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**Systems and Software
Verification Laboratory**

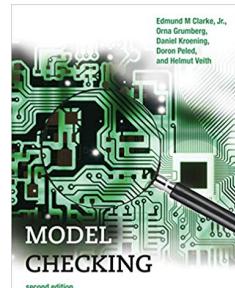


Detection of Software Vulnerabilities: Static Analysis

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Static Analysis

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 - Office: 2.28
 - Office hours: 15-16 Tuesday, 14-15 Wednesday
- Textbook:
 - *Model checking* (Chapter 14)
 - *Software model checking*. ACM Comput. Surv., 2009
 - *The Cyber Security Body of Knowledge*, 2019
 - *Software Engineering* (Chapters 8, 13)



Motivating Example

- Functionality demanded increased significantly
 - Peer reviewing and testing

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void *threadA(void *arg) {  
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    if (x == 1) lock(&lock);  
    unlock(&mutex);  
    lock(&mutex);  
    x--;  
    if (x == 0) unlock(&lock);  
    unlock(&mutex);  
}
```

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Deadlock

Intended learning outcomes

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- **Validation:** "*Are we building the right product*"
 - The software should do what the **user requires**
- Verification and validation must be applied at **each stage in the software process**
 - The **discovery of defects** in a system
 - The assessment of whether or not the system is **usable in an operational situation**

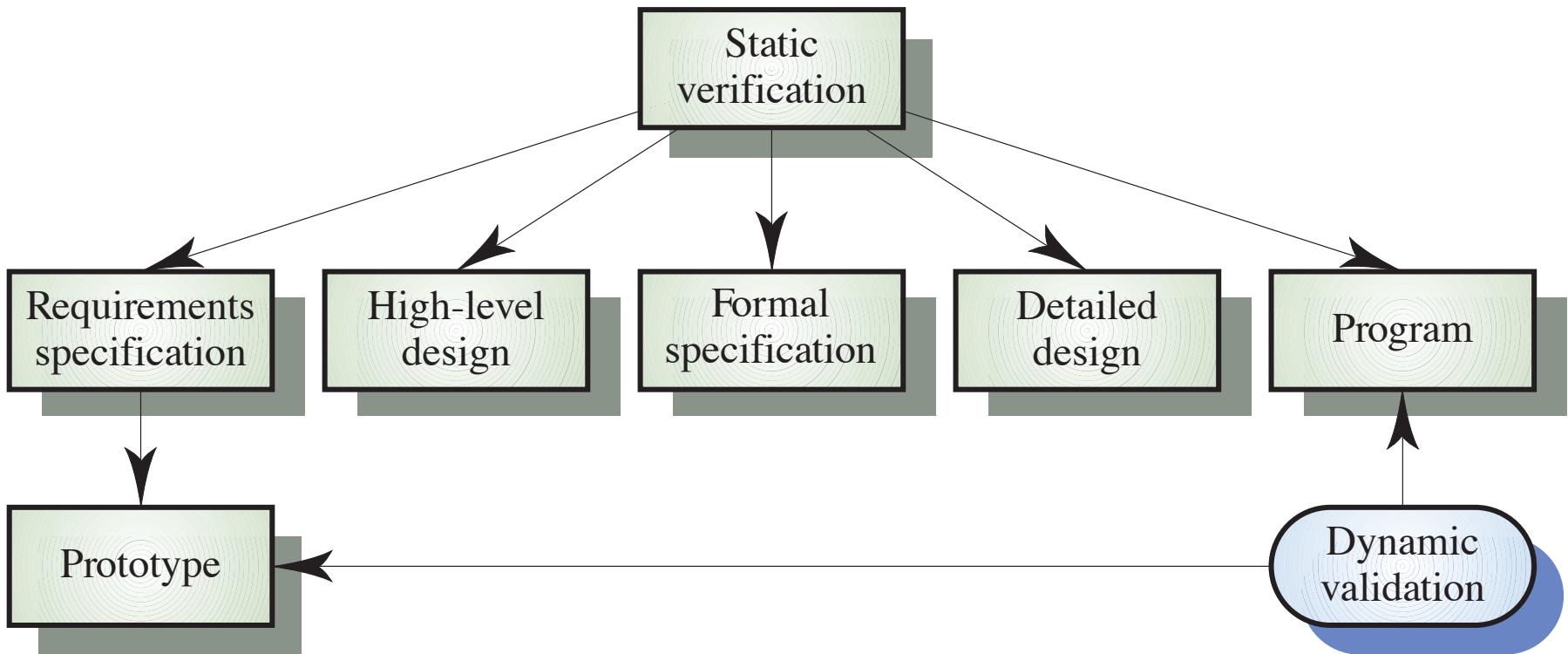
Static and Dynamic Verification

- **Software inspections** are concerned with the analysis of the static system representation to discover problems (**static verification**)
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 - **Code analysis** can prove the absence of errors but might subject to **incorrect results**
- **Software testing** is concerned with exercising and observing product behaviour (**dynamic verification**)
 - The system is executed with **test data**
 - **Operational behaviour is observed**
 - Can reveal the presence of errors **NOT their absence**

Static and Dynamic Verification



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V & V planning

- **Careful planning** is required to get the most out of **dynamic and static verification**
 - Planning should start **early in the development process**
 - The plan should identify the **balance between static and dynamic verification**

V & V planning

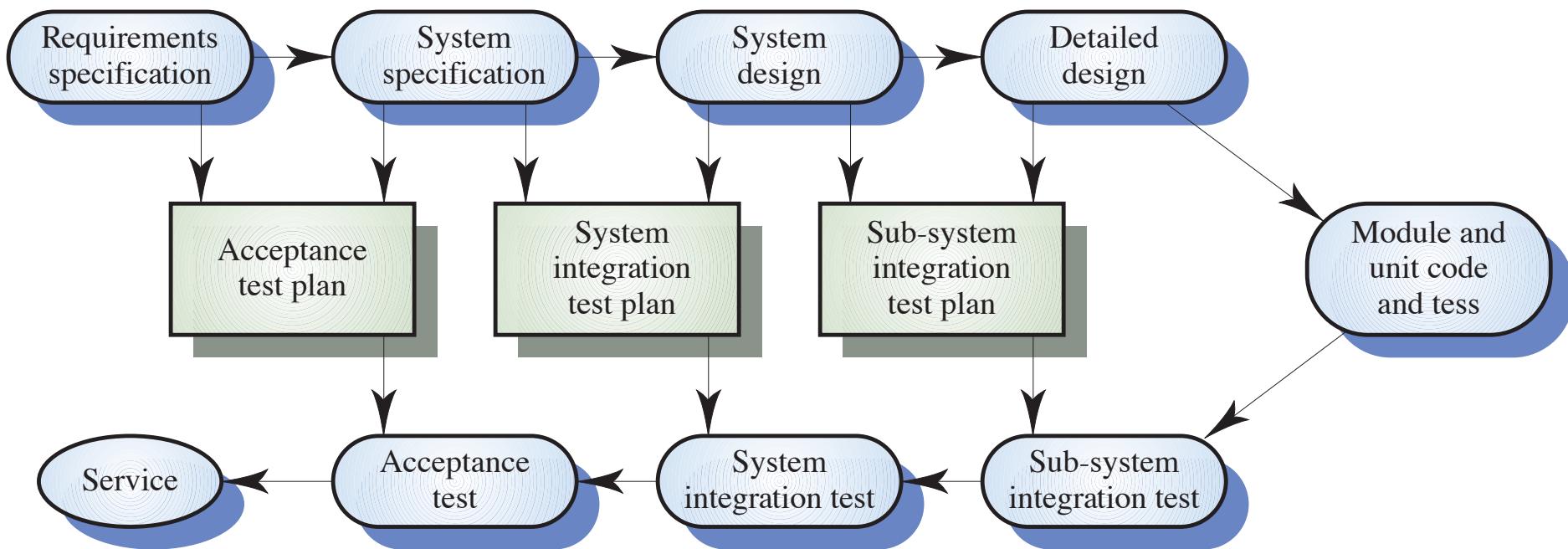
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V & V planning depends on system's purpose, user expectations and marketing environment

The V-model of development



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 - A detection technique is **complete** for a given category, if any vulnerability it finds is an actual vulnerability
 - An incomplete detection technique may have **false positives**, i.e., it may detect issues that do not turn out to be actual vulnerabilities

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 - This can be done by static checking of the program code while making suitable abstractions of the executions
- Achieving **completeness** can be done by performing actual, **concrete executions** of a program that are witnesses to any vulnerability reported
 - The analysis technique has to come up with concrete inputs for the program that triggers a vulnerability
 - A typical dynamic approach is software testing: the tester writes test cases with concrete inputs and specific checks for the outputs

Detection of Vulnerabilities

Detection tools can use a **hybrid combination of static and dynamic analysis** techniques to achieve a good trade-off between **soundness and completeness**

Detection of Vulnerabilities

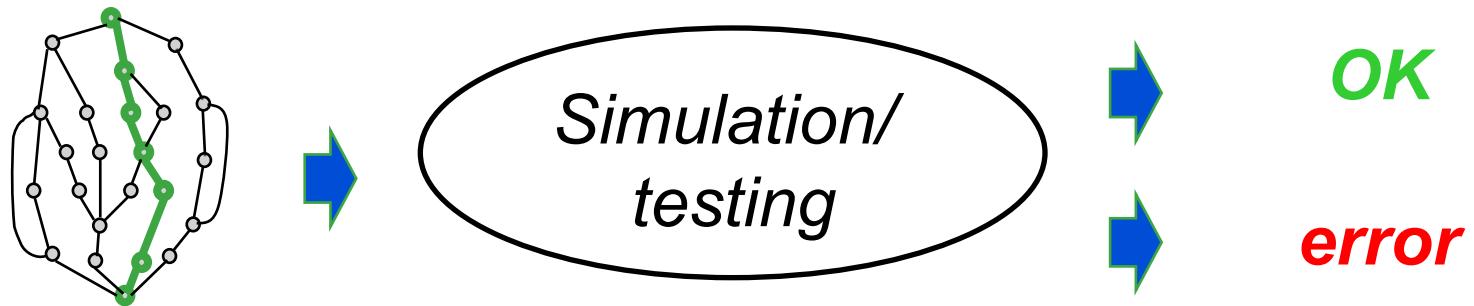
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Dynamic verification should be used in conjunction with **static verification** to provide **full code coverage**

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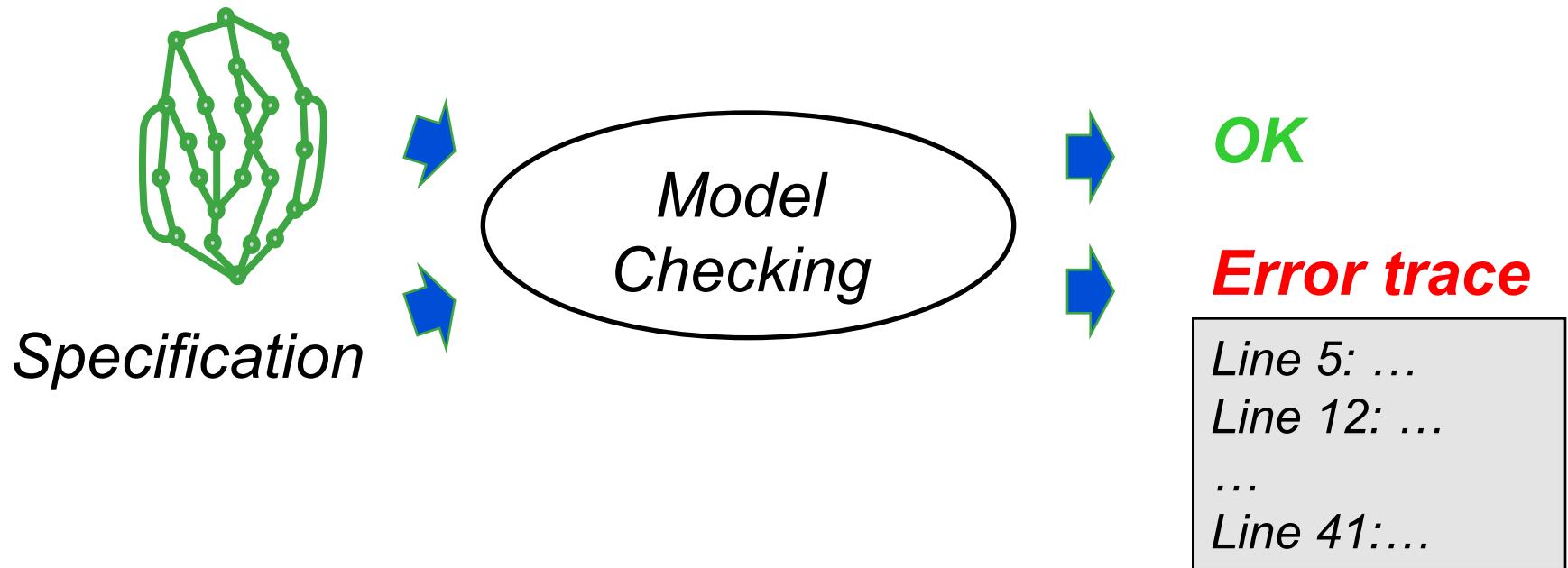
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Static analysis vs Testing/ Simulation



- **Checks only some of the system executions**
 - May **miss errors**
- **A successful execution** is an execution that **discovers one or more errors**

Static analysis vs Testing/ Simulation

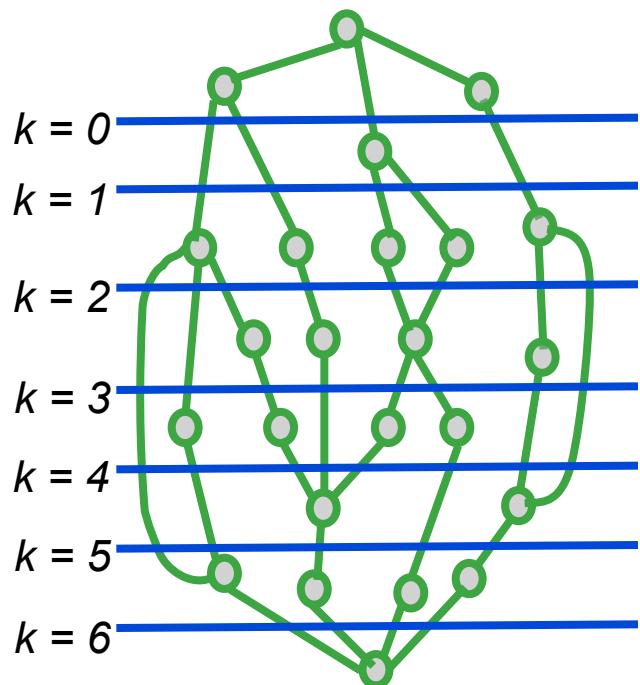


- **Exhaustively explores all executions**
- Report errors as **traces**
- May produce **incorrect results**

Avoiding state space explosion

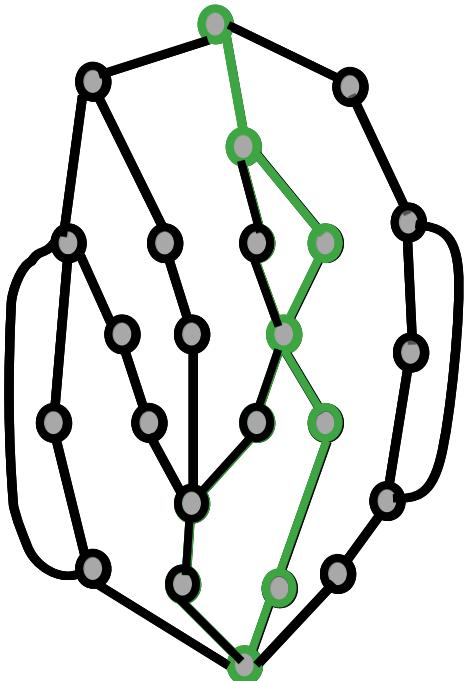
- Bounded Model Checking (BMC)
 - Breadth-first search (BFS) approach
- Symbolic Execution
 - Depth-first search (DFS) approach

Bounded Model Checking



- Bounded model checkers explore the state space in depth
- Can only prove correctness if all states are reachable within the bound

Symbolic Execution



- Symbolic execution explores all paths individually
- Can only prove correctness if all paths are explored

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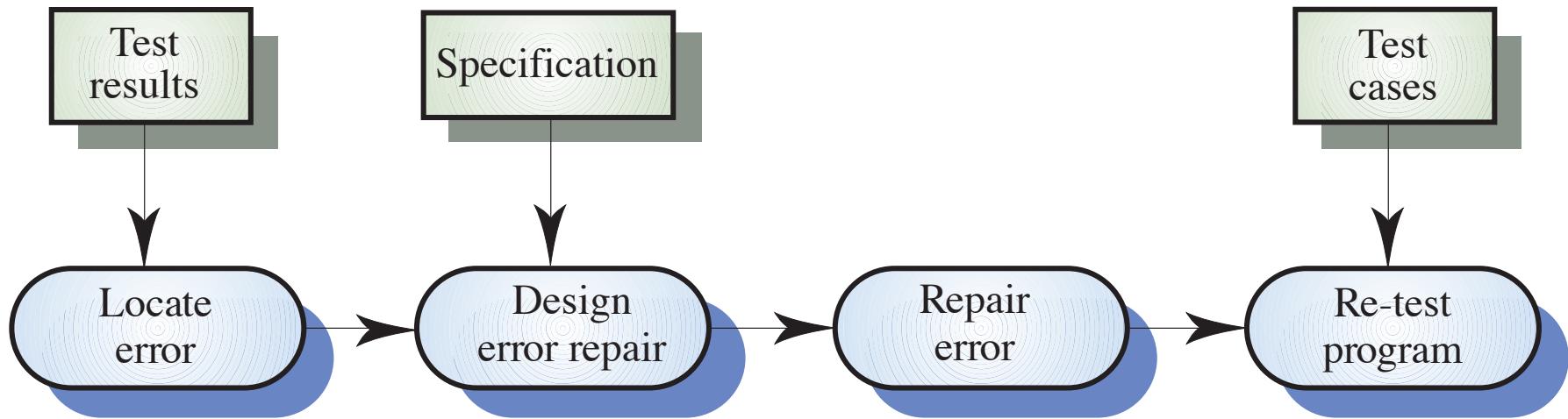
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- Debugging is concerned with two main tasks
 - Locating and
 - Repairing these errors
- Debugging involves
 - Formulating a hypothesis about program behaviour
 - Test these hypotheses to find the system error

The debugging process



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Intended learning outcomes

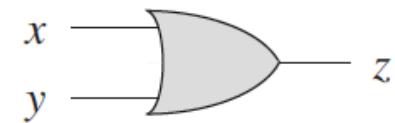
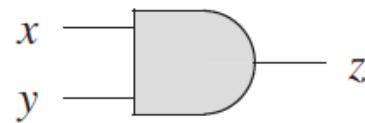
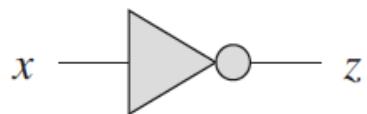
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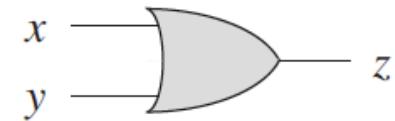
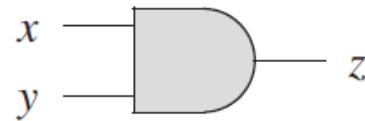
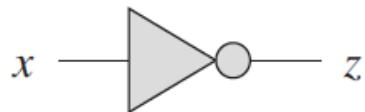
x	$\neg x$
0	1
1	0

x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

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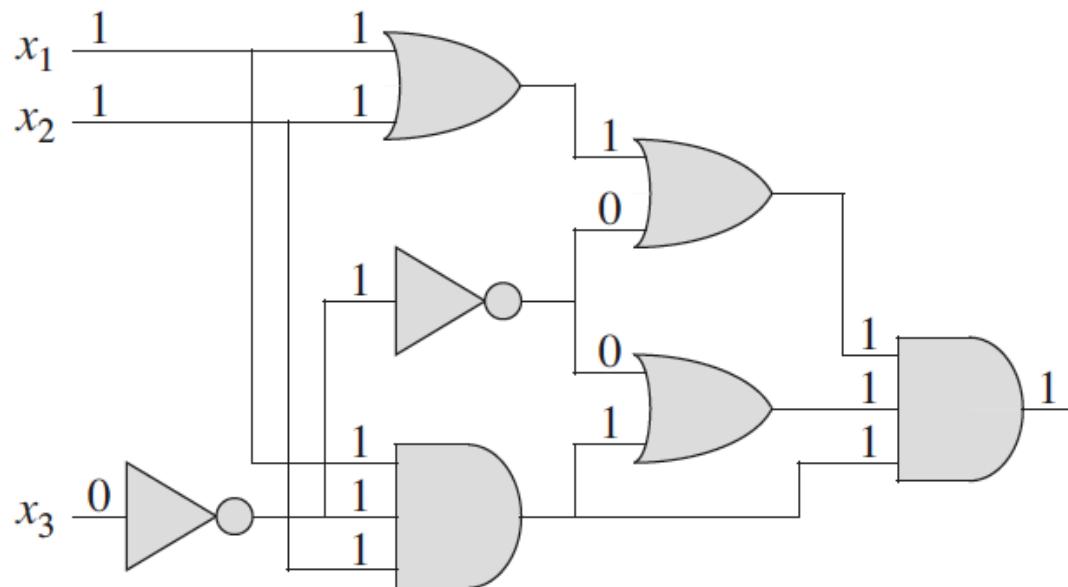
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0	1	1
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- A Boolean formula is **SAT** if there exists some assignment to its variables that **evaluates it to 1**

Circuit Satisfiability

- A **Boolean combinational circuit** consists of one or more **Boolean combinational elements** interconnected by **wires**



SAT: $\langle x_1 = 1, x_2 = 1, x_3 = 0 \rangle$

Circuit-Satisfiability Problem

- Given a **Boolean combinational circuit** of AND, OR, and NOT gates, is it **satisfiable**?

CIRCUIT-SAT = { $\langle C \rangle : C$ is a satisfiable Boolean combinational circuit}

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- When the **size of C is polynomial in k** , checking each one takes **$\Omega(2^k)$**
 - o Super-polynomial in the size of k

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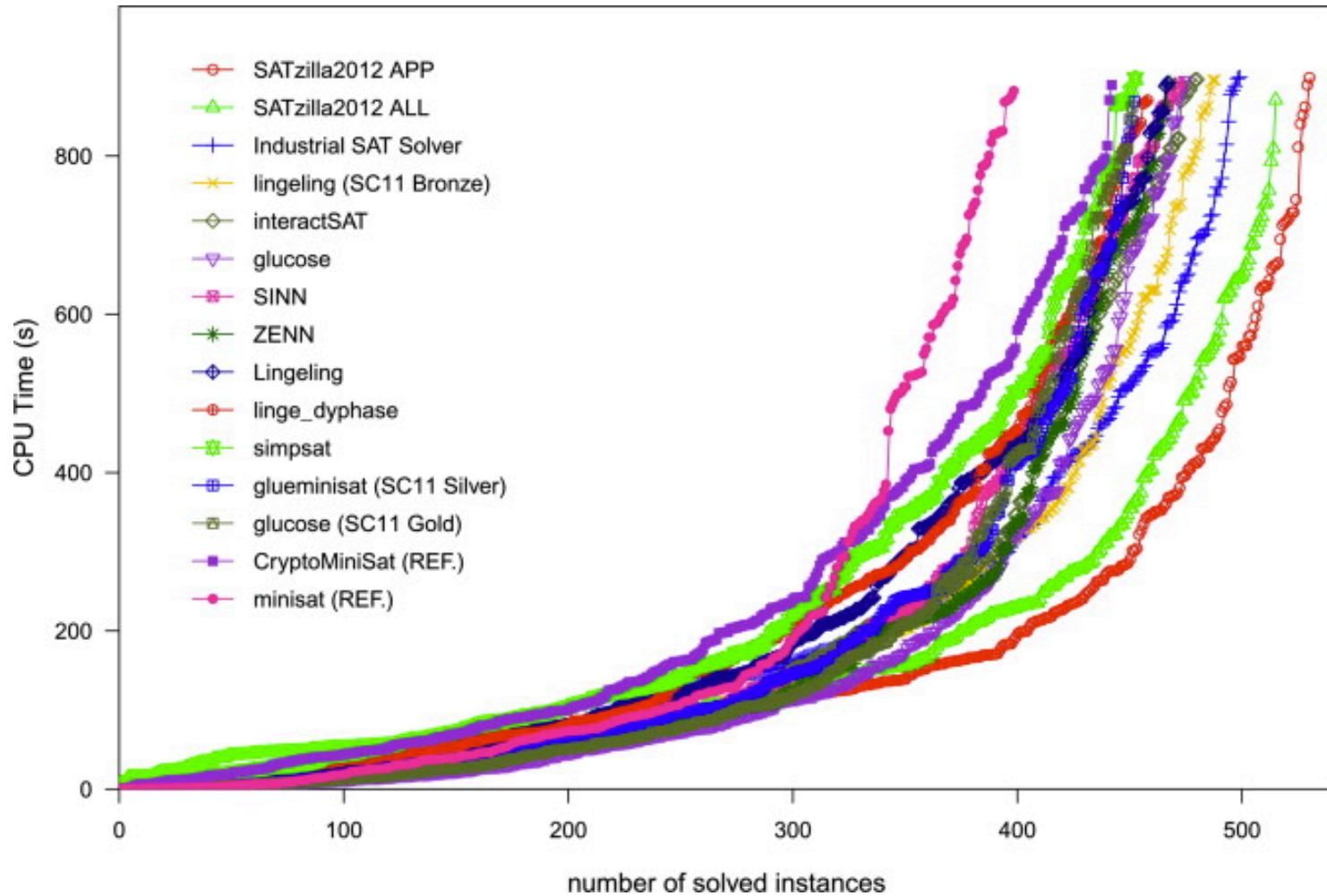
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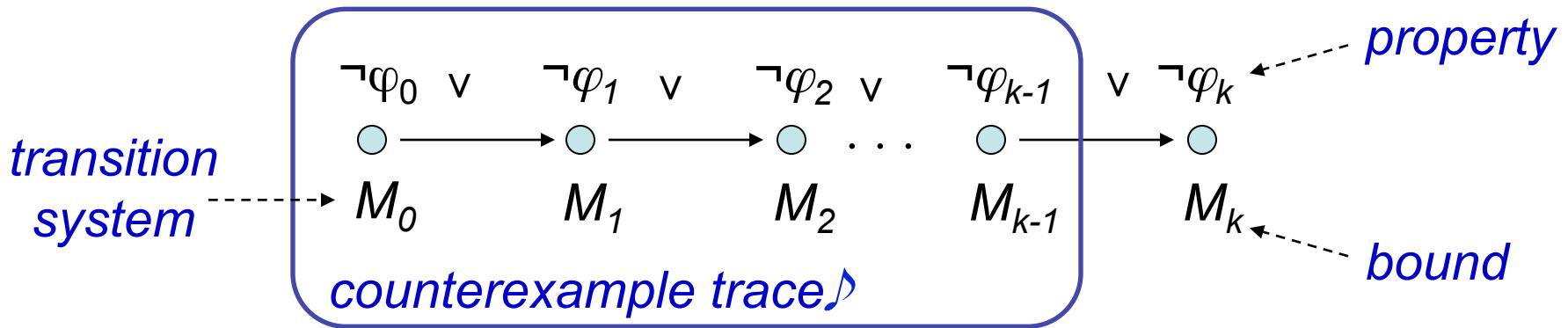
- $\Phi = 1$

SAT Competition



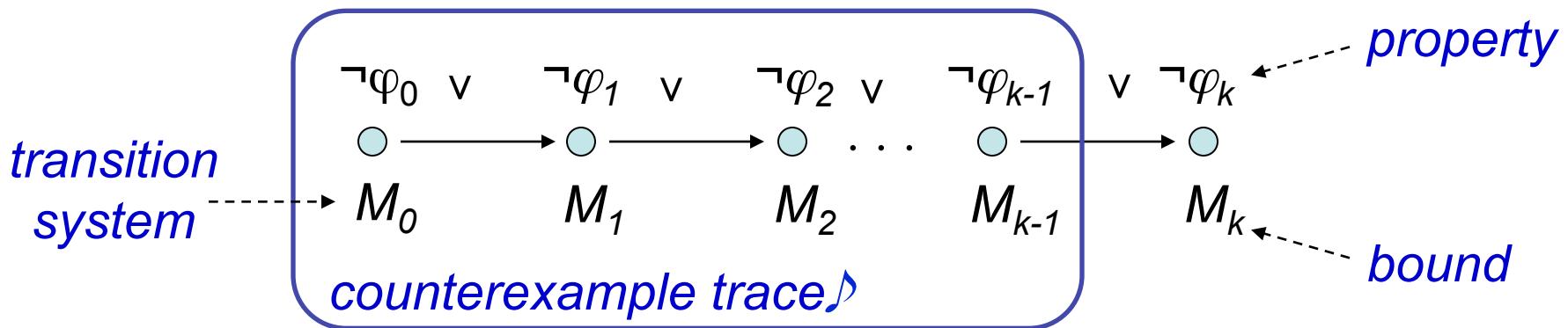
Bounded Model Checking

Basic Idea: check negation of given property up to given depth



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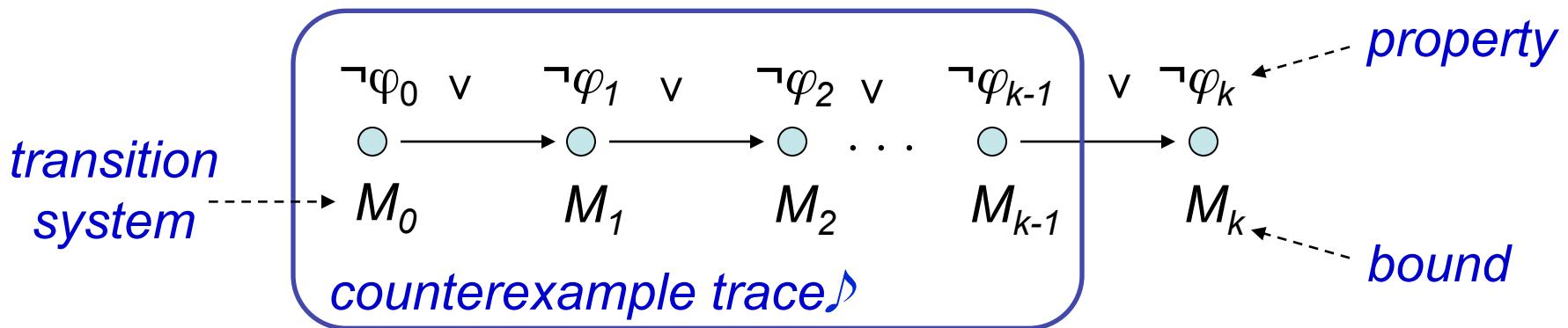
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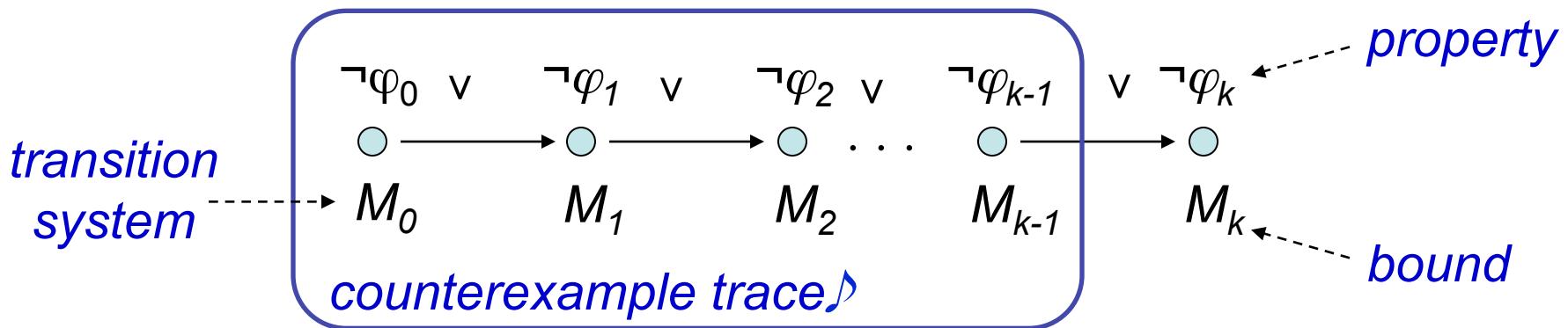
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- has been applied successfully to verify HW/SW systems

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Arrays	$(j = k \wedge a[k] = 2) \Rightarrow a[j] = 2$

Satisfiability Modulo Theories (1)

SMT decides the **satisfiability** of first-order logic formulae using the combination of different **background theories** (building-in operators)

Theory	Example
Equality	$x_1 = x_2 \wedge \neg(x_1 = x_3) \Rightarrow \neg(x_1 = x_3)$
Bit-vectors	$(b >> i) \& 1 = 1$
Linear arithmetic	$(4y_1 + 3y_2 \geq 4) \vee (y_2 - 3y_3 \leq 3)$
Arrays	$(j = k \wedge a[k] = 2) \Rightarrow a[j] = 2$
Combined theories	$(j \leq k \wedge a[j] = 2) \Rightarrow a[i] < 3$

Satisfiability Modulo Theories (2)

- Given
 - a decidable Σ -theory T
 - a quantifier-free formula φ
- φ **is T -satisfiable** iff $T \cup \{\varphi\}$ is satisfiable, i.e., there exists a structure that satisfies both formula and sentences of T

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φ is a **T -consequence of Γ** ($\Gamma \models_T \varphi$) iff every model of $T \cup \Gamma$ is also a model of φ
- Checking $\Gamma \models_T \varphi$ can be reduced in the usual way to checking the T -satisfiability of $\Gamma \cup \{\neg\varphi\}$

Satisfiability Modulo Theories (3)

- let **a** be an array, **b**, **c** and **d** be signed bit-vectors of width 16, 32 and 32 respectively, and let **g** be an unary function.

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↓ **b'** extends **b** to the signed equivalent bit-vector of size 32

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↓ replace **b'** by **c-3** in the inequality

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$$step\ 2: g(select(store(a,c,12), c - 3 + 3)) \neq g(c - 3 - c + 4) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$$

↓ using facts about bit-vector arithmetic

$$step\ 3: g(select(store(a,c,12), c)) \neq g(1) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$$

Satisfiability Modulo Theories (4)

step 3: $g(\text{select}(\text{store}(a, c, 12), c)) \neq g(1) \wedge c - 3 = c - 3 \wedge c + 1 = d - 4$

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 The function g implies that for all x and y ,
 if $x = y$, then $g(x) = g(y)$ (*congruence rule*).

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- SMT solvers also apply:
 - standard algebraic reduction rules
 - contextual simplification

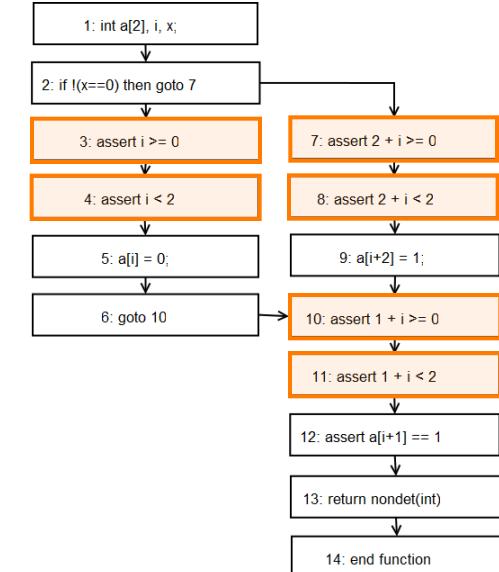
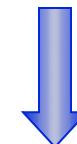
$$r \wedge \text{false} \mapsto \text{false}$$

$$a = 7 \wedge p(a) \mapsto a = 7 \wedge p(7)$$

BMC of Software

- program modelled as state transition system
 - state: program counter and program variables
 - derived from control-flow graph
 - checked safety properties give extra nodes
 - program unfolded up to given bounds
 - loop iterations
 - context switches
 - unfolded program optimized to reduce blow-up
 - constant propagation
 - forward substitutions
- } crucial

```
int main() {  
    int a[2], i, x;  
    if (x==0)  
        a[i]=0;  
    else  
        a[i+2]=1;  
    assert(a[i+1]==1);  
}
```



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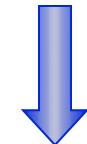
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$$\begin{aligned} g_1 &= x_1 == 0 \\ a_1 &= a_0 \text{ WITH } [i_0:=0] \\ a_2 &= a_0 \\ a_3 &= a_2 \text{ WITH } [2+i_0:=1] \\ a_4 &= g_1 ? a_1 : a_3 \\ t_1 &= a_4[1+i_0] == 1 \end{aligned}$$

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- front-end converts unrolled and optimized program into SSA
- extraction of constraints C and properties P
 - specific to selected SMT solver, uses theories
- satisfiability check of $C \wedge \neg P$

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}
```



$$C := \left[\begin{array}{l} g_1 := (x_1 = 0) \\ \wedge a_1 := \text{store}(a_0, i_0, 0) \\ \wedge a_2 := a_0 \\ \wedge a_3 := \text{store}(a_2, 2 + i_0, 1) \\ \wedge a_4 := \text{ite}(g_1, a_1, a_3) \end{array} \right]$$

$$P := \left[\begin{array}{l} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge 2 + i_0 \geq 0 \wedge 2 + i_0 < 2 \\ \wedge 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge \text{select}(a_4, i_0 + 1) = 1 \end{array} \right]$$

Encoding of Numeric Types

- SMT solvers typically provide different encodings for numbers:
 - abstract domains (**Z**, **R**)
 - fixed-width bit vectors (`unsigned int`, ...)
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- majority of VCs solved faster if numeric types are modelled by abstract domains but possible loss of precision
- ESBMC supports both types of encoding and also combines them to improve scalability and precision

Encoding Numeric Types as Bitvectors

Bitvector encodings need to handle

- type casts and implicit conversions
 - arithmetic conversions implemented using word-level functions (part of the bitvector theory: Extract, SignExt, ...)
 - different conversions for every pair of types
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 - $t = \text{ite}(v \neq k, \text{true}, \text{false})$ //conversion to bool
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 - standard requires modulo-arithmetic for unsigned integer
 - $\text{unsigned_overflow} \Leftrightarrow (r - (r \bmod 2^w)) < 2^w$
 - define error literals to detect over- / underflow for other types
 - $\text{res_op} \Leftrightarrow \neg \text{overflow}(x, y) \wedge \neg \text{underflow}(x, y)$
 - similar to conversions

Floating-Point Numbers

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 - encode the integral (i) and fractional (f) parts

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 - encode the integral (i) and fractional (f) parts
- **Binary encoding:** get a new bit-vector $b = i @ f$ with the same bitwidth before and after the radix point of a.

$$i = \begin{cases} Extract(b, n_b + m_a - 1, n_b) & : m_a \leq m_b \\ SignExt(Extract(b, t_b - 1, n_b), m_a - m_b) & : \text{otherwise} \end{cases} \quad // m = \text{number of bits of } i$$
$$f = \begin{cases} Extract(b, n_b - 1, n_b - n_b) & : n_a \leq n_b \\ Extract(b, n_b, 0) @ SignExt(b, n_a - n_b) & : \text{otherwise} \end{cases} \quad // n = \text{number of bits of } f$$

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- **Rational encoding:** convert a to a rational number

$$a = \begin{cases} \frac{\left(i * p + \left(\frac{f * p}{2^n} + 1 \right) \right)}{p} & : f \neq 0 \\ i & : \text{otherwise} \end{cases} \quad // p = \text{number of decimal places}$$

Floating-point SMT Encoding

- The SMT floating-point theory is an addition to the SMT standard, proposed in 2010 and formalises:
 - Floating-point arithmetic
 - Positive and negative infinities and zeroes,
 - NaNs,
 - Comparison operators,
 - Five rounding modes: round nearest with ties choosing the even value, round nearest with ties choosing away from zero, round towards zero, round towards positive infinity and round towards negative infinity

Floating-point SMT Encoding

- Missing from the standard:
 - Floating-point exceptions
 - Signaling NaNs
- Two solvers currently support the standard:
 - Z3: implements all operators
 - MathSAT: implements all but two operators
 - o *fp.rem*: remainder: $x - y * n$, where n in \mathbb{Z} is nearest to x/y
 - o *fp.fma*: fused multiplication and addition; $(x * y) + z$
- Both solvers offer non-standard functions:
 - *fp_as_ieeebv*: converts floating-point to bitvectors
 - *fp_from_ieeebv*: converts bitvectors to floating-point

How to encode Floating-Point programs?

- Most operations performed at program-level to encode FP numbers have a **one-to-one conversion to SMT**
- Special cases being casts to boolean types and the fp.eq operator
 - Usually, cast operations are encoded using **extend/extract operation**
 - Extending floating-point numbers is non-trivial because of the format

```
int main()
{
    _Bool c;

    double b = 0.0f;
    b = c;
    assert(b != 0.0f);

    c = b;
    assert(c != 0);

}
```

Cast to/from booleans

- Simpler solutions:
 - Casting **booleans** to **floating-point numbers** can be done using an `ite` operator

```
(assert (= (ite |main::c|
                (fp #b0 #b11111111111 #x00000000000000)
                (fp #b0 #b00000000000 #x00000000000000))
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```

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```
(assert (= (ite |main::c|  
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           (fp #b0 #b000000000000 #x00000000000000))  
        |main::b|))
```

If true, assign 1f to b

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Cast to/from booleans

- Simpler solutions:
 - Casting **floating-point numbers** to **booleans** can be done using an equality and one not:

```
(assert (= (not (fp.eq |main::b|
                  (fp #b0 #b00000000000 #x00000000000000)))
           |main::c|))
```

:note

"(fp.eq x y) evaluates to true if x evaluates to -zero and y to +zero, or vice versa. fp.eq and all the other comparison operators evaluate to false if one of their arguments is NaN."

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```
(assert (= (not (fp.eq |main::b|  
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           |main::c|))
```

true when the floating is not 0.0

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*otherwise, the result is
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Illustrative Example

```
int main()
{
    float x;
    float y = x;
    assert( x==y );
    return 0;
}
```

Illustrative Example

```
; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
(declare-fun |main::y| () (_ FloatingPoint 8 24))

; symbol created to represent a nondeterministic number
(declare-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24))

; Global guard, used for checking properties
(declare-fun |execution_statet::\\guard_exec| () Bool)

; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|))

; assert x == y
(assert (let ((a!1 (not (=> true
                           (=> |execution_statet::\\guard_exec|
                               (fp.eq |main::x| |main::y|)))))))
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```

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Variable declarations

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```

Nondeterministic symbol
declaration (optional)

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```

Guard used to check
satisfiability

Illustrative Example

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```

Assignment of
nondeterministic
value to x

Illustrative Example

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; declaration of x and y
(declare-fun |main::x| () (_ FloatingPoint 8 24))
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; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|)) ← Assignment x to y

; assert x == y
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(declare-fun |execution_statet::\\guard_exec| () Bool)

; assign the nondeterministic symbol to x
(assert (= |nondet_symex::nondet0| |main::x|))

; assign x to y
(assert (= |main::x| |main::y|))

; assert x == y
(assert (let ((a!1 (not (=> true
                           (=> |execution_statet::\\guard_exec|
                                (fp.eq |main::x| |main::y|)))))))
        (or a!1)))
```

Check if the comparison
satisfies the guard



Illustrative Example

- Z3 produces:

```
sat
(model
  (define-fun |main::x| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |main::y| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |nondet_symex::nondet0| () (_ FloatingPoint 8 24)
    (_ NaN 8 24))
  (define-fun |execution_statet::\\\\\\guard_exec| () Bool
    true)
)
```

Illustrative Example

- MathSAT produces:

```
sat
( (|main::x| (_ NaN 8 24))
  (|main::y| (_ NaN 8 24))
  (|nondet_symex::nondet0| (_ NaN 8 24))
  (|execution_statet::\\guard_exec| true) )
```

Illustrative Example

Counterexample:

State 1 file main3.c line 3 function main thread 0
main

State 2 file main3.c line 4 function main thread 0
main

State 3 file main3.c line 5 function main thread 0
main

Violated property:

```
file main3.c line 5 function main
assertion
(_Bool)(x == y)
```

VERIFICATION FAILED

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
 - $p.o \triangleq$ representation of underlying object
 - $p.i \triangleq$ index (if pointer used as array base)

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```
int main() {
    int a[2], i, x, *p;
    p=a;
    if (x==0)
        a[i]=0;
    else
        a[i+1]=1;
    assert(*(p+2)==1);
}
```

Encoding of Pointers

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int main() {  
    int a[2], i, x, *p;  
    p=a;  
    if (x==0)  
        a[i]=0;  
    else  
        a[i+1]=1;  
    assert(*(p+2)==1);  
}
```



$$C := \left\{ \begin{array}{l} p_1 := \text{store}(p_0, 0, \&a[0]) \\ \wedge p_2 := \text{store}(p_1, 1, 0) \\ \wedge g_2 := (x_2 == 0) \\ \wedge a_1 := \text{store}(a_0, i_0, 0) \\ \wedge a_2 := a_0 \\ \wedge a_3 := \text{store}(a_2, 1 + i_0, 1) \\ \wedge a_4 := \text{ite}(g_1, a_1, a_3) \\ \wedge p_3 := \text{store}(p_2, 1, \text{select}(p_2, 1) + 2) \end{array} \right\}$$

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- pointers modelled as tuples
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Store object at position 0

```
int main() {  
    int a[2], i, x, *p;  
    p=a;  
    if (x==0)  
        a[i]=0;  
    else  
        a[i+1]=1;  
    assert(*(p+2)==1);  
}
```

$$C := \left\{ \begin{array}{l} p_1 := \text{store}(p_0, 0, \&a[0]) \\ \wedge p_2 := \text{store}(p_1, 1, 0) \\ \wedge g_2 := (x_2 == 0) \\ \wedge a_1 := \text{store}(a_0, i_0, 0) \\ \wedge a_2 := a_0 \\ \wedge a_3 := \text{store}(a_2, 1 + i_0, 1) \\ \wedge a_4 := \text{ite}(g_1, a_1, a_3) \\ \wedge p_3 := \text{store}(p_2, 1, \text{select}(p_2, 1) + 2) \end{array} \right\}$$

Encoding of Pointers

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 - $p.o \triangleq$ representation of underlying object
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```
int main() {  
    int a[2], i, x, *p;  
    p=a;  
    if (x==0)  
        a[i]=0;  
    else  
        a[i+1]=1;  
    assert(*(p+2)==1);  
}
```

$$C := \left\{ \begin{array}{l} p_1 := \text{store}(p_0, 0, \&a[0]) \\ \wedge p_2 := \text{store}(p_1, 1, 0) \\ \wedge g_2 := (x_2 == 0) \\ \wedge a_1 := \text{store}(a_0, i_0, 1) \\ \wedge a_2 := a_0 \\ \wedge a_3 := \text{store}(a_2, 1 + i_0, 1) \\ \wedge a_4 := \text{ite}(g_1, a_1, a_3) \\ \wedge p_3 := \text{store}(p_2, 1, \text{select}(p_2, 1) + 2) \end{array} \right\}$$

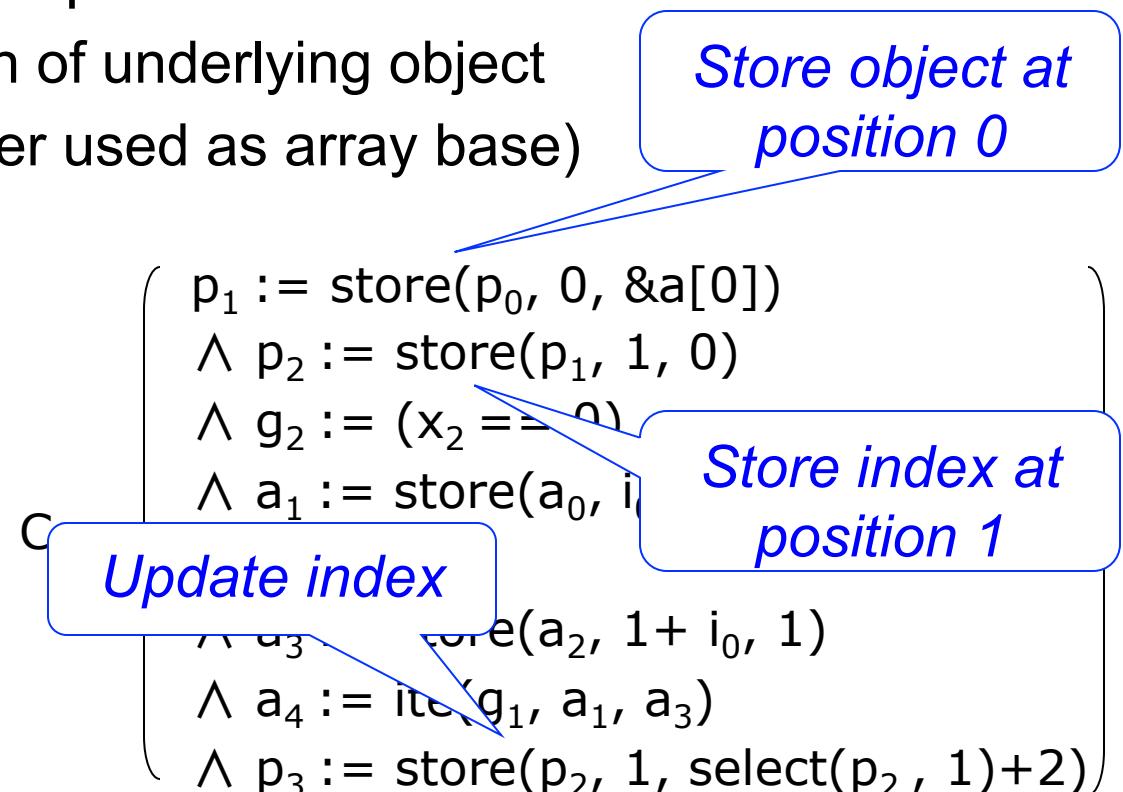
Store object at position 0

Store index at position 1

Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
 - $p.o \triangleq$ representation of underlying object
 - $p.i \triangleq$ index (if pointer used as array base)

```
int main() {  
    int a[2], i, x, *p;  
    p=a;  
    if (x==0)  
        a[i]=0;  
    else  
        a[i+1]=1;  
    assert(*(p+2)==1);  
}
```



Encoding of Pointers

- arrays and records / tuples typically handled directly by SMT-solver
- pointers modelled as tuples
 - $p.o \triangleq$ representation of underlying object
 - $p.i \triangleq$ index (if pointer used as array base)

```
int main() {  
    int a[2], i, x, *p;  
    p=a;  
    if (x==0)  
        a[i]=0;  
    else  
        a[i+1]=1;  
    assert(*(p+2)==1);  
}
```



$$P := \left\{ \begin{array}{l} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge \text{select}(p_3, 0) = \&a[0] \\ \wedge \text{select}(\text{select}(p_3, 0), \\ \quad \quad \quad \text{select}(p_3, 1)) = 1 \end{array} \right\}$$

*negation satisfiable
(a[2] unconstrained)
⇒ assert fails*

Encoding of Memory Allocation

- model memory just as an array of bytes (array theories)
 - read and write operations to the memory array on the logic level

Encoding of Memory Allocation

- model memory just as an array of bytes (array theories)
 - read and write operations to the memory array on the logic level
- each dynamic object d_o consists of
 - $m \triangleq$ memory array
 - $s \triangleq$ size in bytes of m
 - $\rho \triangleq$ unique identifier
 - $v \triangleq$ indicate whether the object is still alive
 - $l \triangleq$ the location in the execution where m is allocated

Encoding of Memory Allocation

- model memory just as an array of bytes (array theories)
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- each dynamic object d_o consists of
 - $m \triangleq$ memory array
 - $s \triangleq$ size in bytes of m
 - $\rho \triangleq$ unique identifier
 - $v \triangleq$ indicate whether the object is still alive
 - $l \triangleq$ the location in the execution where m is allocated
- to detect invalid reads/writes, we check whether
 - d_o is a dynamic object
 - i is within the bounds of the memory array

$$l_{is_dynamic_object} \Leftrightarrow \left(\bigvee_{j=1}^k d_o.\rho = j \right) \wedge (0 \leq i < n)$$

Encoding of Memory Allocation

- to check for invalid objects, we
 - set v to true when the function malloc is called (d_o is alive)
 - set v to false when the function free is called (d_o is not longer alive)

$$I_{valid_object} \Leftrightarrow (I_{is_dynamic_object} \Rightarrow d_o \cdot v)$$

Encoding of Memory Allocation

- to check for invalid objects, we
 - set v to true when the function malloc is called (d_o is alive)
 - set v to false when the function free is called (d_o is not longer alive)

$$I_{valid_object} \Leftrightarrow (I_{is_dynamic_object} \Rightarrow d_o \cdot v)$$

- to detect forgotten memory, at the end of the (unrolled) program we check
 - whether the d_o has been deallocated by the function free

$$I_{deallocated_object} \Leftrightarrow (I_{is_dynamic_object} \Rightarrow \neg d_o \cdot v)$$

Example of Memory Allocation

```
#include <std  
void main() {  
    char *p =  
    char *q = malloc(5); // p =  
    p=q;  
    free(p)  
    p = malloc(5);      // p = 3  
    free(p)  
}
```

*memory leak: pointer
reassignment makes d_{o1}.v
to become an orphan*

Example of Memory Allocation

```
#include <stdlib.h>
```

```
void main() {
```

```
    char *p = malloc(5); // ρ = 1
```

```
    char *q = malloc(5); // ρ = 2
```

```
p=q;
```

```
free(p)
```

```
p = malloc(5); // ρ = 3
```

```
free(p)
```

```
}
```



$$P := (\neg d_{o1}.v \wedge \neg d_{o2}.v \wedge \neg d_{o3}.v)$$

↓

$$C := \left\{ \begin{array}{l} d_{o1}.\rho=1 \wedge d_{o1}.s=5 \wedge d_{o1}.v=true \wedge p=d_{o1} \\ \wedge d_{o2}.\rho=2 \wedge d_{o2}.s=5 \wedge d_{o2}.v=true \wedge q=d_{o2} \\ \wedge p=d_{o2} \wedge d_{o2}.v=false \\ \wedge d_{o3}.\rho=3 \wedge d_{o3}.s=5 \wedge d_{o3}.v=true \wedge p=d_{o3} \\ \wedge d_{o3}.v=false \end{array} \right\}$$

Example of Memory Allocation

```
#include <stdlib.h>
```

```
void main() {
```

```
    char *p = malloc(5); // ρ = 1
```

```
    char *q = malloc(5); // ρ = 2
```

```
p=q;
```

```
free(p)
```

```
p = malloc(5); // ρ = 3
```

```
free(p)
```

```
}
```

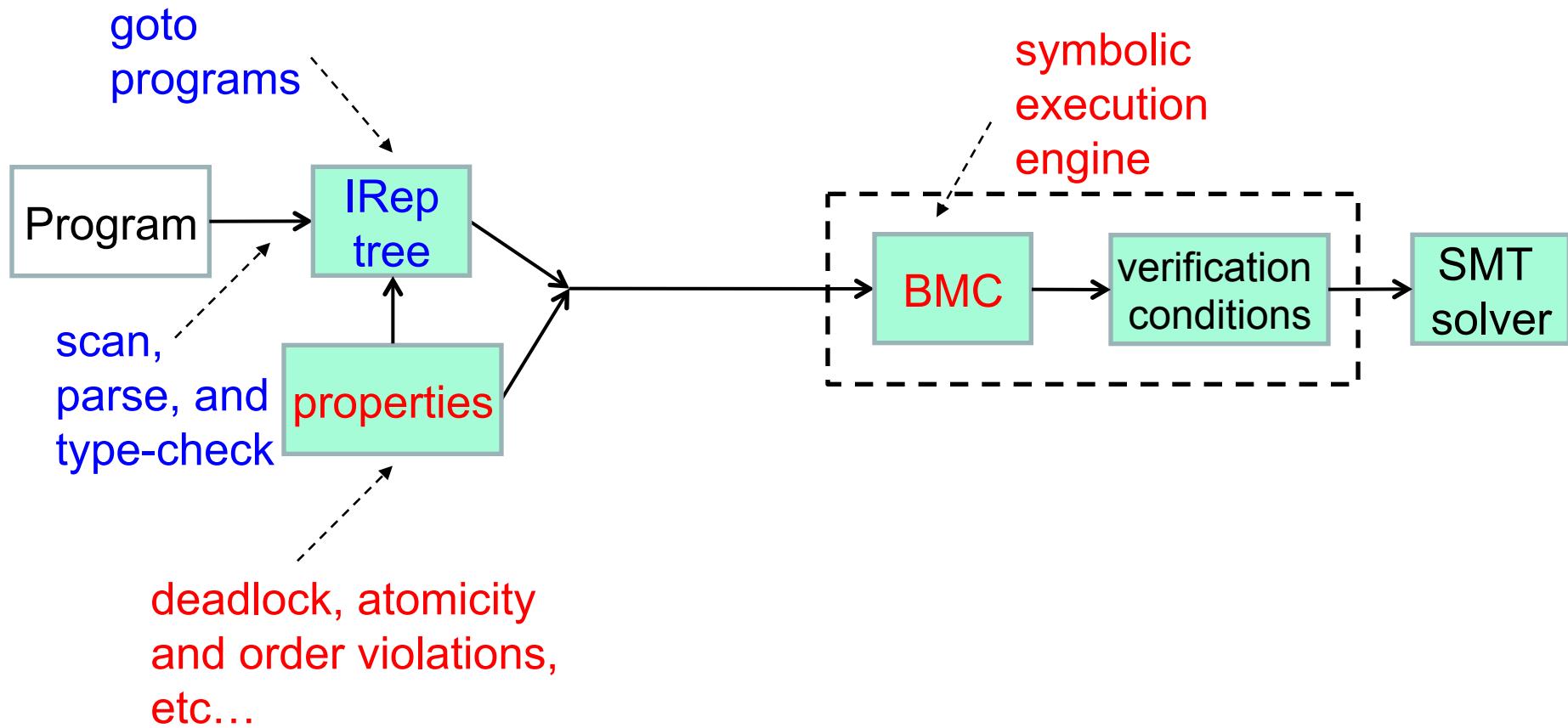


$$P := (\neg d_{o1}.v \wedge \neg d_{o2}.v \wedge \neg d_{o3}.v)$$

C :=
$$\left\{ \begin{array}{l} d_{o1}.\rho=1 \wedge d_{o1}.s=5 \wedge \textcolor{red}{d_{o1}.v=true} \wedge p=d_{o1} \\ \wedge d_{o2}.\rho=2 \wedge d_{o2}.s=5 \wedge d_{o2}.v=true \wedge q=d_{o2} \\ \wedge p=d_{o2} \wedge d_{o2}.v=false \\ \wedge d_{o3}.\rho=3 \wedge d_{o3}.s=5 \wedge d_{o3}.v=true \wedge p=d_{o3} \\ \wedge d_{o3}.v=false \end{array} \right\}$$

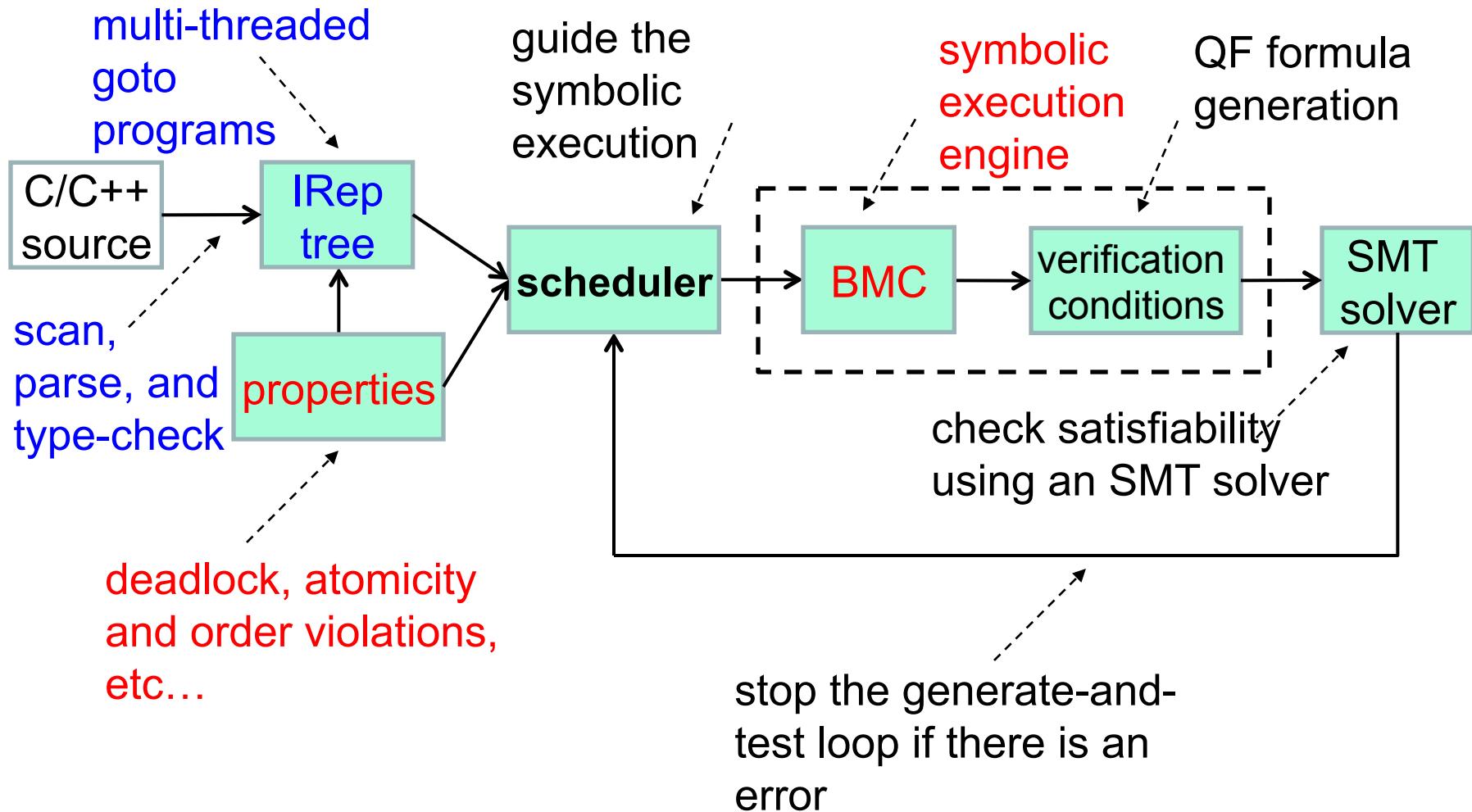
BMC Architecture

- A typical BMC architecture for verifying programs



BMC of Multi-threaded Software

Idea: iteratively generate all possible interleavings and call the BMC procedure on each interleaving



Running Example

- the program has sequences of operations that need to be protected together to avoid atomicity violation
 - requirement: the region of code (val1 and val2) should execute atomically

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

program counter: 0

mutexes: m1=0; m2=0;

global variables: val1=0; val2=0;

local variables: t1= -1; t2= -1;

A state $s \in S$ consists of the value of the program counter pc and the values of all program variables

```
7: unlock(m1);
8: lock(m2);
9: t2 = val2;
10: unlock(m2);
11: assert(t2==(t1+1));
```

Lazy exploration: interleaving I_s

statements:

val1-access:

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 0

mutexes: m1=0; m2=0;

global variables: val1=0; val2=0;

local variables: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1

val1-access:

val2-access:

Thread twoStage

- 1: *lock(m1);*
- 2: *val1 = 1;*
- 3: *unlock(m1);*
- 4: *lock(m2);*
- 5: *val2 = val1 + 1;*
- 6: *unlock(m2);*

Thread reader

- 7: *lock(m1);*
- 8: *if (val1 == 0) {*
- 9: *unlock(m1);*
- 10: *return NULL; }*
- 11: *t1 = val1;*
- 12: *unlock(m1);*
- 13: *lock(m2);*
- 14: *t2 = val2;*
- 15: *unlock(m2);*
- 16: *assert(t2==(t1+1));*

program counter: 1

mutexes: **m1=1**; m2=0;

global variables: val1=0; val2=0;

local variables: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2

val1-access: W_{twoStage, 2}

val2-access:

write access to the shared variable val1 in statement 2 of the thread twoStage

Thread twoStage
1: lock(m1);
● 2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);

Thread reader
7: lock(m1);
8: if (val1 == 0) {
9: unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));

program counter: 2

mutexes: m1=1; m2=0;

global variables: val1=1; val2=0;

local variables: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3

val1-access: $W_{\text{twoStage}, 2}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
● 3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 3

mutexes: **m1=0**; m2=0;

global variables: val1=1; val2=0;

local variables: t1= -1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7

val1-access: $W_{\text{twoStage}, 2}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 7

mutexes: m1=1; m2=0;

global variables: val1=1; val2=0;

local variables: t1= -1; t2= -1;

Lazy exploration: interleaving 1

statements: 1-2-3-7-8

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8}$

val2-access:

read access to the shared variable *val1* in statement 8 of the thread *reader*

Thread twoStage
1: *lock(m1)*;
2: *val1 = 1*;
3: *unlock(m1)*;
4: *lock(m2)*;
5: *val2 = val1 + 1*;
6: *unlock(m2)*;

CS1

Thread reader
7: *lock(m1)*;
8: *if (val1 == 0) {*
9: *unlock(m1)*;
10: *return NULL; }*
11: *t1 = val1*;
12: *unlock(m1)*;
13: *lock(m2)*;
14: *t2 = val2*;
15: *unlock(m2)*;
16: *assert(t2==(t1+1))*;

program counter: 8

mutexes: $m1=1; m2=0$;

global variables: $val1=1; val2=0$;

local variables: $t1= -1; t2= -1$;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11

val1-access: $W_{\text{twoStage}, 2} - R_{\text{reader}, 8} - R_{\text{reader}, 11}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
```

11: ***t1 = val1;***

```
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 11

mutexes: m1=1; m2=0;

global variables: val1=1; val2=0;

local variables: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1); 12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 12

mutexes: **m1=0**; m2=0;

global variables: val1=1; val2=0;

local variables: t1= 1; t2= -1;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);           CS1
5: val2 = val1 + 1;    CS2
6: unlock(m2);
```

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 4

mutexes: $m1=0; m2=0;$

global variables: $val1=1; val2=0;$

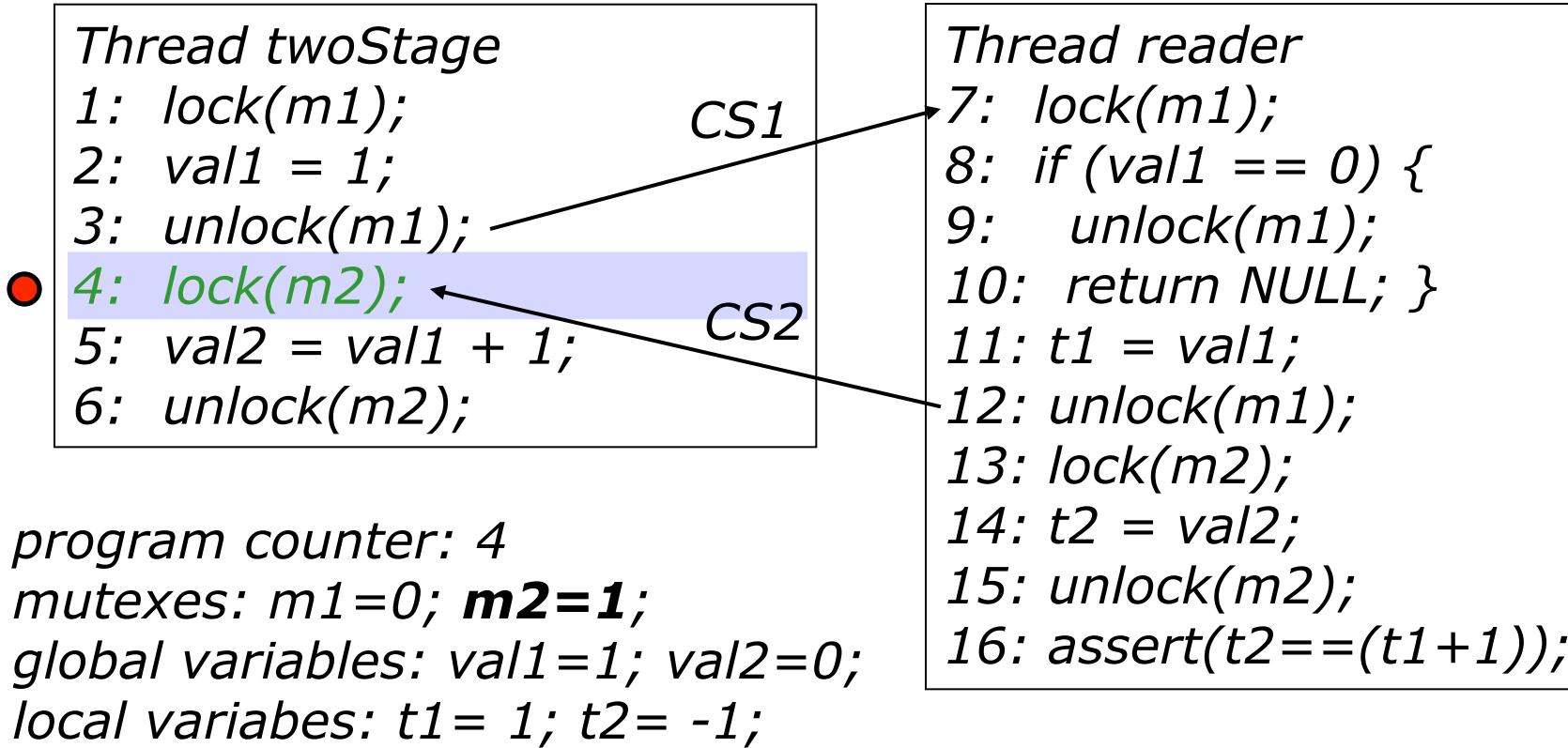
local variables: $t1= 1; t2= -1;$

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4

val1-access: $W_{\text{twoStage}, 2} - R_{\text{reader}, 8} - R_{\text{reader}, 11}$

val2-access:

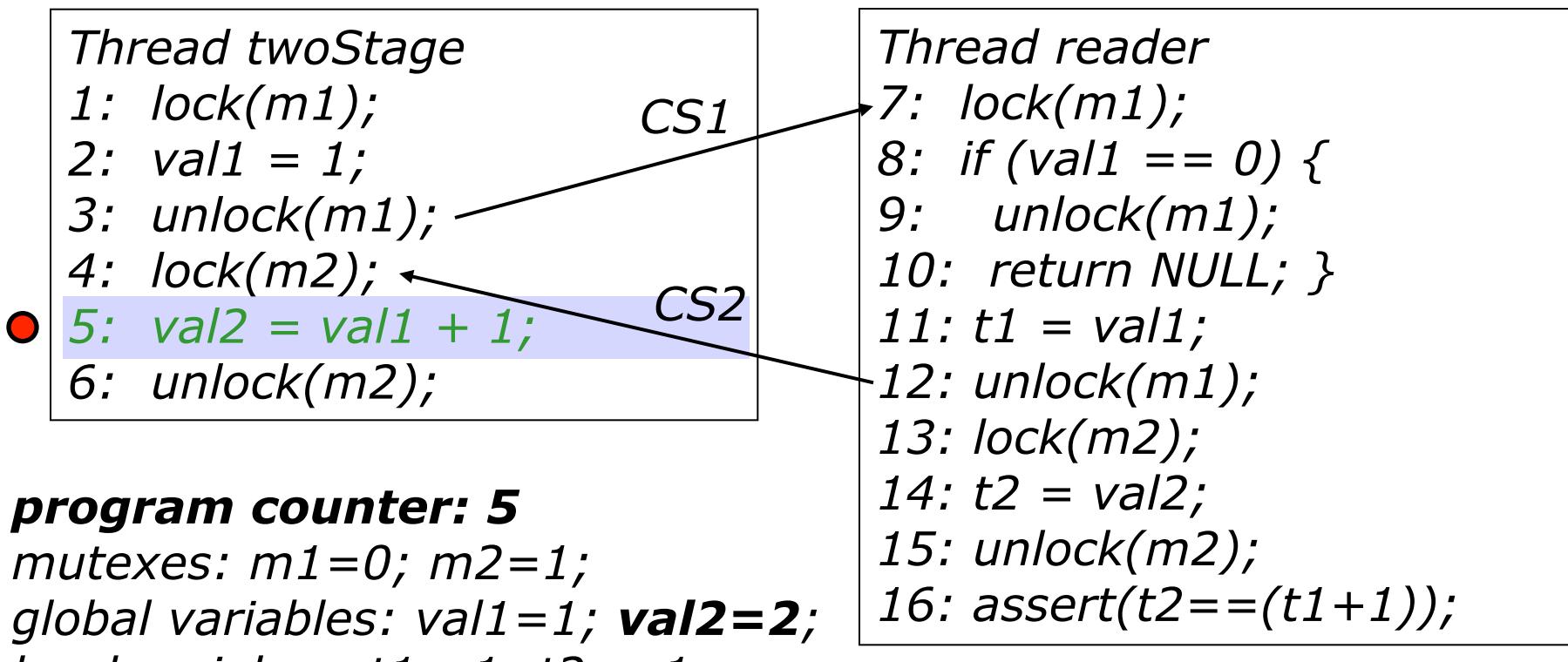


Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5}$

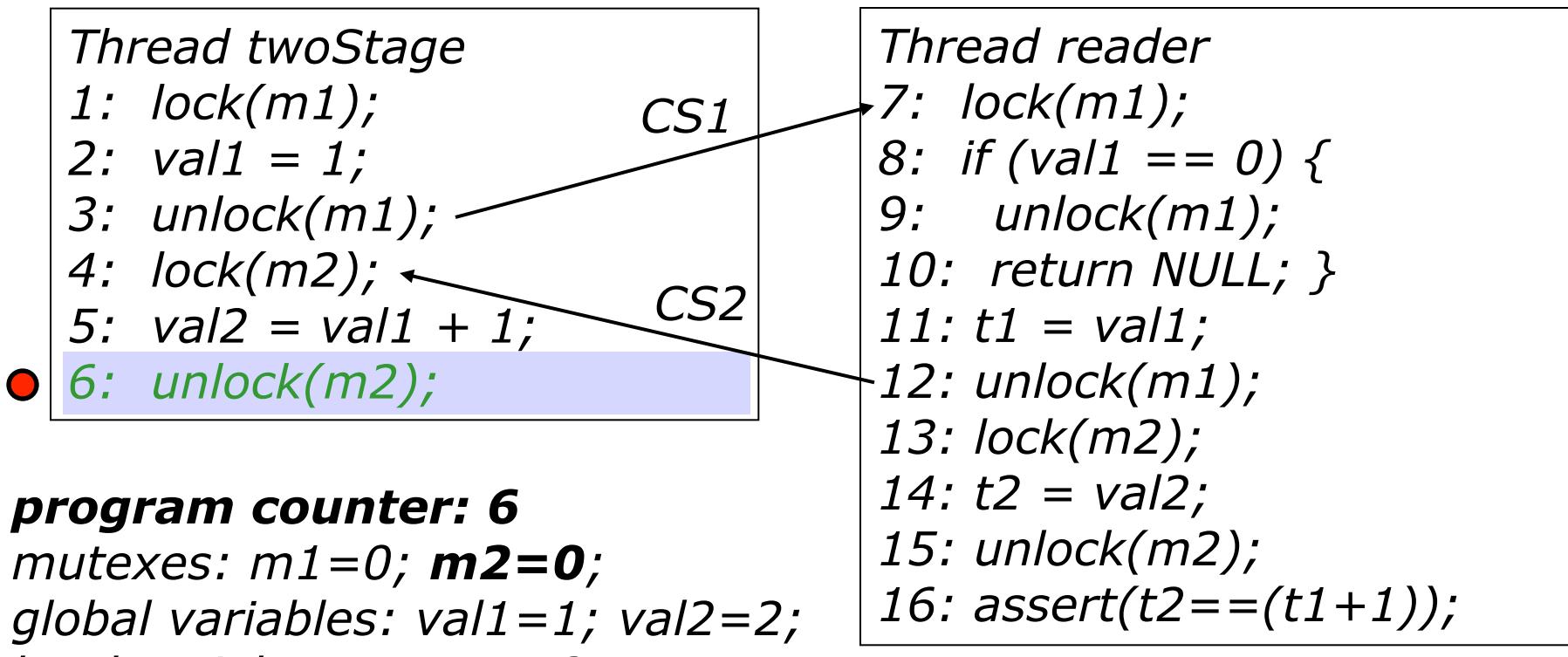


Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5}$



Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

$CS1$

$CS2$

$CS3$

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 13

mutexes: $m1=0; m2=0;$

global variables: $val1=1; val2=2;$

local variables: $t1= 1; t2= -1;$

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

$CS1$

$CS2$

$CS3$

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
```

13: lock(m2);

```
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 13

mutexes: $m1=0$; **$m2=1$** ;

global variables: $val1=1$; $val2=2$;

local variables: $t1= 1$; $t2= -1$;

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5} - R_{\text{reader},14}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

CS2

CS3

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
```

14: **t2 = val2;**

```
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 14

mutexes: $m1=0; m2=1;$

global variables: $val1=1; val2=2;$

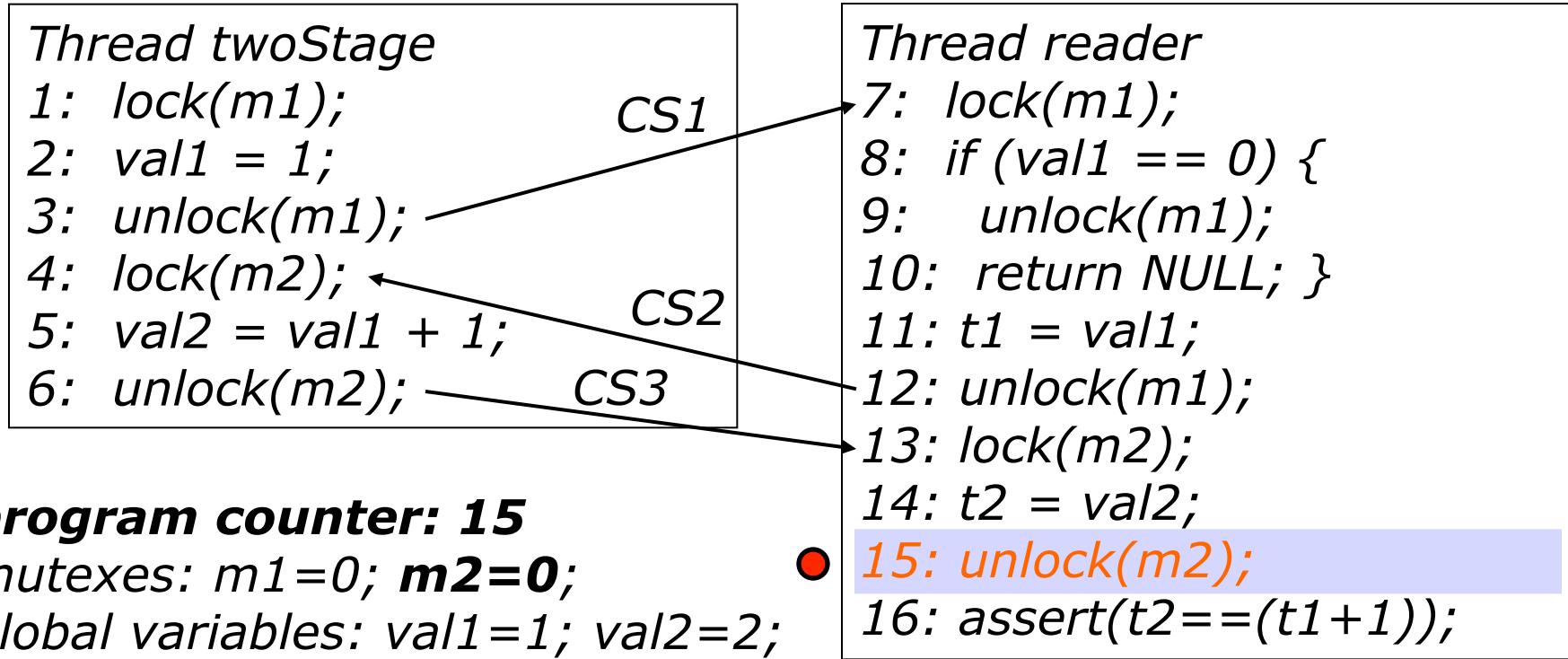
local variables: $t1= 1; t2= 2;$

Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5} - R_{\text{reader},14}$

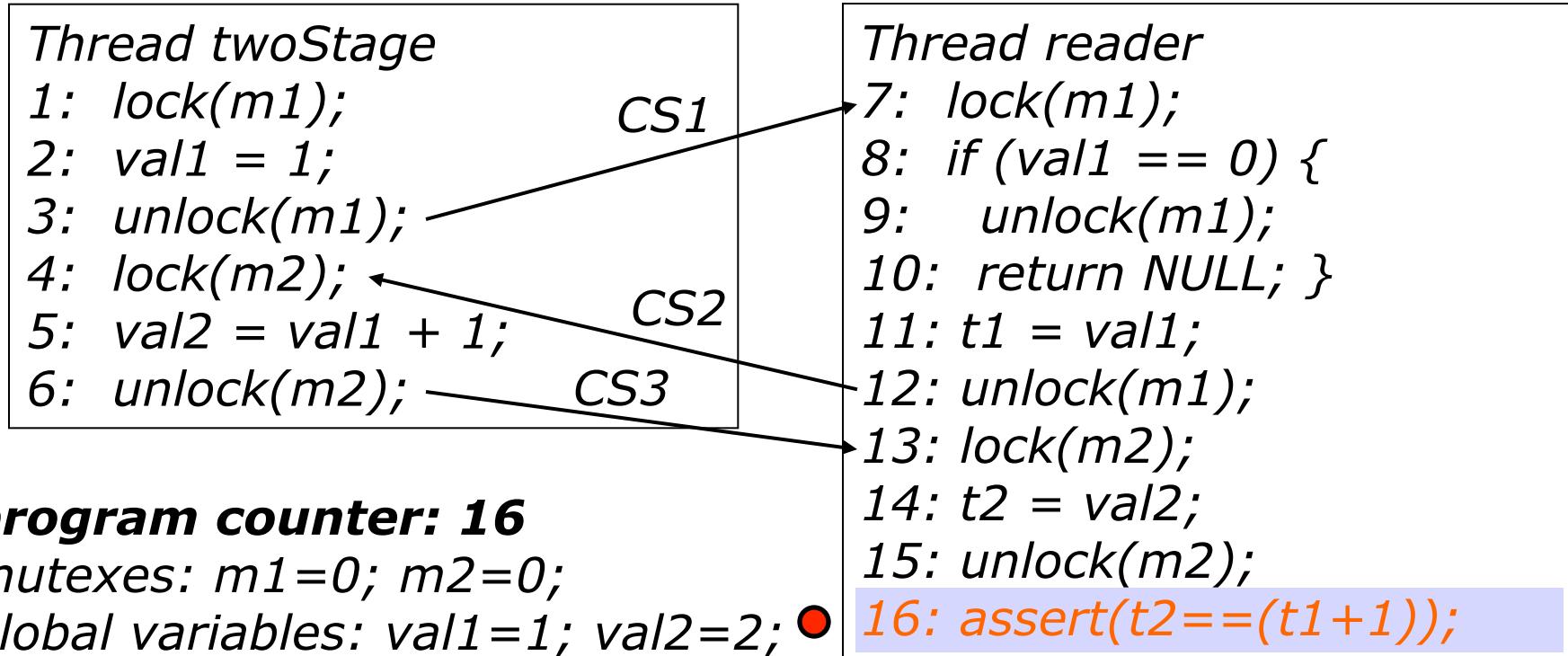


Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5} - R_{\text{reader},14}$

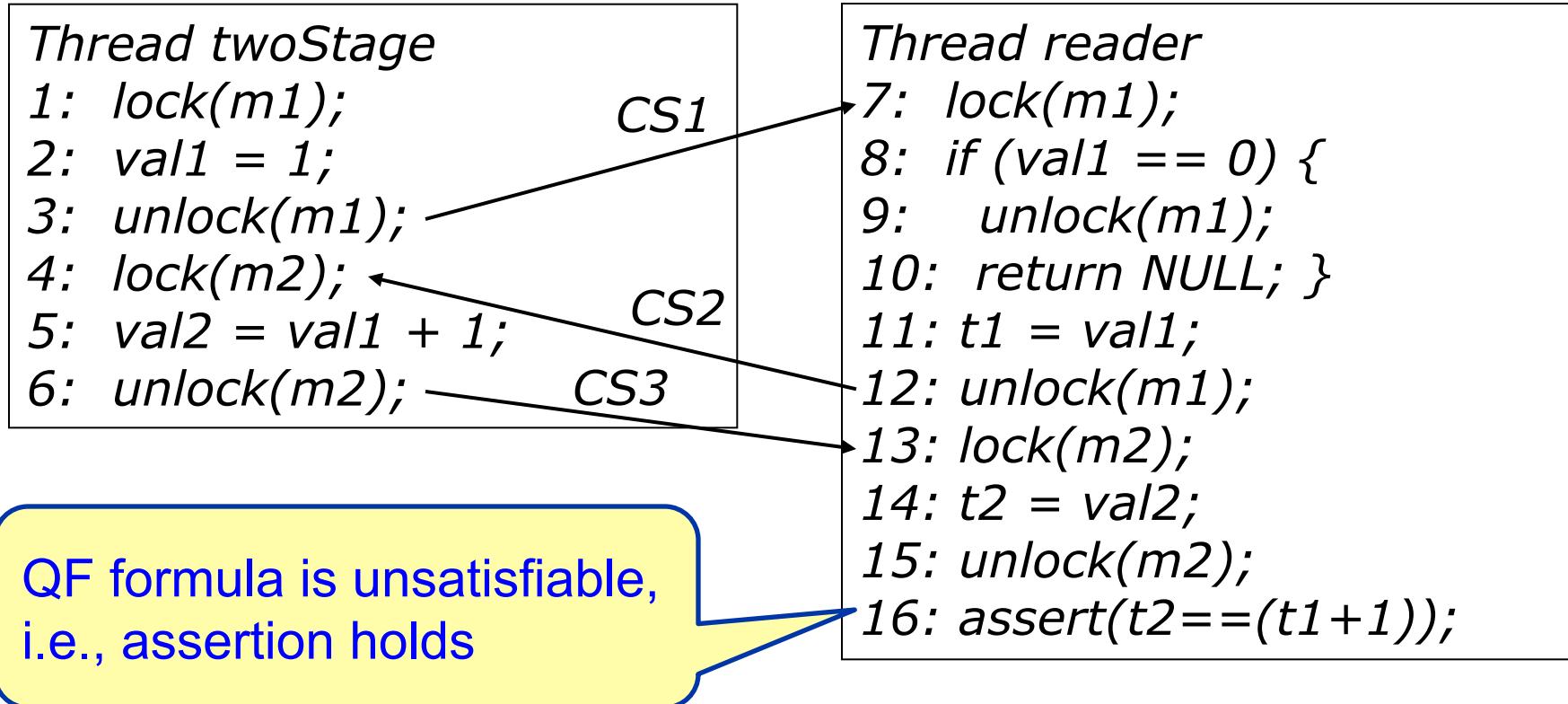


Lazy exploration: interleaving I_s

statements: 1-2-3-7-8-11-12-4-5-6-13-14-15-16

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $W_{\text{twoStage},5} - R_{\text{reader},14}$



Lazy exploration: interleaving I_f

statements:

val1-access:

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

program counter: 0

mutexes: m1=0; m2=0;

global variables: val1=0; val2=0;

local variables: t1= -1; t2= -1;

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

Lazy exploration: interleaving I_f

statements: 1-2-3

val1-access: $W_{\text{twoStage}, 2}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 3

mutexes: $m1=0$; $m2=0$;

global variables: **val1=1**; $val2=0$;

local variables: $t1 = -1$; $t2 = -1$;

Lazy exploration: interleaving I_f

statements: 1-2-3

val1-access: $W_{\text{twoStage}, 2}$

val2-access:

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
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11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 7

mutexes: $m1=0$; $m2=0$;

global variables: $val1=1$; $val2=0$;

local variables: $t1= -1$; $t2= -1$;

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11}$

val2-access: $R_{\text{reader},14}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
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11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

program counter: 16

mutexes: $m1=0; m2=0;$

global variables: $val1=1; val2=0;$

local variables: **$t1= 1; t2= 0;$**

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11}$

val2-access: $R_{\text{reader},14}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
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```

CS1

Thread reader

```
7: lock(m1);
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13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

CS2

program counter: 4

mutexes: $m1=0; m2=0;$

global variables: $val1=1; val2=0;$

local variables: $t1= 1; t2= 0;$

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $R_{\text{reader},14} - W_{\text{twoStage},5}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

Thread reader

```
7: lock(m1);
8: if (val1 == 0) {
9:   unlock(m1);
10: return NULL; }
11: t1 = val1;
12: unlock(m1);
13: lock(m2);
14: t2 = val2;
15: unlock(m2);
16: assert(t2==(t1+1));
```

CS2

program counter: 6

mutexes: $m1=0; m2=0;$

global variables: $\text{val1}=1; \text{val2}=2;$

local variables: $t1= 1; t2= 0;$

Lazy exploration: interleaving I_f

statements: 1-2-3-7-8-11-12-13-14-15-16-4-5-6

val1-access: $W_{\text{twoStage},2} - R_{\text{reader},8} - R_{\text{reader},11} - R_{\text{twoStage},5}$

val2-access: $R_{\text{reader},14} - W_{\text{twoStage},5}$

Thread twoStage

```
1: lock(m1);
2: val1 = 1;
3: unlock(m1);
4: lock(m2);
5: val2 = val1 + 1;
6: unlock(m2);
```

CS1

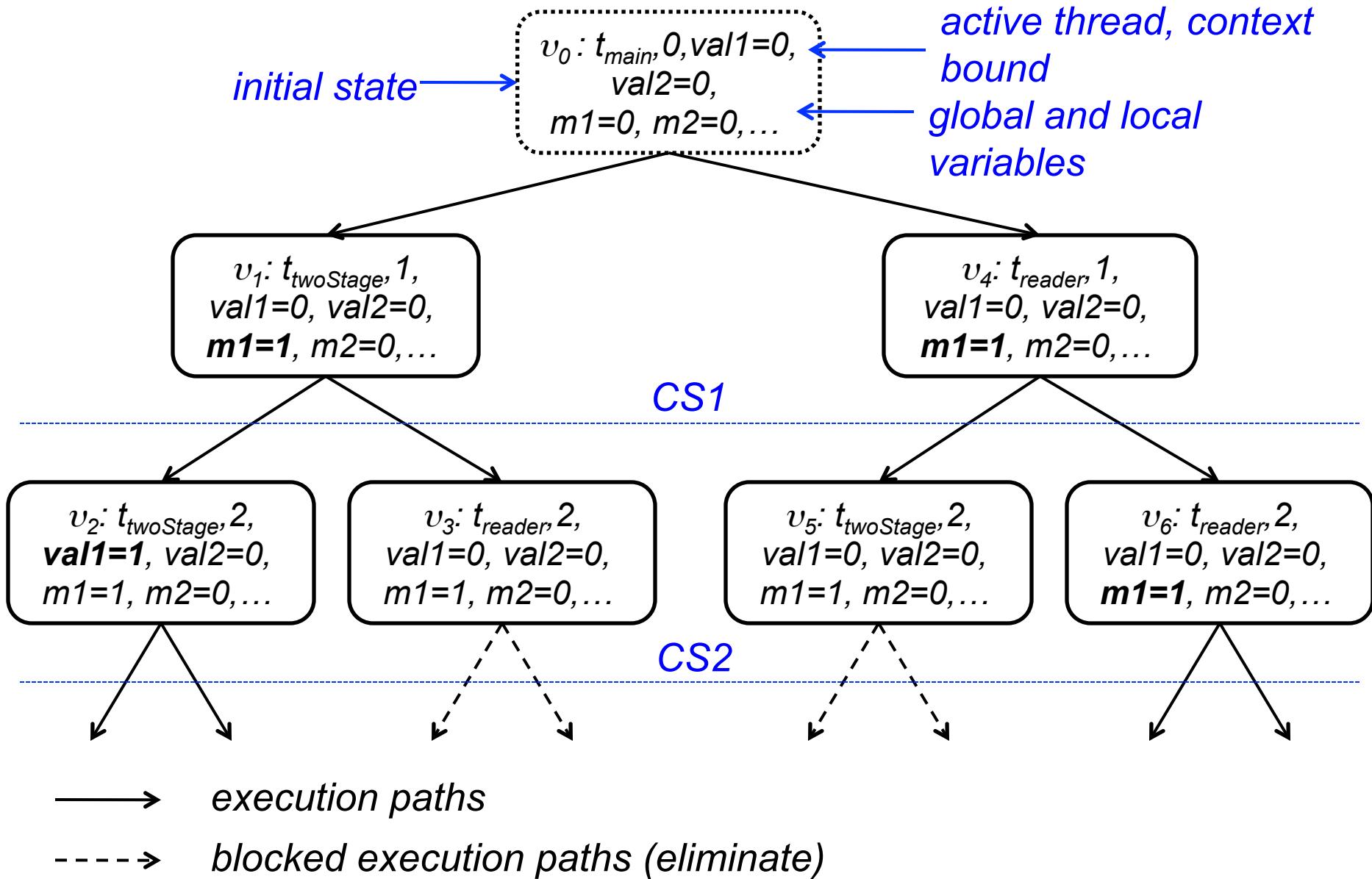
Thread reader

```
7: lock(m1);
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12: unlock(m1);
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```

CS2

*QF formula is satisfiable,
i.e., assertion does not hold*

Lazy Approach: State Transitions



Exploring the Reachability Tree

- Use a reachability tree (RT) to describe reachable states of a multi-threaded program

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 - A_i represents the currently active thread
 - C_i represents the context switch number

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 - A_i represents the currently active thread
 - C_i represents the context switch number
 - s_i represents the current state
 - l_i^j represents the current location of thread j
 - G_i^j represents the control flow guards accumulated in thread j along the path from l_0^j to l_i^j

Expansion Rules of the RT

R1 (assign): If l is an assignment, we execute l , which generates s_{i+1} . We add as child to v a new node v'

$$v' = \left(A_i, C_i, \underline{s_{i+1}}, \overbrace{\left\langle l_{i+1}^j, G_i^j \right\rangle}_{i+1} \right) \xrightarrow{l_{i+1}^{A_i} = l_i^{A_i} + 1}$$

- we have fully expanded v if
 - l within an atomic block; or
 - l contains no global variable; or
 - the upper bound of context switches ($C_i = C$) is reached
- if v is not fully expanded, for each thread $j \neq A_i$ where G_i^j is enabled in s_{i+1} , we thus create a new child node

$$v'_j = \left(\underline{j}, \underline{C_i + 1}, \underline{s_{i+1}}, \left\langle \underline{l_i^j}, \underline{G_i^j} \right\rangle \right)_{i+1}$$

Expansion Rules of the RT

R2 (skip): If l is a *skip*-statement with target l , we increment the location of the current thread and continue with it. We explore no context switches:

$$v' = \left(A_i, C_i, s_i, \overbrace{\left\langle l_{i+1}^j, G_i^j \right\rangle}^{l_{i+1}^j} \right)_{i+1} \rightarrow l_{i+1}^j = \begin{cases} l_i^j + 1 & : j = A_i \\ l_i^j & : otherwise \end{cases}$$

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R3 (unconditional goto): If l is an unconditional *goto*-statement with target l , we set the location of the current thread and continue with it. We explore no context switches:

$$v' = \left(A_i, C_i, s_i, \overbrace{\left\langle l_{i+1}^j, G_i^j \right\rangle}^{l_{i+1}^j} \right)_{i+1} \rightarrow l_{i+1}^j = \begin{cases} l & : j = A_i \\ l_i^j & : otherwise \end{cases}$$

Expansion Rules of the RT

R4 (conditional goto): If l is a conditional *goto*-statement with test c and target l , we create two child nodes v' and v'' .

- for v' , we assume that c is *true* and proceed with the target instruction of the jump:

$$v' = \left(A_i, C_i, s_i, \overbrace{\left\langle l_{i+1}^j, c \wedge G_i^j \right\rangle}_{i+1} \right)$$
$$l_{i+1}^j = \begin{cases} l & : j = A_i \\ l_i^j & : \text{otherwise} \end{cases}$$

- for v'' , we add $\neg c$ to the guards and continue with the next instruction in the current thread

$$v'' = \left(A_i, C_i, s_i, \overbrace{\left\langle l_{i+1}^j, \neg c \wedge G_i^j \right\rangle}_{i+1} \right)$$
$$l_{i+1}^j = \begin{cases} l_i^j + 1 & : j = A_i \\ l_i^j & : \text{otherwise} \end{cases}$$

- prune one of the nodes if the condition is determined statically

Expansion Rules of the RT

R5 (assume): If l is an *assume*-statement with argument c , we proceed similar to R1.

- we continue with the unchanged state s , but add c to all guards, as described in R4
- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

Expansion Rules of the RT

R5 (assume): If $/$ is an *assume*-statement with argument c , we proceed similar to R1.

- we continue with the unchanged state s , but add c to all guards, as described in R4
- If $c \wedge G_i^j$ evaluates to *false*, we prune the execution path

R6 (assert): If $/$ is an *assert*-statement with argument c , we proceed similar to R1.

- we continue with the unchanged state s , but add c to all guards, as described in R4
- we generate a verification condition to check the validity of c

Expansion Rules of the RT

R5 (start_thread): If l is a *start_thread* instruction, we add the indicated thread to the set of active threads:

$$v' = \left(A_i, C_i, s_i, \underbrace{\left\langle l_{i+1}^j, G_{i+1}^j \right\rangle}_{j=1}^{n+1} \right)_{i+1}$$

- where l_{i+1}^{n+1} is the initial location of the thread and $G_{i+1}^{n+1} = G_i^{A_i}$
- the thread starts with the guards of the currently active thread

Expansion Rules of the RT

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- the thread starts with the guards of the currently active thread

R6 (join_thread): If l is a *join_thread* instruction with argument Id , we add a child node:

$$v' = \left(A_i, C_i, s_i, \underbrace{\left\langle l_{i+1}^j, G_i^j \right\rangle}_{j=1} \right)_{i+1}$$

- where $l_{i+1}^j = l_i^{A_i} + 1$ only if the joining thread Id has exited

Lazy exploration of interleavings

- Main steps of the algorithm:

1. Initialize the stack with the initial node v_0 and the initial path $\pi_0 = \langle v_0 \rangle$
2. If the stack is empty, terminate with “no error”.
3. Pop the current node v and current path π off the stack and compute the set v' of successors of v using rules R1-R8.
4. If v' is empty, derive the VC φ_k^π for π and call the SMT solver on it. If φ_k^π is satisfiable, terminate with “error”; otherwise, goto step 2.
5. If v' is not empty, then for each node $v \in v'$, add v to π , and push node and extended path on the stack. goto step 3.

$$\pi = \{v_1, \dots, v_n\}$$
$$\varphi_k^\pi = \overbrace{I(s_0) \wedge R(s_0, s_1) \wedge \dots \wedge R(s_{k-1}, s_k)}^{\text{constraints}} \wedge \overbrace{\neg \phi_k}^{\text{property}}$$

\nearrow computation path
 \searrow bound

Observations about the lazy approach

- naïve but useful:
 - bugs usually manifest with few context switches
[Qadeer&Rehof'05]

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 - bound the number of preemptions (C) allowed per threads
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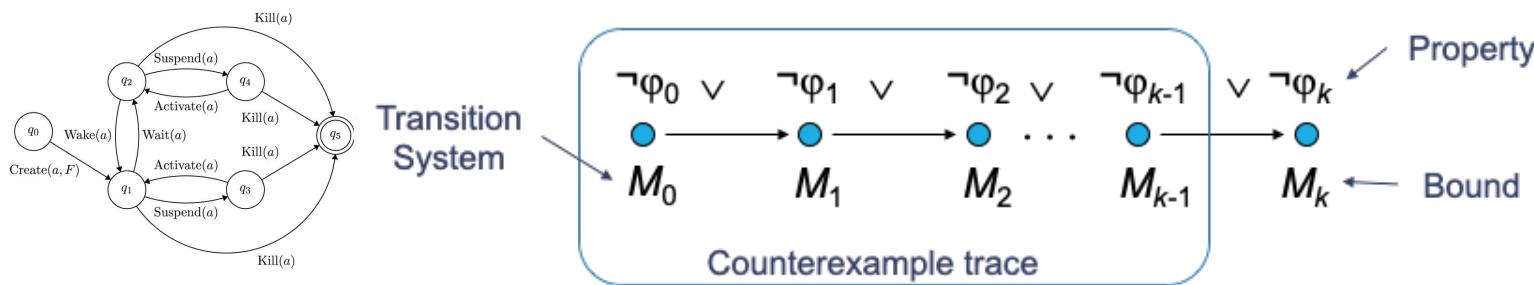
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 - ▷ *number of executions:* $O(n^c)$
 - as each formula corresponds to one possible path only, its size is relatively small
- can suffer performance degradation:
 - in particular for correct programs where we need to invoke the SMT solver once for each possible execution path

Intended learning outcomes

- Understand **soundness** and **completeness** concerning **detection techniques**
- Emphasize the difference between **static analysis** and **testing / simulation**
- Explain **bounded model checking** of software
- Explain **unbounded model checking** of software

Revisiting BMC

- Basic Idea: given a transition system M , check negation of a given property φ up to given depth k :



- Translated into a VC ψ such that: ψ satisfiable iff φ has counterexample of max. depth k

BMC is aimed at finding bugs; it cannot prove correctness, unless the bound k safely reaches all program states

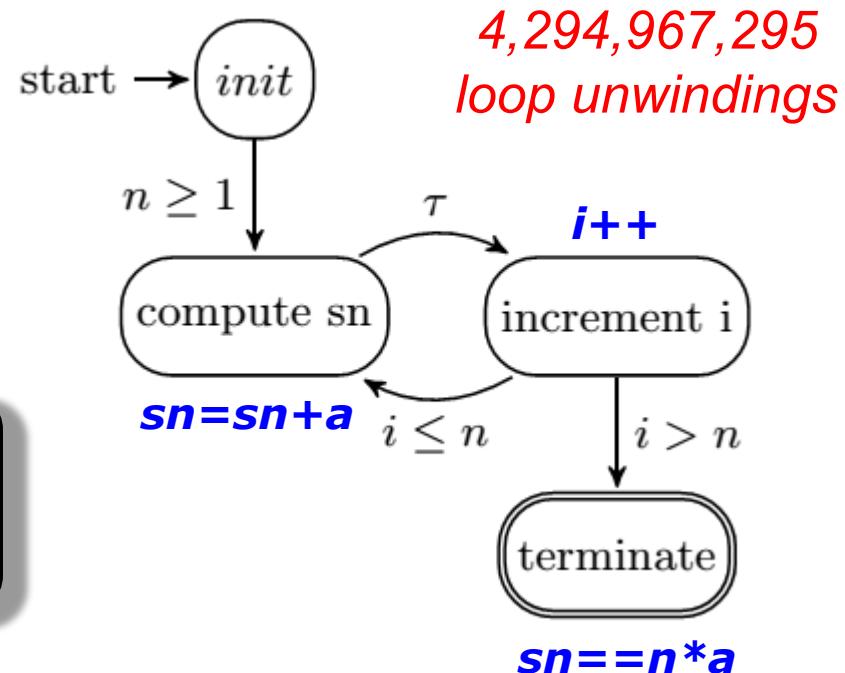
Difficulties in proving the correctness of programs with loops in BMC

- BMC techniques can falsify properties up to a given depth k
 - they can prove correctness only if an upper bound of k is known (**unwinding assertion**)
 - » BMC tools typically fail to verify programs that contain bounded and unbounded loops

$$S_n = \sum_{i=1}^n a = na, n \geq 1$$



the loop will be unfolded 2^{n-1} times
(in the worst case, 2^{32-1} times on 32 bits integer)



Induction-Based Verification

k -induction checks...

- **base case (base_k)**: find a counter-example with up to k loop unwindings (plain BMC)
 - **forward condition (fwd_k)**: check that P holds in all states reachable within k unwindings
 - **inductive step (step_k)**: check that whenever P holds for k unwindings, it also holds after next unwinding
 - havoc state
 - run k iterations
 - assume invariant
 - run final iteration
- ⇒ iterative deepening if inconclusive

The k -induction algorithm

k =initial bound

while *true* **do**

if $base_k$ **then**

return *trace* $s[0..k]$

else if fwd_k

return *true*

else if $step_k$ **then**

return *true*

end if

$k = k + 1$

end

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return $trace\ s[0..k]$

else if fwd_k

return $true$

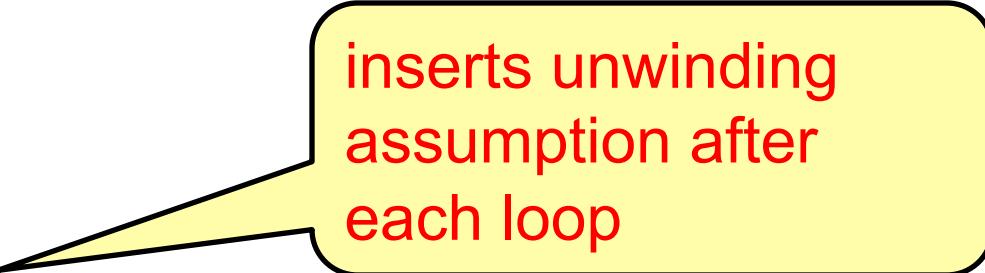
else if $step_k$ **then**

return $true$

end if

$k=k+1$

end



inserts unwinding
assumption after
each loop

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$k = \text{initial bound}$

while true **do**

if base_k **then**

return $\text{trace } s[0..k]$

else if fwd_k

return true

else if step_k **then**

return true

end if

$k = k + 1$

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havoc variables that occur in the loop's termination condition

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else if $step_k$ **then**

return *true*

end if

$k = k + 1$

end

inserts unwinding assumption after each loop

inserts unwinding assertion after each loop

havoc variables that occur in the loop's termination condition

unable to falsify or prove the property

Running example

Prove that $S_n = \sum_{i=1}^n a = na$ for $n \geq 1$

```
unsigned int nondet_uint();
int main() {
    unsigned int i, n=nondet_uint(), sn=0;
    assume (n>=1);
    for(i=1; i<=n; i++)
        sn = sn + a;
    assert(sn==n*a);
}
```

Running example: *base case*

Insert an **unwinding assumption** consisting of the termination condition after the loop

- find a counter-example with k loop unwindings

```
unsigned int nondet_uint();
int main() {
    unsigned int i, n=nondet_uint(), sn=0;
    assume (n>=1);
    for(i=1; i<=n; i++)
        sn = sn + a;
    assume(i>n);
    assert(sn==n*a);
}
```

Running example: *forward condition*

Insert an **unwinding assertion** consisting of the termination condition after the loop

- check that P holds in all states reachable with k unwindings

```
unsigned int nondet_uint();
int main() {
    unsigned int i, n=nondet_uint(), sn=0;
    assume (n>=1);
    for(i=1; i<=n; i++)
        sn = sn + a;
    assert(i>n);
    assert(sn==n*a);
}
```

Running example: *inductive step*

Havoc (only) the variables that occur in the loop's termination and branch conditions

```
unsigned int nondet_uint();
typedef struct state {
    unsigned int i, n, sn;
} statet;
int main() {
    unsigned int i, n=nondet_uint(), sn=0, k;
    assume(n>=1);
    statet cs, s[n];
    cs.i=nondet_uint();
    cs.sn=nondet_uint();
    cs.n=n;
```

Running example: *inductive step*

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```

define the type of the program state

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define the type of the program state

state vector

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    cs.i=nondet_uint();  
    cs.sn=nondet_uint();  
    cs.n=n;
```

define the type of the program state

state vector

explore all possible values implicitly

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {  
    s[i-1]=cs;  
    sn = sn + a;  
    cs.i=i;  
    cs.sn=sn;  
    cs.n=n;  
    assume(s[i-1]!=cs);  
}  
  
assume(i>n);  
assert(sn == n*a);  
}
```

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {  
    s[i-1]=cs;  
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    cs.sn=sn;  
    cs.n=n;  
    assume(s[i-1]!=cs);  
}  
  
assume(i>n);  
assert(sn == n*a);  
}
```

capture the state *cs* before the iteration

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

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    s[i-1]=cs;  
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    cs.sn=sn;  
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    assume(s[i-1]!=cs);  
}  
  
assume(i>n);  
assert(sn == n*a);  
}
```

capture the state *cs*
before the iteration

capture the state *cs*
after the iteration

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {  
    s[i-1]=cs;  
    sn = sn + a;  
    cs.i=i;  
    cs.sn=sn;  
    cs.n=n;  
    assume(s[i-1]!=cs);  
}  
assume(i>n);  
assert(sn == n*a);  
}
```

capture the state *cs*
before the iteration

capture the state *cs*
after the iteration

constraints are
included by means
of assumptions

Running example: *inductive step*

BMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {  
    s[i-1]=cs;  
    sn = sn + a;  
    cs.i=i;  
    cs.sn=sn;  
    cs.n=n;  
    assume(s[i-1]!=cs);  
}  
assume(i>n);  
assert(sn == n)  
}
```

capture the state *cs*
before the iteration

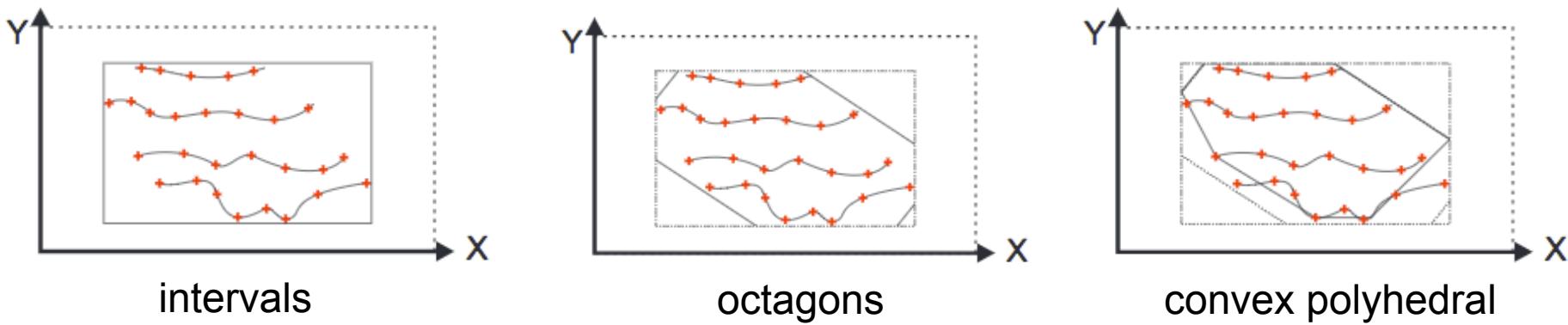
capture the state *cs*
after the iteration

constraints are
included by means
of assumptions

insert unwinding
assumption

Automatic Invariant Generation

- Infer invariants using **intervals**, **octagons**, and **convex polyhedral** constraints for the inductive step
 - e.g., $a \leq x \leq b$; $x \leq a$, $x-y \leq b$; and $ax + by \leq c$



- Use existing libraries to discover linear/polynomial relations among integer/real variables to infer **loop invariants**
 - compute **pre-** and **post-conditions**

Running Example: Plain BMC

- Plain BMC unrolls this *while*-loop 100 times...

```
int main() {  
    int x=0, t=0, phase=0;  
    while(t<100) {  
        if(phase==0) x=x+2;  
        if(phase==1) x=x-1;  
        phase=1-phase;  
        t++;  
    }  
    assert(x<=100);  
    return 0;  
}
```

```
$esbmc example.c --clang-frontend  
ESBMC version 4.2.0 64-bit x86_64 macos  
file example.c: Parsing  
Converting  
Type-checking example  
Generating GOTO Program  
GOTO program creation time: 0.232s  
GOTO program processing time: 0.001s  
Starting Bounded Model Checking  
Unwinding loop 1 iteration 1 file example.c line 5 function  
main  
Unwinding loop 1 iteration 2 file example.c line 5 function  
main  
...  
Unwinding loop 1 iteration 100 file example.c line 5 function  
main  
Symex completed in: 0.340s (313 assignments)  
Slicing time: 0.000s  
Generated 1 VCC(s), 0 remaining after simplification  
VERIFICATION SUCCESSFUL  
BMC program time: 0.340s
```

Running Example: k -induction + invariants

- Inductive step proves correctness for k -step 2...

```
int main() {
    int x=0, t=0, phase=0;
    while(t<100) {
        assume(-2*x+t+3*phase == 0);
        assume(3-2*x+t >= 0);
        assume(-x+2*t >= 0);
        assume(147+x-2*t >= 0);
        assume(2*x-t >= 0);
        if(phase==0) x=x+2;
        if(phase==1) x=x-1;
        phase=1-phase;
        t++;
    }
    assert(x<=100);
    return 0;
}
```

```
$esbmc example.c --clang-frontend --k-induction
*** K-Induction Loop Iteration 2 ***
*** Checking inductive step
Starting Bounded Model Checking
Unwinding loop 1 iteration 1 file example_pagai.c line 6 function main
Unwinding loop 1 iteration 2 file example_pagai.c line 6 function main
Symex completed in: 0.002s (53 assignments)
Slicing time: 0.000s
Generated 1 VCC(s), 1 remaining after simplification
No solver specified; defaulting to Boolector
Encoding remaining VCC(s) using bit-vector arithmetic
Encoding to solver time: 0.001s
Solving with solver Boolector 2.4.0
Encoding to solver time: 0.001s
Runtime decision procedure: 0.144s
VERIFICATION SUCCESSFUL
BMC program time: 0.148s
Solution found by the inductive step (k = 2)
```

inductive invariants

reuse k-induction counterexamples to speed-up bug finding
reuse results of previous steps (caching SMT queries)

Summary

- Described the difference between **soundness** and **completeness** concerning **detection techniques**
 - **False positive** and **false negative**
- Pointed out the difference between **static analysis** and **testing / simulation**
 - **hybrid combination** of static and dynamic analysis techniques to achieve a good trade-off between **soundness** and **completeness**
- Explained **bounded** and **unbounded model checking of software**
 - they have been applied successfully to verify **single- and multi-threaded software**