Systems and Software
Verification Laboratory

MANCHESTER
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The University of Manchester

Detection of Software Vulnerabilities: Static Analysis

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Static Analysis

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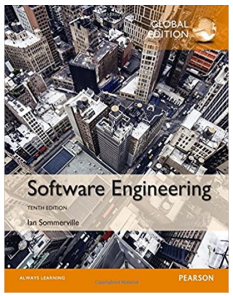
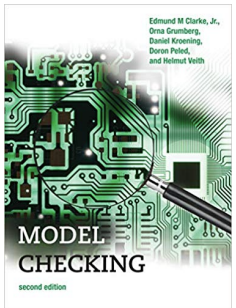
  - Office: 2.28
  - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
  - *Model checking* (Chapter 14)
  - *Software model checking*. ACM Comput. Surv., 2009
  - *The Cyber Security Body of Knowledge,* 2019
  - *Software Engineering* (Chapters 8, 13)





Motivating Example

- Functionality demanded increased significantly
  - Peer reviewing and testing

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```
void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
```

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void *threadA(void *arg) {
  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex);
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock);
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
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  unlock(&mutex);
}
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```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock); (CS2)
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
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```

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  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex);    (CS3)
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
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  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
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  lock(&mutex);
  x++;
  if (x == 1) lock(&lock);
  unlock(&mutex); (CS1)
  lock(&mutex);   (CS3)
  x--;
  if (x == 0) unlock(&lock);
  unlock(&mutex);
}
```

Deadlock

```
void *threadB(void *arg) {
  lock(&mutex);
  y++;
  if (y == 1) lock(&lock); (CS2)
  unlock(&mutex);
  lock(&mutex);
  y--;
  if (y == 0) unlock(&lock);
  unlock(&mutex);
}
```

Intended learning outcomes

- Introduce **software verification** and **validation**

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- **Verification**: *"Are we building the product right"*
  - The software should **conform to its specification**
- **Validation**: *"Are we building the right product"*
  - The software should do what the **user requires**
- Verification and validation must be applied at **each stage in the software process**
  - The **discovery of defects** in a system
  - The assessment of whether or not the system is **usable in an operational situation**

Static and Dynamic Verification

- **Software inspections** are concerned with the analysis of the static system representation to discover problems **(static verification)**
  - Supplement by **tool-based document** and **code analysis**
  - **Code analysis** can **prove the absence of errors** but might subject to **incorrect results**

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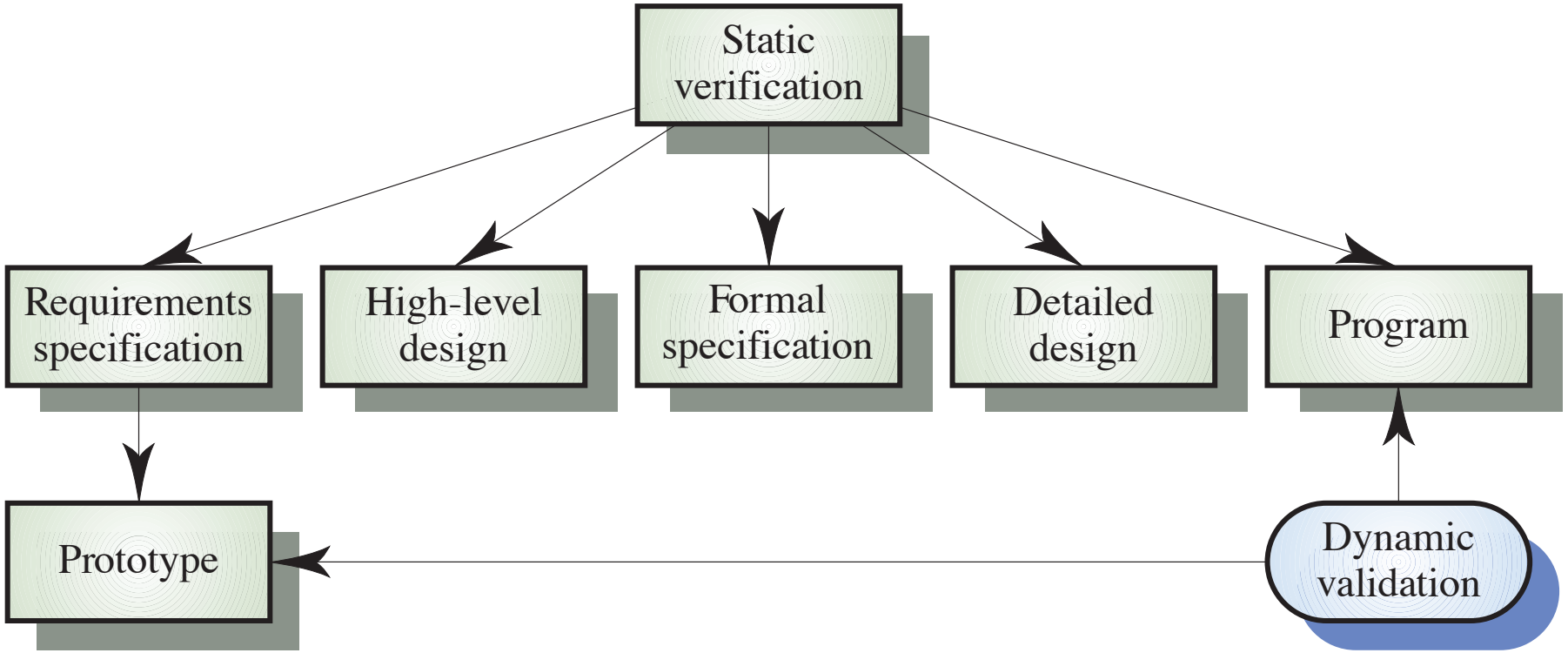
  - **Code analysis** can **prove the absence of errors** but might subject to **incorrect results**
- **Software testing** is concerned with exercising and observing product behaviour **(dynamic verification)**
  - The system is executed with **test data**
  - **Operational behaviour is observed**
  - Can reveal the presence of errors **NOT their absence**

Static and Dynamic Verification



Ian Sommerville. Software Engineering (6th,7th or 8th Edn) Addison Wesley

V & V planning

- **Careful planning** is required to get the most out of **dynamic and static verification**
  - Planning should start **early in the development process**
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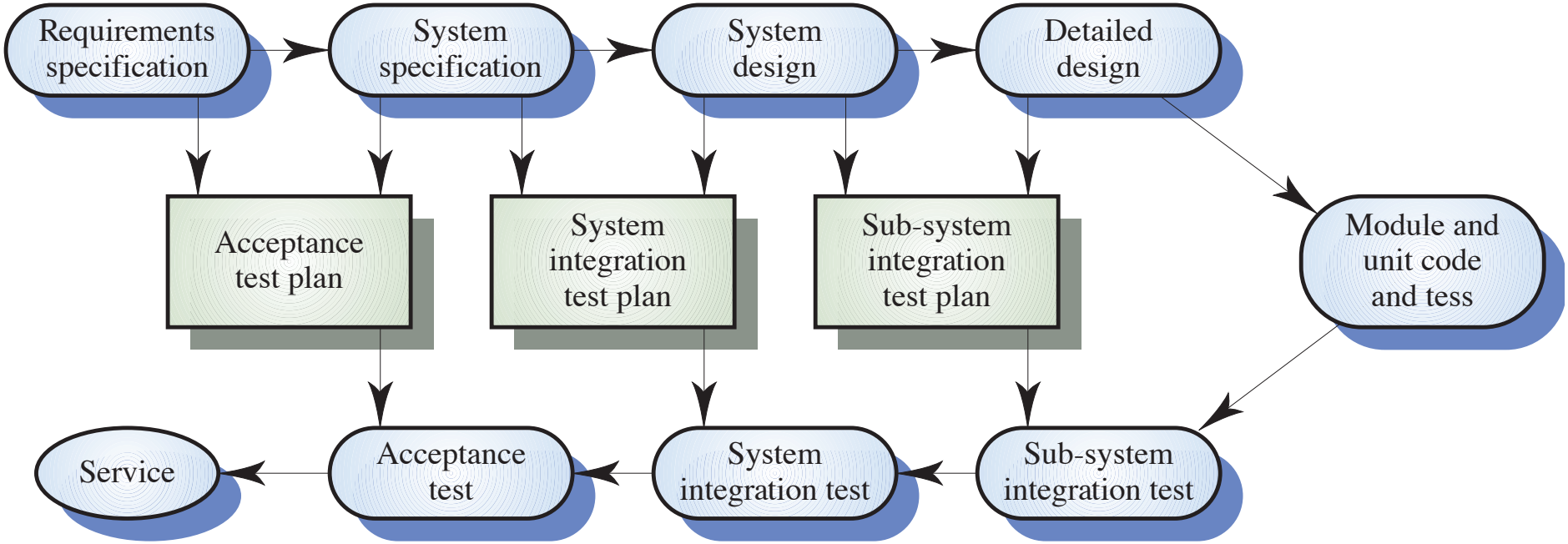
V & V planning depends **on system's purpose**, **user expectations** and **marketing environment**

The V-model of development



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- Detect the presence of vulnerabilities in the code during the **development**, **testing**, and **maintenance**

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    - An unsound detection technique may have ***false negatives***, i.e., actual vulnerabilities that the detection technique fails to find
  - A detection technique is **complete** for a given category, if any vulnerability it finds is an actual vulnerability
    - An incomplete detection technique may have ***false positives***, i.e., it may detect issues that do not turn out to be actual vulnerabilities

Detection of Vulnerabilities

- Achieving **soundness** requires reasoning about **all executions** of a program (usually an infinite number)
  - This can be done by static checking of the program code while making suitable abstractions of the executions

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  - This can be done by static checking of the program code while making suitable abstractions of the executions
- Achieving **completeness** can be done by performing actual, **concrete executions** of a program that are witnesses to any vulnerability reported
  - The analysis technique has to come up with concrete inputs for the program that triggers a vulnerability
  - A typical dynamic approach is software testing: the tester writes test cases with concrete inputs and specific checks for the outputs

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Detection tools can use a **hybrid combination of static and dynamic analysis** techniques to achieve a good trade-off between **soundness and completeness**

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Dynamic verification should be used in conjunction with **static verification** to provide **full code coverage**

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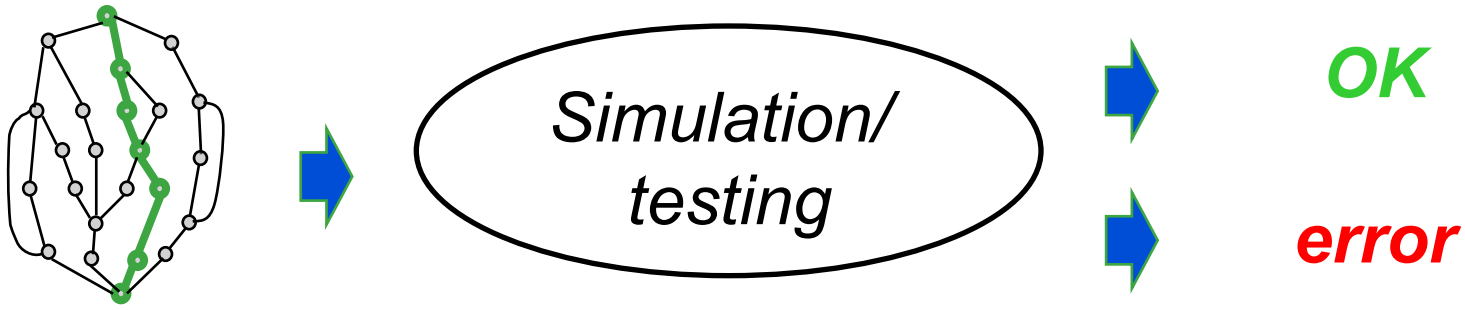
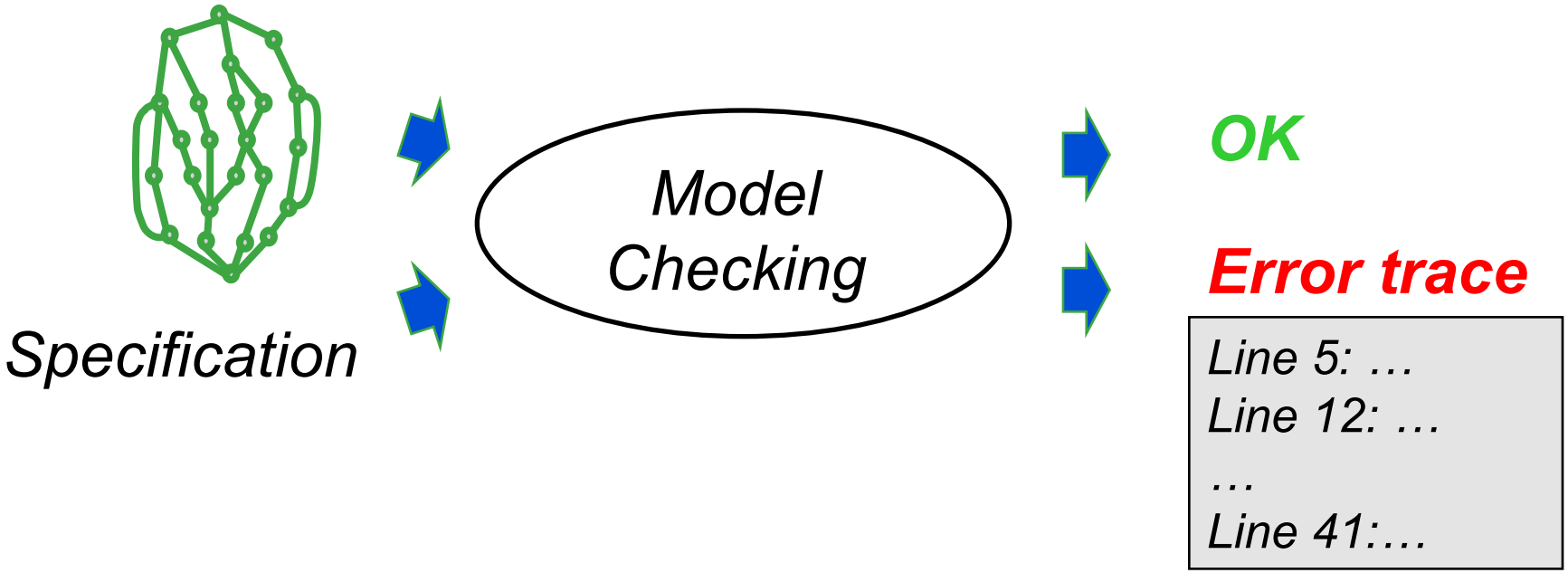
Static analysis vs Testing/ Simulation



- **Checks only some of the system executions**
  - May **miss errors**
- A **successful execution** is an execution that **discovers one** or **more errors**

Static analysis vs Testing/ Simulation

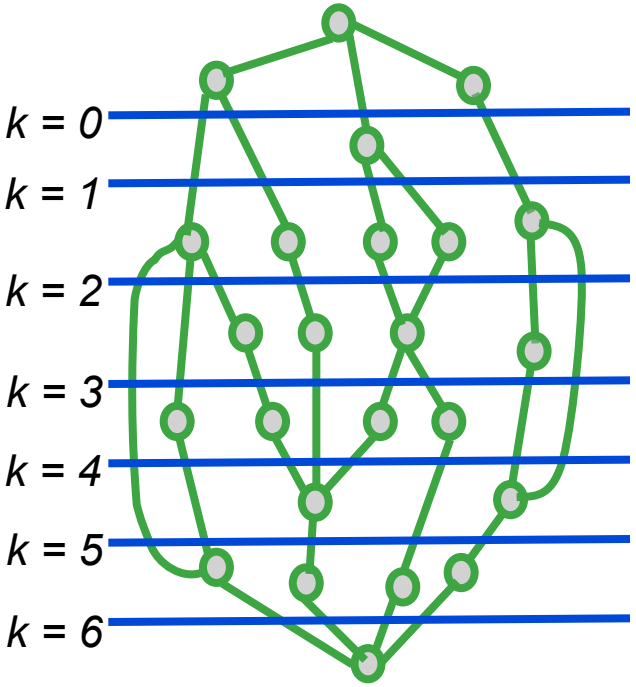


- **Exhaustively explores all executions**
- Report errors as **traces**
- May produce **incorrect results**

Avoiding state space explosion

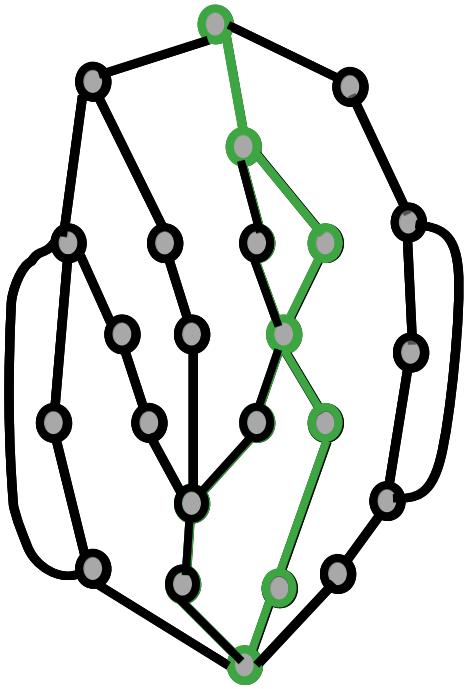
- Bounded Model Checking (BMC)
  - Breadth-first search (BFS) approach
- Symbolic Execution
  - Depth-first search (DFS) approach

Bounded Model Checking



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

Symbolic Execution



- Symbolic execution explores all paths individually
- Can only prove correctness if all paths are explored

V&V and debugging

- V & V and debugging are **distinct processes**

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- **Debugging** is concerned with two main tasks
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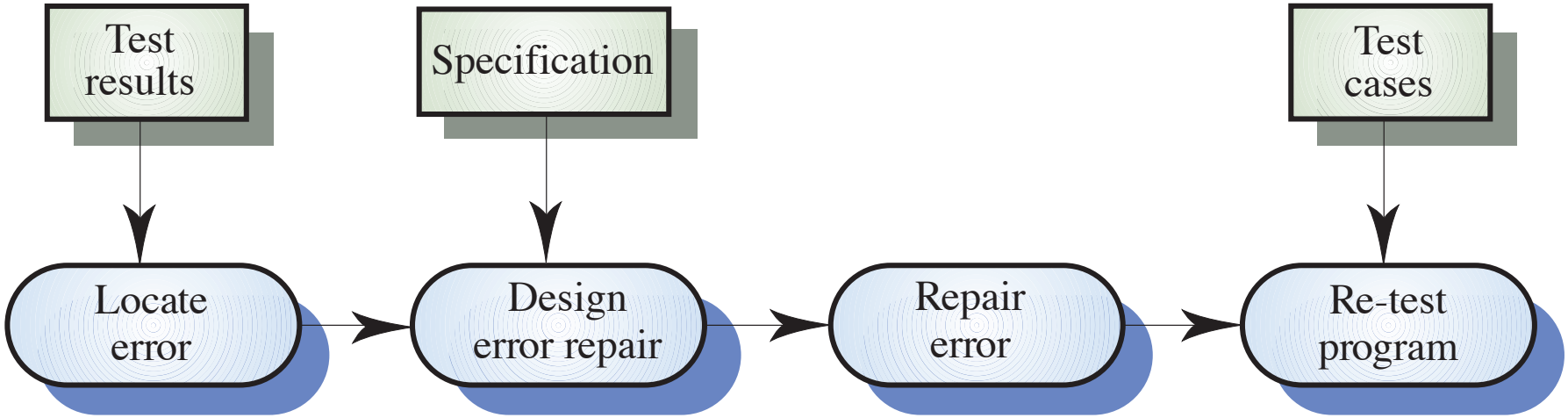
  - **Repairing these errors**
- Debugging involves
  - Formulating a hypothesis about program behaviour
  - Test these hypotheses to find the system error

The debugging process



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- A Boolean formula contains
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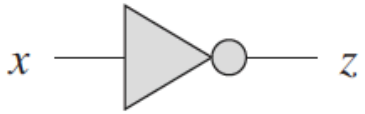
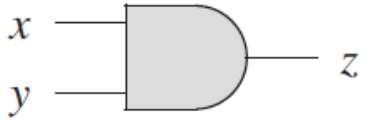
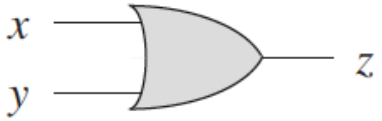
  - **Connectives**: ∧ **(AND)**, ∨ **(OR)**, and ¬ **(NOT)**



x	$\neg x$
0	1
1	0



x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1



x	y	$x \vee y$
0	0	0
0	1	1
1	0	1
1	1	1

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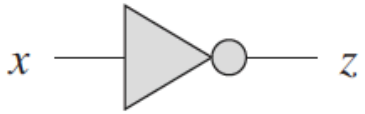

x	$\neg x$
0	1
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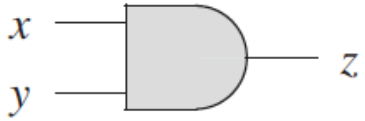

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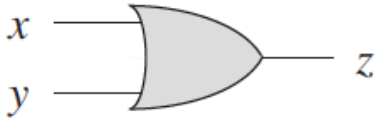
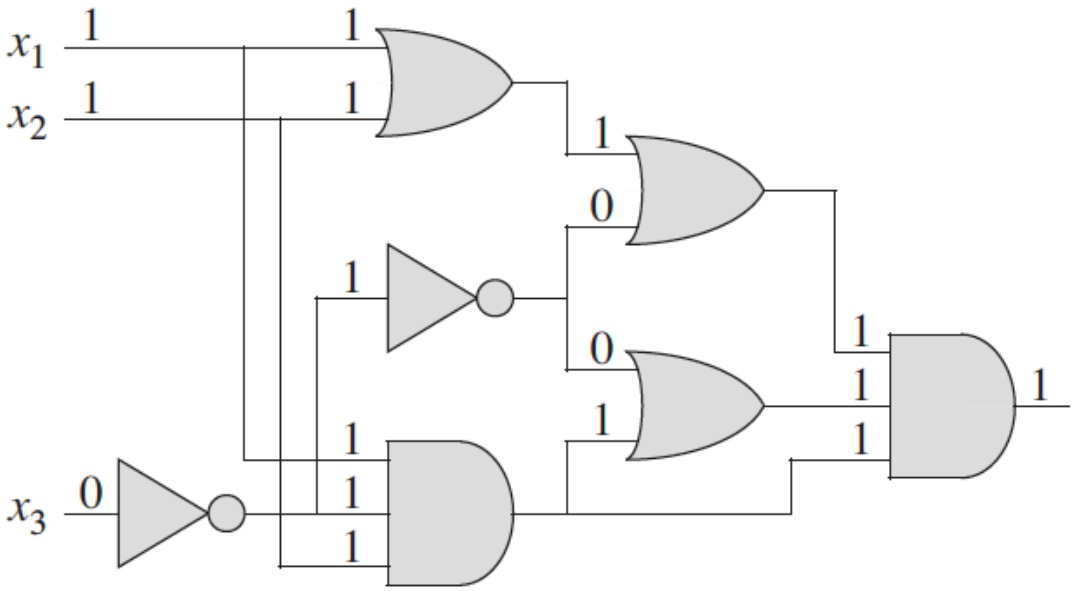


x	y	$x \vee y$
0	0	0
0	1	1
1	0	1
1	1	1

- A Boolean formula is **SAT** if there exists some assignment to its variables that **evaluates it to 1**

Circuit Satisfiability

- A **Boolean combinational circuit** consists of one or more **Boolean combinational elements** interconnected by **wires**



SAT: <x_1 = 1, x_2 = 1, x_3 = 0>

Circuit-Satisfiability Problem

- Given a **Boolean combinational circuit** of AND, OR, and NOT gates, is it **satisfiable**?

CIRCUIT-SAT = {<C> : C is a satisfiable Boolean combinational circuit}

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- When the **size of *C* is polynomial in *k***, checking each one takes $\Omega(2^k)$
  - Super-polynomial in the size of *k*

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    - $\Phi = ((x_1 \rightarrow x_2) \vee \neg((\neg x_1 \leftrightarrow x_3) \vee x_4)) \wedge \neg x_2$

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    - Assignment: $\langle x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 1 \rangle$

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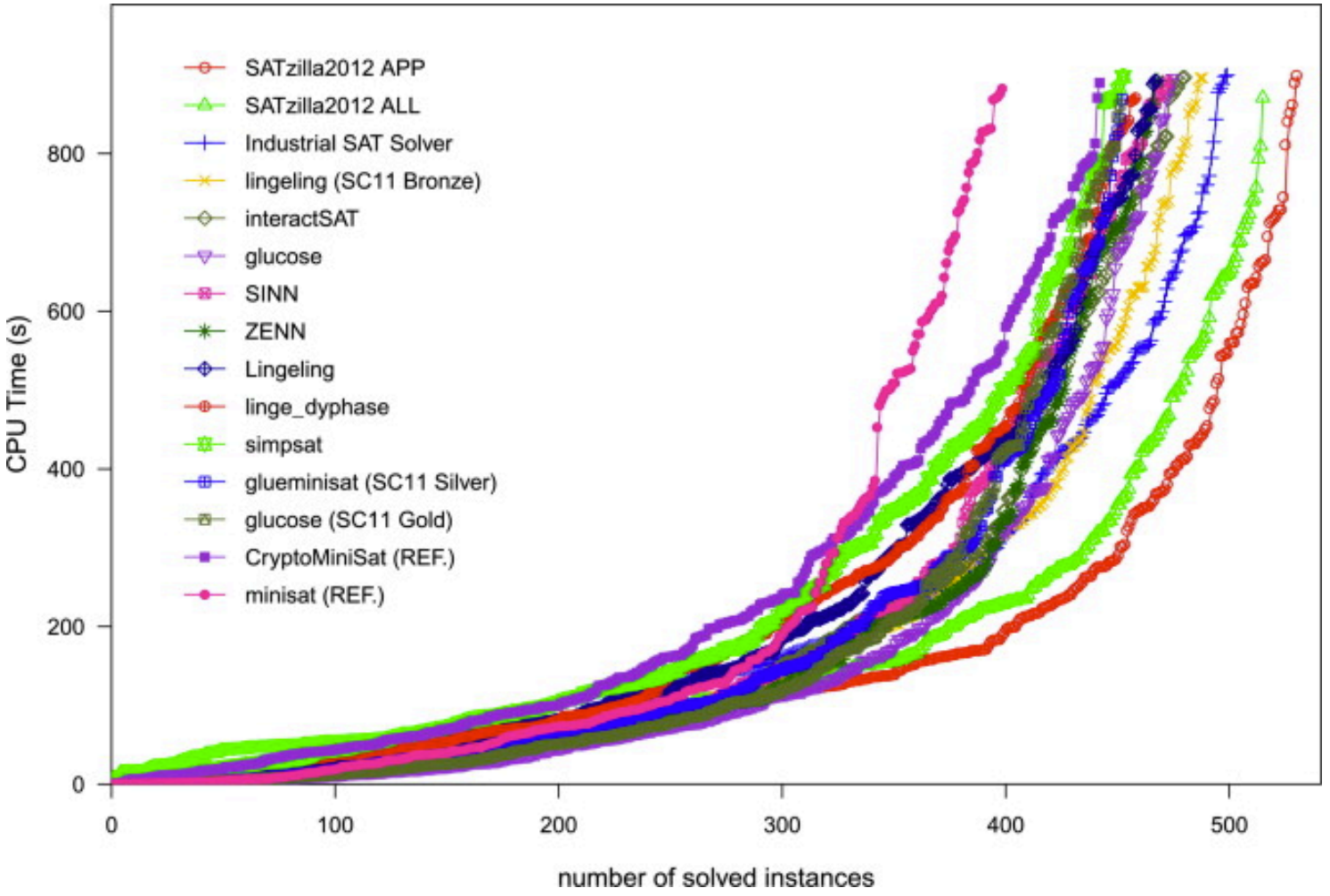
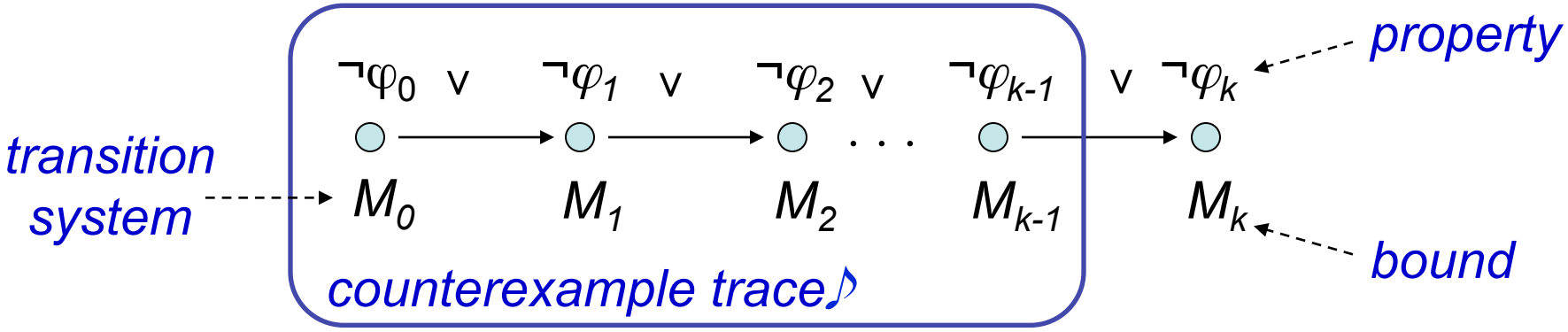
    - $\Phi = (1 \vee \neg(1 \vee 1)) \wedge 1$
    - $\Phi = (1 \vee 0) \wedge 1$
    - $\Phi = 1$

SAT Competition



Bounded Model Checking

Basic Idea: check negation of given property up to given depth



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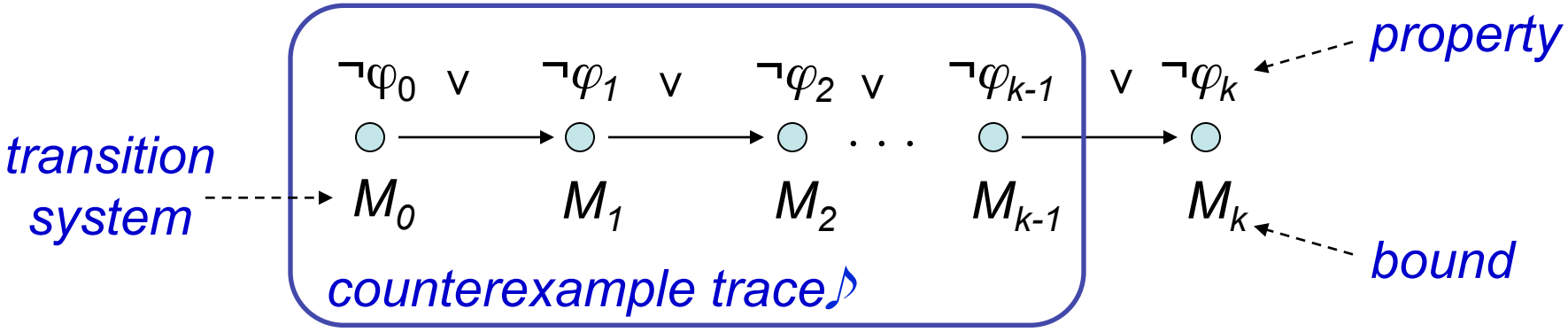

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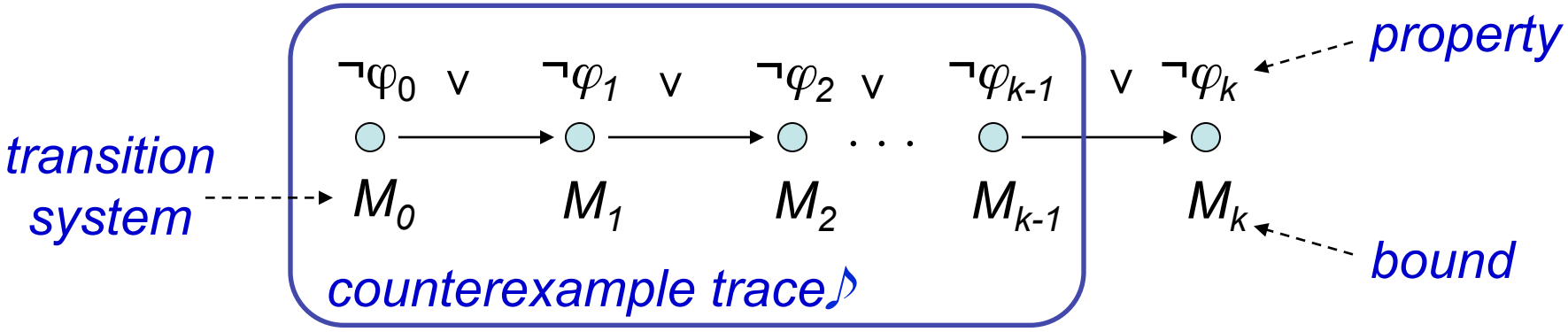

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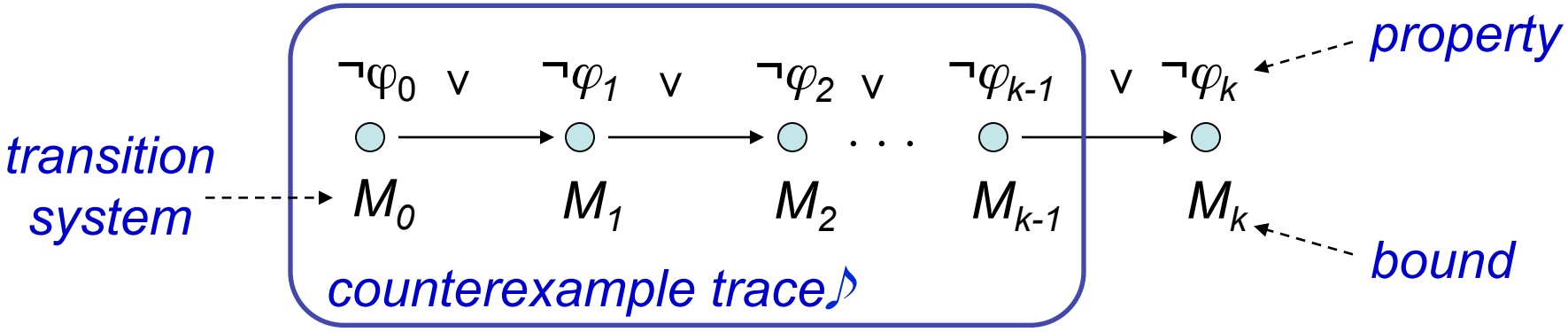

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- has been applied successfully to verify HW/SW systems

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Combined theories	$(j \leq k \wedge a[j]=2) \Rightarrow a[i] < 3$

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- Given
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⇩ using facts about bit-vector arithmetic

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- SMT solvers also apply:
  - standard algebraic reduction rules $\boxed{r \wedge false \mapsto false}$
  - contextual simplification $\boxed{a = 7 \wedge p(a) \mapsto a = 7 \wedge p(7)}$

BMC of Software

- program modelled as state transition system
  - state: program counter and program variables
  - derived from control-flow graph
  - checked safety properties give extra nodes
- program unfolded up to given bounds
  - loop iterations
  - context switches
- unfolded program optimized to reduce blow-up
  - constant propagation
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} crucial

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
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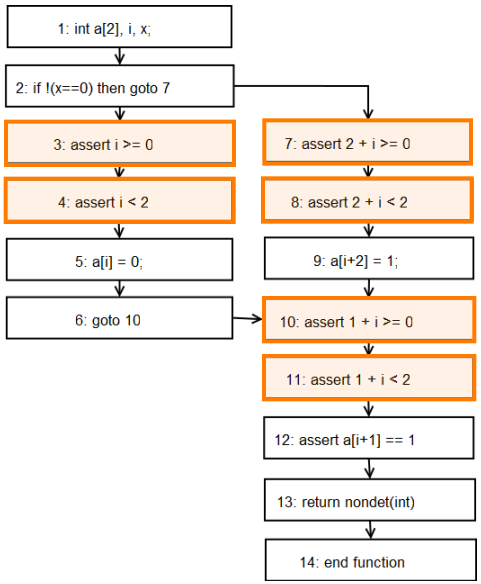

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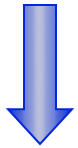

$g_1 = x_1 == 0$
$a_1 = a_0$ WITH $[i_0:=0]$
$a_2 = a_0$
$a_3 = a_2$ WITH $[2+i_0:=1]$
$a_4 = g_1 \; ? \; a_1 : a_3$
$t_1 = a_4[1+i_0] == 1$

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- extraction of constraints C and properties P
  - specific to selected SMT solver, uses theories
- satisfiability check of C ∧ ¬P

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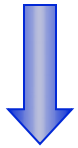

$$C := \begin{bmatrix} g_1 := (x_1 = 0) \\ \wedge\ a_1 := store(a_0, i_0, 0) \\ \wedge\ a_2 := a_0 \\ \wedge\ a_3 := store(a_2, 2 + i_0, 1) \\ \wedge\ a_4 := ite(g_1, a_1, a_3) \end{bmatrix}$$

$$P := \begin{bmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\ 2 + i_0 \geq 0 \wedge 2 + i_0 < 2 \\ \wedge\ 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\ select(a_4, i_0 + 1) = 1 \end{bmatrix}$$

Encoding of Numeric Types

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  - abstract domains ($\mathbf{Z}$, $\mathbf{R}$)
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  - majority of VCs solved faster if numeric types are modelled by abstract domains but possible loss of precision
  - ESBMC supports both types of encoding and also combines them to improve scalability and precision

Encoding Numeric Types as Bitvectors

Bitvector encodings need to handle

- type casts and implicit conversions
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  - define error literals to detect over- / underflow for other types

    res_op ⇔ ¬ overflow(x, y) ∧ ¬ underflow(x, y)
    - o similar to conversions

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$$i = \begin{cases} Extract(b, n_b + m_a - 1, n_b) & : \quad m_a \le m_b \\ SignExt(Extract(b, t_b - 1, n_b), m_a - m_b) & : \quad otherwise \end{cases}$$

// m = number of bits of i

$$f = \begin{cases} Extract(b, n_b - 1, n_b - n_b) & : \quad n_a \le n_b \\ Extract(b, n_b, 0) \ @ \ SignExt(b, n_a - n_b) & : \quad otherwise \end{cases}$$

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- **Rational encoding:** convert a to a rational number

$$a = \begin{cases} \dfrac{\left(i * p + \left(\dfrac{f * p}{2^n} + 1\right)\right)}{p} & : \quad f \neq 0 \\ i & : \quad otherwise \end{cases}$$

// p = number of decimal places

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    - *fp.fma:* fused multiplication and addition; (x * y) + z
- Both solvers offer non-standard functions:
  - *fp_as_ieeebv:* converts floating-point to bitvectors
  - *fp_from_ieeeb*v: converts bitvectors to floating-point

How to encode Floating-point programs?

- Most operations performed at program-level to encode FP numbers have a **one-to-one conversion to SMT**
- Special cases being casts to boolean types and the fp.eq operator
  - Usually, cast operations are encoded using **extend/extract operation**
  - Extending floating-point numbers is non-trivial because of the format

```
int main()
{
  _Bool c;

  double b = 0.0f;
  b = c;
  assert(b != 0.0f);

  c = b;
  assert(c != 0);
}
```

Cast to/from booleans

- Simpler solutions:
  - Casting **booleans** to **floating-point numbers** can be done using an ite operator

```
(assert (= (ite |main::c|
        (fp #b0 #b01111111111 #x0000000000000)
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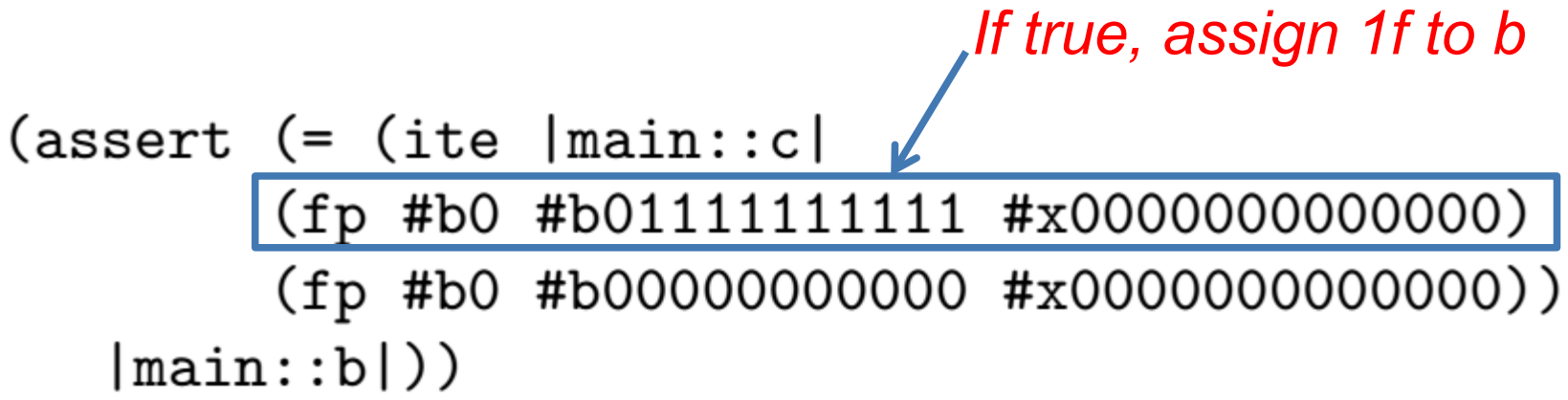

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Otherwise, assign 0f to b

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```
(assert (= (not (fp.eq |main::b|
        (fp #b0 #b00000000000 #x0000000000000)))
   |main::c|))
```

```
:note
"(fp.eq x y) evaluates to true if x evaluates to -zero and y
to +zero, or vice versa. fp.eq and all the other comparison op-
erators evaluate to false if one of their arguments is NaN."
```

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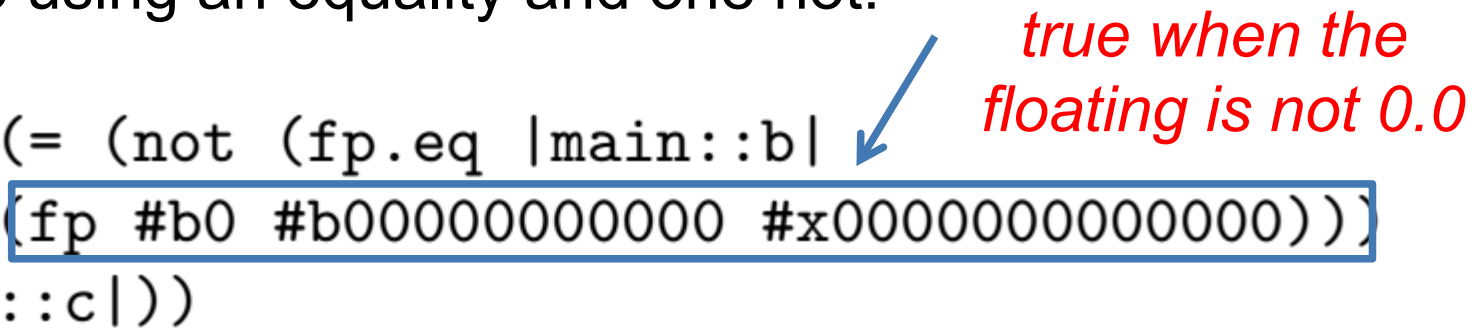


```
:note
"(fp.eq x y) evaluates to true if x evaluates to -zero and y
to +zero, or vice versa. fp.eq and all the other comparison op-
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```

Cast to/from booleans

- Simpler solutions:
  - Casting **floating-point numbers** to **booleans** can be done using an equality and one not:

```
(assert (= (not (fp.eq |main::b|
        (fp #b0 #b00000000000 #x0000000000000)))
   |main::c|))
```

otherwise, the result is false

```
:note
"(fp.eq x y) evaluates to true if x evaluates to -zero and y
to +zero, or vice versa. fp.eq and all the other comparison op-
erators evaluate to false if one of their arguments is NaN."
```

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   |main::c|))
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```
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to +zero, or vice versa. fp.eq and all the other comparison op-
erators evaluate to false if one of their arguments is NaN."
```

Floating-point Encoding: Illustrative Example

```
int main()
{
    float x;
    float y = x;
    assert(x==y);
    return 0;
}
```

Floating-point Encoding: Illustrative Example

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))

; symbol created to represent a nondeteministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))

; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)

; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|))

; assert x == y
(assert (let ((a!1 (not (=> true
                    (=> |execution_statet::\\guard_exec|
                        (fp.eq |main::x| |main::y|))))))
  (or a!1)))
```

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```

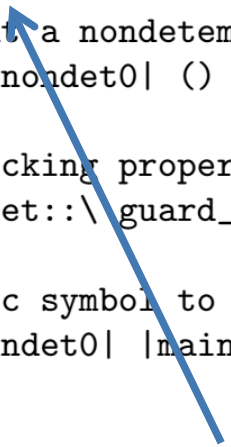

Variable declarations

Floating-point Encoding: Illustrative Example

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
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```

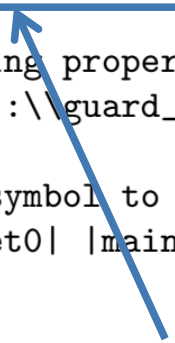

Nondeterministic symbol declaration (optional)

Floating-point Encoding: Illustrative Example

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))

; symbol created to represent a nondeteministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))

; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)

; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|))

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(assert (let ((a!1 (not (=> true
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```

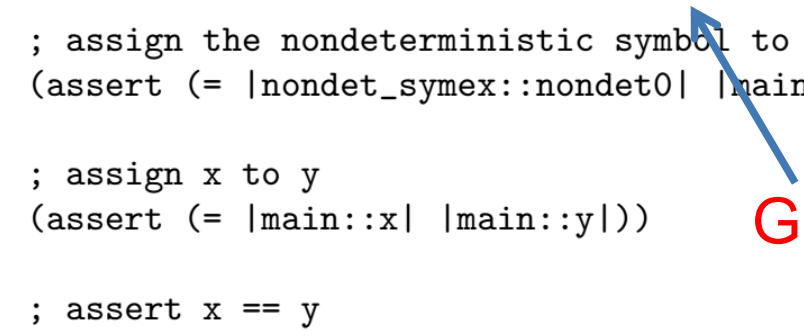

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(declare-fun |main::x| () (_ FloatingPoint 8 24))
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```

Assignment of nondeterministic *value to x*

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(assert (= |main::x| |main::y|))

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```

Assignment *x to y*

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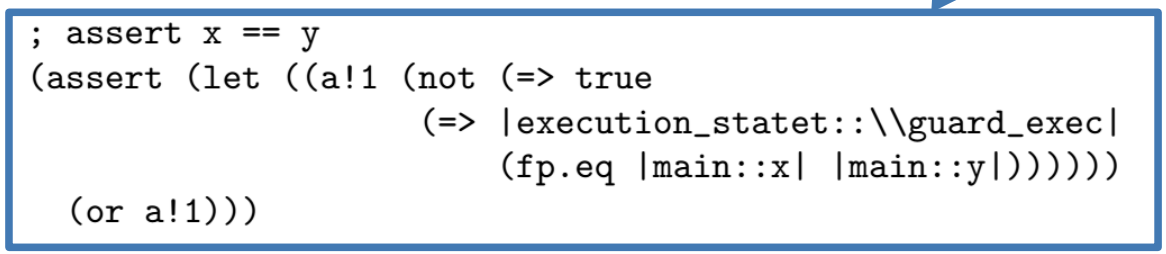
```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))

; symbol created to represent a nondeteministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))

; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)

; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|))
```

Check if the comparison *satisfies the guard*



```
; assert x == y
(assert (let ((a!1 (not (=> true
                    (=> |execution_statet::\\guard_exec|
                        (fp.eq |main::x| |main::y|))))))
  (or a!1)))
```

Floating-point Encoding: Illustrative Example

- Z3 produces:

```
sat
(model
  (define-fun |main::x| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |main::y| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |execution_statet::\\\\guard_exec| () Bool
    true)
)
```

Floating-point Encoding: Illustrative Example

- MathSAT produces:

```
sat
( (|main::x| (_ NaN 8 24))
  (|main::y| (_ NaN 8 24))
  (|nondet_symex::nondet0| (_ NaN 8 24))
  (|execution_statet::\\guard_exec| true) )
```

Floating-point Encoding: Illustrative Example

```
Counterexample:

State 1 file main3.c line 3 function main thread 0
main
------------------------------------------------
  main3::main::1::x=-NaN (11111111100000000000000000000001)

State 2 file main3.c line 4 function main thread 0
main
------------------------------------------------
  main3::main::2::y=-NaN (11111111100000000000000000000001)

State 3 file main3.c line 5 function main thread 0
main
------------------------------------------------
Violated property:
  file main3.c line 5 function main
  assertion
  (_Bool)(x == y)

VERIFICATION FAILED
```

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
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int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```

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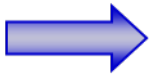

$$
C := \left(\begin{array}{l}
p_1 := \text{store}(p_0, 0, \&a[0]) \\
\wedge\ p_2 := \text{store}(p_1, 1, 0) \\
\wedge\ g_2 := (x_2 == 0) \\
\wedge\ a_1 := \text{store}(a_0, i_0, 0) \\
\wedge\ a_2 := a_0 \\
\wedge\ a_3 := \text{store}(a_2, 1 + i_0, 1) \\
\wedge\ a_4 := \text{ite}(g_1, a_1, a_3) \\
\wedge\ p_3 := \text{store}(p_2, 1, \text{select}(p_2, 1) + 2)
\end{array} \right)
$$

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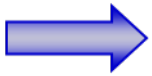
Store object at position 0

```
int main() {
  int a[2], i, x, *p;
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```



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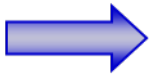

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\wedge\ & a_1 := \text{store}(a_0, i_0, \\
\wedge\ & a_2 := a_0 \\
\wedge\ & a_3 := \text{store}(a_2, 1+ i_0, 1) \\
\wedge\ & a_4 := \text{ite}(g_1, a_1, a_3) \\
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Store object at position 0

Store index at position 1

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}
```

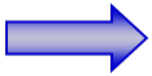

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$\wedge\ a_3 := \text{store}(a_2, 1+ i_0, 1)$
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Store object at position 0

Store index at position 1

Update index

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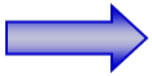
  p=a;
  if (x==0)
    a[i]=0;
  else
    a[i+1]=1;
  assert(*(p+2)==1);
}
```



$$P := \begin{pmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\ 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\ \text{select}(p_3, 0) = \&a[0] \\ \wedge\ \text{select}(\text{select}(p_3, 0), \\ \qquad \text{select}(p_3, 1)) == 1 \end{pmatrix}$$

negation satisfiable (a[2] unconstrained) ⇒ assert fails

Encoding of Memory Allocation

- model memory just as an array of bytes (array theories)
  - read and write operations to the memory array on the logic level

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  - read and write operations to the memory array on the logic level
- each dynamic object d_o consists of
  - m ≜ memory array
  - s ≜ size in bytes of m
  - ρ ≜ unique identifier
  - υ ≜ indicate whether the object is still alive
  - l ≜ the location in the execution where m is allocated

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  - ρ ≙ unique identifier
  - υ ≙ indicate whether the object is still alive
  - l ≙ the location in the execution where m is allocated
- to detect invalid reads/writes, we check whether
  - d_o is a dynamic object
  - i is within the bounds of the memory array

$$l_{is\_dynamic\_object} \Leftrightarrow \left(\bigvee_{j=1}^{k} d_o.\rho = j \right) \wedge \left(0 \leq i < n \right)$$

Encoding of Memory Allocation

- to check for invalid objects, we
  - set υ to true when the function malloc is called (d_o is alive)
  - set υ to false when the function free is called (d_o is not longer alive)

$$I_{valid\_object} \Leftrightarrow (I_{is\_dynamic\_object} \Rightarrow d_o.\upsilon)$$

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- to check for invalid objects, we
  - set υ to true when the function malloc is called (d_o is alive)
  - set υ to false when the function free is called (d_o is not longer alive)

$$l_{valid\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow d_o.\upsilon)$$

- to detect forgotten memory, at the end of the (unrolled) program we check
  - whether the d_o has been deallocated by the function free

$$l_{deallocated\_object} \Leftrightarrow (l_{is\_dynamic\_object} \Rightarrow \neg\, d_o.\upsilon)$$

Example of Memory Allocation

```
#include <std
void main() {
  char *p =
  char *q = malloc(5);  // p
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

memory leak: pointer reassignment makes $d_{o1}.\upsilon$ to become an orphan

Example of Memory Allocation

```
#include <stdlib.h>
void main() {
  char *p = malloc(5);  // ρ = 1
  char *q = malloc(5);  // ρ = 2
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

$$P := \left(\neg d_{o1}.\upsilon \wedge \neg d_{o2}.\upsilon \; \neg d_{o3}.\upsilon \right)$$

$$C := \left(\begin{array}{l} d_{o1}.\rho=1 \wedge d_{o1}.s=5 \wedge d_{o1}.\upsilon=\mathit{true} \wedge p=d_{o1} \\ \wedge \; d_{o2}.\rho=2 \wedge d_{o2}.s=5 \wedge d_{o2}.\upsilon=\mathit{true} \wedge q=d_{o2} \\ \wedge \; p=d_{o2} \wedge d_{o2}.\upsilon=\mathit{false} \\ \wedge \; d_{o3}.\rho=3 \wedge d_{o3}.s=5 \wedge d_{o3}.\upsilon=\mathit{true} \wedge p=d_{o3} \\ \wedge \; d_{o3}.\upsilon=\mathit{false} \end{array} \right)$$

Example of Memory Allocation

```
#include <stdlib.h>
void main() {
  char *p = malloc(5);  // ρ = 1
  char *q = malloc(5);  // ρ = 2
  p=q;
  free(p)
  p = malloc(5);        // ρ = 3
  free(p)
}
```

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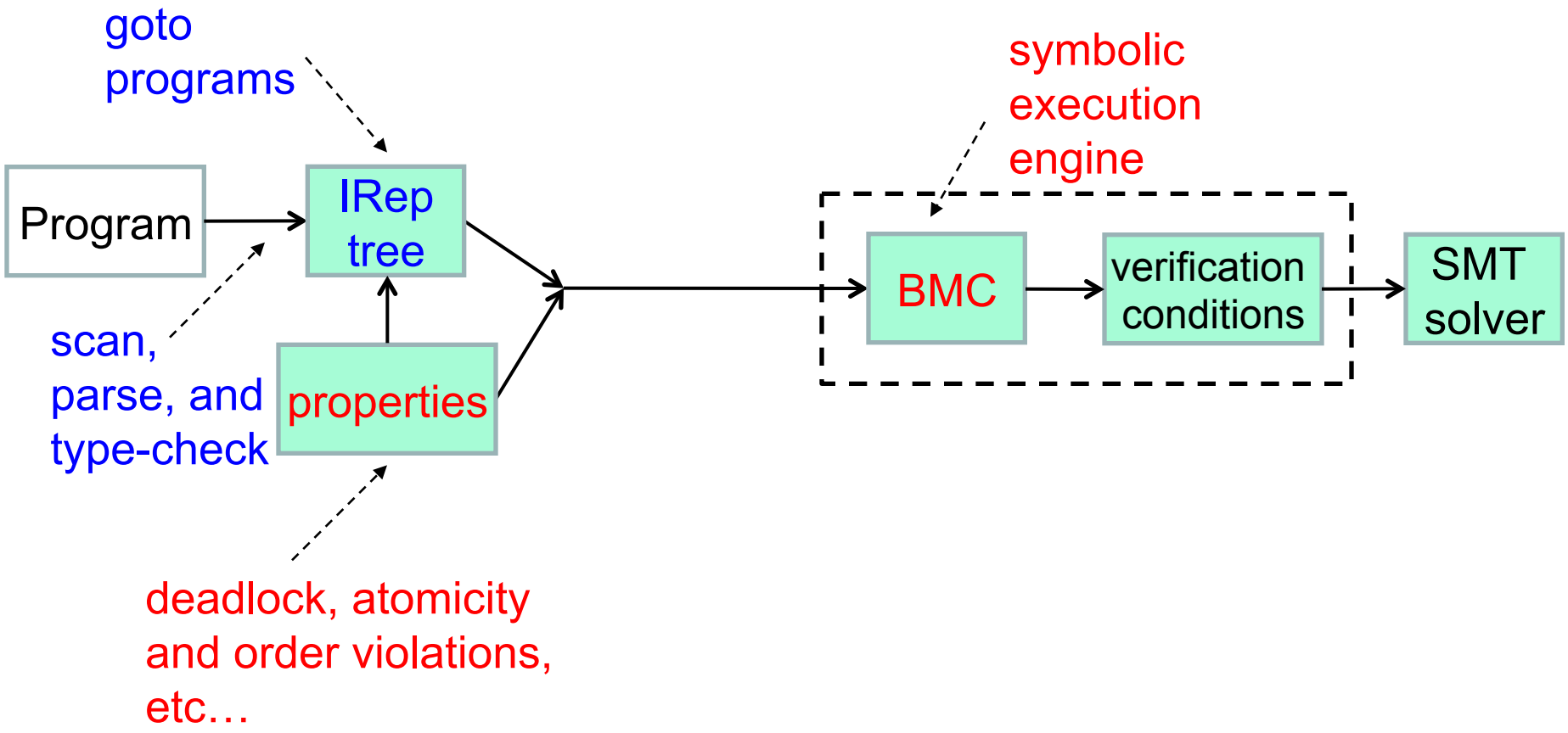
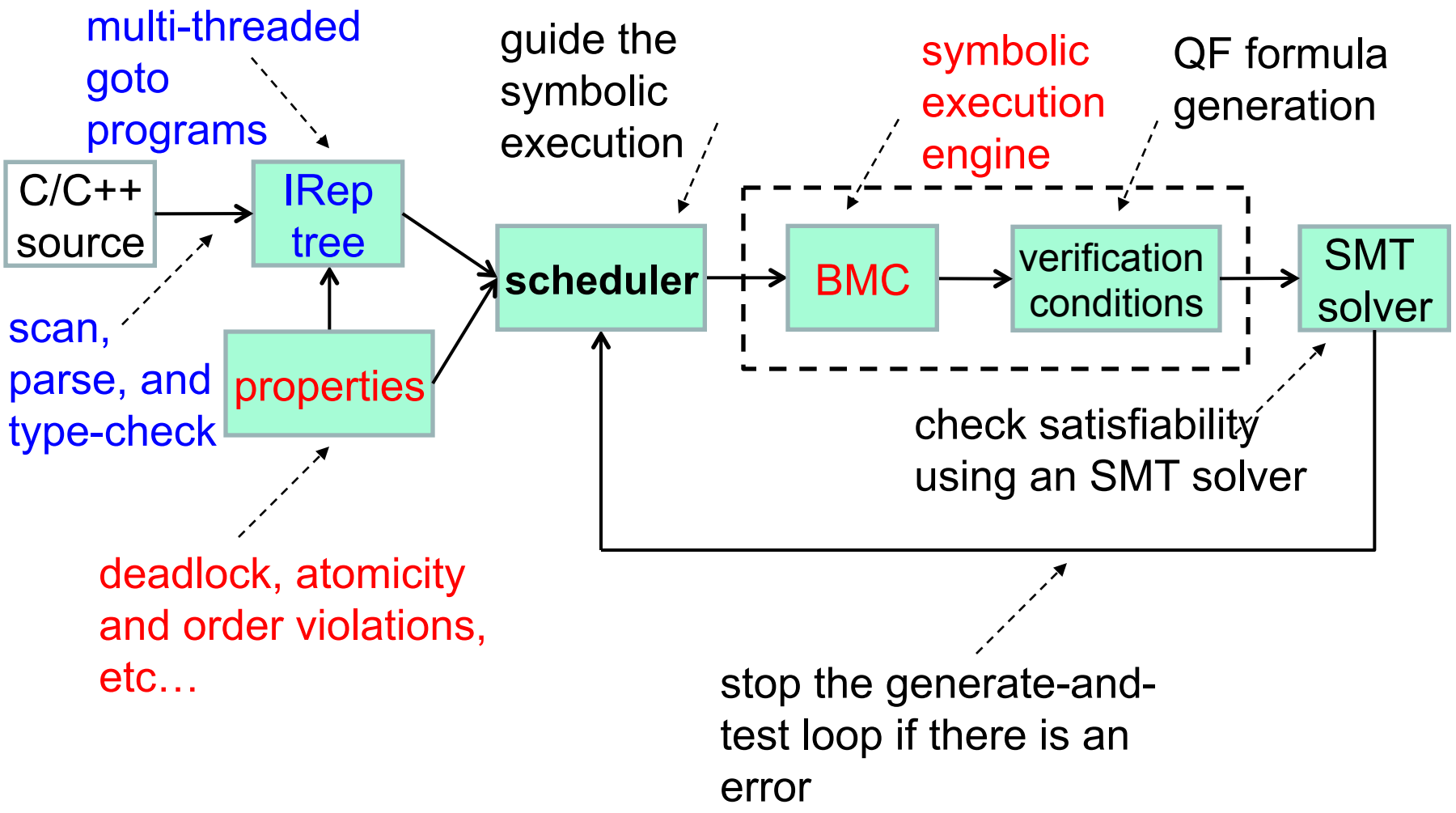
BMC Architecture

- A typical BMC architecture for verifying programs



BMC of Multi-threaded Software

Idea: iteratively generate all possible interleavings and call the BMC procedure on each interleaving



Running Example

- the program has sequences of operations that need to be protected together to avoid atomicity violation
  - requirement: the region of code (val1 and val2) should execute atomically

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

A state $s \in S$ consists of the value of the program counter pc and the values of all program variables

```
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

```
program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;
```

Lazy exploration: interleaving l_s

statements:

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 1
mutexes: ***m1=1****; m2=0;*
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2

val1-access: $W_{twoStage,2}$

val2-access:

write access to the shared variable val1 in statement 2 of the thread twoStage

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 2
mutexes: m1=1; m2=0;
*global variables: **val1=1**; val2=0;*
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1-2-3

val1-access: $W_{twoStage,2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 3
mutexes: ***m1=0****; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7

val1-access: $W_{twoStage,2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 7
mutexes: ***m1=1****; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l

statements: 1-2-3-7-8

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$

val2-access:

read access to the shared variable val1 in statement 8 of the thread reader

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

CS1

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 8

mutexes: m1=1; m2=0;

global variables: val1=1; val2=0;

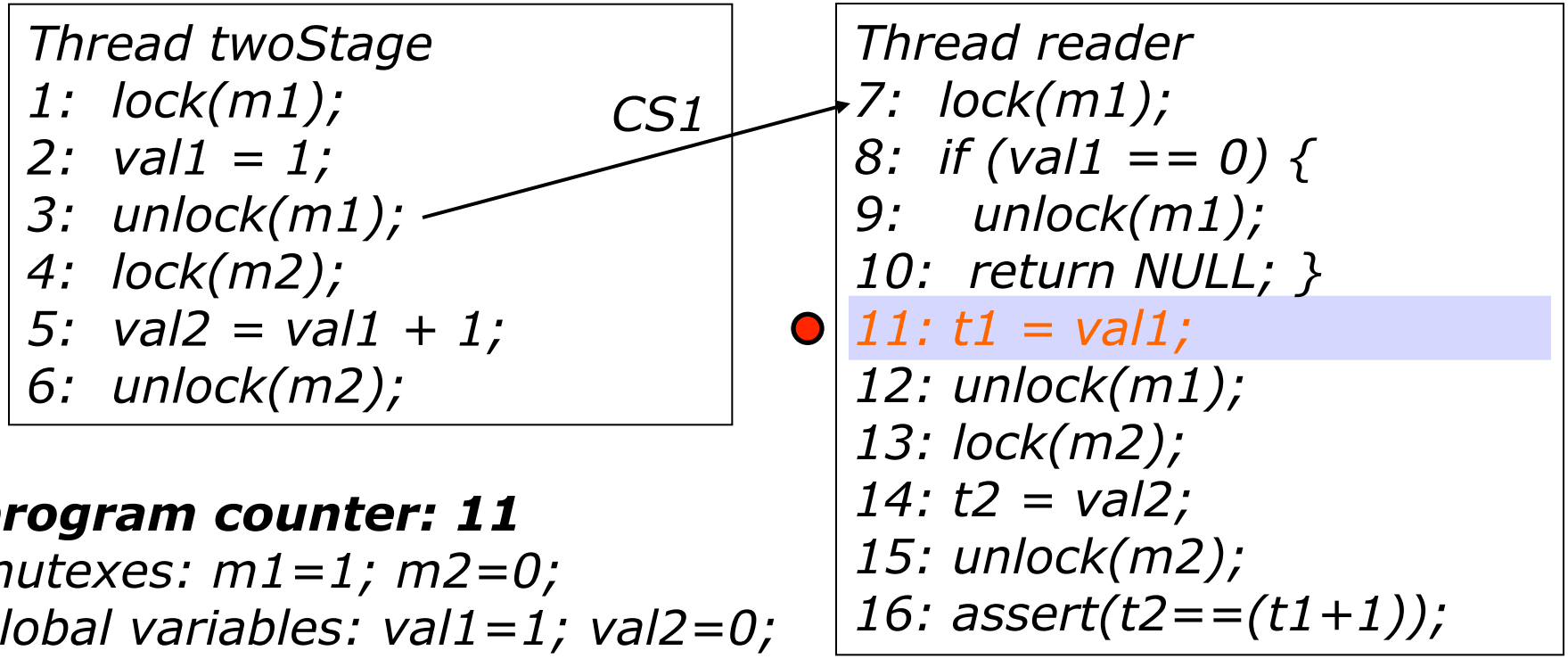
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



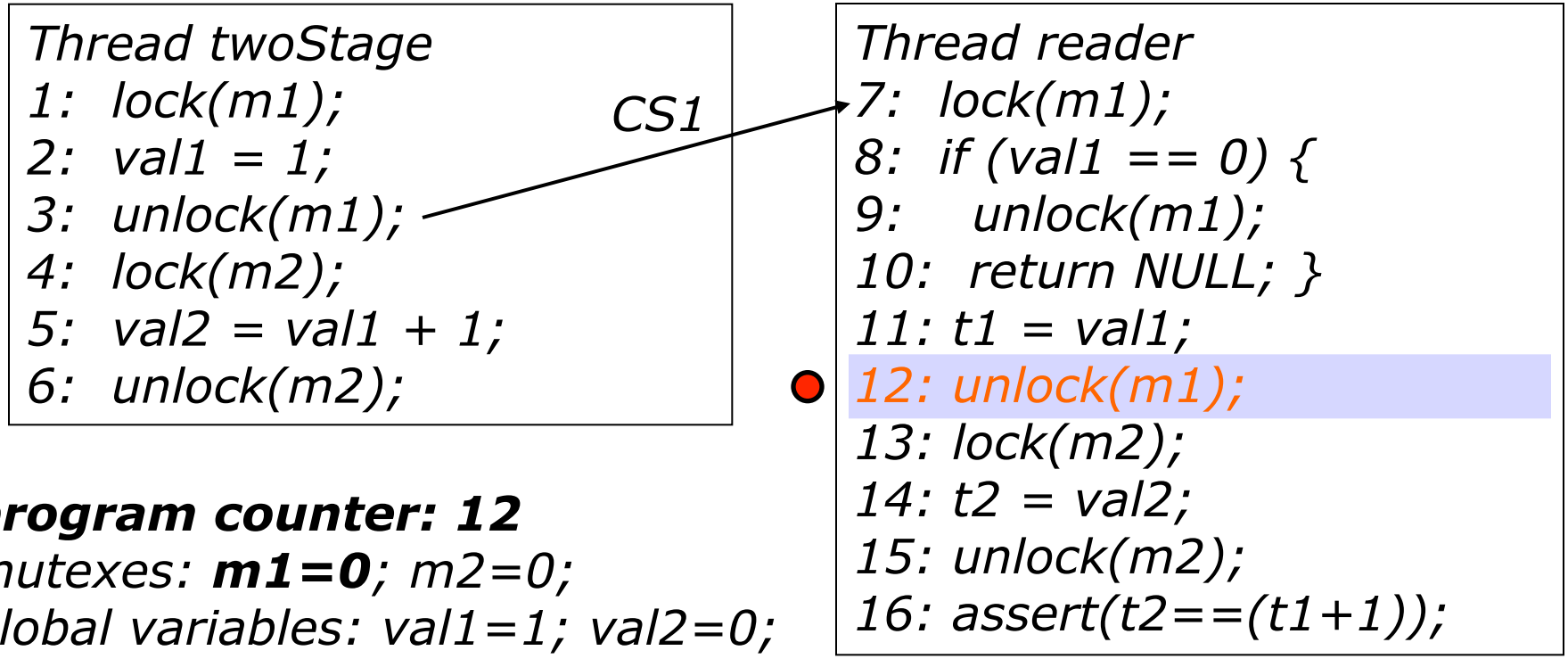
program counter: 11
mutexes: m1=1; m2=0;
global variables: val1=1; val2=0;
local variabes: ***t1= 1****; t2= -1;*

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



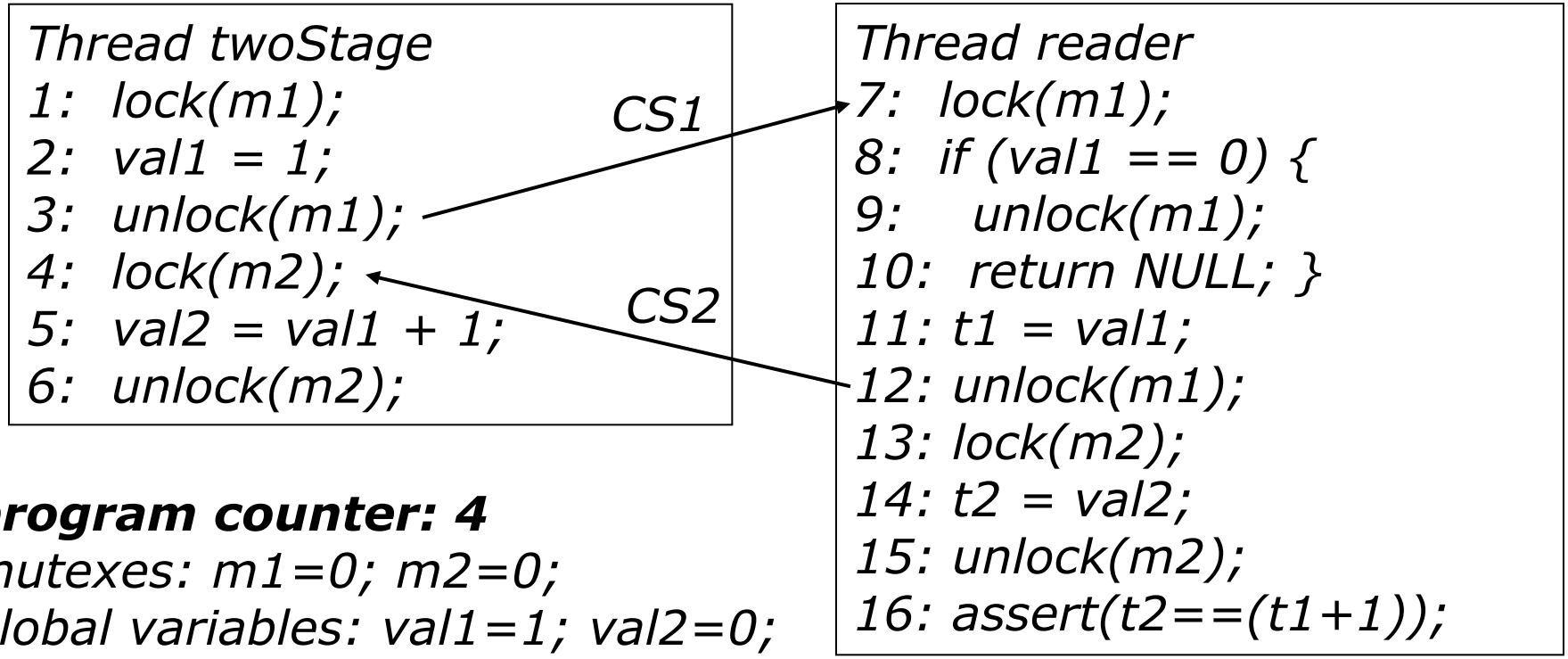
program counter: 12
*mutexes: **m1=0**; m2=0;*
global variables: val1=1; val2=0;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



program counter: 4

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

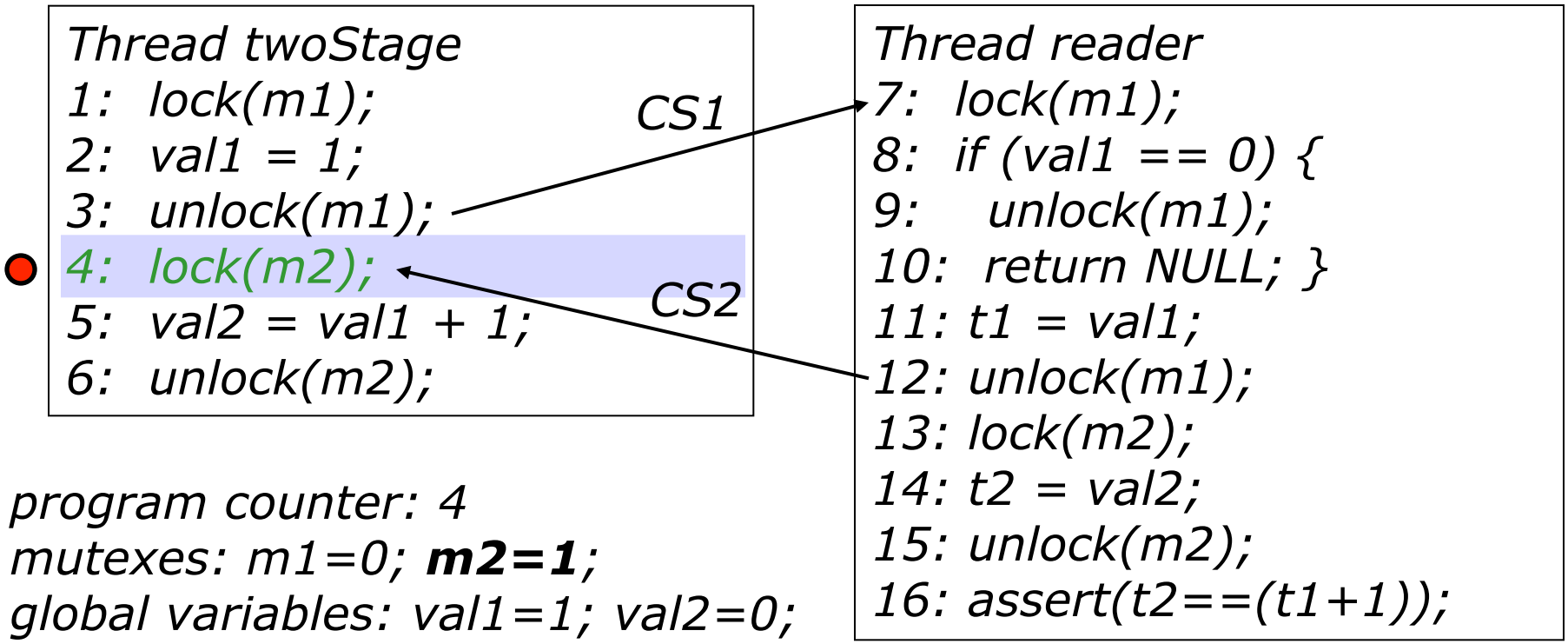
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$

val2-access:



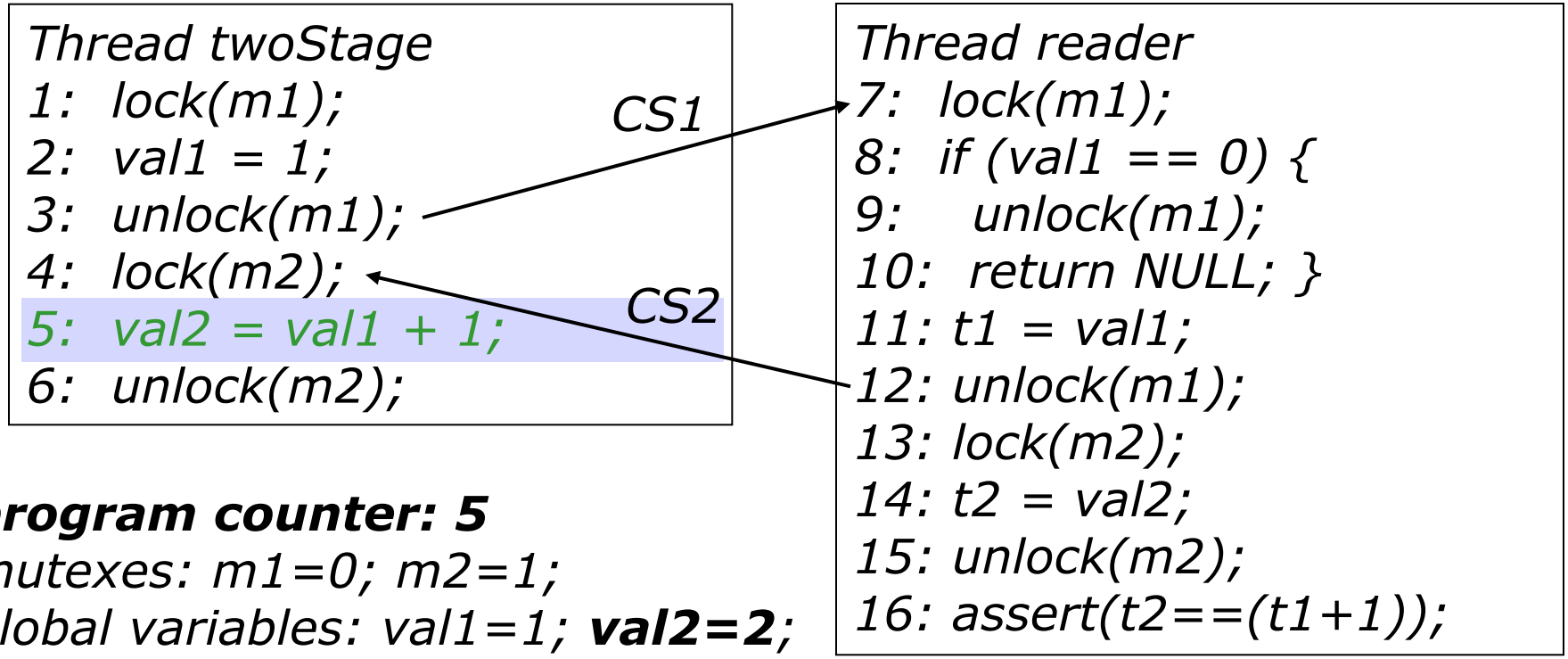
program counter: 4
*mutexes: m1=0; **m2=1**;*
global variables: val1=1; val2=0;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



program counter: 5

mutexes: m1=0; m2=1;

global variables: val1=1; ***val2=2****;*

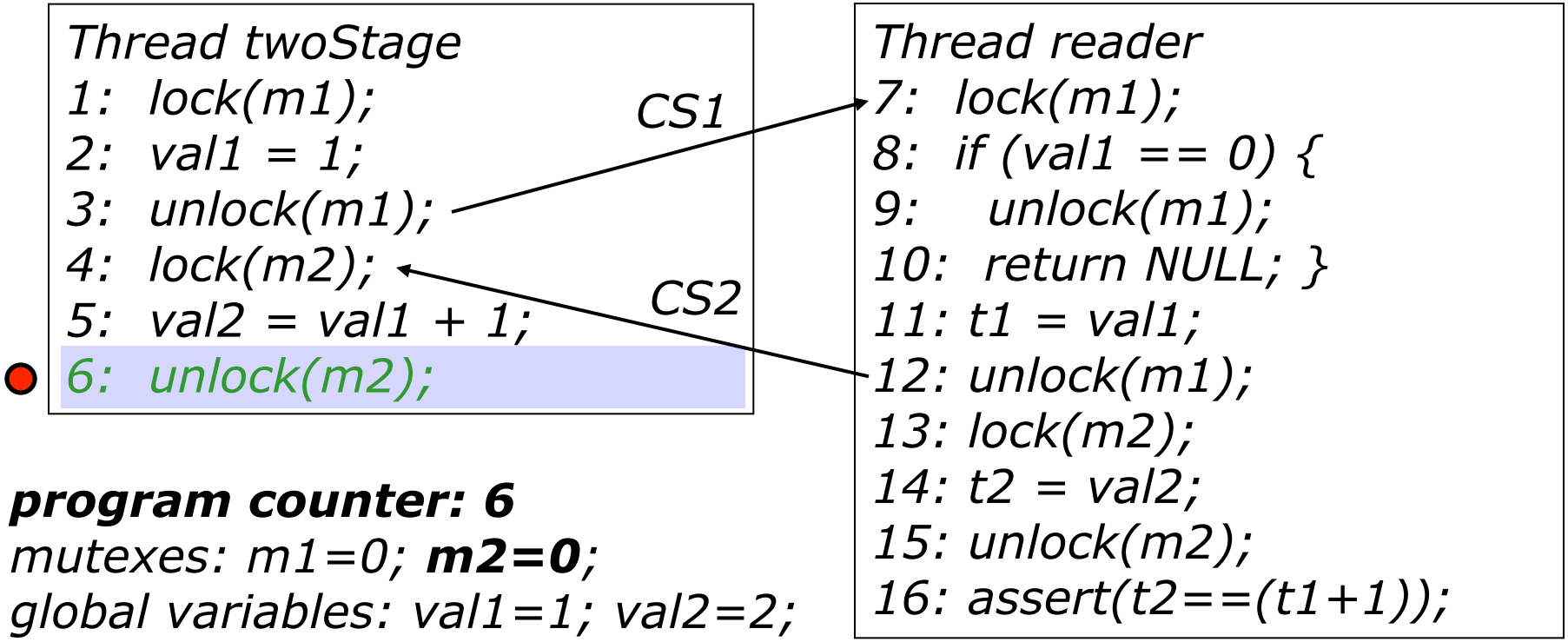
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



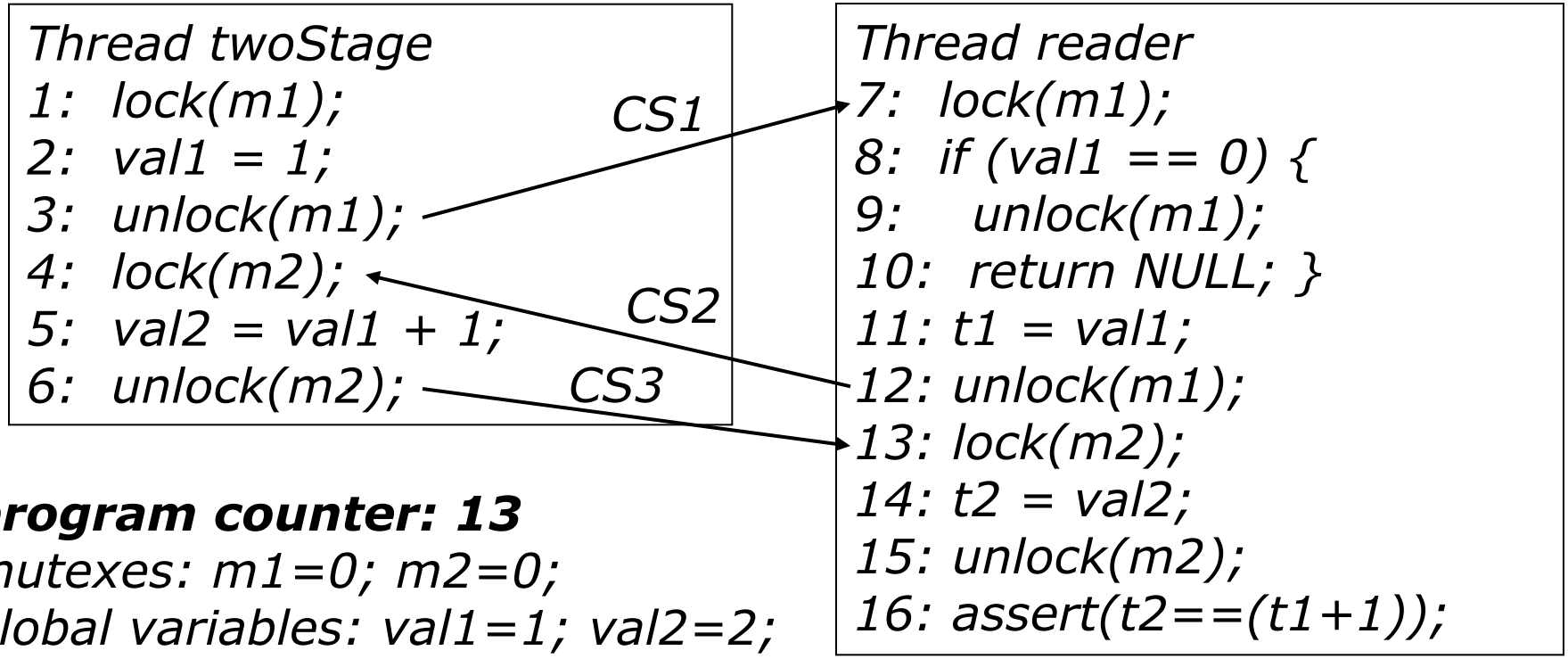
program counter: 6
*mutexes: m1=0; **m2=0**;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



program counter: 13

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2;

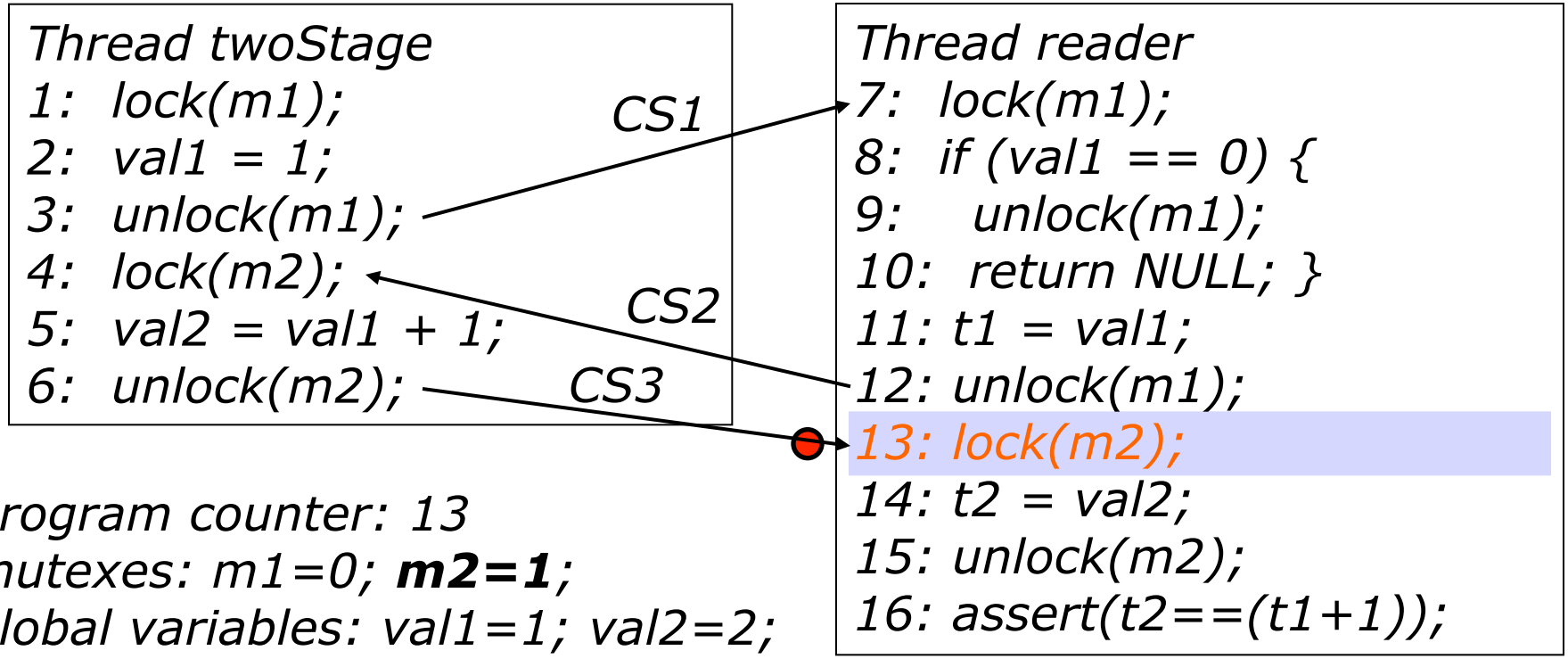
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$



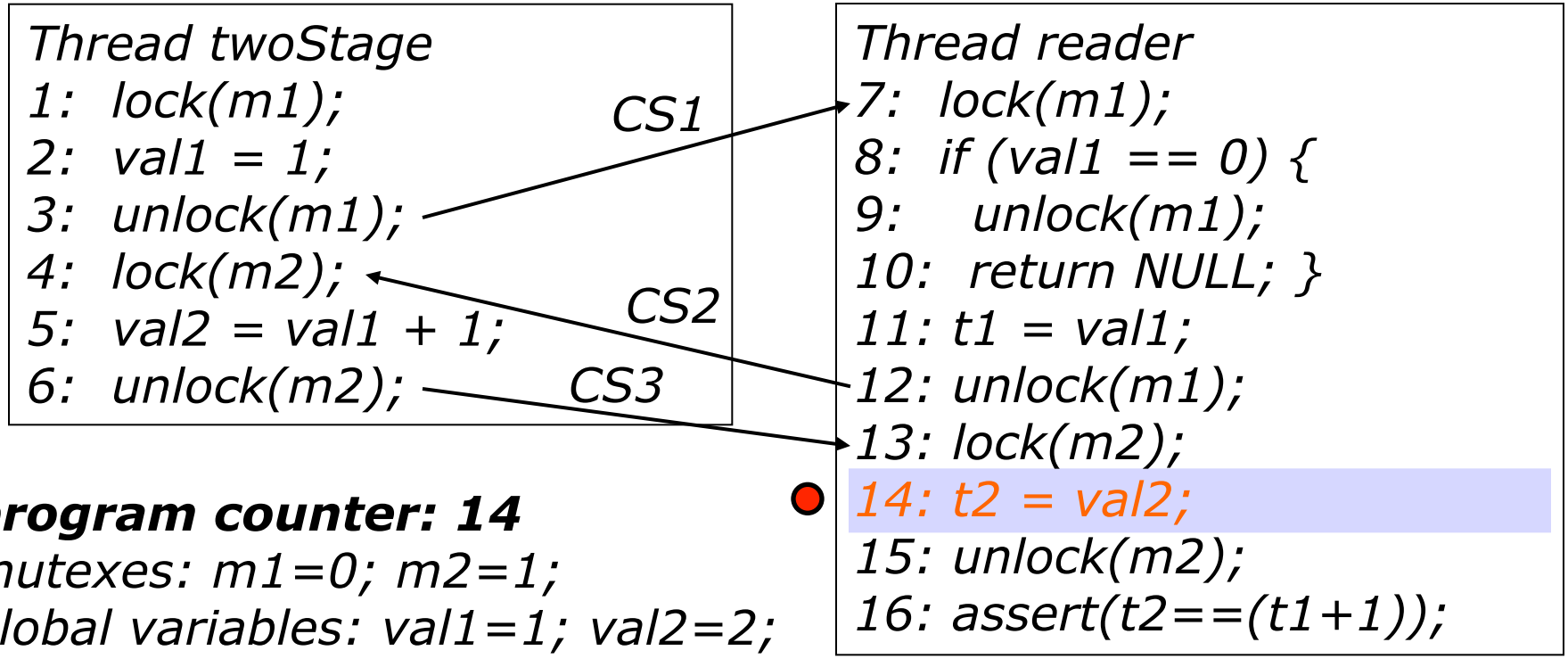
program counter: 13
*mutexes: m1=0; **m2=1**;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= -1;

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



program counter: 14

mutexes: m1=0; m2=1;

global variables: val1=1; val2=2;

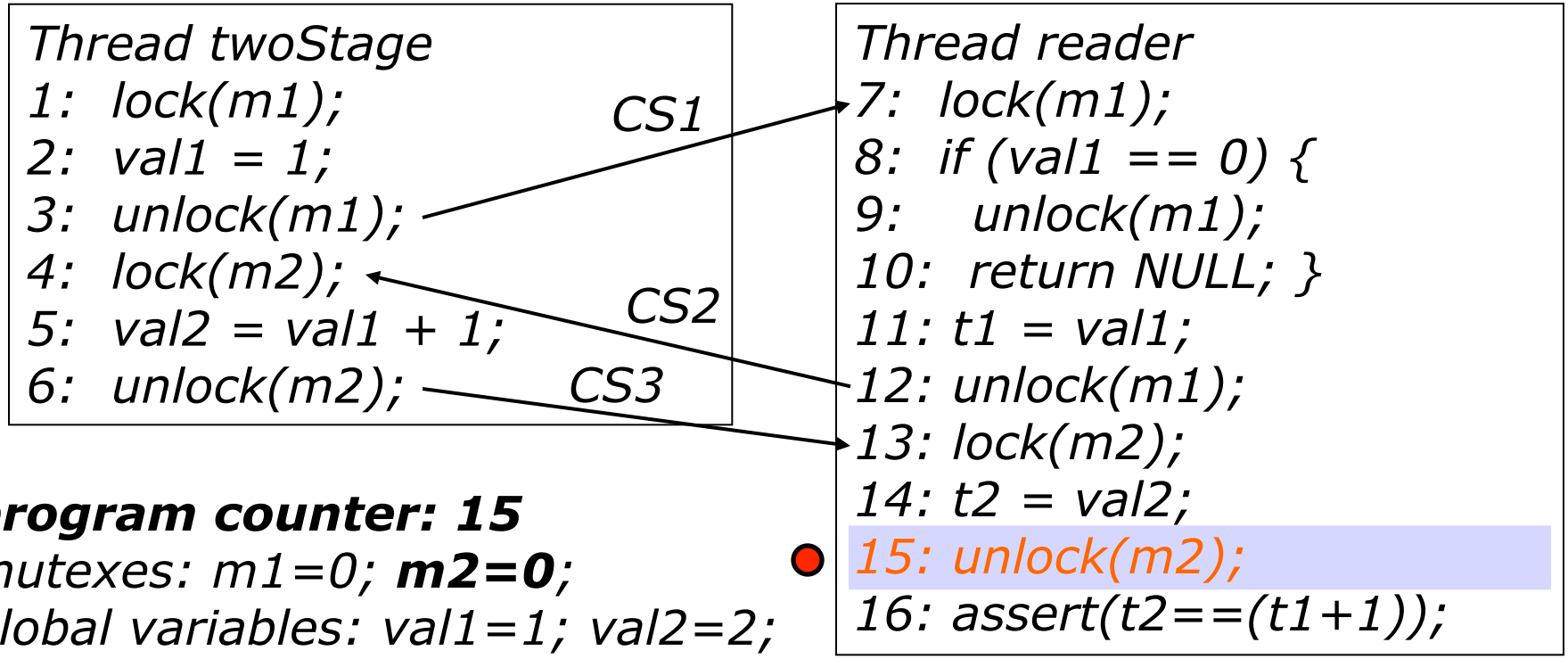
local variabes: t1= 1; ***t2= 2;***

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



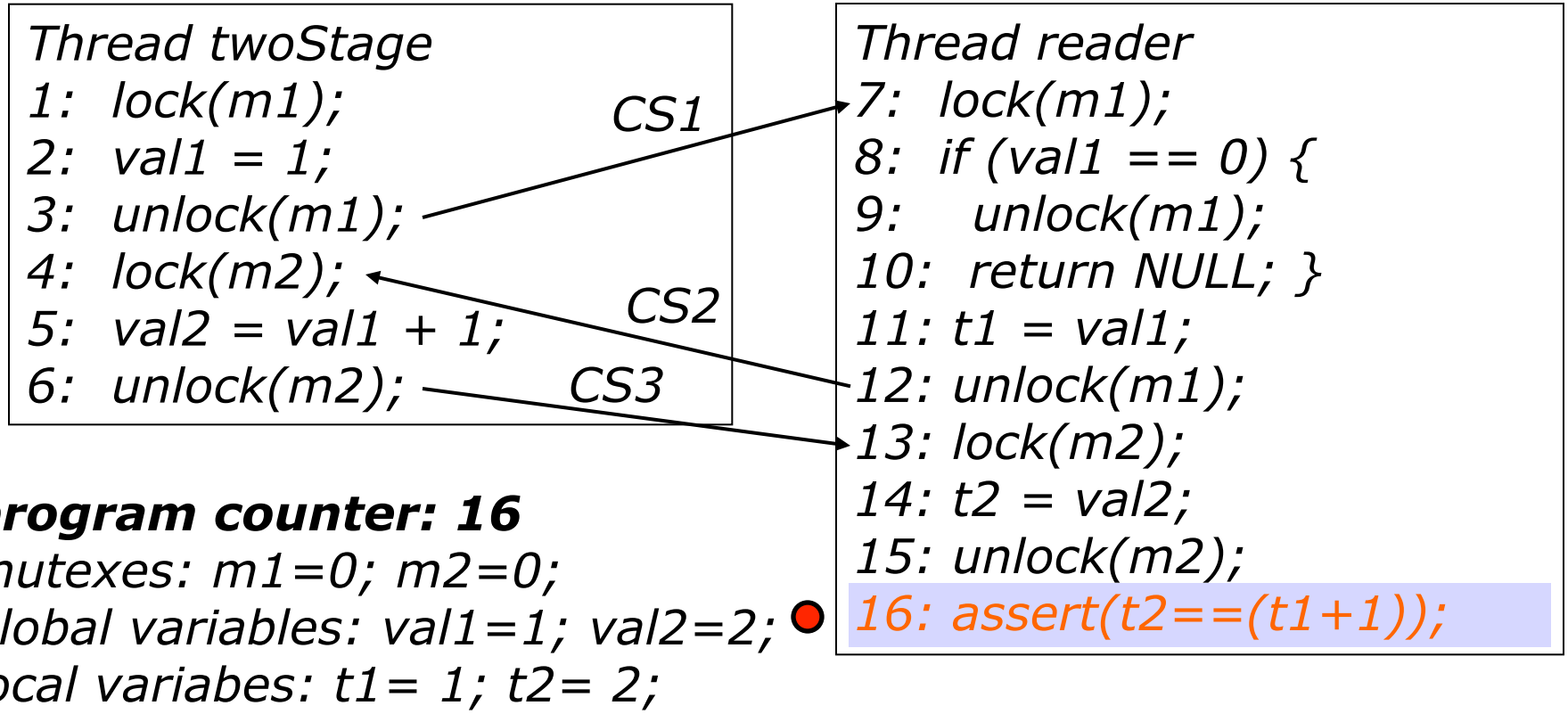
program counter: 15
*mutexes: m1=0; **m2=0**;*
global variables: val1=1; val2=2;
local variabes: t1= 1; t2= 2;

Lazy exploration: interleaving l_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



program counter: 16

mutexes: m1=0; m2=0;

global variables: val1=1; val2=2;

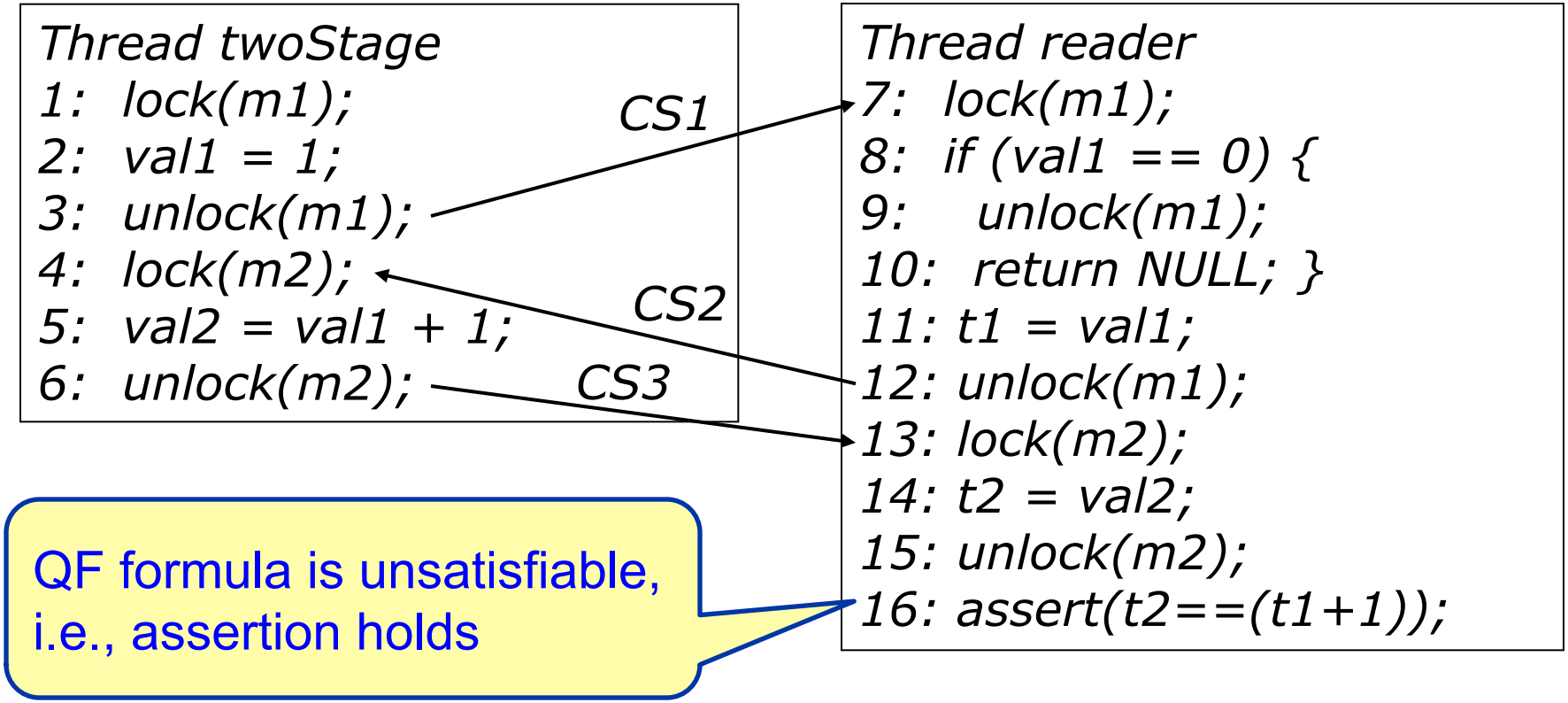
local variabes: t1= 1; t2= 2;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{twoStage,2}$ - $R_{reader,8}$ - $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $W_{twoStage,5}$ - $R_{reader,14}$



Lazy exploration: interleaving l_f

statements:

val1-access:

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0
mutexes: m1=0; m2=0;
global variables: val1=0; val2=0;
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_f

statements: 1-2-3

val1-access: $W_{twoStage,2}$

val2-access:

```
Thread twoStage
1:  lock(m1);
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

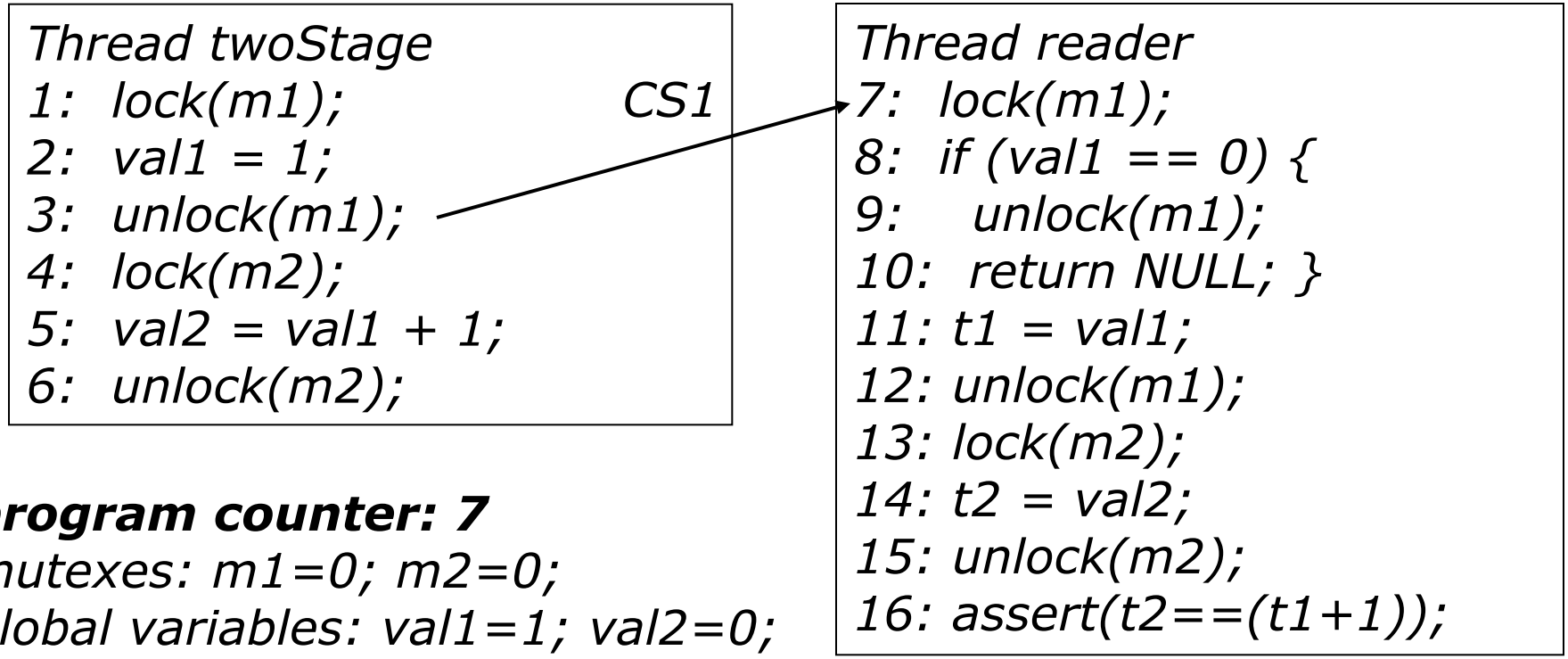
program counter: 3
mutexes: m1=0; m2=0;
*global variables: **val1=1**; val2=0;*
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving l_f

statements: 1-2-3

val1-access: $W_{twoStage,2}$

val2-access:



program counter: 7

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

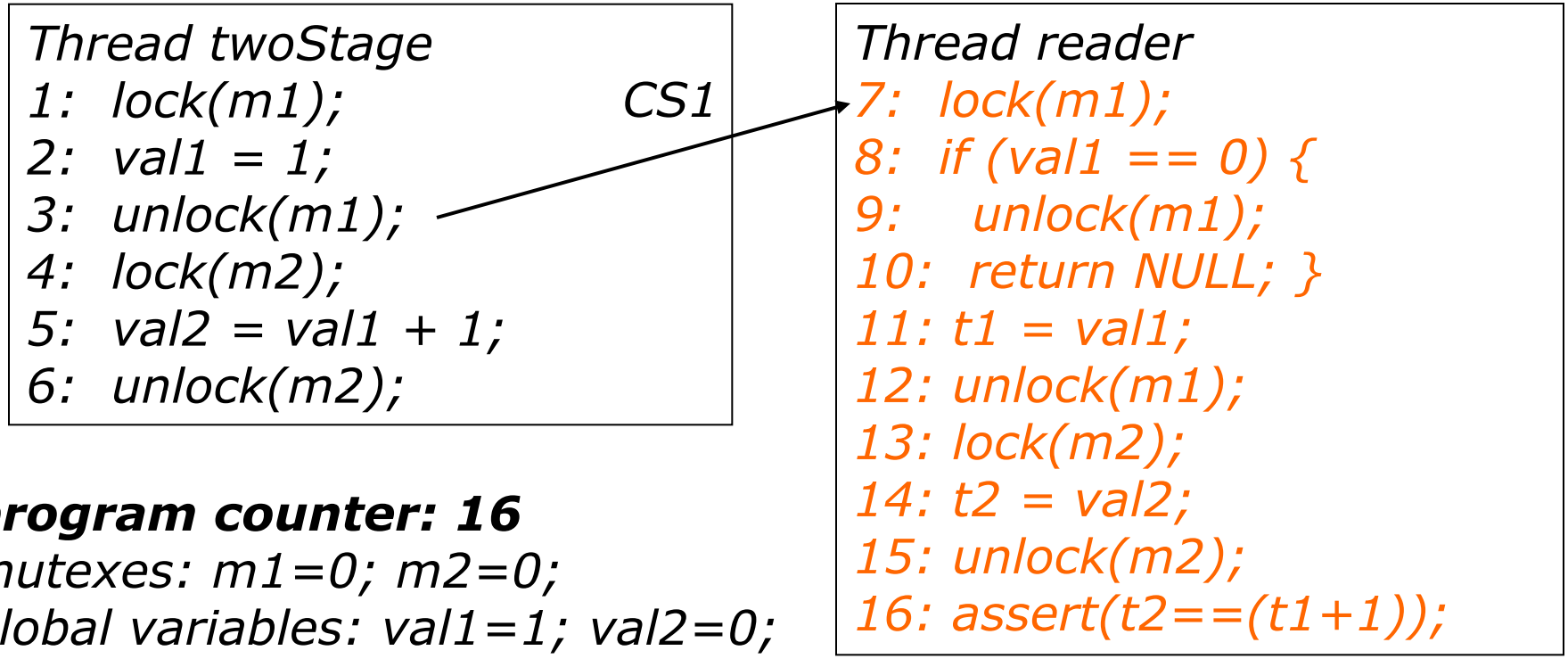
local variabes: t1= -1; t2= -1;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$

val2-access: $R_{reader,14}$



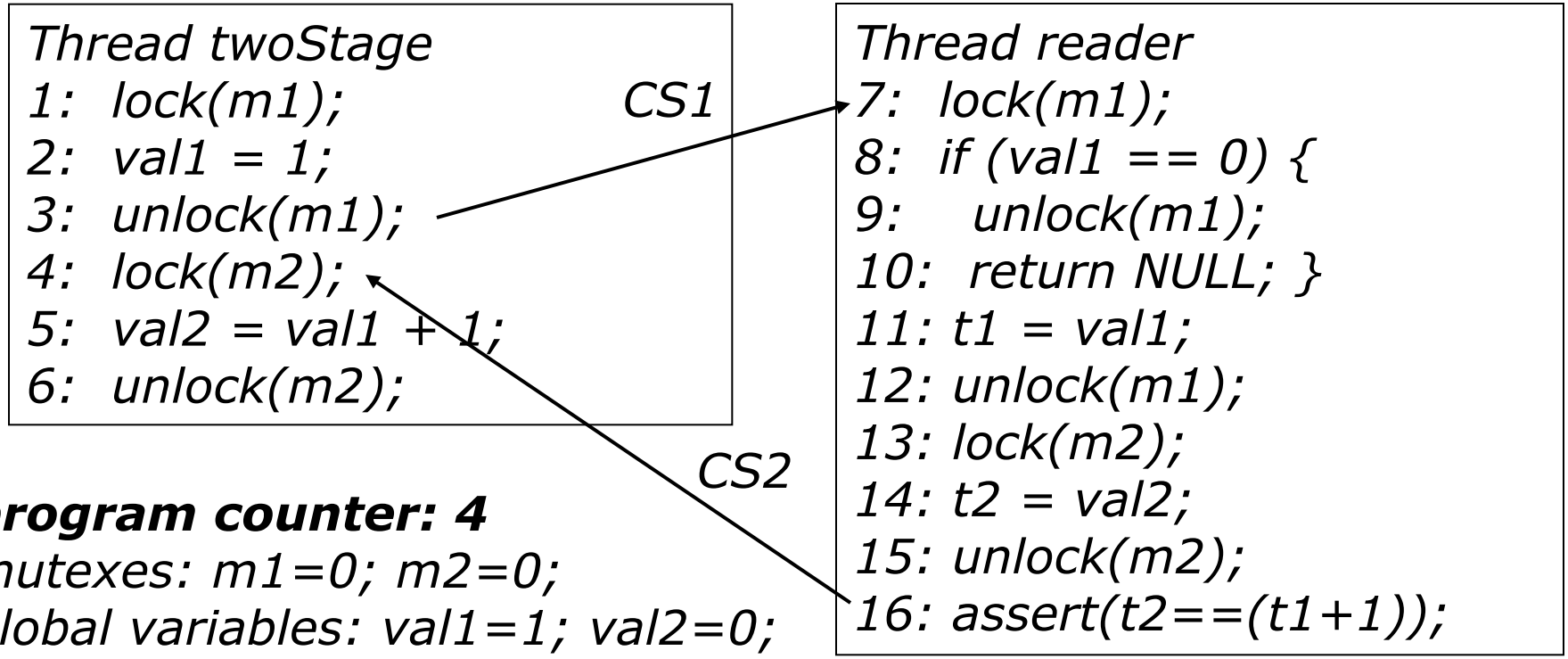
program counter: 16
mutexes: m1=0; m2=0;
global variables: val1=1; val2=0;
local variabes: ***t1= 1; t2= 0;***

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$

val2-access: $R_{reader,14}$



program counter: 4

mutexes: m1=0; m2=0;

global variables: val1=1; val2=0;

local variabes: t1= 1; t2= 0;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $R_{reader,14}$- $W_{twoStage,5}$

```
Thread twoStage
1:  lock(m1);                CS1
2:  val1 = 1;
3:  unlock(m1);
4:  lock(m2);
5:  val2 = val1 + 1;
6:  unlock(m2);
```

```
Thread reader
7:  lock(m1);
8:  if (val1 == 0) {
9:    unlock(m1);
10:  return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

CS2

program counter: 6

mutexes: m1=0; m2=0;

global variables: val1=1; ***val2=2;***

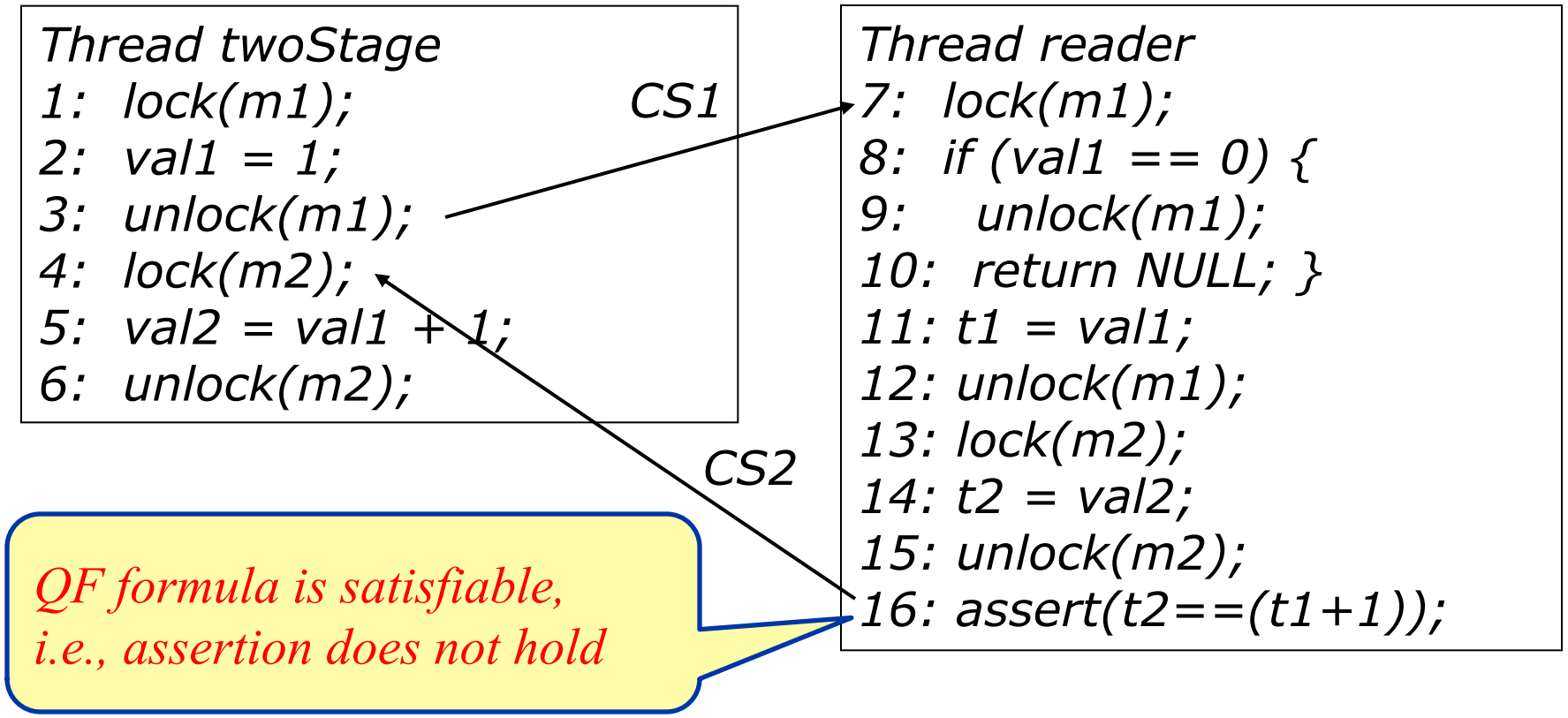
local variabes: t1= 1; t2= 0;

Lazy exploration: interleaving I_f

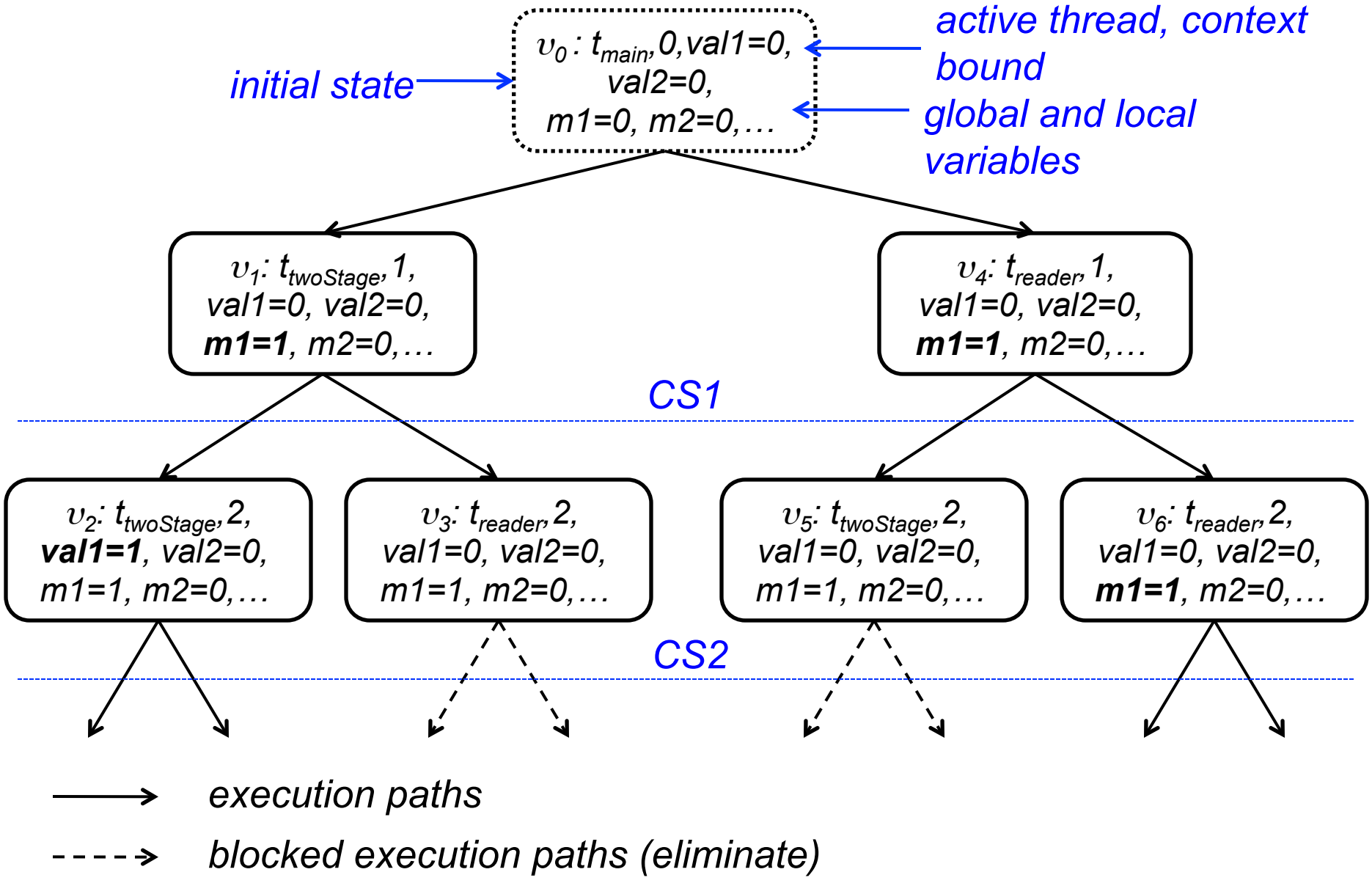
statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{twoStage,2}$- $R_{reader,8}$- $R_{reader,11}$ - $R_{twoStage,5}$

val2-access: $R_{reader,14}$- $W_{twoStage,5}$



Lazy Approach: State Transitions



Exploring the Reachability Tree

- Use a reachability tree (RT) to describe reachable states of a multi-threaded program

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- Each node in the RT is a tuple $\upsilon = \left(A_i, C_i, s_i, \left\langle l_i^j, G_i^j \right\rangle_{j=1}^{n} \right)_i$ for a given time step i, where:

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  - C_i represents the context switch number

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  - s_i represents the current state

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  - l_i^j represents the current location of thread *j*

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  - A_i represents the currently active thread
  - C_i represents the context switch number
  - s_i represents the current state
  - l_i^j represents the current location of thread *j*
  - G_i^j represents the control flow guards accumulated in thread *j* along the path from l_0^j to l_i^j

Expansion Rules of the RT

R1 (assign): If *l* is an assignment, we execute *l*, which generates s_{i+1}. We add as child to υ a new node υ'

$$\upsilon' = \left(A_i, C_i, s_{i+1}, \left\langle l_{i+1}^{j}, G_i^{j} \right\rangle\right)_{i+1} \qquad l_{i+1}^{A_i} = l_i^{A_i} + 1$$

- we have fully expanded υ if
  - I within an atomic block; or
  - *l* contains no global variable; or
  - the upper bound of context switches ($C_i = C$) is reached
- if υ is not fully expanded, for each thread $j \neq A_i$ where G_i^j is enabled in s_{i+1}, we thus create a new child node

$$\upsilon_j' = \left(j, C_i + 1, s_{i+1}, \left\langle l_i^j, G_i^j \right\rangle \right)_{i+1}$$

Expansion Rules of the RT

R2 (skip): If *l* is a *skip*-statement with target *l*, we increment the location of the current thread and continue with it. We explore no context switches:

$$v' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l_i^j + 1 & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

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R3 (unconditional goto): If *l* is an unconditional *goto*-statement with target *l*, we set the location of the current thread and continue with it. We explore no context switches:

$$v' = \left(A_i, C_i, s_i, \left\langle \underline{l_{i+1}^j}, G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

Expansion Rules of the RT

R4 (conditional goto): If *l* is a conditional *goto*-statement with test *c and* target *l*, we create two child nodes υ' and υ''.

- for υ', we assume that *c* is *true* and proceed with the target instruction of the jump:

$$\upsilon' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^j, c \wedge G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

- for υ'', we add $\neg c$ to the guards and continue with the next instruction in the current thread

$$\upsilon'' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^j, \neg c \wedge G_i^j \right\rangle\right)_{i+1} \longrightarrow l_{i+1}^j = \begin{cases} l_i^j + 1 & : \quad j = A_i \\ l_i^j & : \quad otherwise \end{cases}$$

- prune one of the nodes if the condition is determined statically

Expansion Rules of the RT

R5 (assume): If *l* is an *assume*-statement with argument *c*, we proceed similar to R1.

- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

Expansion Rules of the RT

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- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

R6 (assert): If *l* is an *assert*-statement with argument *c*, we proceed similar to R1.

- we continue with the unchanged state s_i but add *c* to all guards, as described in R4
- we generate a verification condition to check the validity of *c*

Expansion Rules of the RT

R5 (start_thread): If *l* is a *start_thread* instruction, we add the indicated thread to the set of active threads:

$$v' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^{j}, G_{i+1}^{j} \right\rangle_{j=1}^{n+1} \right]_{i+1}$$

- where l_{i+1}^{n+1} is the initial location of the thread and $G_{i+1}^{n+1} = G_i^{A_i}$
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R6 (join_thread): If *l* is a *join_thread* instruction with argument *ld*, we add a child node:

$$v' = \left(A_i, C_i, s_i, \left\langle l_{i+1}^{j}, G_i^{j} \right\rangle \right)_{i+1}$$

- where $l_{i+1}^{j} = l_i^{A_i} + 1$ only if the joining thread Id has exited

Lazy exploration of interleavings

- Main steps of the algorithm:

  1. Initialize the stack with the initial node ν_0 and the initial path $\pi_0 = \langle \upsilon_0 \rangle$

  2. If the stack is empty, terminate with "no error".

  3.Pop the current node υ and current path π off the stack and compute the set υ' of successors of υ using rules R1-R8.

  4. If υ' is empty, derive the VC φ_k^π for π and call the SMT solver on it. If φ_k^π is satisfiable, terminate with "error"; otherwise, goto step 2.

  5. If υ' is not empty, then for each node $\upsilon \in \upsilon'$, add ν to π, and push node and extended path on the stack. goto step 3.

computation path

$$\pi = \{\upsilon_1, \ldots \upsilon_n\}$$

$$\varphi_k^\pi = \overbrace{I(s_0) \wedge R(s_0, s_1) \wedge \ldots \wedge R(s_{k-1}, s_k)}^{\text{constraints}} \wedge \overbrace{\neg \phi_k}^{\text{property}}$$

bound

Observations about the lazy approach

- naïve but useful:
  - bugs usually manifest with few context switches [Qadeer&Rehof'05]

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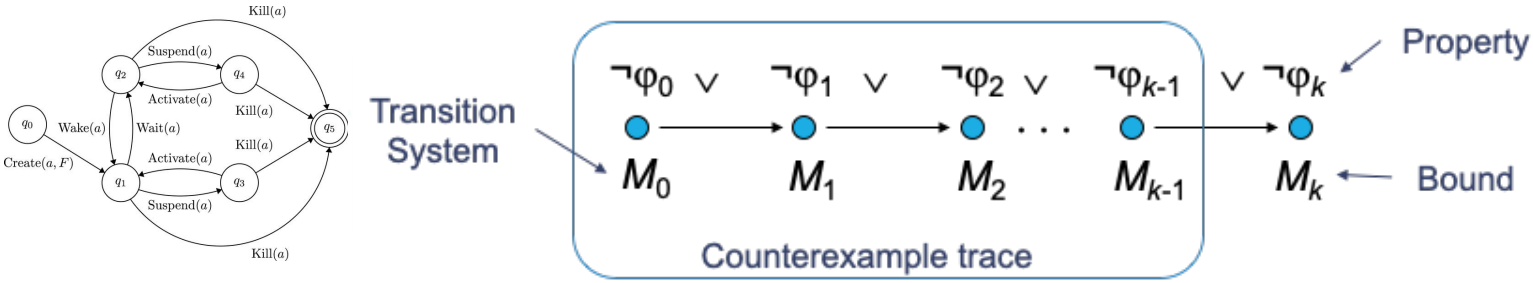
    - ▷ *number of executions:* $O(n^c)$
  - as each formula corresponds to one possible path only, its size is relatively small
- can suffer performance degradation:
  - in particular for correct programs where we need to invoke the SMT solver once for each possible execution path

Intended learning outcomes

- Understand **soundness** and **completeness** concerning **detection techniques**
- Emphasize the difference between **static analysis** and **testing / simulation**
- Explain **bounded model checking of software**
- Explain **unbounded model checking of software**

Revisiting BMC

- Basic Idea: given a transition system *M*, check negation of a given property φ up to given depth *k:*



- Translated into a VC ψ such that: *ψ satisfiable iff φ has counterexample of max. depth k*

BMC is aimed at finding bugs; it cannot prove correctness, unless the bound *k* safely reaches all program states

Difficulties in proving the correctness of programs with loops in BMC

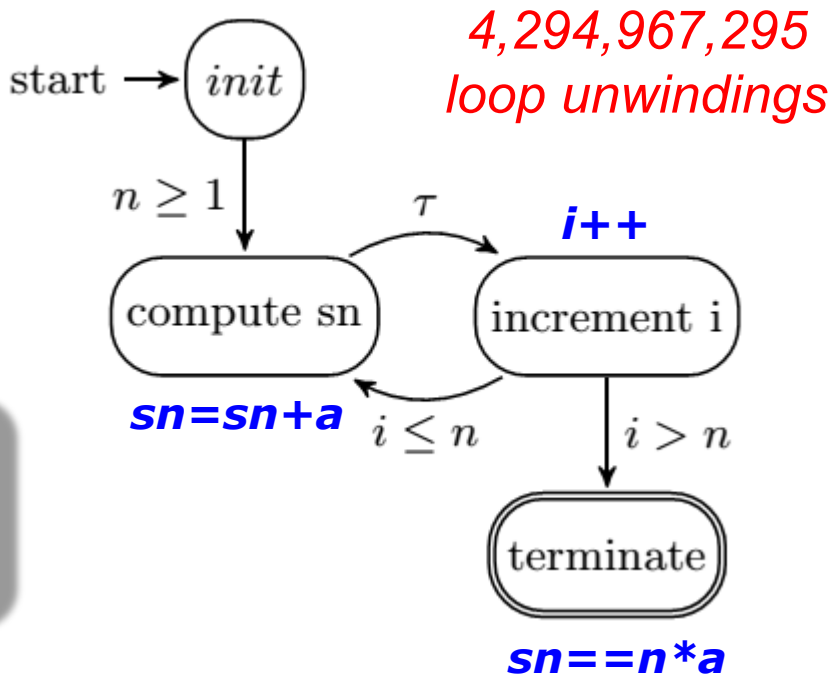
- BMC techniques can falsify properties up to a given depth k

– they can prove correctness only if an upper bound of k is known (**unwinding assertion**)

» BMC tools typically fail to verify programs that contain bounded and unbounded loops

$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

the loop will be unfolded 2^{n-1} times (in the worst case, 2^{32-1} times on 32 bits integer)



Induction-Based Verification

***k*-induction** checks...

- **base case ($base_k$):** find a counter-example with up to *k* loop unwindings (plain BMC)
- **forward condition (fwd_k):** check that *P* holds in all states reachable within *k* unwindings
- **inductive step ($step_k$):** check that whenever *P* holds for *k* unwindings, it also holds after next unwinding
  - havoc state
  - run *k* iterations
  - assume invariant
  - run final iteration

⇒ iterative deepening if inconclusive

The *k*-induction algorithm

k=initial bound
while *true* **do**
    if $base_k$ **then**
        return *trace s[0..k]*
    else if fwd_k
        return *true*
    else if $step_k$ **then**
        return *true*
    end if
    k=k+1
end

The *k*-induction algorithm

inserts unwinding assumption after each loop

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inserts unwinding assumption after each loop

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havoc variables that occur in the loop's termination condition

unable to falsify or prove the property

Running example

Prove that $S_n = \sum_{i=1}^{n} a = na$ for $n \geq 1$

```
unsigned int nondet_uint();
int main() {
  unsigned int i, n=nondet_uint(), sn=0;
  assume (n>=1);
  for(i=1; i<=n; i++)
    sn = sn + a;
  assert(sn==n*a);
}
```

Running example: *base case*

Insert an **unwinding assumption** consisting of the termination condition after the loop

– find a counter-example with *k* loop unwindings

```
unsigned int nondet_uint();
int main() {
  unsigned int i, n=nondet_uint(), sn=0;
  assume (n>=1);
  for(i=1; i<=n; i++)
    sn = sn + a;
  assume(i>n);
  assert(sn==n*a);
}
```

Running example: *forward condition*

Insert an **unwinding assertion** consisting of the termination condition after the loop

– check that *P* holds in all states reachable with k unwindings

```
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int main() {
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  assume (n>=1);
  for(i=1; i<=n; i++)
    sn = sn + a;
  assert(i>n);
  assert(sn==n*a);
}
```

Running example: *inductive step*

Havoc (only) the variables that occur in the loop's termination and branch conditions

```
unsigned int nondet_uint();
typedef struct state {
    unsigned int i, n, sn;
} statet;
int main() {
    unsigned int i, n=nondet_uint(), sn=0, k;
    assume(n>=1);
    statet cs, s[n];
    cs.i=nondet_uint();
    cs.sn=nondet_uint();
    cs.n=n;
```

Running example: *inductive step*

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define the type of the program state

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define the type of the program state

state vector

explore all possible values implicitly

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {
    s[i-1]=cs;
    sn = sn + a;
    cs.i=i;
    cs.sn=sn;
    cs.n=n;
    assume(s[i-1]!=cs);
  }
assume(i>n);
assert(sn ==  n*a);
}
```

Running example: *inductive step*

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capture the state *cs* before the iteration

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```

capture the state *cs* before the iteration

capture the state *cs* after the iteration

constraints are included by means of assumptions

Running example: *inductive step*

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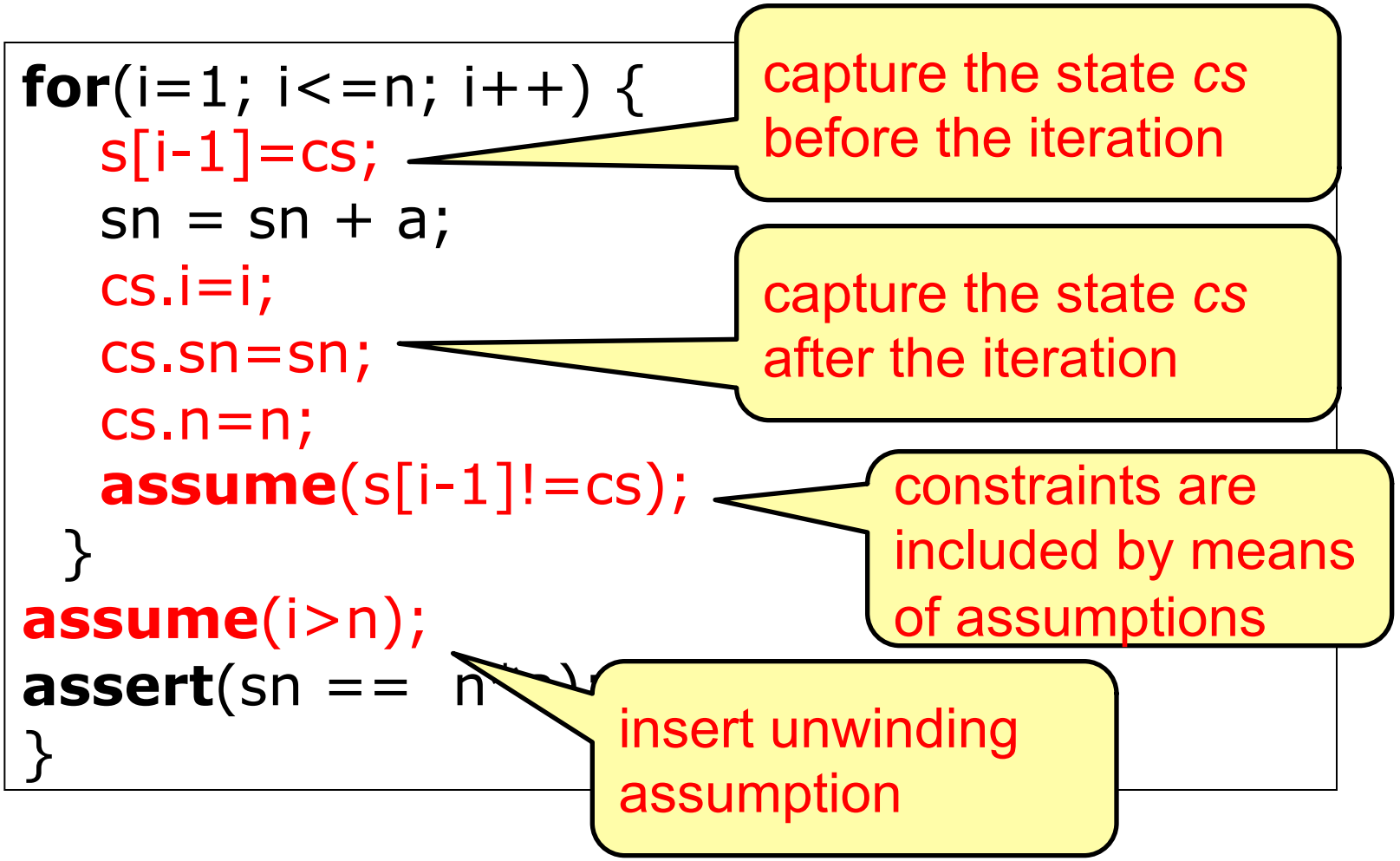
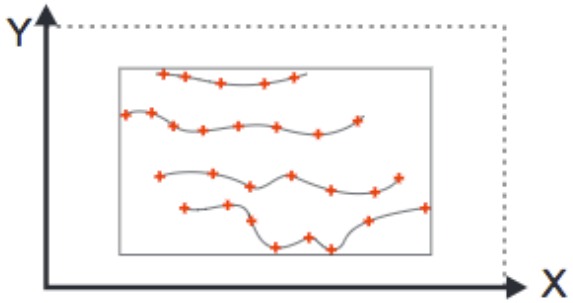
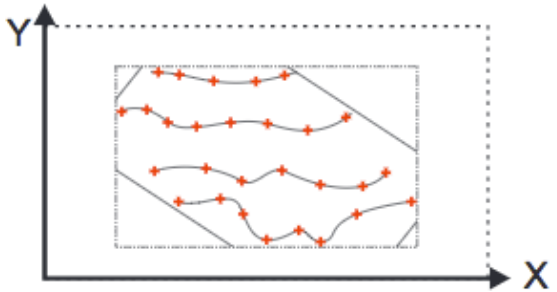


Automatic Invariant Generation

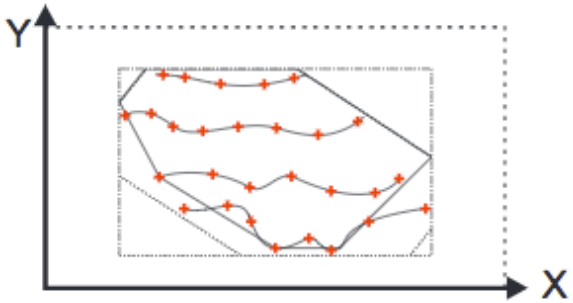
- Infer invariants using **intervals**, **octagons**, and **convex polyhedral** constraints for the inductive step
  - *e.g.*, $a \le x \le b$; $x \le a$, $x-y \le b$; and $ax + by \le c$



intervals



octagons



convex polyhedral

- Use existing libraries to discover linear/polynomial relations among integer/real variables to infer **loop invariants**
  - compute **pre-** and **post-conditions**

Running Example: Plain BMC

- Plain BMC unrolls this *while*-loop 100 times…

```
int main() {
  int x=0, t=0, phase=0;
  while(t<100) {
    if(phase==0) x=x+2;
    if(phase==1) x=x-1;
    phase=1-phase;
    t++;
  }
  assert(x<=100);
  return 0;
}
```

```
$esbmc example.c --clang-frontend
ESBMC version 4.2.0 64-bit x86_64 macos
file example.c: Parsing
Converting
Type-checking example
Generating GOTO Program
GOTO program creation time: 0.232s
GOTO program processing time: 0.001s
Starting Bounded Model Checking
Unwinding loop 1 iteration 1 file example.c line 5 function
main
Unwinding loop 1 iteration 2 file example.c line 5 function
main
…
Unwinding loop 1 iteration 100 file example.c line 5 function
main
Symex completed in: 0.340s (313 assignments)
Slicing time: 0.000s
Generated 1 VCC(s), 0 remaining after simplification
VERIFICATION SUCCESSFUL
BMC program time: 0.340s
```

Running Example: *k*-induction + invariants

- Inductive step proves correctness for *k*-step 2…

```
int main() {
  int x=0, t=0, phase=0;
  while(t<100) {
    assume(-2*x+t+3*phase == 0);
    assume(3-2*x+t >= 0);
    assume(-x+2*t >= 0);
    assume(147+x-2*t >= 0);
    assume(2*x-t >= 0);
    if(phase==0) x=x+2;
    if(phase==1) x=x-1;
    phase=1-phase;
    t++;
  }
  assert(x<=100);
  return 0;
}
```

```
$esbmc example.c --clang-frontend --k-induction
*** K-Induction Loop Iteration 2 ***
*** Checking inductive step
Starting Bounded Model Checking
Unwinding loop 1 iteration 1 file example_pagai.c line 6 function main
Unwinding loop 1 iteration 2 file example_pagai.c line 6 function main
Symex completed in: 0.002s (53 assignments)
Slicing time: 0.000s
Generated 1 VCC(s), 1 remaining after simplification
No solver specified; defaulting to Boolector
Encoding remaining VCC(s) using bit-vector arithmetic
Encoding to solver time: 0.001s
Solving with solver Boolector 2.4.0
Encoding to solver time: 0.001s
Runtime decision procedure: 0.144s
VERIFICATION SUCCESSFUL
BMC program time: 0.148s
Solution found by the inductive step (k = 2)
```

inductive invariants

reuse k-induction counterexamples to speed-up bug finding

reuse results of previous steps (caching SMT queries)

Summary

- Described the difference between **soundness** and **completeness** concerning **detection techniques**
  - **False positive** and **false negative**
- Pointed out the difference between **static analysis** and **testing / simulation**
  - **hybrid combination** of static and dynamic analysis techniques to achieve a good trade-off between **soundness** and **completeness**
- Explained **bounded** and **unbounded model checking of software**
  - they have been applied successfully to verify **single- and multi-threaded software**