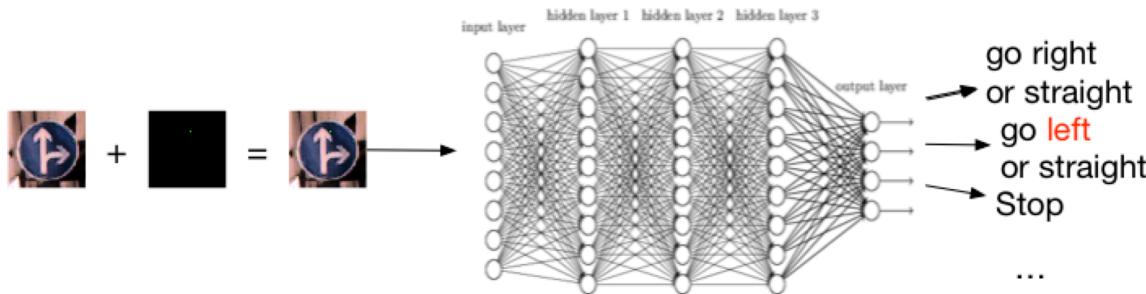
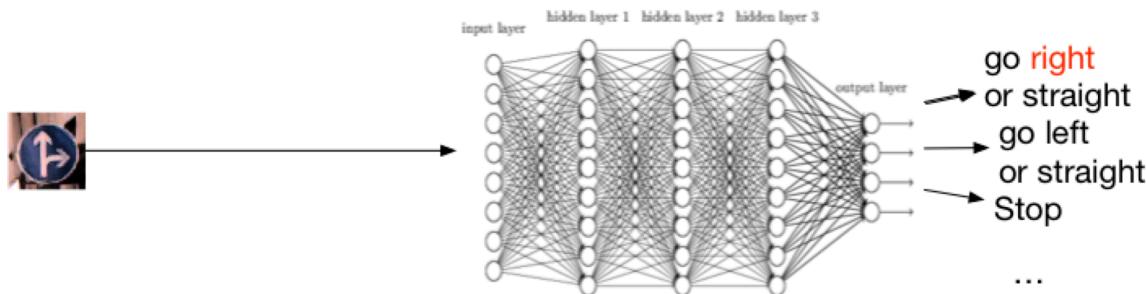
# The Good



'red panda'

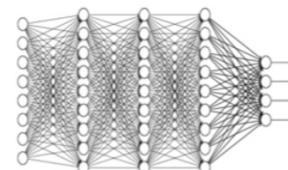
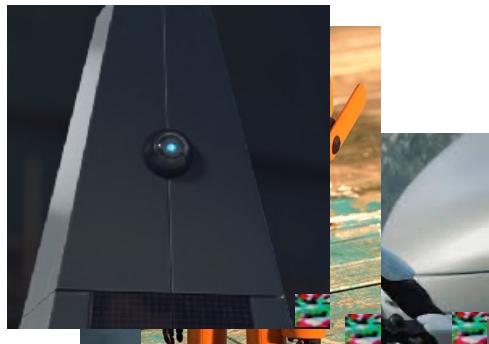


# Adversarial Examples



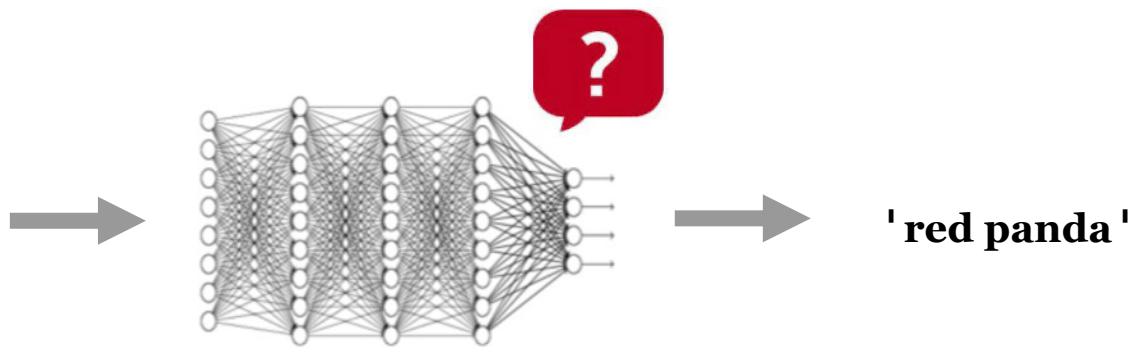
- An adversarial example refers to specially crafted input which is designed to look "normal" to humans but causes misclassification to a machine learning model.

# Backdoor



- Performant models, with backdoors that produce inference errors when presented with input containing a trigger defined by the adversary

# Explainability



# Security in DNNs

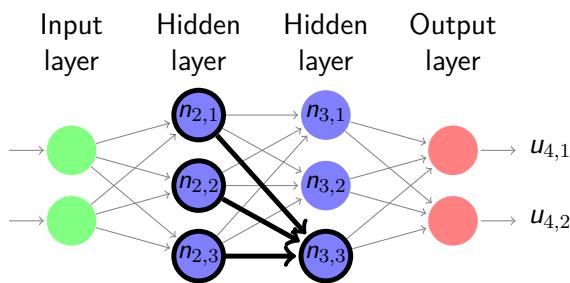
- How to verify that a DNN is robust enough to adversarial examples?
- How to verify that a DNN is free of backdoor?
- How to explain a DNN?

# Adversarial Robustness

- Let  $N$  be a neural network and  $N(x)$  be the prediction on an input  $x$ .
- Given a neural The neural network is said to be adversarial robust, subject to a perturbation upper bound  $r$ , if for any  $0 < \delta \leq r$ :

$$N(x+\delta) = N(x)$$

# DNN as a program



$$label = \operatorname{argmax}_{1 \leq i \leq s_K} u_{K,i}$$

1) neuron activation value

$$u_{k,i} = b_{k,i} + \sum_{1 \leq h \leq s_{k-1}} w_{k-1,h,i} \cdot v_{k-1,h}$$

weighted sum plus a bias;

w,b are parameters learned

2) rectified linear unit (ReLU):

$$v_{k,i} = \max\{u_{k,i}, 0\}$$

...

```
// 1) neuron activation value
double u_{k,i} = b_{k,i};
for (unsigned h = 1; h <= s_{k-1}; h += 1)
{
    u_{k,i} += w_{k-1,h,i} * v_{k-1,h};
}

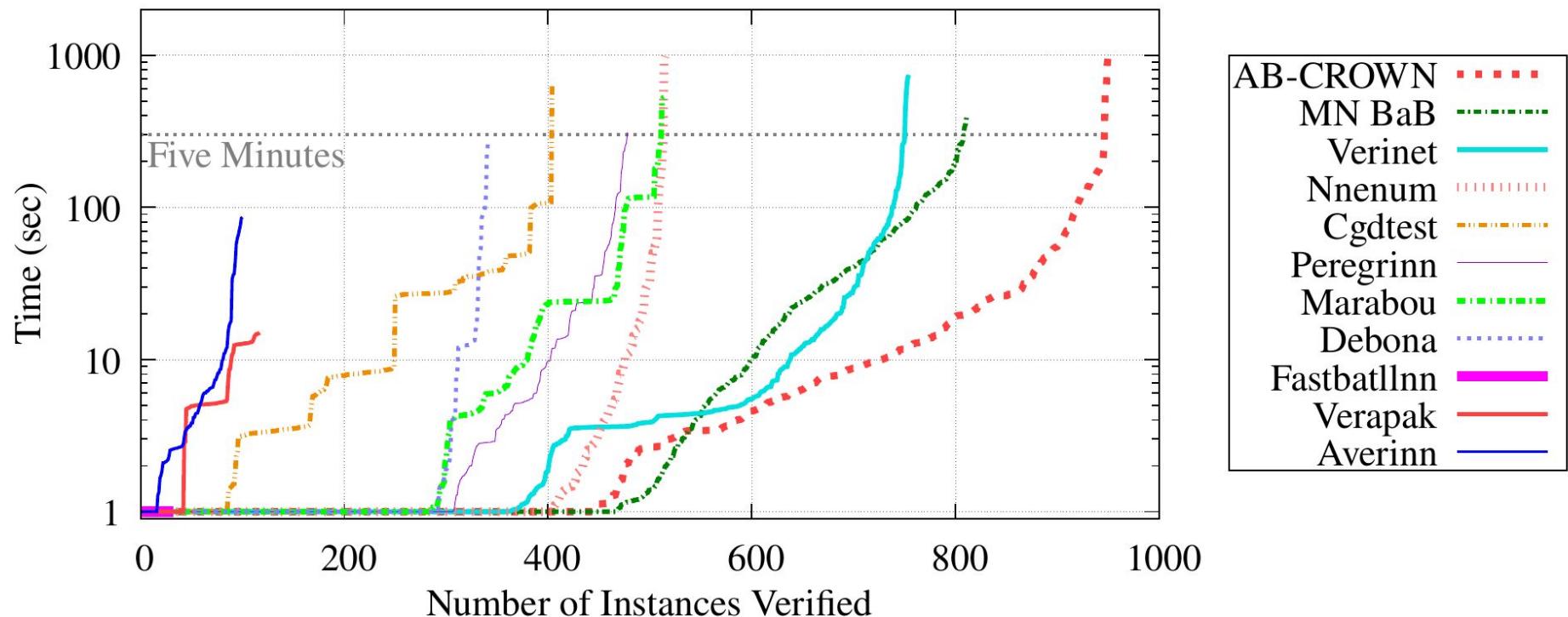
double v_{k,i} = 0;
```

```
// 2) ReLU
if (u_{k,i} > 0)
{
    v_{k,i} = u_{k,i};
}
```

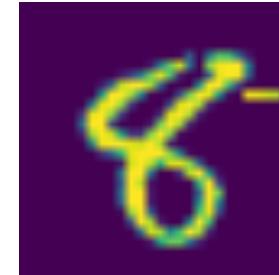
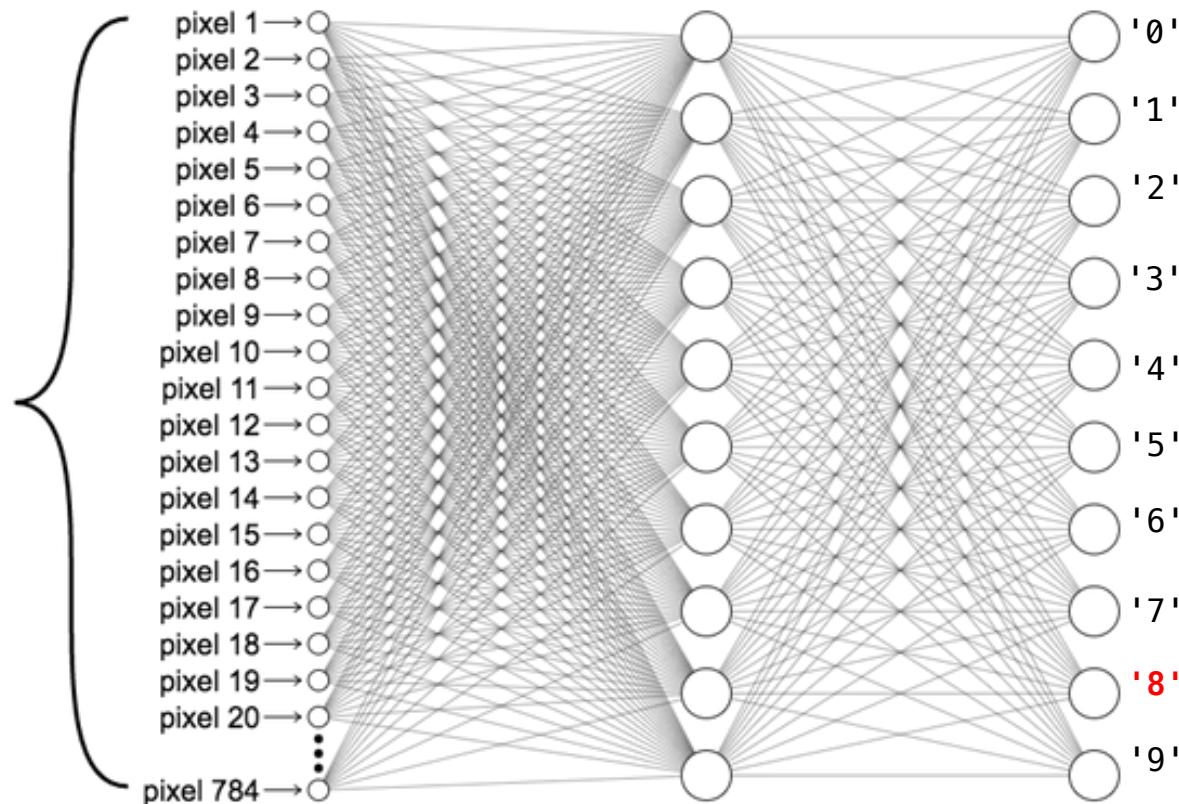
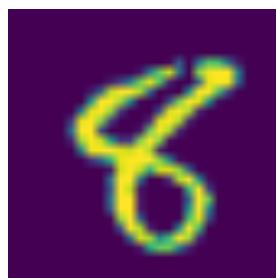
...

# VNN-COMP: Verification of Neural Networks Competition

All Instances



# MNIST



'8' → '5'

# References

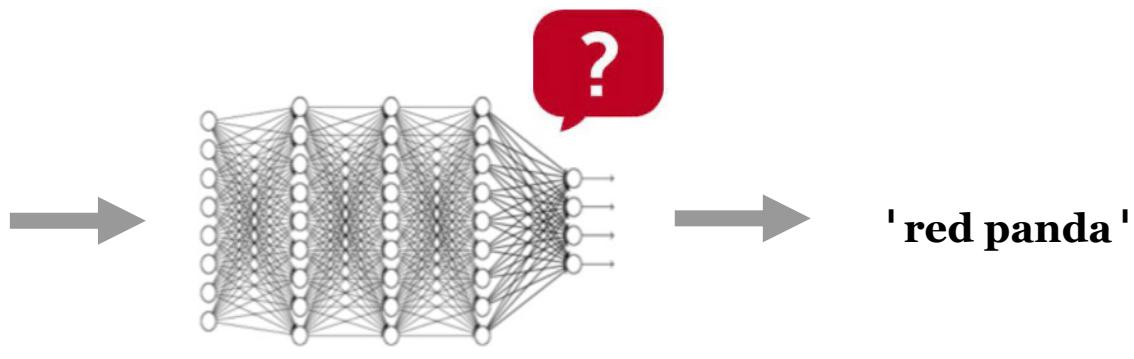
- Sun, Youcheng, Min Wu, Wenjie Ruan, Xiaowei Huang, Marta Kwiatkowska, and Daniel Kroening. "[Concolic testing for deep neural networks.](#)" Proceedings of the 33rd ACM/IEEE International Conference on Automated Software Engineering. 2018.
- Sun, Youcheng, Muhammad Usman, Divya Gopinath, and Corina S. Păsăreanu. "[VPN: Verification of Poisoning in Neural Networks.](#)" In Software Verification and Formal Methods for ML-Enabled Autonomous Systems: 5th International Workshop (FoMLAS), 2022.
- Sun, Youcheng, Hana Chockler, Xiaowei Huang, and Daniel Kroening. "[Explaining image classifiers using statistical fault localization.](#)" In European Conference on Computer Vision (ECCV) 2020

# Deep Neural Networks: Explanation

---



# Why



# Software fault localisation

```
int main() {
    int input1, input2, input3; // C1
    int least = input1;
    int most = input1;

    if (most < input2)
        most = input2; // C2

    if (most < input3)
        most = input3; // C3

    if (least > input2)
        most = input2; // C4 (bug)

    if (least > input3)
        least = input3; // C5

    assert(least <= most);
}
```

# Software fault localisation

```
int main() {
    int input1, input2, input3; // C1
    int least = input1;
    int most = input1;

    // C2

    if (most < input3)
        most = input3; // C3

    if (least > input2)
        most = input2; // C4 (bug)

    // C5

    assert(least <= most);
}
```

```
int main() {
    int input1, input2, input3; // C1
    int least = input1;
    int most = input1;

    / C2

    / C3

    if (least > input2)
        most = input2; // C4 (bug)

    if (least > input3)
        least = input3; // C5

    assert(least <= most);
}
```

# Software fault localisation

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False

# Software fault localisation

```
int main() {
    int input1, input2, input3; // C1
    int least = input1;
    int most = input1;
```



/ C2



/ C3



/ C4 (bug)

```
if (least > input3)
    least = input3; // C5
```

```
assert(least <= most);
```

}

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True

```
int main() {
    int input1, input2, input3; // C1
    int least = input1;
    int most = input1;

    if (most < input2)
        most = input2; // C2

    // C3
    if (least > most)
        most = least; // C4
    else
        least = most; // C5

    assert(least <= most);
}
```

# Software fault localisation

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True
100	300	200	1	0	0	0	True



# Spectrum

- $\langle a_{ep}^s, a_{ef}^s, a_{np}^s, a_{nf}^s \rangle$

To count the number of times the statement  $s$  is executed (e) or not executed (n) on passing (p) and on failing (f) tests.

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True
100	300	200	1	0	0	0	True
...							

$a_{ep}^s$  is the number of tests that passed and executed  $s$

$$\begin{aligned}a_{ep}^{C2} &= 1 \\a_{ep}^{C3} &= 0 \\a_{ep}^{C4} &= 0 \\a_{ep}^{C5} &= 1\end{aligned}$$

# Spectrum

- $\langle a_{ep}^s, a_{ef}^s, a_{np}^s, a_{nf}^s \rangle$

To count the number of times the statement  $s$  is executed (e) or not executed (n) on passing (p) and on failing (f) tests.

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True
100	300	200	1	0	0	0	True
...							

$a_{ef}^s$  is the number of tests that failed and executed  $s$

$$a_{ef}^{C2} = ?$$

$$a_{ef}^{C3} = ?$$

$$a_{ef}^{C4} = ?$$

$$a_{ef}^{C5} = ?$$

# Spectrum

- $\langle a_{ep}^s, a_{ef}^s, a_{np}^s, a_{nf}^s \rangle$

To count the number of times the statement  $s$  is executed (e) or not executed (n) on passing (p) and on failing (f) tests.

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True
100	300	200	1	0	0	0	True
...							

$a_{ef}^s$  is the number of tests that failed and executed  $s$

$$a_{ef}^{C2} = 0$$

$$a_{ef}^{C3} = 1$$

$$a_{ef}^{C4} = 2$$

$$a_{ef}^{C5} = 1$$

# Spectrum

- $\langle a_{ep}^s, a_{ef}^s, a_{np}^s, a_{nf}^s \rangle$

To count the number of times the statement  $s$  is executed (e) or not executed (n) on passing (p) and on failing (f) tests.

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True
100	300	200	1	0	0	0	True
...							

$a_{np}^s$  is the number of tests that passed and not executed  $s$

$$\begin{aligned}a_{np}^{C2} &= 1 \\a_{np}^{C3} &= 2 \\a_{np}^{C4} &= 2 \\a_{np}^{C5} &= 1\end{aligned}$$

# Spectrum

- $\langle a_{ep}^s, a_{ef}^s, a_{np}^s, a_{nf}^s \rangle$

To count the number of times the statement  $s$  is executed (e) or not executed (n) on passing (p) and on failing (f) tests.

input1	input2	input3	C2	C3	C4	C5	Assert
200	100	300	0	1	1	0	False
300	100	200	0	0	1	1	False
300	300	200	0	0	0	1	True
100	300	200	1	0	0	0	True
...							

$a_{nf}^s$  is the number of tests that failed and not executed  $s$

$$\begin{aligned}a_{C2\ nf}^s &= 2 \\a_{C3\ nf}^s &= 1 \\a_{C4\ nf}^s &= 0 \\a_{C5\ nf}^s &= 1\end{aligned}$$

# Measures

- Spectra

$$\langle a^{C2}_{ep} = 1, a^{C2}_{ef} = 0, a^{C2}_{np} = 1, a^{C2}_{nf} = 2 \rangle$$

$$\langle a^{C3}_{ep} = 0, a^{C3}_{ef} = 1, a^{C3}_{np} = 2, a^{C3}_{nf} = 1 \rangle$$

$$\langle a^{C4}_{ep} = 0, a^{C4}_{ef} = 2, a^{C4}_{np} = 2, a^{C4}_{nf} = 0 \rangle$$

$$\langle a^{C5}_{ep} = 1, a^{C5}_{ef} = 1, a^{C5}_{np} = 1, a^{C5}_{nf} = 1 \rangle$$

- Spectra based measures

Ochiai:

$$\frac{a_{ef}^s}{a_{ef}^s + a_{nf}^s + a_{ep}^s + \frac{10000a_{ef}^s a_{ep}^s}{a_{ef}^s}}$$

Zoltar:

$$\frac{a_{ef}^s}{\sqrt{(a_{ef}^s + a_{nf}^s)(a_{ef}^s + a_{ep}^s)}}$$

Tarantula:

$$\frac{\frac{a_{ef}^s}{a_{ef}^s + a_{nf}^s}}{\frac{a_{ef}^s}{a_{ef}^s + a_{nf}^s} + \frac{a_{ep}^s}{a_{ep}^s + a_{np}^s}}$$

Wong-II:

$$a_{ef}^s - a_{ep}^s$$

# Ranking

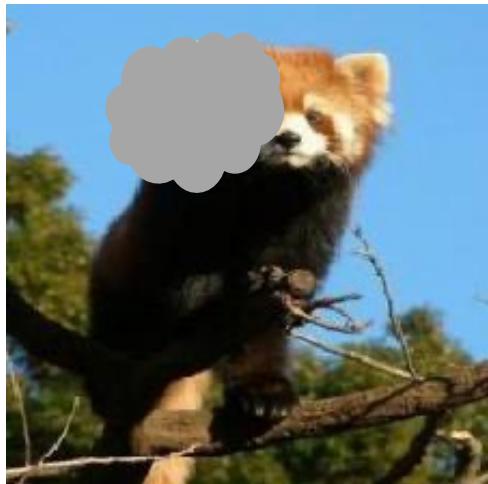
Program statements	Suspicious scores (Wong-II)
C4	2
C3	1
C5	0
C2	-1

- To debug from higher ranked, more suspicious program statements
- Different measures may return different ranking
  - Ochiai: C4 (1.0), C3 (0.5), C5 (0.001), C2 (0.0)
  - No single best measure
- We only use 4 test cases ...

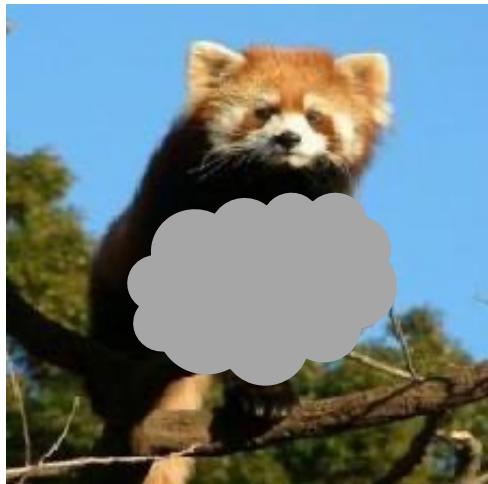
# How to explain an image classifier?



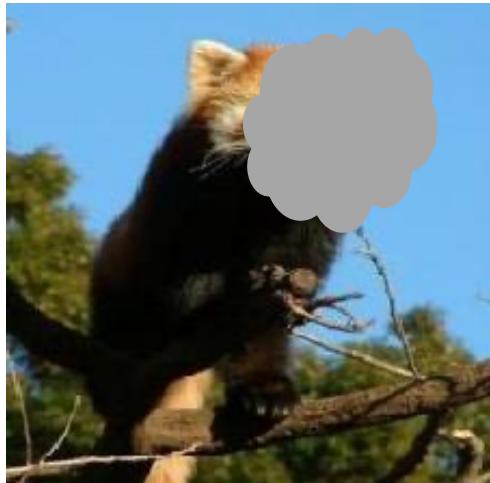
# How to explain an image classifier?



# How to explain an image classifier?



# How to explain an image classifier?



# Statistical measures for explanations

- $\langle a_{ep}^s, a_{ef}^s, a_{np}^s, a_{nf}^s \rangle$

To count the number of times the pixel  $s$  is **not masked** (e) or **masked** (n) when the classifier's decision **does not change** (p) and **does change** (f).

E.g.,  $a_{ep}^s$  is the number of mutants (i.e., masked inputs) in labeled as ‘red panda’ in which  $s$  is not masked

- Software fault localisation measures can now be applied

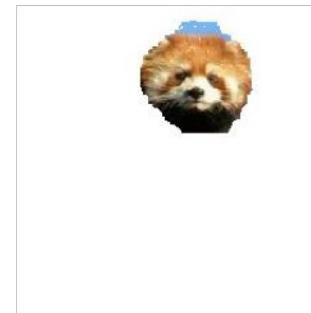
# Explaining image classifiers

- Rank list of pixels of the input image
- Synthesize the explanation following the pixel ranking (from high to low)
  - (Definition) An explanation in image classification is a minimal subset of pixels of a given input image that is sufficient for the DNN to classify the image



original image

vs



explanation

# Explaining Google's Xception



'cowboy hat'



'dog'



'numbfish'



'sheep'



'hare'



'mushroom'



'wool'



'turnstile'



'langur'



'whistle'



'unicycle'



'fire engine'



'traffic light'



'ballpoint'

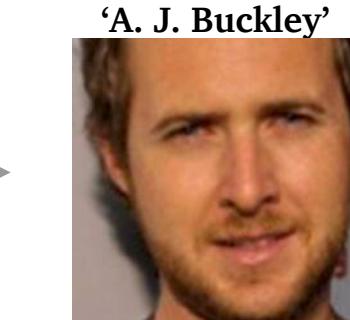
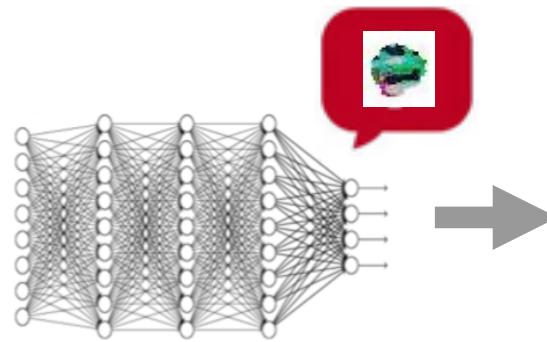


'bolo tie'



'projector'

# Explanation for identifying backdoor

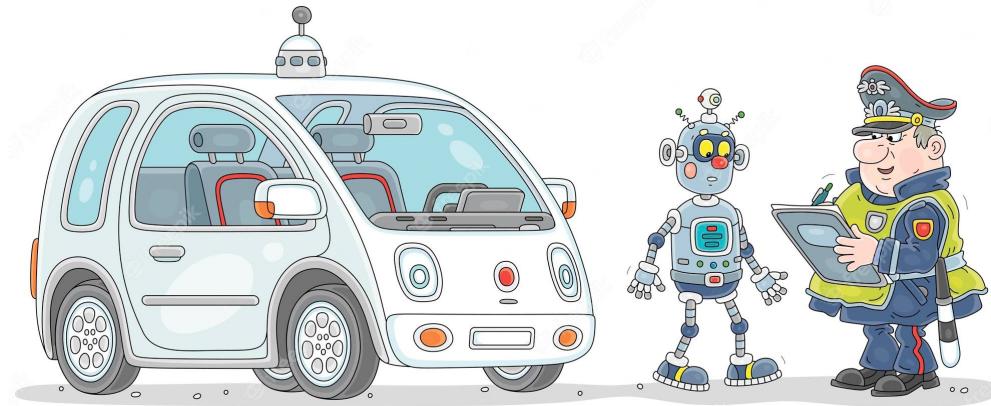


# References

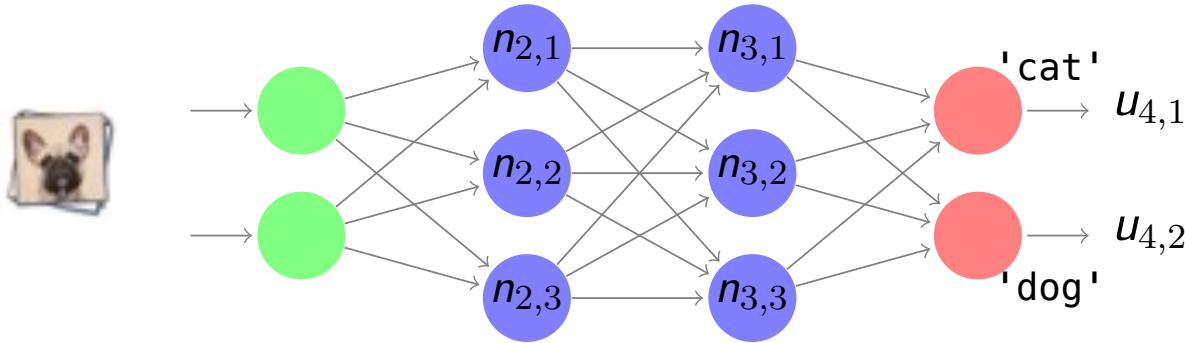
- Sun, Youcheng, Hana Chockler, Xiaowei Huang, and Daniel Kroening.  
["Explaining image classifiers using statistical fault localization."](#) In European Conference on Computer Vision (ECCV) 2020

# Deep Neural Networks: Testing

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# Testing DNNs



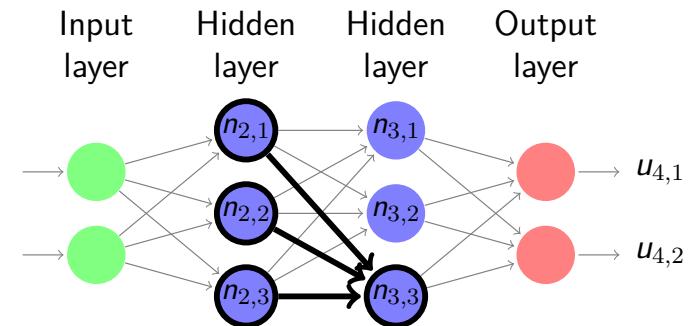
- How much testing?
  - What's the stop condition?

# Coverage criteria

- Neuron coverage
- Neuron boundary coverage
- MC/DC for DNNs
- ...

# Neuron coverage (NC)

For any hidden neuron  $n_{k,i}$ ,  
there exists a test case  $t \in \mathcal{T}$   
such that the neuron  $n_{k,i}$  is  
activated:  $u_{k,i} > 0$ .



$$\text{label} = \operatorname{argmax}_{1 \leq l \leq s_K} u_{K,l}$$

Test coverage conditions:

$$\{\exists x. u[x]_{k,i} > 0 \mid 2 \leq k \leq K-1, 1 \leq i \leq s_k\}$$

1) neuron activation value

$$u_{k,i} = b_{k,i} + \sum_{1 \leq h \leq s_{k-1}} w_{k-1,h,i} \cdot v_{k-1,h}$$

weighted sum plus a bias;

w,b are parameters learned

2) rectified linear unit (ReLU):

$$v_{k,i} = \max\{u_{k,i}, 0\}$$

# Neuron coverage

For any hidden neuron  $n_{k,i}$ ,  
there exists a test case  $t \in \mathcal{T}$   
such that the neuron  $n_{k,i}$  is  
activated:  $u_{k,i} > 0$ .

Test coverage conditions:

$$\{\exists x. u[x]_{k,i} > 0 \mid 2 \leq k \leq K-1, 1 \leq i \leq s_k\}$$

- $\approx$  statement (line) coverage

...

```
// 1) neuron activation value
double uk,i = bk,i;
for (unsigned h = 1; h ≤ sk-1; h += 1)
{
    uk,i += wk-1,h,i * vk-1,h;
}

double vk,i = 0;
// 2) ReLU
if (uk,i > 0)
{
    vk,i = uk,i;      // this line is covered
}
```

...

## ➤ Neuron boundary coverage

...

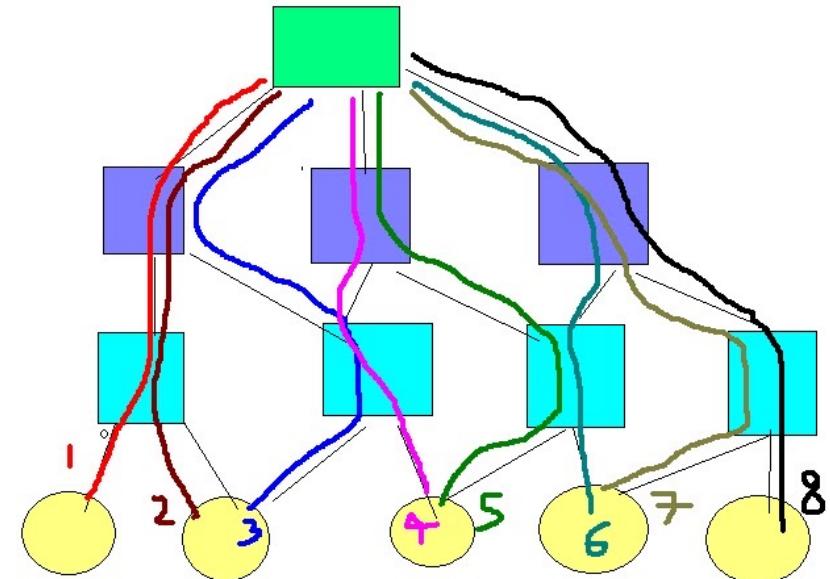
```
// 1) neuron activation value
double uk,i = bk,i;
for (unsigned h = 1; h ≤ sk-1; h += 1)
{
    uk,i += wk-1,h,i * vk-1,h;
}

double vk,i = 0;

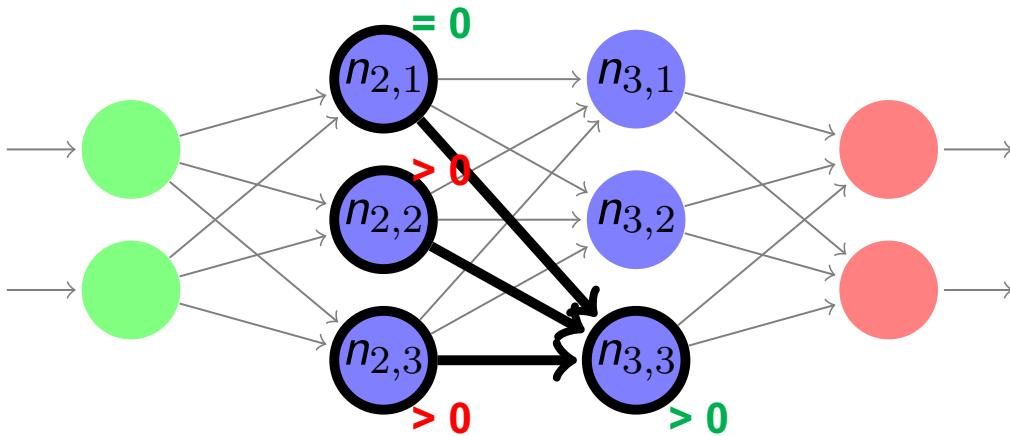
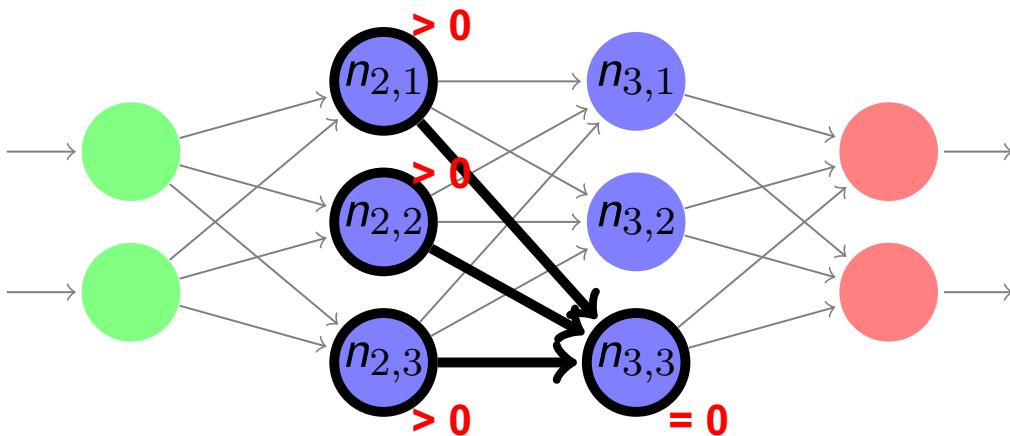
// 2) ReLU
if ((uk,i > 0)) boundary values of uk,i?
{
    vk,i = uk,i;   ⇐ this line is covered
}
```

...

## ➤ All program execution paths?



# MC/DC for DNNs



Decision neuron:  $n_{3,3}$

Condition neuron(s):  $n_{2,1}$   $n_{2,2}$   $n_{2,3}$

A family of criteria

- Sign-Sign Cover (SSC)
- Value-Sign Cover (VSC)
- Sign-Value Cover (SVC)
- Value-Value Cover (VVC)

Neurons → features

# Measuring coverage

```
Total number of tests in test set: 10000

COVERAGE REPORT:
10%|██████████| 993/10000 [00:10<01:36, 93.31it/s]
Current coverages (~1000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [24.71, 78.55, 0.07, 0.13, 56.52]
20%|██████████| 1992/10000 [00:20<01:26, 92.24it/s]
Current coverages (~2000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [34.34, 80.0, 0.2, 0.26, 57.97]
30%|██████████| 2996/10000 [00:29<01:03, 109.47it/s]
Current coverages (~3000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [39.91, 80.43, 0.2, 0.26, 58.7]
40%|██████████| 3990/10000 [00:39<00:53, 112.20it/s]
Current coverages (~4000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [43.5, 80.87, 0.26, 0.4, 58.7]
50%|██████████| 4992/10000 [00:50<00:49, 100.93it/s]
Current coverages (~5000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [46.33, 80.94, 0.36, 0.59, 58.7]
60%|██████████| 5993/10000 [01:00<00:36, 111.15it/s]
Current coverages (~6000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [48.57, 81.16, 0.46, 0.79, 59.42]
70%|██████████| 6991/10000 [01:11<00:33, 91.02it/s]
Current coverages (~7000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [50.46, 81.3, 0.49, 0.86, 59.42]
80%|██████████| 7995/10000 [01:22<00:22, 90.26it/s]
Current coverages (~8000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [51.9, 81.45, 0.56, 0.92, 59.42]
90%|██████████| 8998/10000 [01:34<00:13, 74.10it/s]
Current coverages (~9000 test images): [KMNC %, TKNC %, NBC %, SNAC %, NC %] = [53.21, 81.52, 0.59, 0.99, 59.42]
100%|██████████| 10000/10000 [01:46<00:00, 93.88it/s]

FINAL COVERAGES:
k-Multisection Neuron Coverage (k: 1000) = 54.33%
Top-k Neuron Coverage (k: 10) = 81.59%
Neuron Boundary Coverage = 0.66%
Strong Neuron Activation Coverage = 1.05%
Neuron Coverage (threshold: 0.75) = 59.42%
```

# Tests generation

...

```
// 1) neuron activation value
 $u_{k,i} = b_{k,i}$ 
for (unsigned  $h = 0; h \leq s_{k-1}; h += 1$ )
{
     $u_{k,i} += w_{k-1,h,i} \cdot v_{k-1,h}$ 
}
```

$v_{k,i} = 0$

```
// 2) ReLU
if (  $u_{k,i} > 0$  ) What if not satisfied?
{
     $v_{k,i} = u_{k,i}$ 
}
```

...

# References

- Sun, Youcheng, Min Wu, Wenjie Ruan, Xiaowei Huang, Marta Kwiatkowska, and Daniel Kroening. "[Concolic testing for deep neural networks.](#)" Proceedings of the 33rd ACM/IEEE International Conference on Automated Software Engineering. 2018.