



Incremental Bounded Model Checking of Artificial Neural Networks in CUDA

**Luiz H. Sena, Iury V Bessa, Lucas C. Cordeiro,
Mikhail R. Gadelha and Edjard Mota**

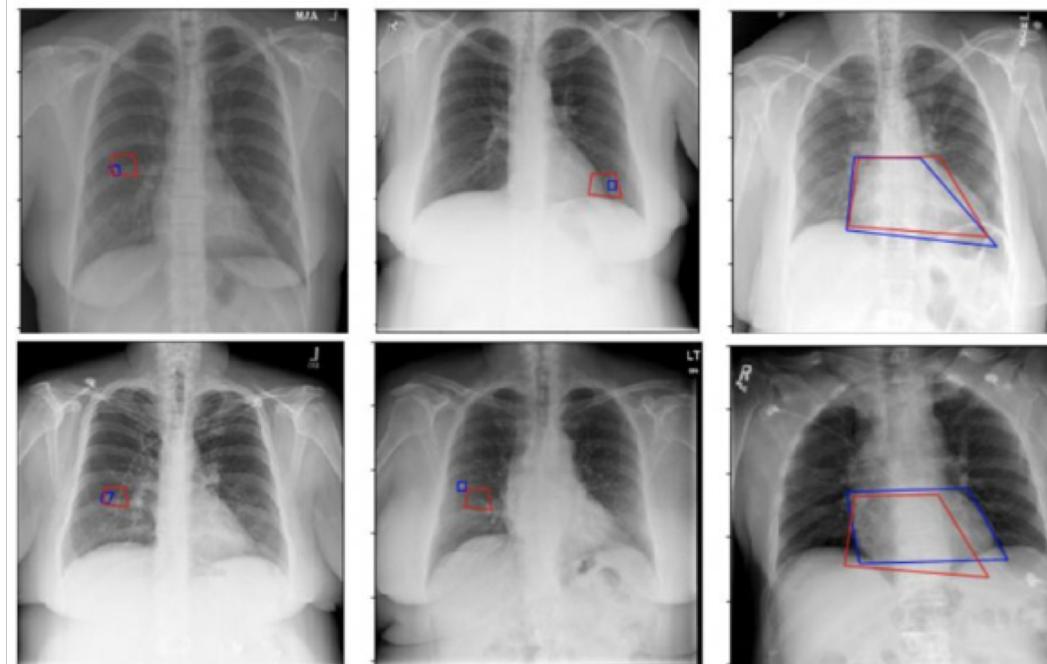


Summary

- Introduction
- Preliminaries
- Incremental BMC of ANNs in CUDA
 - Verification of Covering Methods
 - Verification of Adversarial Cases
- Experimental Evaluation
- Conclusion

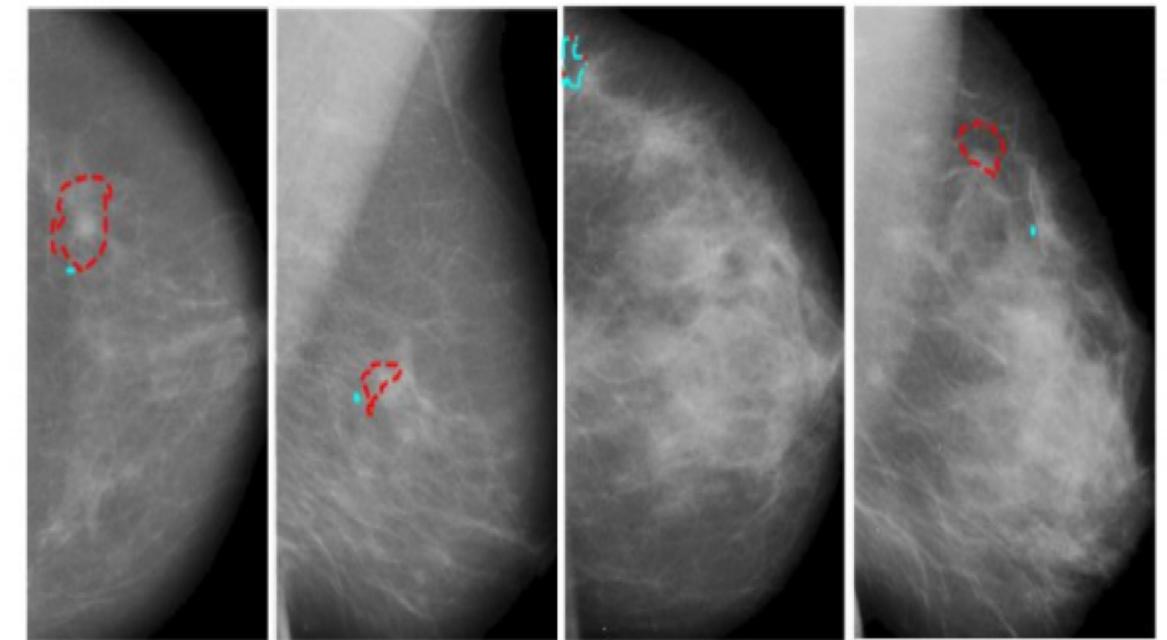
AI in Safety Critical Systems

- Marking regions to examine;



Red quadrilateral is marked by ANN and blue quadrilateral is marked by the radiologist.

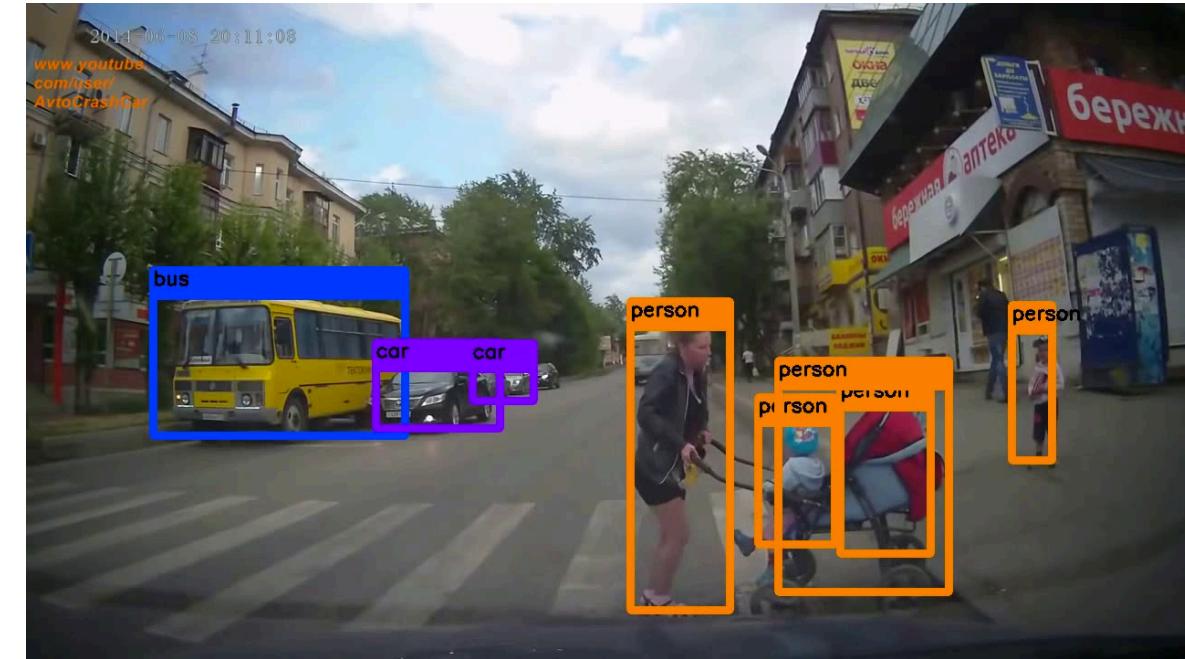
- Mass detection and localization.



Red contours denote ground truth and cyan contours are false positive.

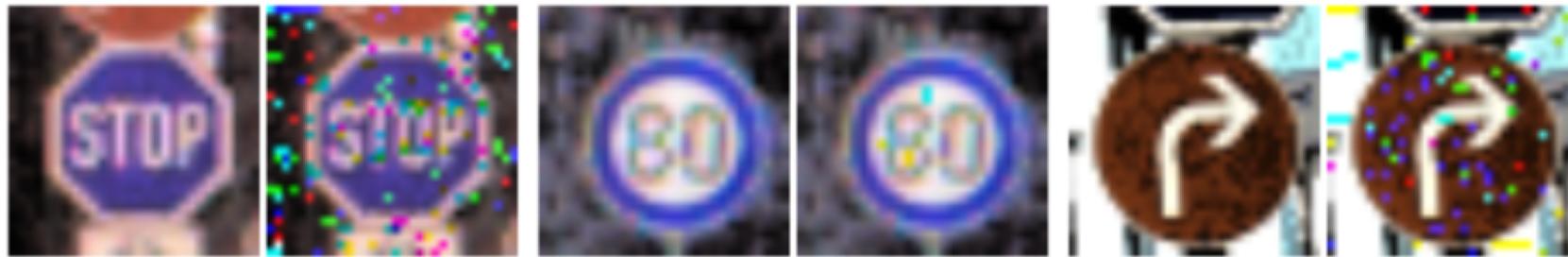
Self-driving car

- Recognizing traffic signs and objects;



Adversarial Cases

- Adversarial Cases are not simple ANN errors, but particular cases where the correct label can be easily classified by humans.



stop

30m
speed
limit

80m
speed
limit

30m
speed
limit

go
right

go
straight

Self-Driving Uber Car Kills Pedestrian in Arizona, Where Robots Roam



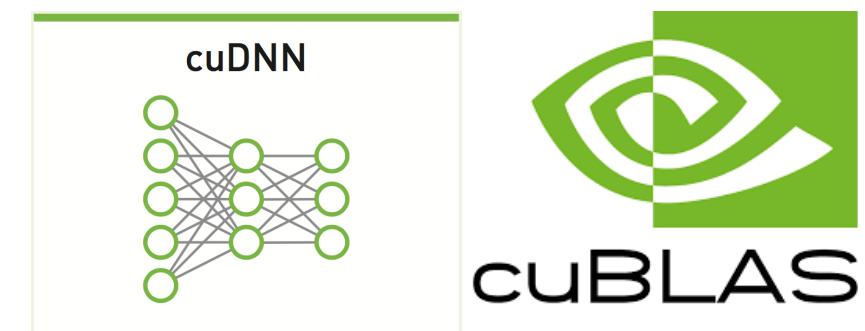
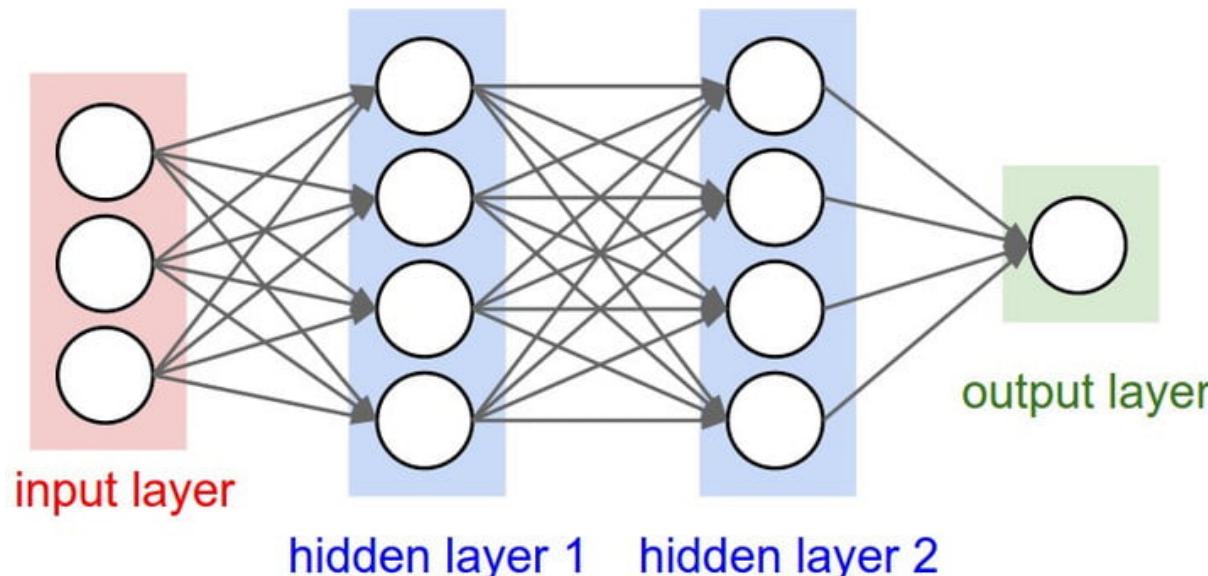
A woman crossing Mill Avenue at its intersection with Curry Road in Tempe, Ariz., on Monday. A pedestrian was struck and killed by a self-driving Uber vehicle at the intersection a night earlier.
Caitlin O'Hara for The New York Times

Objectives

- Showing how unsafe an ANN through:
 - Generated adversarial cases
 - How adversarial is a set of images for the ANN neurons w.r.t. covering methods.

Artificial Neural Networks

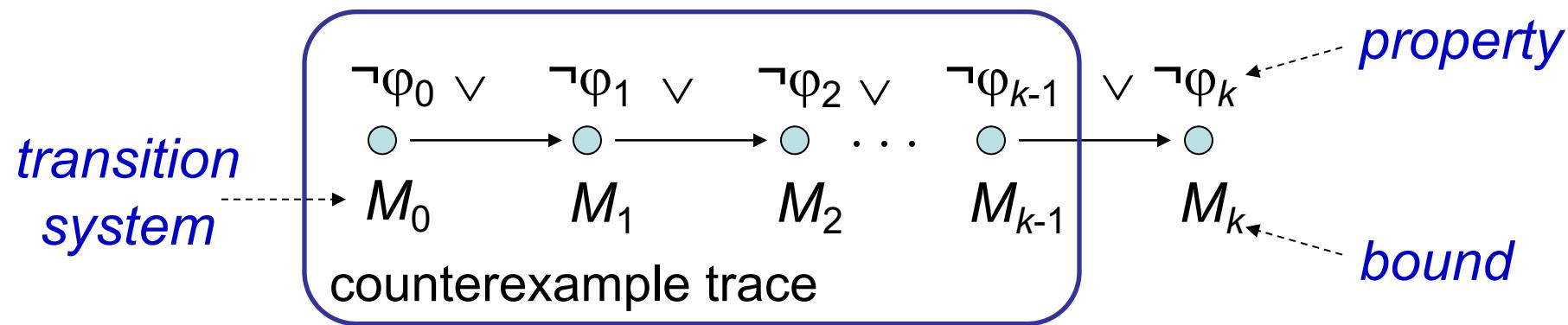
- Artificial Neural Networks (ANNs) are versatile systems capable of generalizing and responding to unexpected inputs/patterns;
- ANNs are based in mathematical operations and learning algorithms.



Bounded Model Checking (BMC)

7

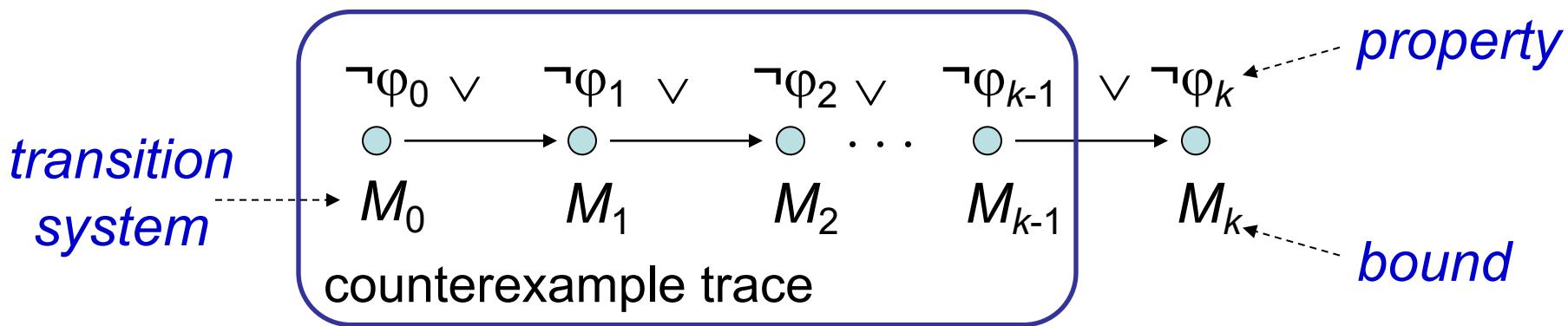
Basic idea: check negation of given property up to given depth



Bounded Model Checking (BMC)

8

Basic idea: check negation of given property up to given depth

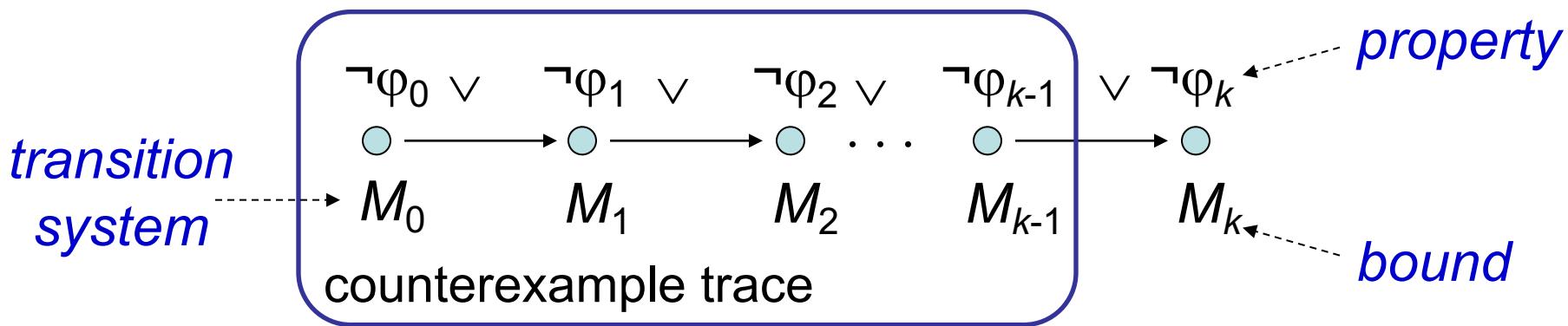


- Transition system M unrolled k times
 - for programs: loops, recursion, ...
- Translated into verification condition ψ such that
 ψ satisfiable iff ϕ has counterexample of max. depth k

Bounded Model Checking (BMC)

9

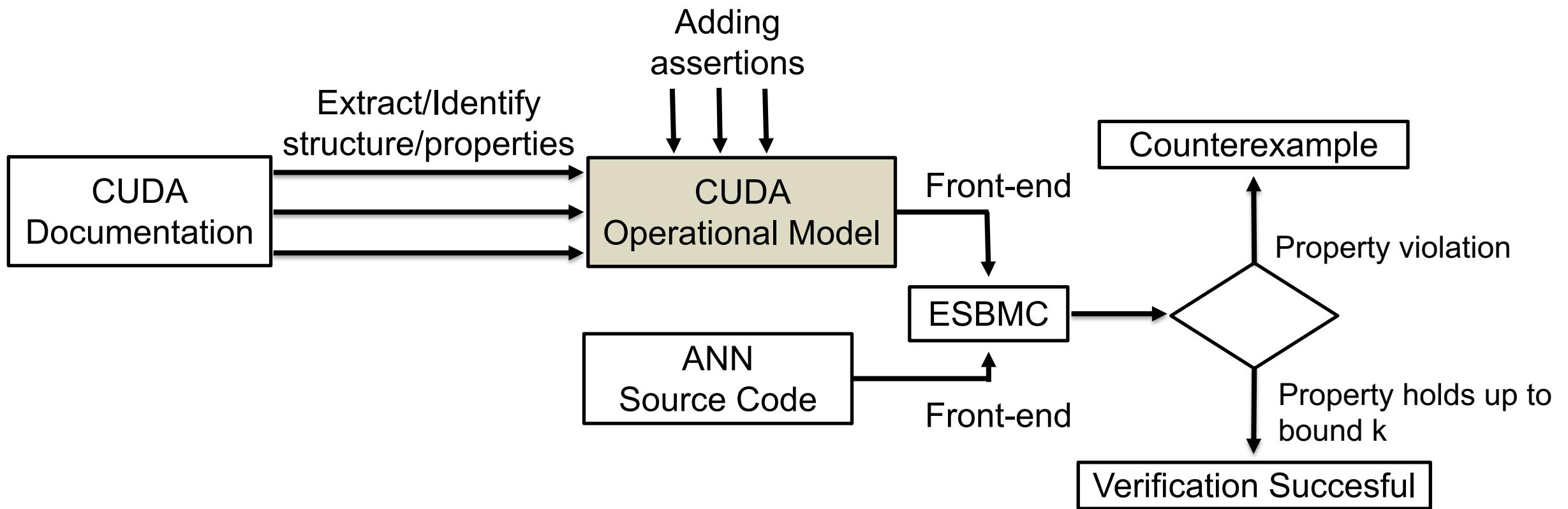
Basic idea: check negation of given property up to given depth



- Transition system M unrolled k times
 - for programs: loops, recursion, ...
- Translated into verification condition ψ such that
 ψ satisfiable iff ϕ has counterexample of max. depth k

BMC has been applied successfully to verify HW and SW

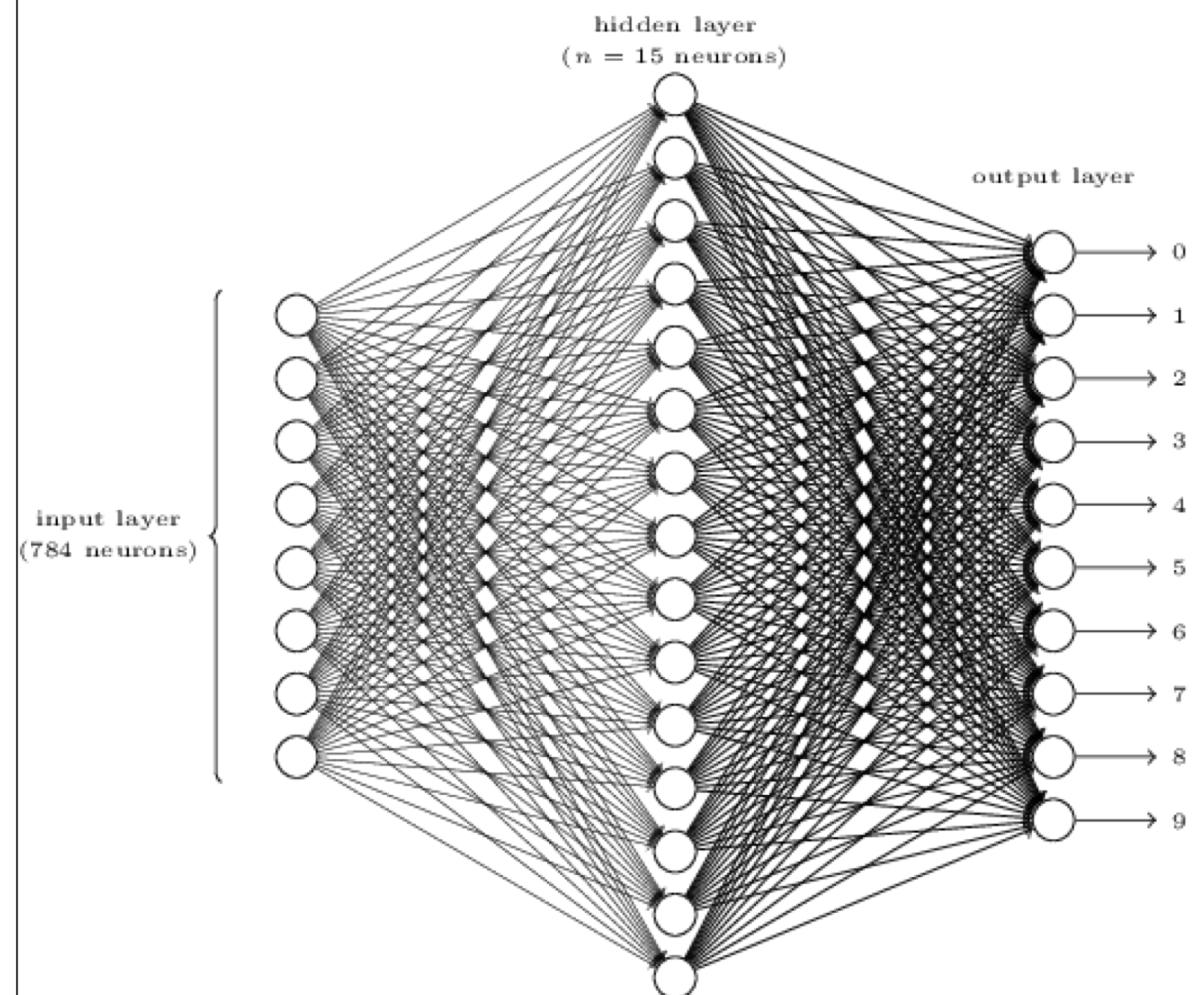
CUDA Operational Model



ANNs in CUDA

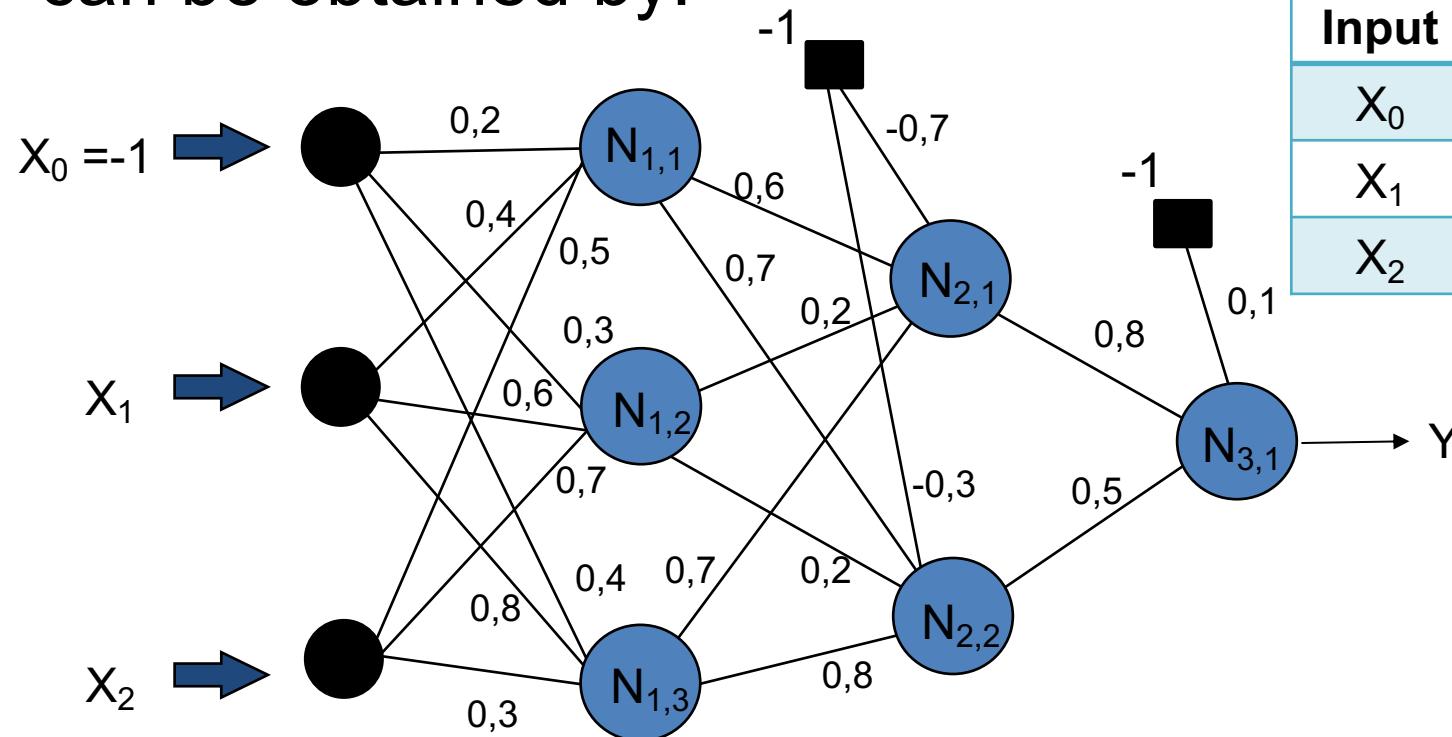
11

```
1 void feedForward(){  
2     cublasSgemm(cublasHandle, CUBLAS_OP_T, CUBLAS_OP_N,  
3     Layer.Outputs, batchSize, Layer.Inputs,  
4     1,  
5     Layer.MatrixBias, Layer.Inputs,  
6     data, Layer.Inputs,  
7     0,  
8     LayerResults,Layer.Outputs);  
9  
10    cublasSgemm(cublasHandle, CUBLAS_OP_N, CUBLAS_OP_N,  
11    Layer.Outputs, batchSize, 1,  
12    0,  
13    BiasVector, Layer.Outputs,  
14    onevec, 1,  
15    1,  
16    LayerResults, Layer.Outputs);  
17    __ESBMC_assert((LayerResults[CorrecLabel] > P) || (  
18        LayerResults[WrongLabel] < P), "The correct label  
19        became a wrong label")
```



Verification of Covering Methods

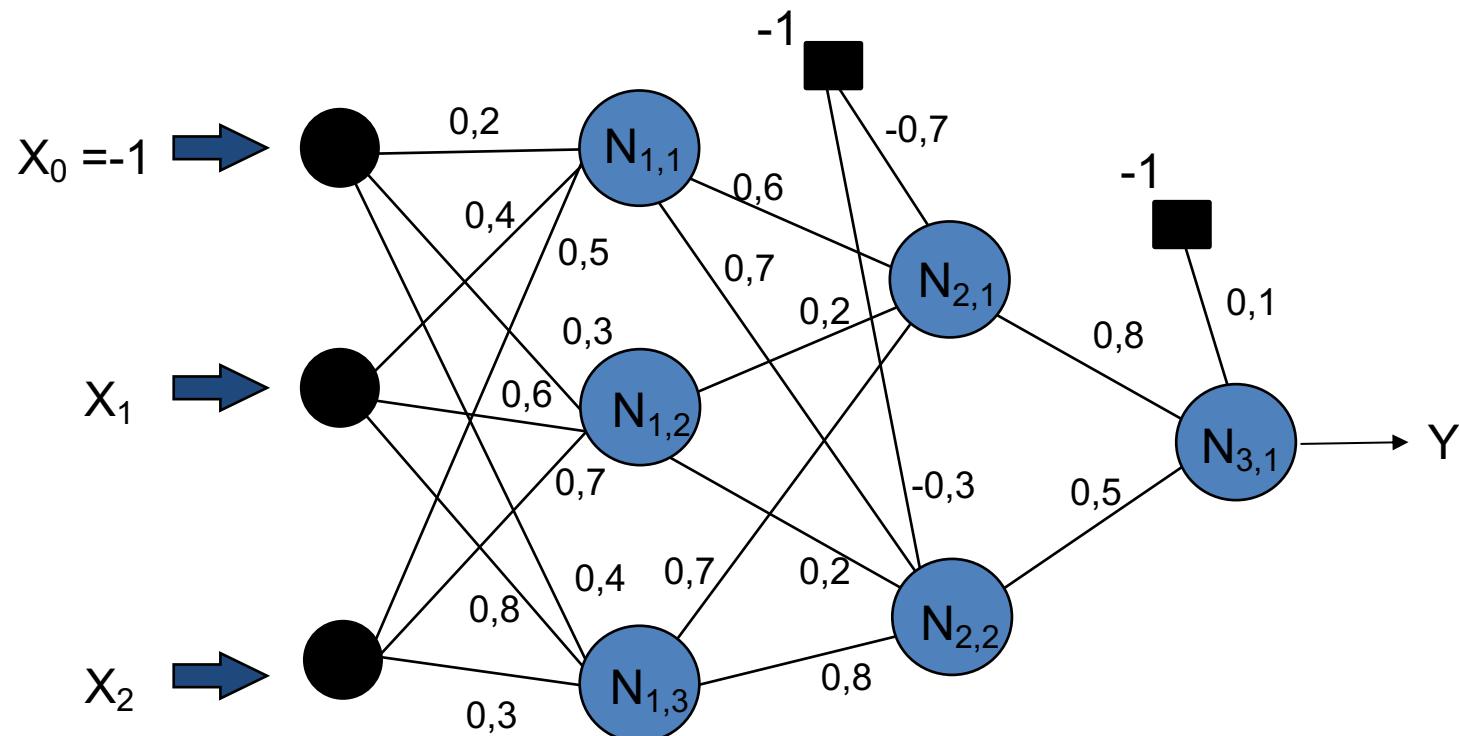
- For a ANN with linear activation function the neuron activation potencial $v_{n,k}$, where n is the neuron index and k is the layer, can be obtained by:



Input	Weight	v
X_0	W^1_{n0}	$X_0 * W^L_{n0} + X_1 * W^L_{n1} + X_2 * W^L_{n2}$
X_1	W^1_{n1}	
X_2	W^1_{n2}	

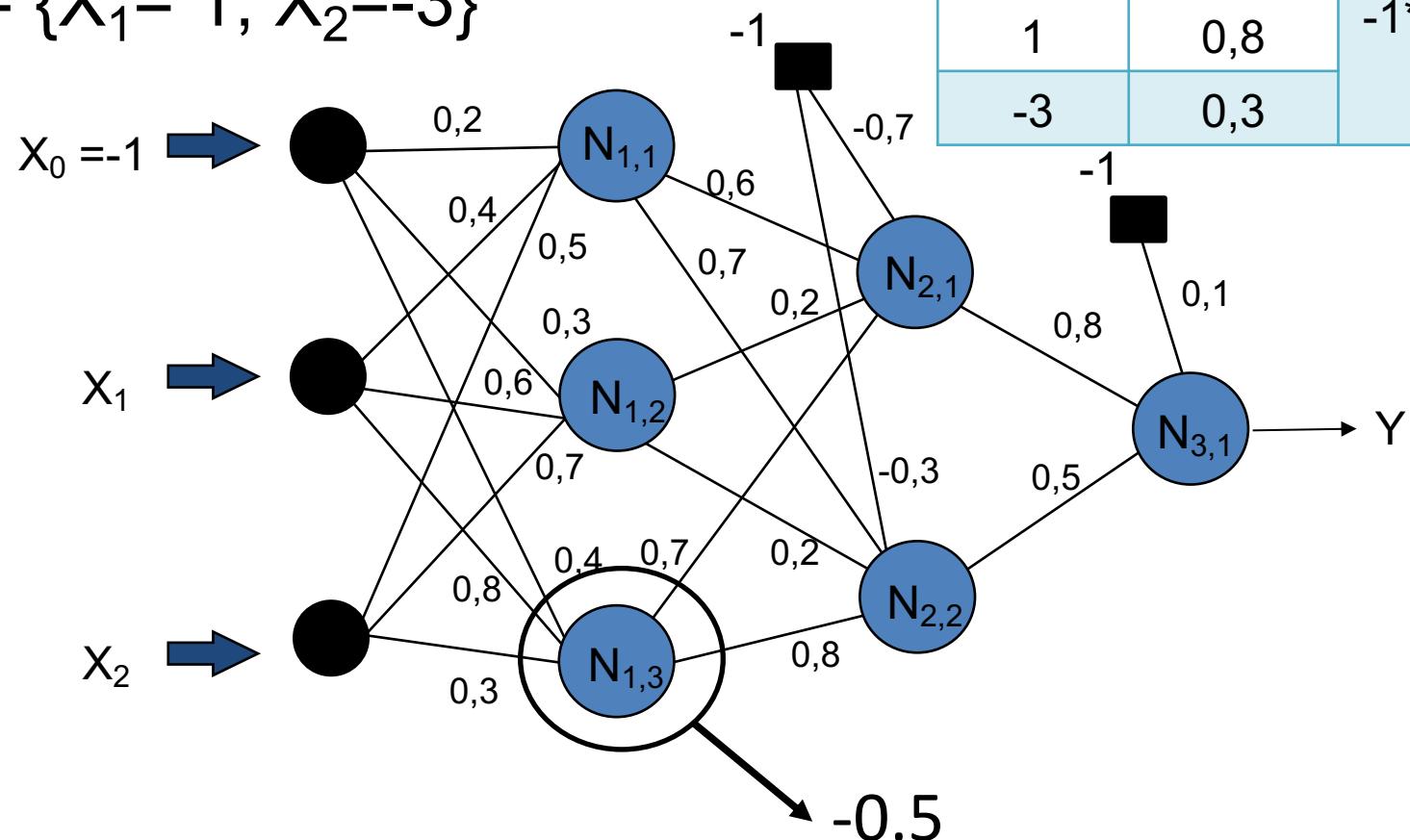
Verification of Covering Methods

- Sign change (sc) occurs when the activation potential of a neuron changes with respect to two different inputs.



Verification of Covering Methods

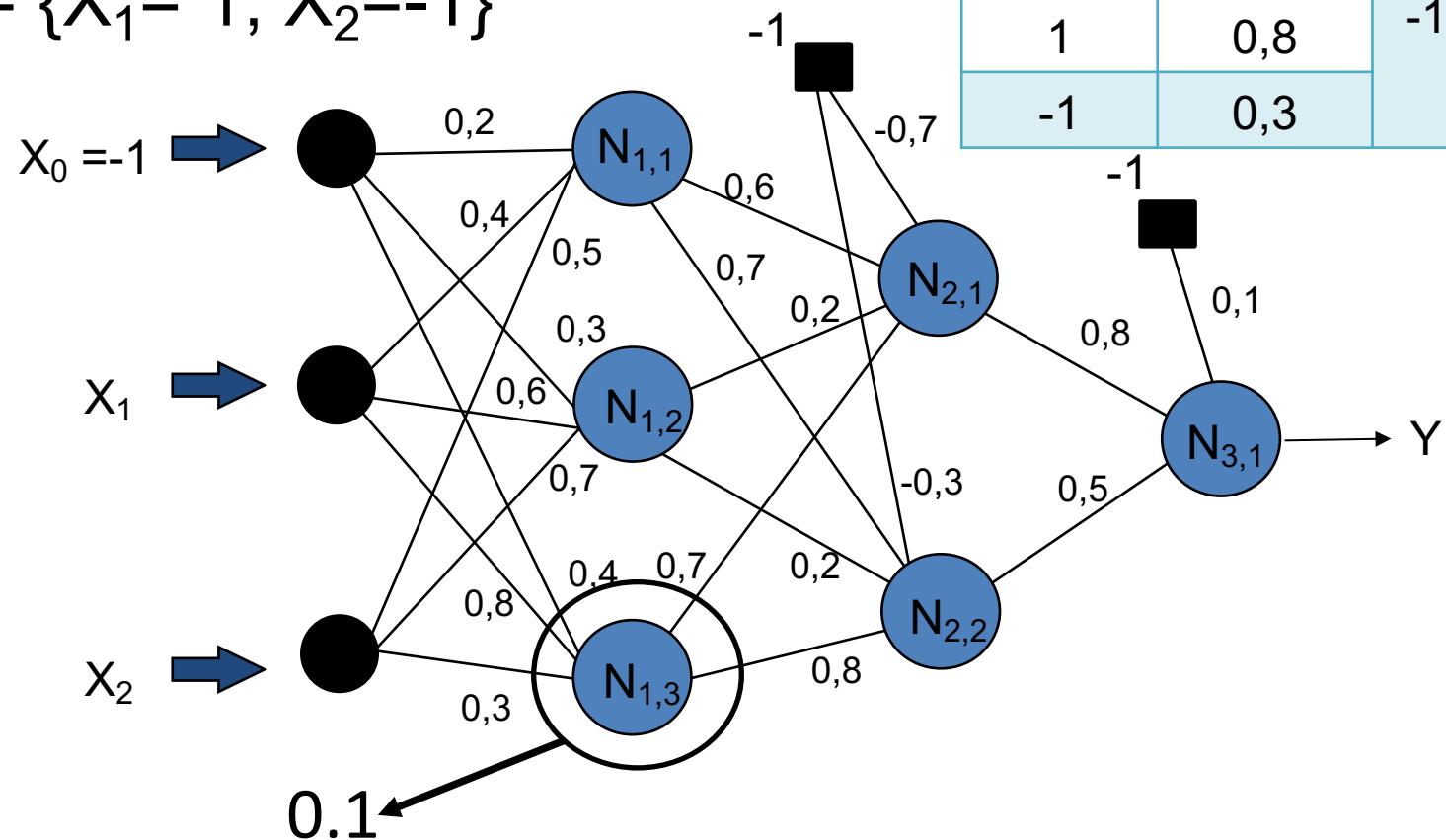
- Sign change (sc):
 - $X_A = \{X_1 = 1; X_2 = -3\}$



Input	Weight	$v_{1,3}$
-1	0,4	$-1*0,4 + 1*0,8 -3*0,3 = -0,5$
1	0,8	
-3	0,3	

Verification of Covering Methods

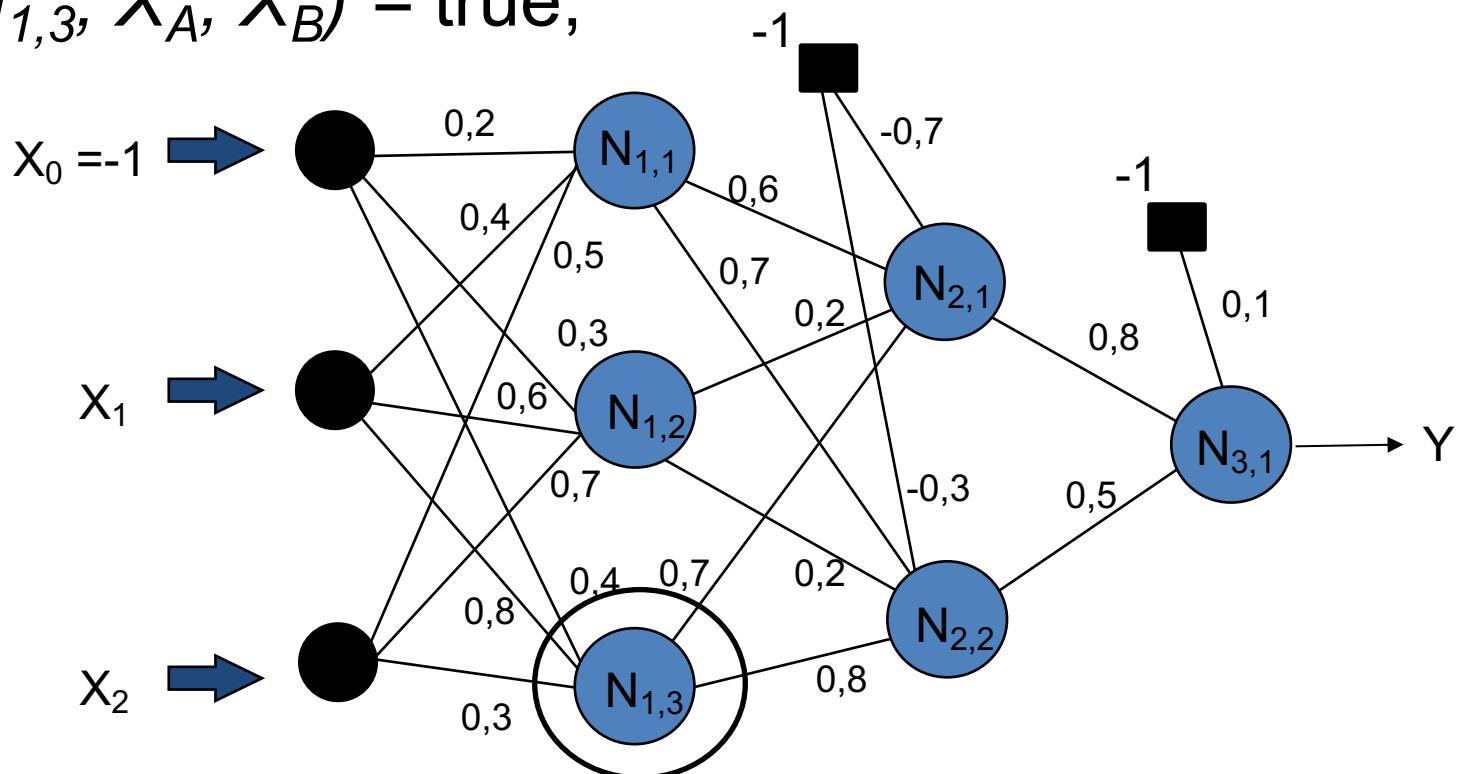
- Sign change (sc):
 - $X_B = \{X_1 = 1; X_2 = -1\}$



Input	Weight	$v_{1,3}$
-1	0,4	$-1*0,4 + 1*0,8 -1*0,3 = 0,1$
1	0,8	
-1	0,3	

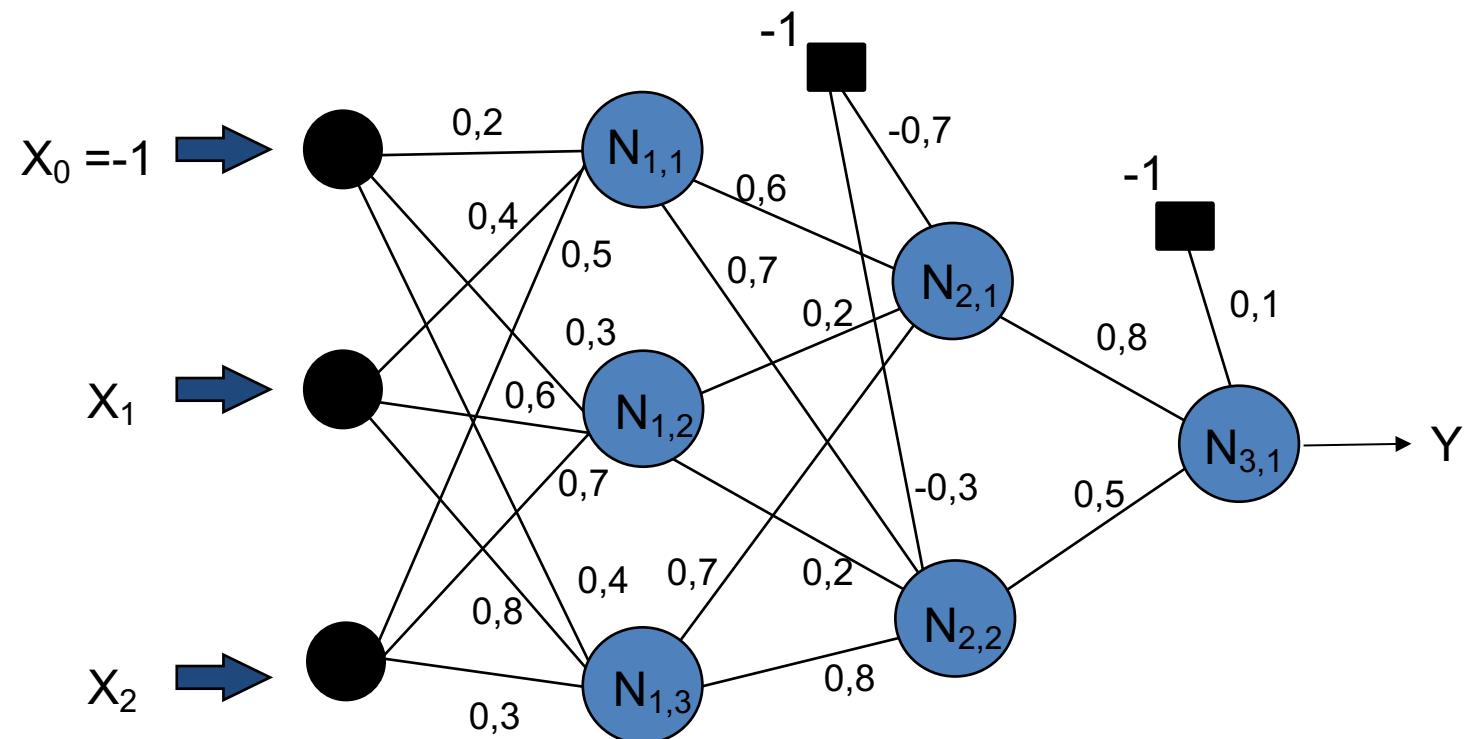
Verification of Covering Methods

- Sign change (sc):
 - $sc(n_{1,3}, X_A, X_B) = \text{true};$



Verification of Covering Methods

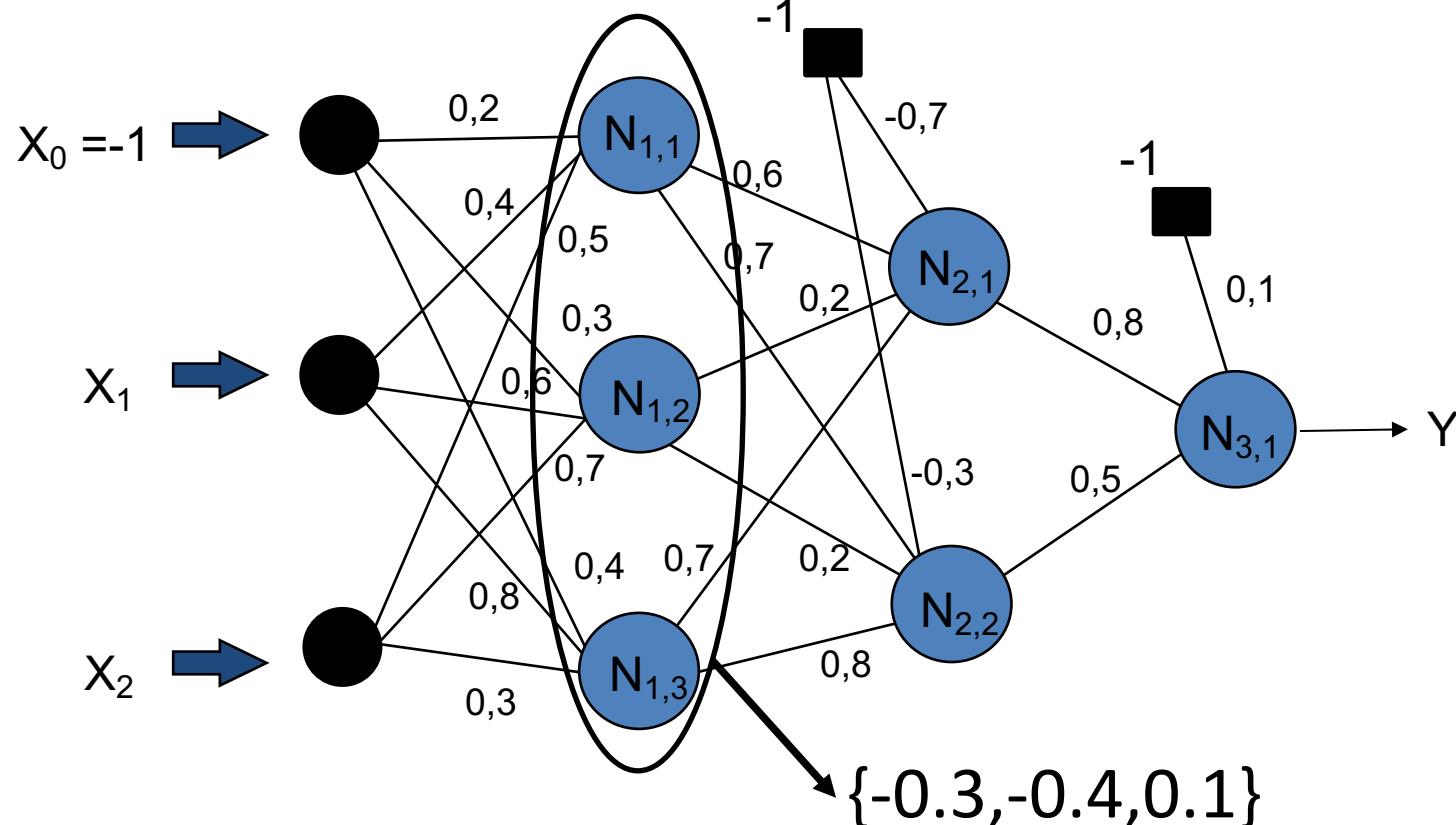
- Value change (vc) occurs when there is no sign change in every neuron of a layer, but there is atleast one who has a value change with respect to a metric h.



Verification of Covering Methods

- Value change (vc):

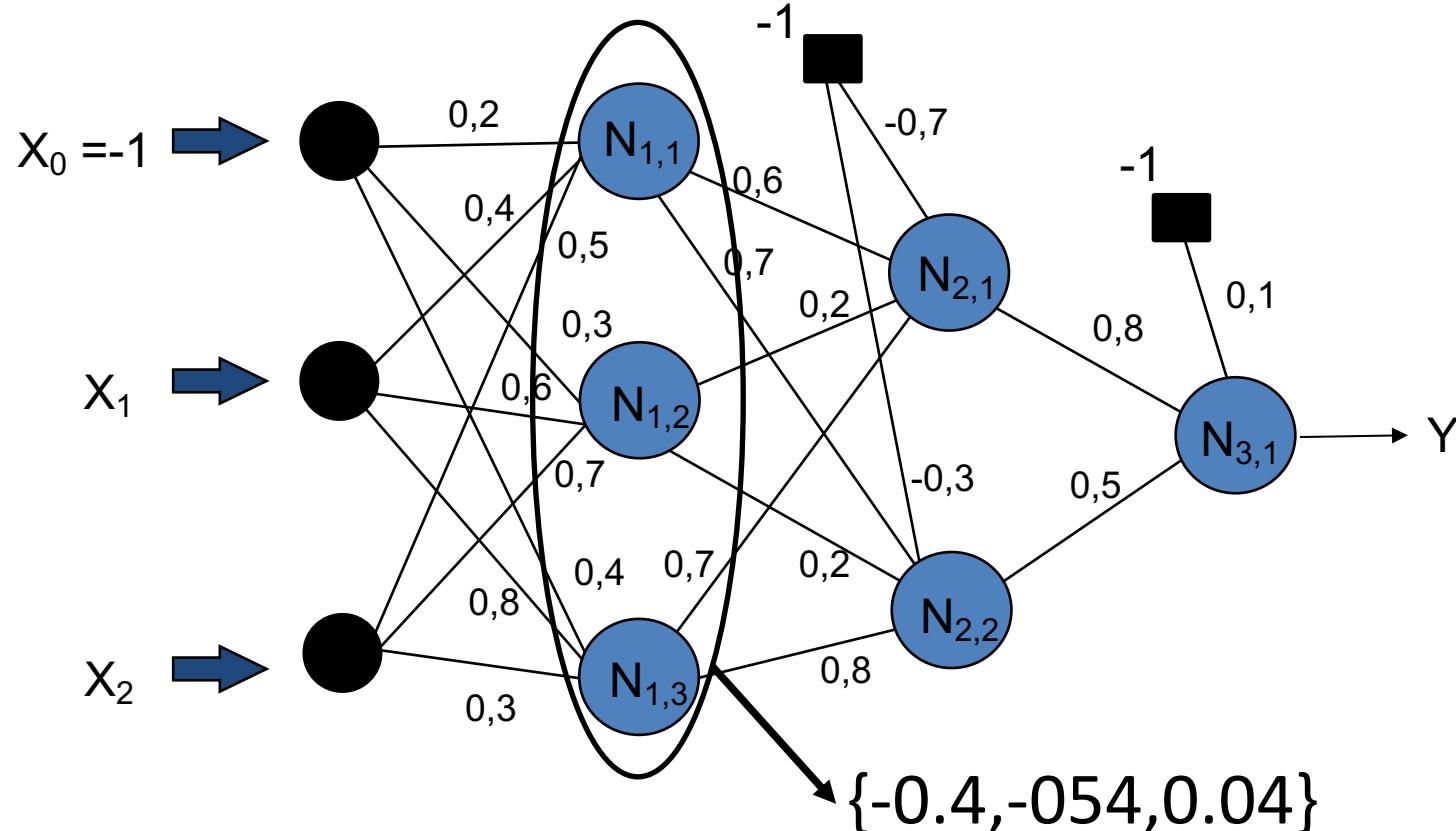
$$X_A = \{X_1 = 1; X_2 = -1\}$$



Verification of Covering Methods

- Value change (vc):

$$X_B = \{X_1 = 1; X_2 = -1.2\}$$



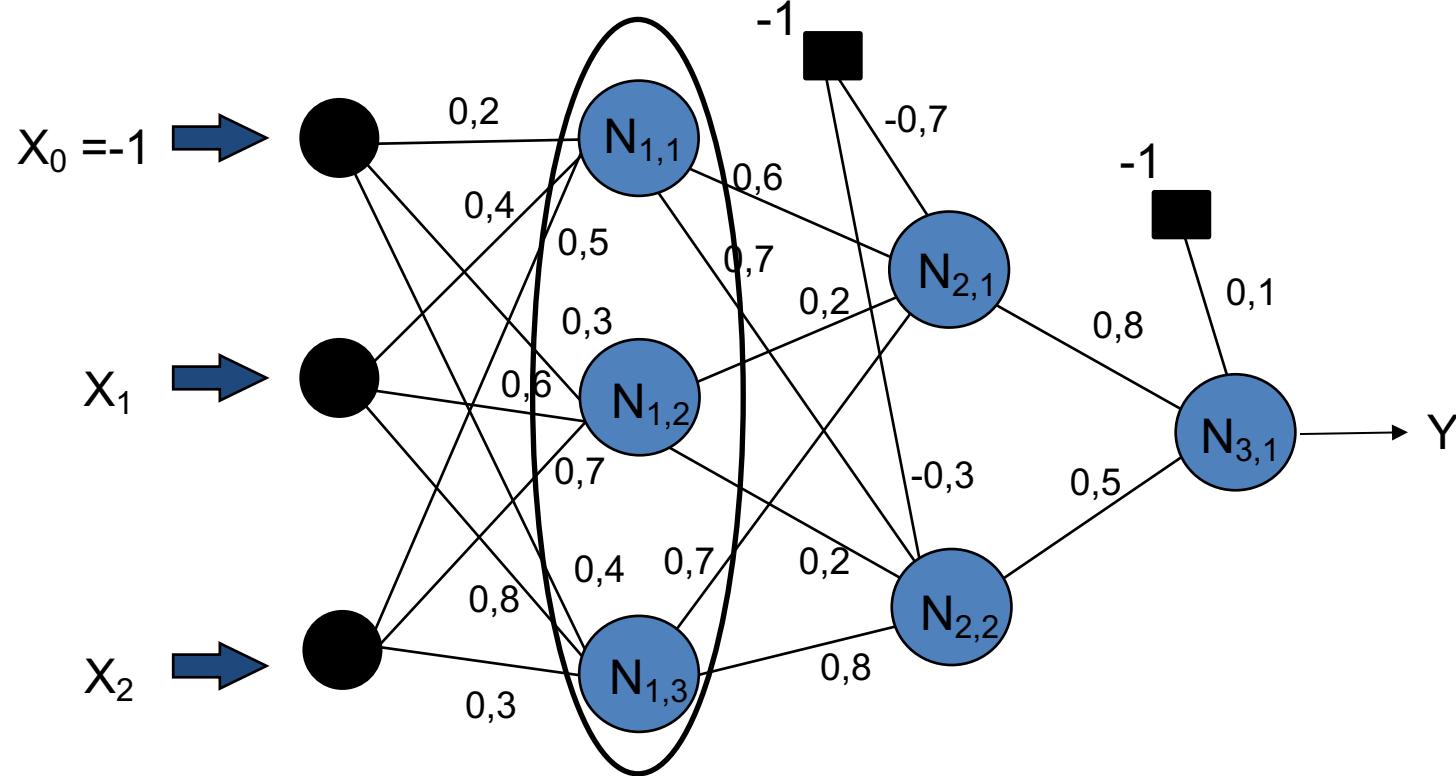
Input	Weight	$v_{1,1}$
-1	0,2	-0,4
1	0,4	
-1,2	0,5	

Input	Weight	$v_{1,2}$
-1	0,3	-0,54
1	0,6	
-1,2	0,7	

Input	Weight	$v_{1,3}$
-1	0,4	0,04
1	0,8	
-1,2	0,3	

Verification of Covering Methods

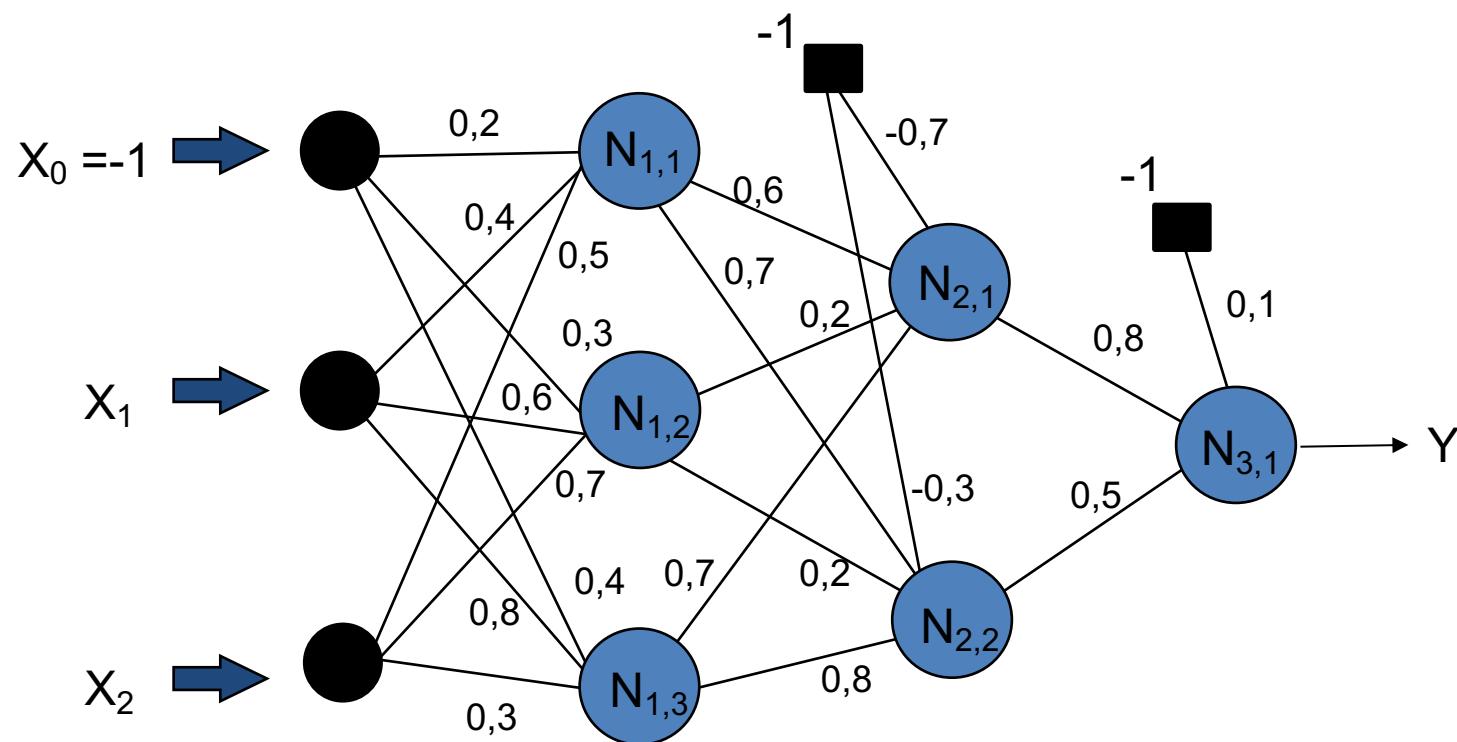
- Value change (vc): Suposing the boolean $h(x,y) = x - y > 0.13$. Then, $vc(h, n_{1,2}, X_A, X_B) = \text{true}$;



$V^1(X_1)$	$V^1(X_2)$	$h(v_{n,l}(X_1), v_{n,l}(X_2))$
-0,3	-0,4	False
-0,4	-0,54	True
0,1	0,04	False

Verification of Covering Methods

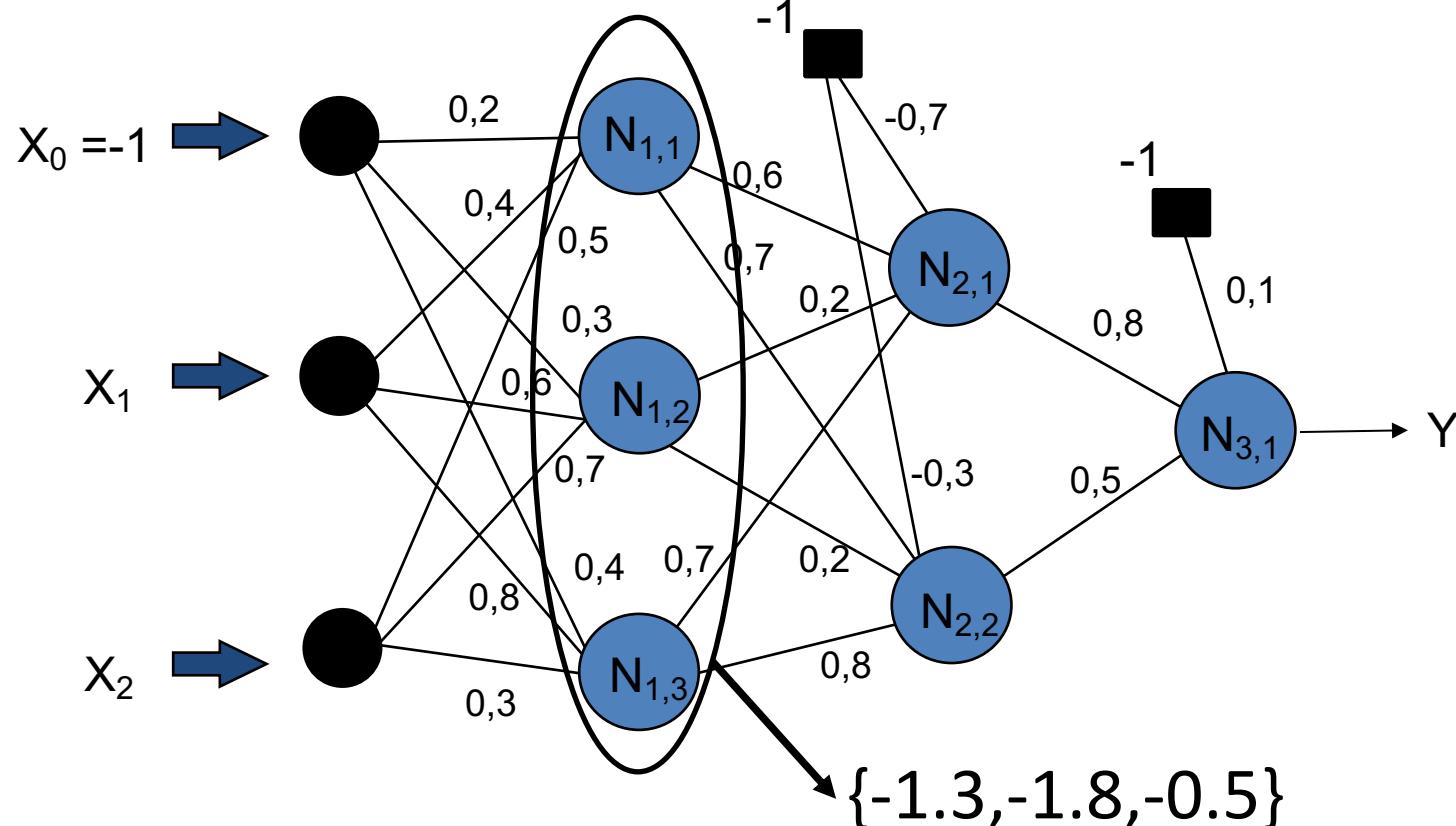
- Distance change (dc) occurs when there is no sign change in every neuron of a layer, but all neurons shows a value change with respect to a metric g.



Verification of Covering Methods

- Distance change (dc):

$$X_A = \{X_1 = 1; X_2 = -3\}$$



Input	Weight	$v_{1,1}$
-1	0,2	$-1,3$
1	0,4	
-3	0,5	

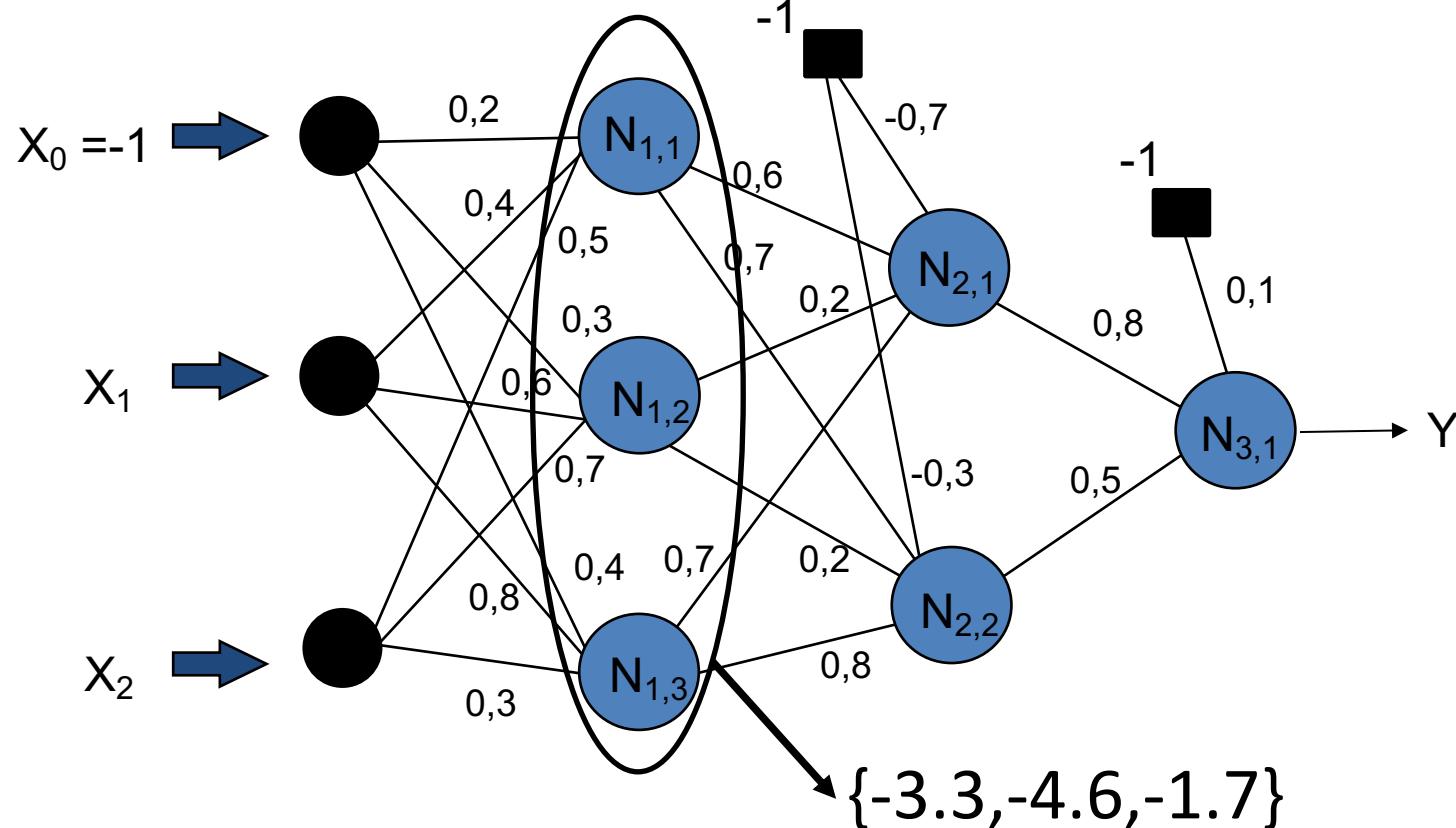
Input	Weight	$v_{1,2}$
-1	0,3	$-1,8$
1	0,6	
-3	0,7	

Input	Weight	$v_{1,3}$
-1	0,4	$-0,5$
1	0,8	
-3	0,3	

Verification of Covering Methods

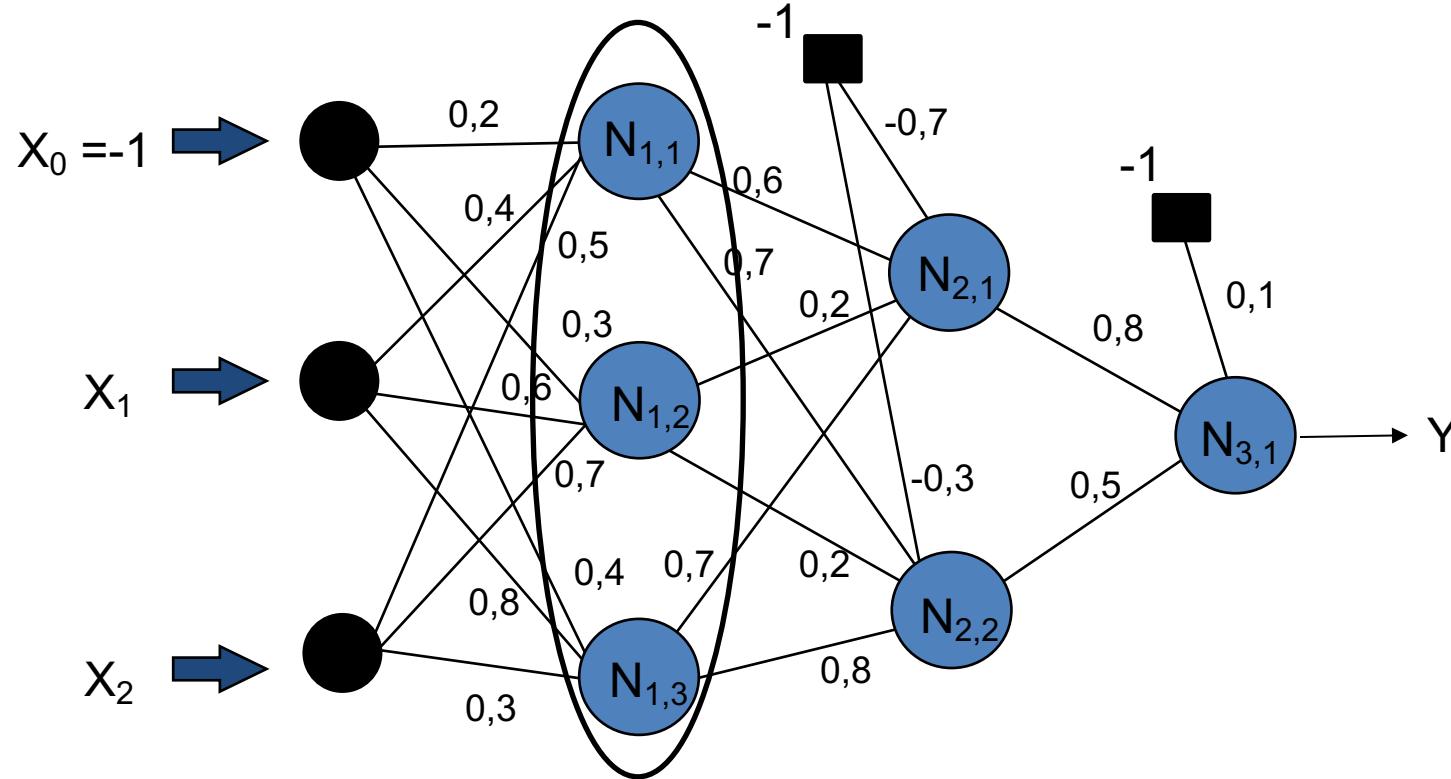
- Distance change (dc):

$$X_B = \{X_1 = 1; X_2 = -7\}$$



Verification of Covering Methods

- Distance change (dc): Suposing the boolean $g(l_k) = \text{EuclidianDistance}(l_k) > 0.1$. Then, $dc(g, l_1, X_A, X_B) = \text{true}$;



$V^1(X_1)$	$V^1(X_2)$	$g(l_1)$
-1,3	-3,3	True
-1,8	-4,6	
-0,5	-1,7	

Verification of Covering Methods

- Covering Methods are used to measure how adversarial can be a pair or a set of inputs to the ANN neurons. They are sectioned in four methods:

- SS-Cover:

$$\begin{aligned} & sc(n_{k,i}, x_1, x_2); \\ & \neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k; \\ & sc(n_{k+1,j}, x_1, x_2); \end{aligned}$$

- DS-Cover:

$$\begin{aligned} & dc(g, k, x_1, x_2); \\ & sc(n_{k+1,j}, x_1, x_2); \end{aligned}$$

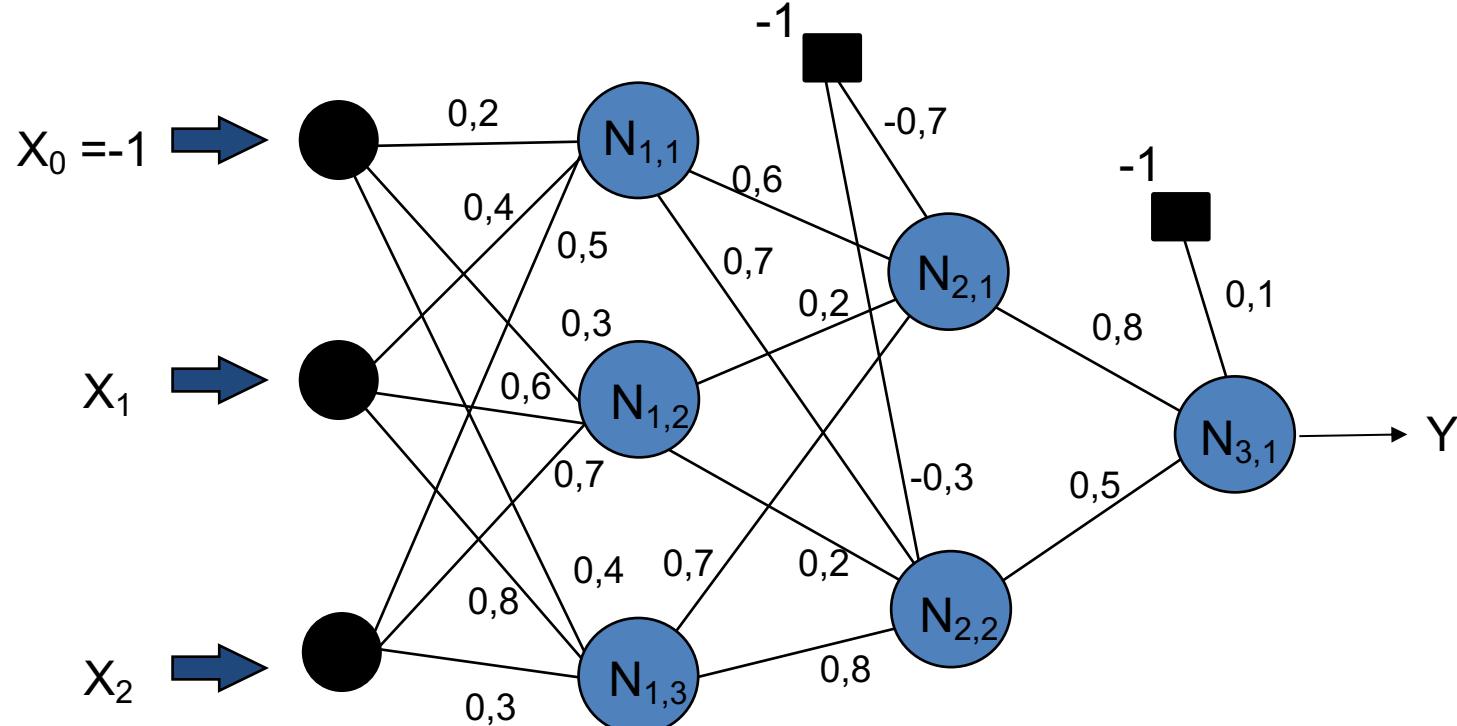
- SV-Cover:

$$\begin{aligned} & sc(n_{k,i}, x_1, x_2); \\ & \neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k; \\ & vc(h, n_{k+1,j}, x_1, x_2); \end{aligned}$$

- DV-Cover:

$$\begin{aligned} & dc(g, k, x_1, x_2); \\ & vc(h, n_{k+1,j}, x_1, x_2); \end{aligned}$$

Verification of Covering Methods



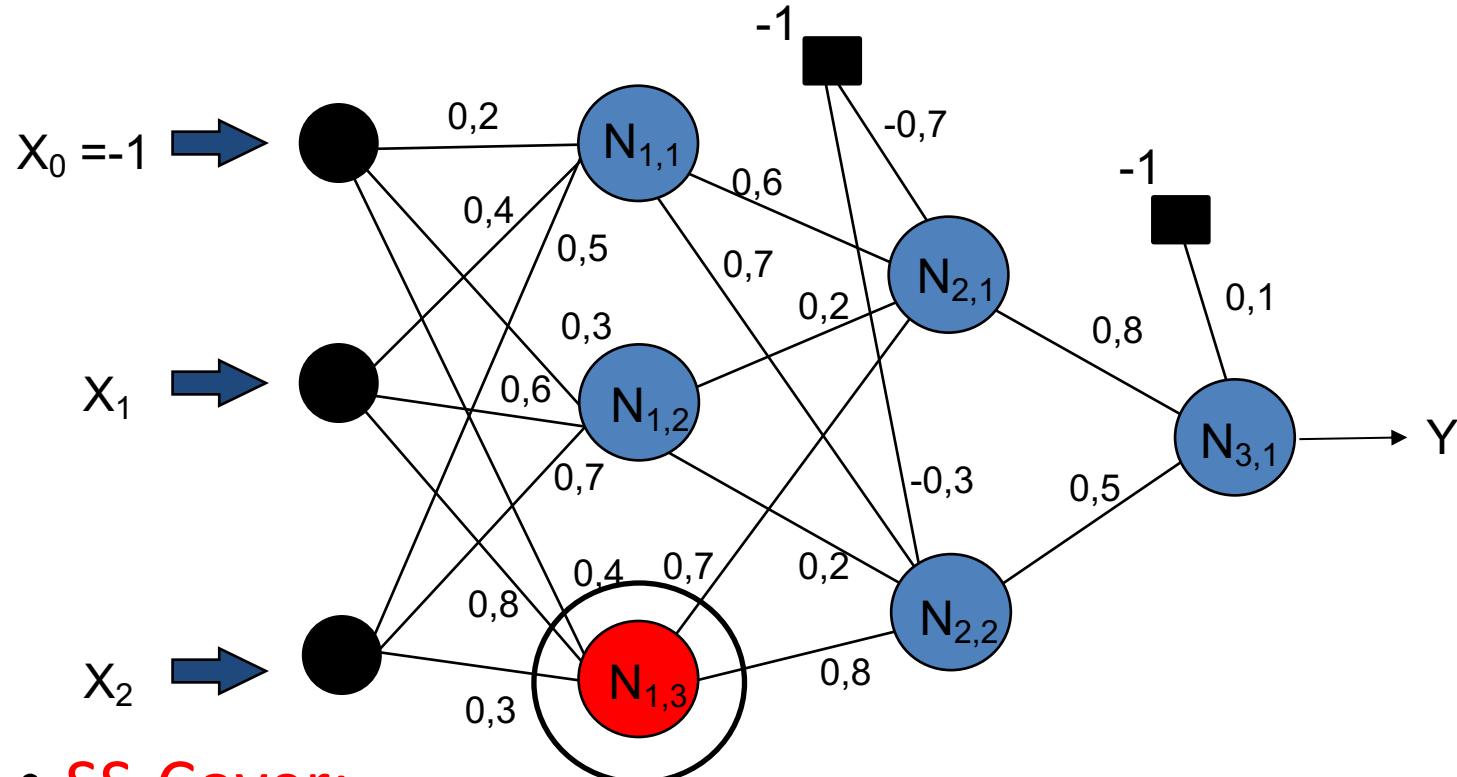
- SS-Cover:

$$\begin{aligned} & sc(n_{k,i}, x_1, x_2); \\ & \neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k; \\ & sc(n_{k+1,j}, x_1, x_2); \end{aligned}$$

- $X_A = \{1, -1\}$
- $X_B = \{1, -3\}$

	X_A	X_B	SC
$N_{1,1}$	-1,3	-0,3	F
$N_{1,2}$	-1,8	-0,4	F
$N_{1,3}$	-0,5	0,1	T
$N_{2,1}$	-0,79	0,51	T
$N_{2,2}$	-1,37	0,09	T
$N_{3,1}$	-1,41	0,35	T

Verification of Covering Methods



- **SS-Cover:**

$$sc(n_{k,j}, x_1, x_2);$$

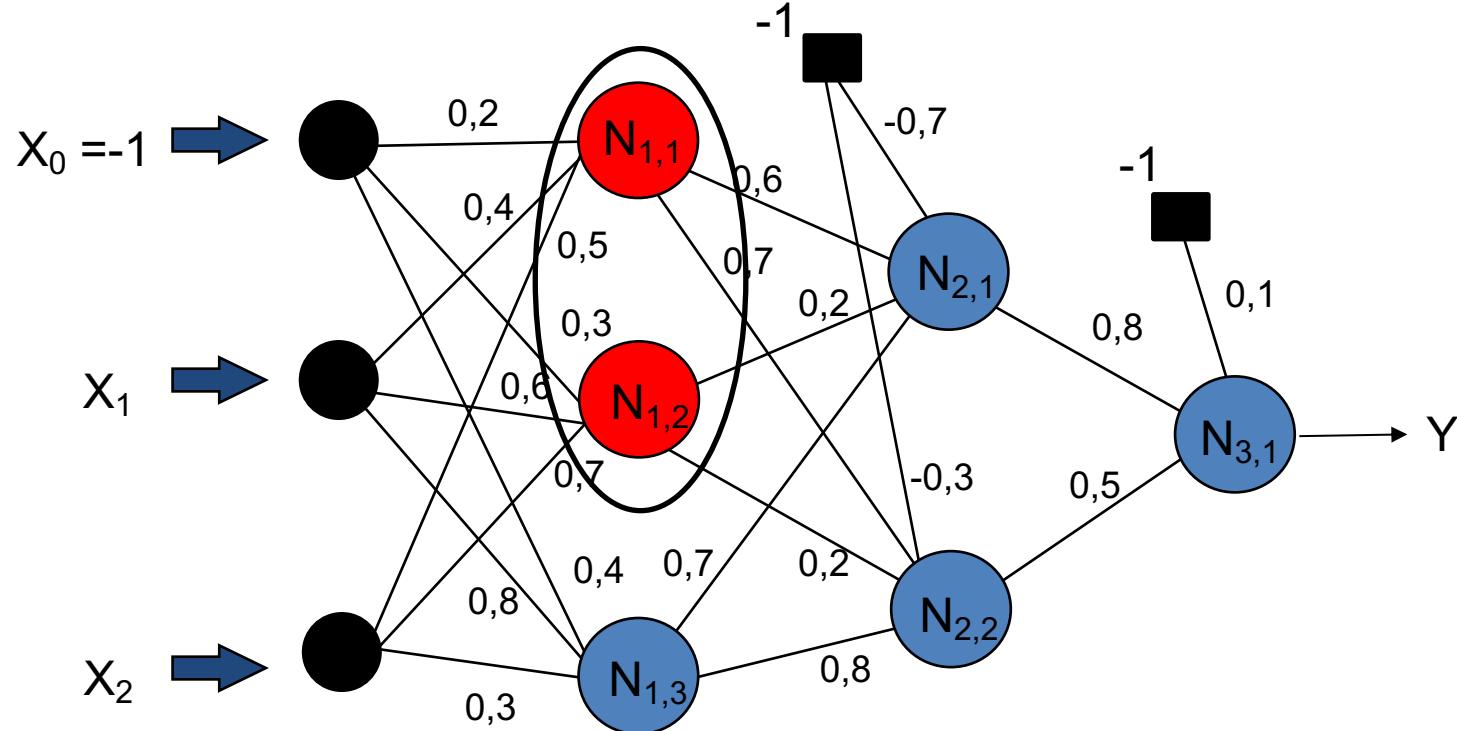
$$\neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k;$$

$$sc(n_{k+1,j}, x_1, x_2);$$

Layer1	x_A	x_B	SC
$N_{1,1}$	-1,3	-0,3	F
$N_{1,2}$	-1,8	-0,4	F
$N_{1,3}$	-0,5	0,1	T

Layer2	x_A	x_B	SC
$N_{2,1}$	-0,79	0,51	T
$N_{2,2}$	-1,37	0,09	T

Verification of Covering Methods



- **SS-Cover:**

$$sc(n_{k,i}, x_1, x_2);$$

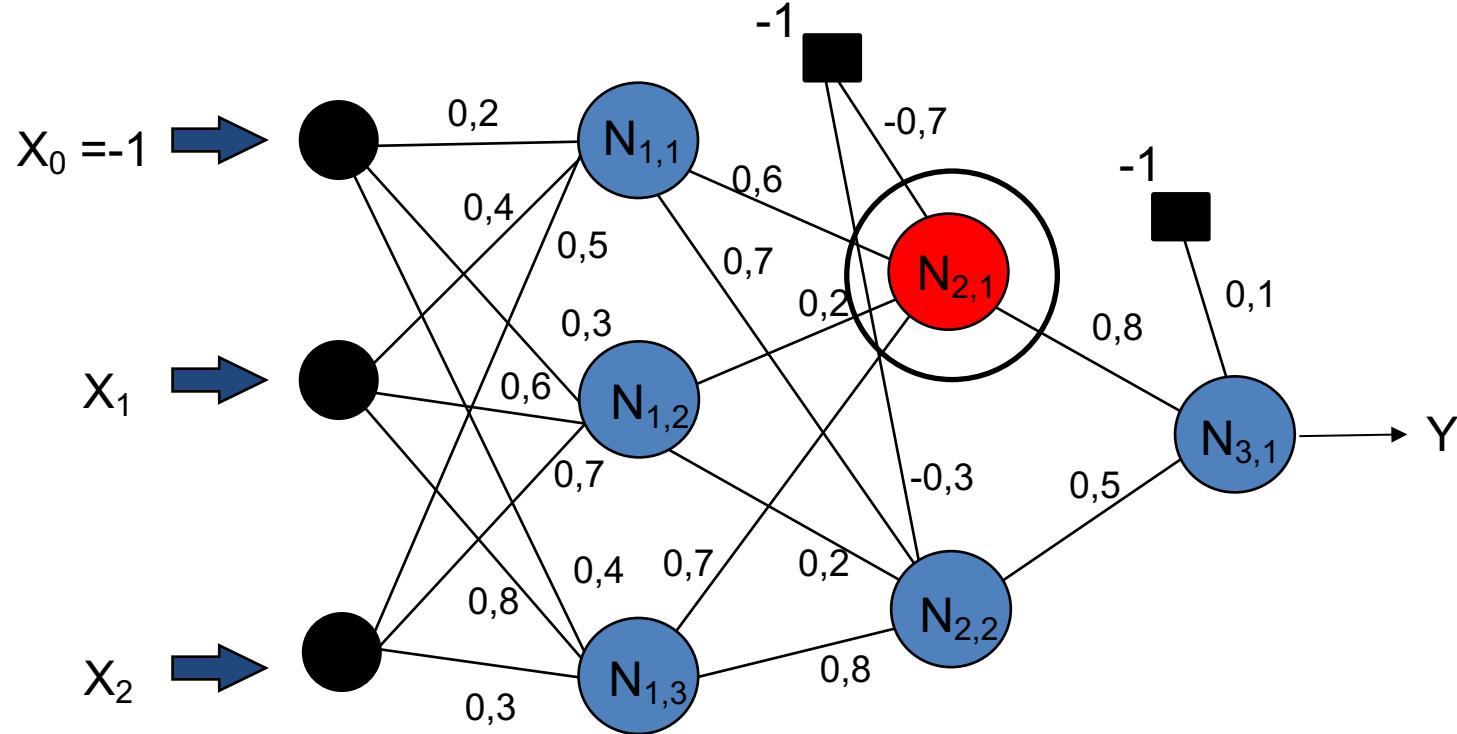
$$\neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k;$$

$$sc(n_{k+1,j}, x_1, x_2);$$

Layer1	x_A	x_B	SC
N _{1,1}	-1,3	-0,3	F
N _{1,2}	-1,8	-0,4	F
N _{1,3}	-0,5	0,1	T

Layer2	x_A	x_B	SC
N _{2,1}	-0,79	0,51	T
N _{2,2}	-1,37	0,09	T

Verification of Covering Methods



- **SS-Cover:**

$$sc(n_{k,i}, x_1, x_2);$$

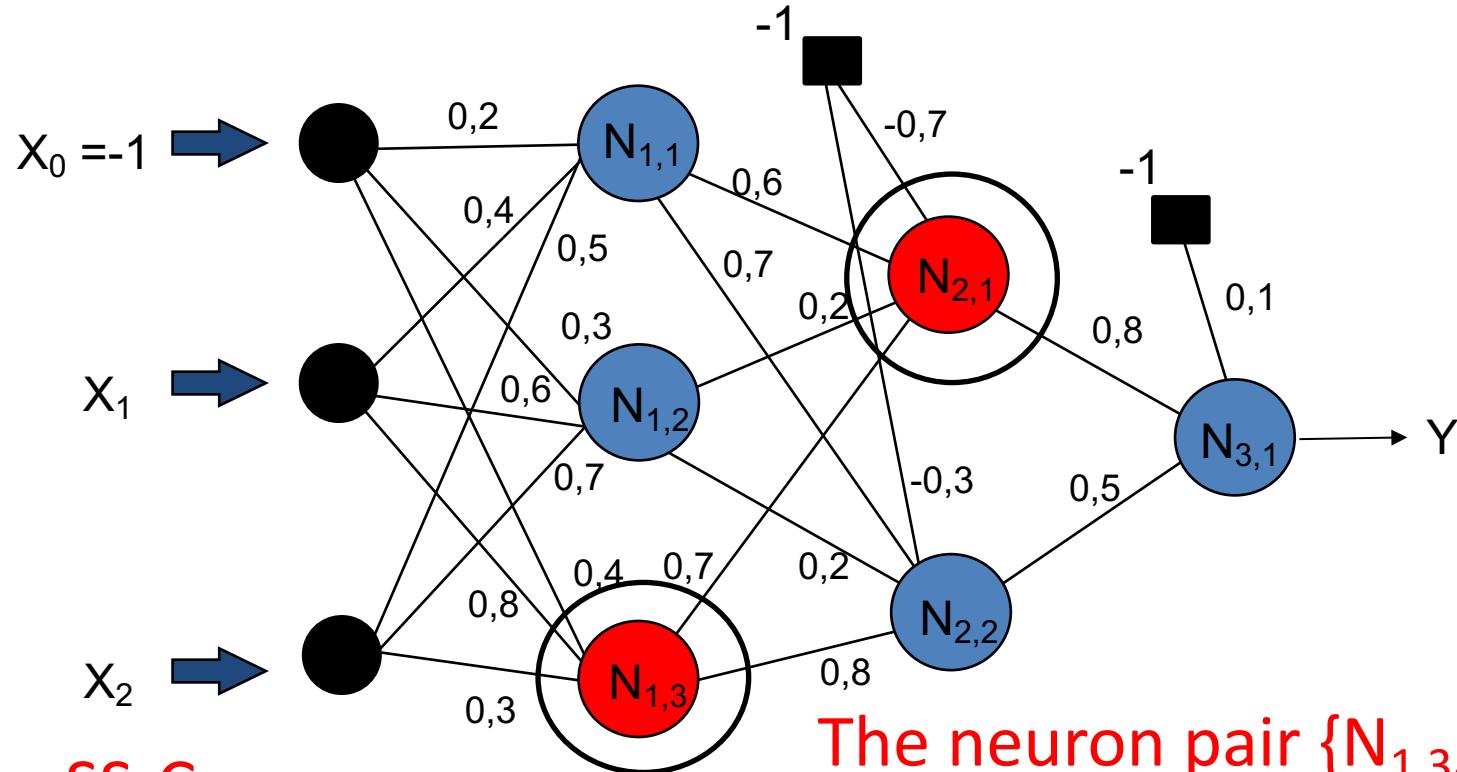
$$\neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k;$$

$$sc(n_{k+1,j}, x_1, x_2);$$

Layer1	x_A	x_B	SC
$N_{1,1}$	-1,3	-0,3	F
$N_{1,2}$	-1,8	-0,4	F
$N_{1,3}$	-0,5	0,1	T

Layer2	x_A	x_B	SC
$N_{2,1}$	-0,79	0,51	T
$N_{2,2}$	-1,37	0,09	T

Verification of Covering Methods



- SS-Cover:

$$sc(n_{k,i}, x_1, x_2);$$

$$\neg sc(n_{k,l}, x_1, x_2) \forall n_{k,l} \in L_k;$$

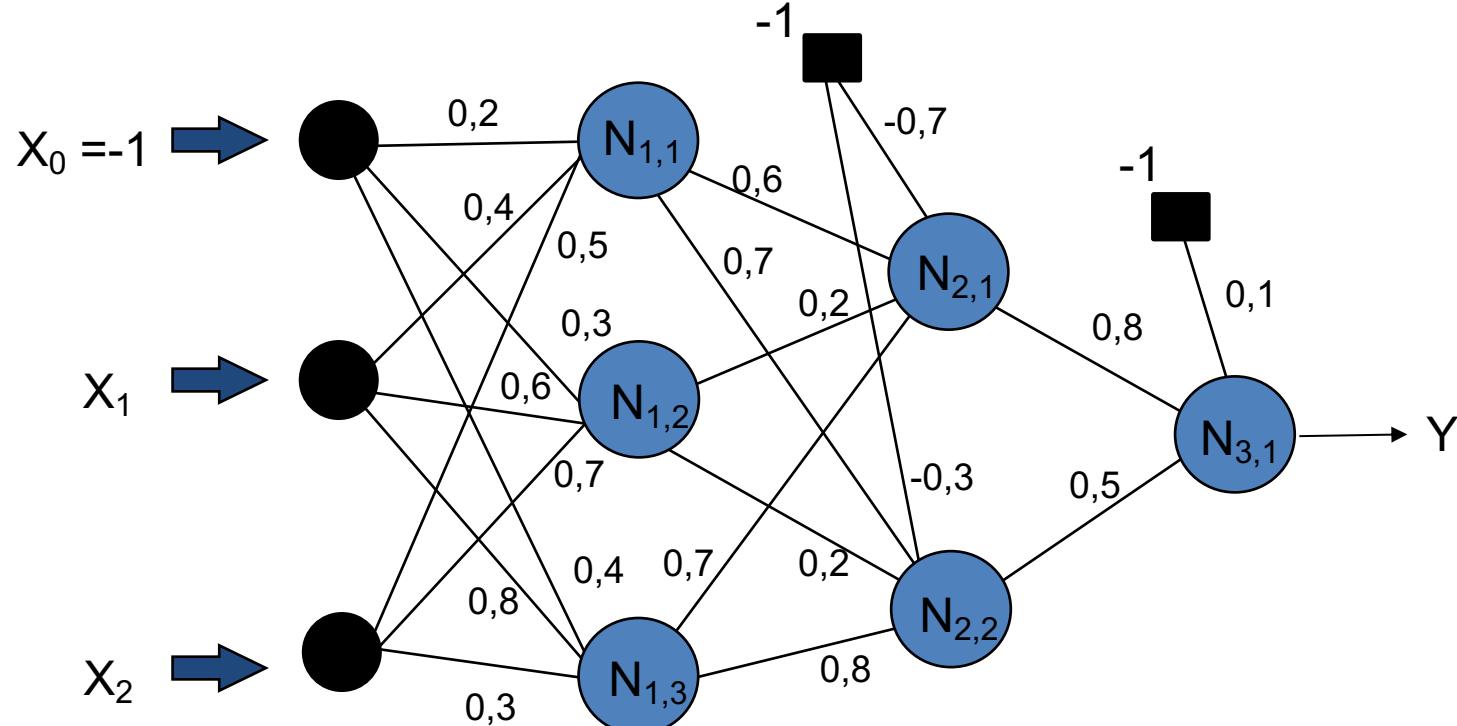
$$sc(n_{k+1,j}, x_1, x_2);$$

The neuron pair $\{N_{1,3}, N_{2,1}\}$ is SS-covered by X_A and X_B

Layer1	X_A	X_B	SC
$N_{1,1}$	-1,3	-0,3	F
$N_{1,2}$	-1,8	-0,4	F
$N_{1,3}$	-0,5	0,1	T

Layer2	X_A	X_B	SC
$N_{2,1}$	-0,79	0,51	T
$N_{2,2}$	-1,37	0,09	T

Verification of Covering Methods



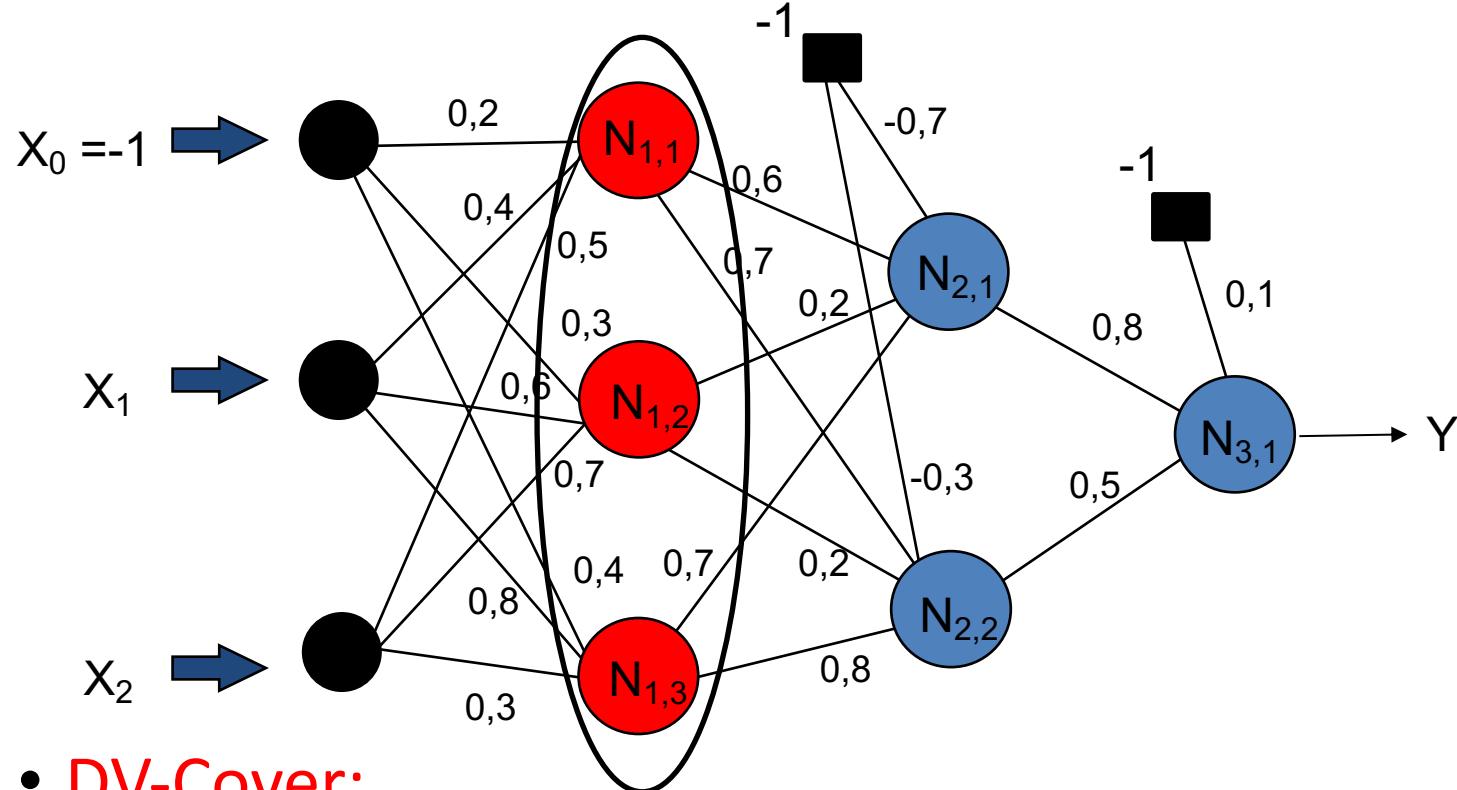
- DV-Cover:

$$\begin{aligned} &dc(g, k, x_1, x_2); \\ &vc(h, n_{k+1,j}, x_1, x_2); \end{aligned}$$

- $X_A = \{1, -3\}$
- $X_B = \{1, -7\}$

	X_A	X_B	SC
$N_{1,1}$	-1,3	-3,3	F
$N_{1,2}$	-1,8	-4,6	F
$N_{1,3}$	-0,5	-1,7	F
$N_{2,1}$	-0,79	-3,39	F
$N_{2,2}$	-1,37	-4,29	F
$N_{3,1}$	-1,41	-4,95	F

Verification of Covering Methods

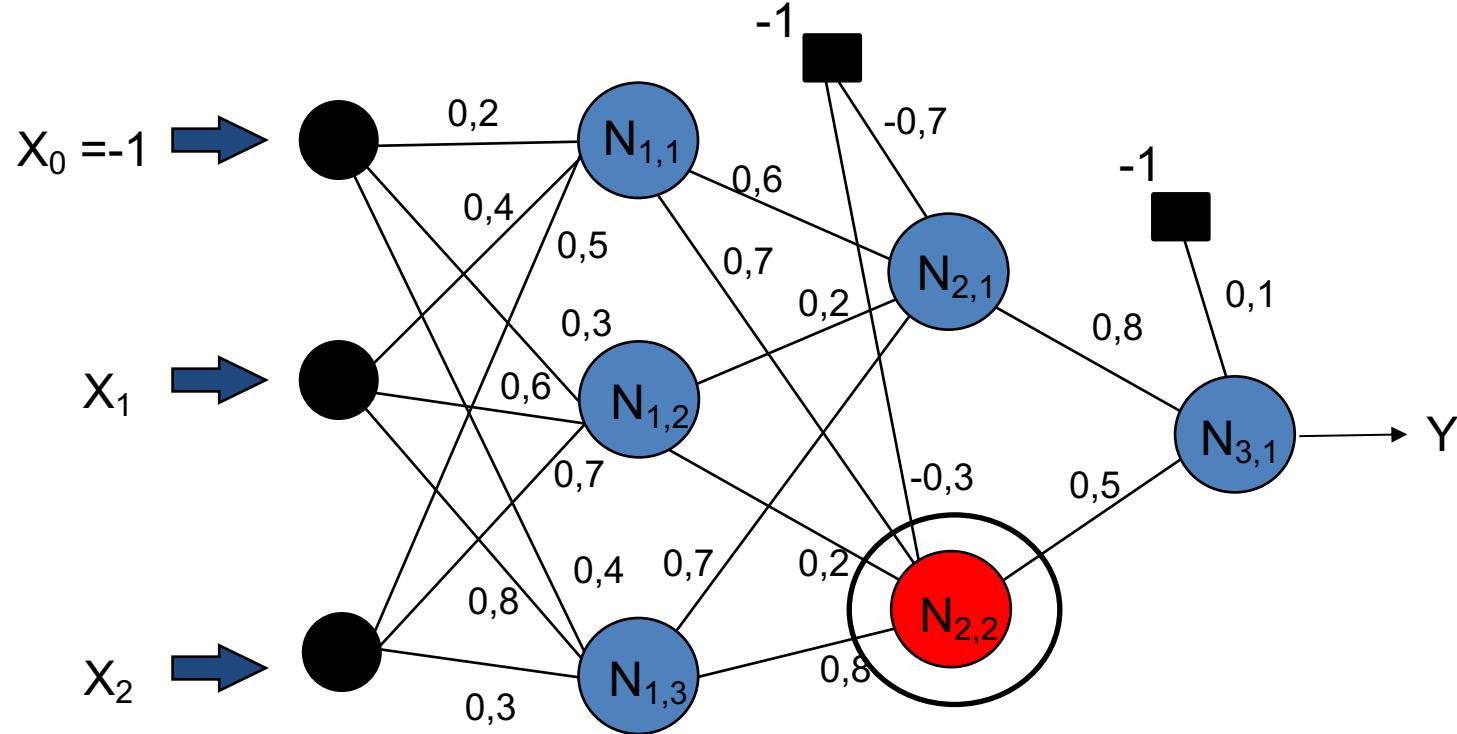


- DV-Cover:
 $dc(g, k, x_1, x_2);$
 $vc(h, n_{k+1,j}, x_1, x_2);$

Layer1	x_A	x_B	SC
$N_{1,1}$	-1,3	-3,3	F
$N_{1,2}$	-1,8	-4,6	F
$N_{1,3}$	-0,5	-1,7	F

Layer2	x_A	x_B	SC
$N_{2,1}$	-0,79	-3,39	F
$N_{2,2}$	-1,37	-4,29	F

Verification of Covering Methods



- **DV-Cover:**

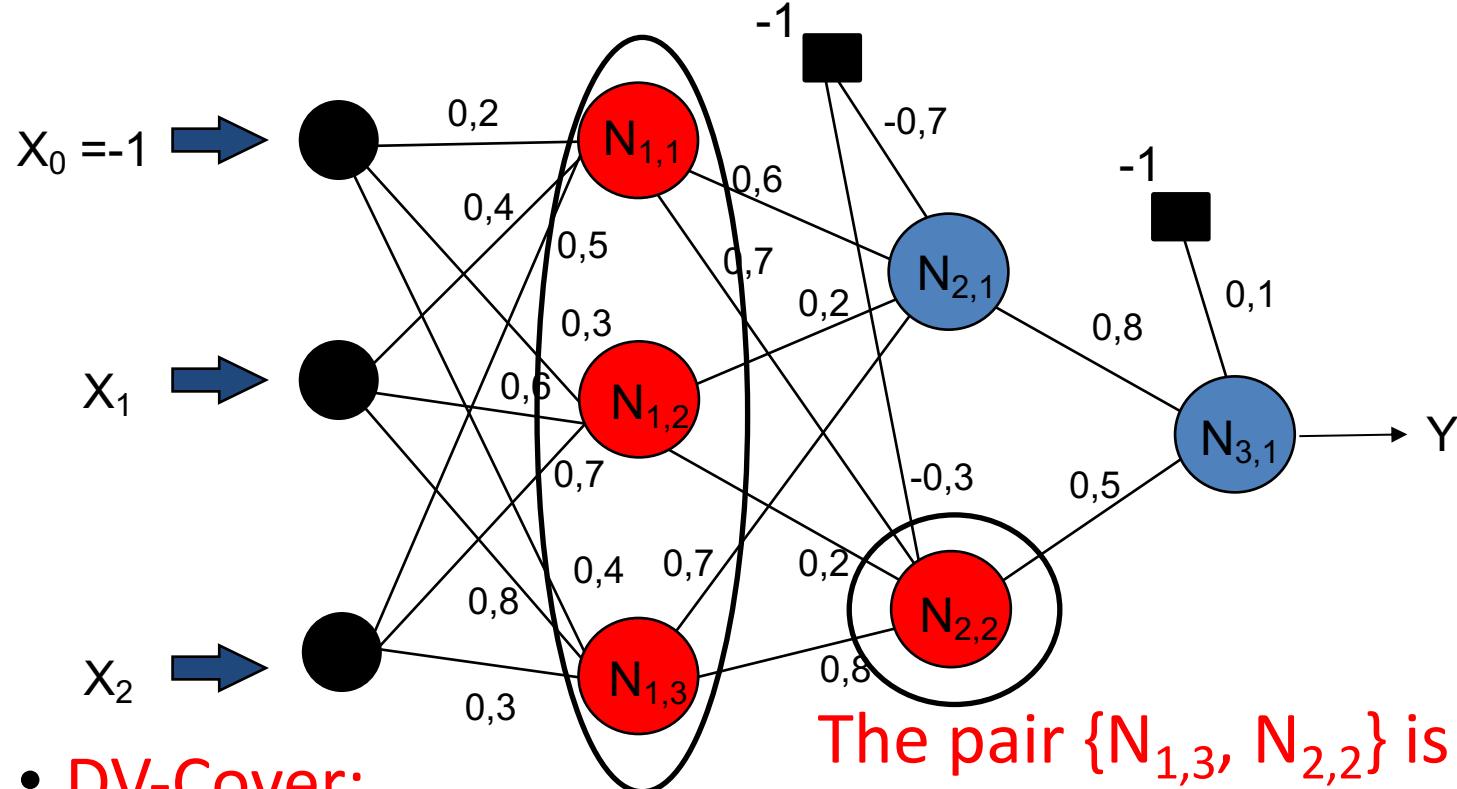
$dc(g, k, x_1, x_2);$

$vc(h, n_{k+1,j}, x_1, x_2);$

Layer1	x_A	x_B	SC
$N_{1,1}$	-1,3	-3,3	F
$N_{1,2}$	-1,8	-4,6	F
$N_{1,3}$	-0,5	-1,7	F

Layer2	x_A	x_B	SC
$N_{2,1}$	-0,79	-3,39	F
$N_{2,2}$	-1,37	-4,29	F

Verification of Covering Methods



- DV-Cover:
 $dc(g, k, x_1, x_2);$
 $vc(h, n_{k+1,j}, x_1, x_2);$

Layer1	x_A	x_B	SC
$N_{1,1}$	-1,3	-3,3	F
$N_{1,2}$	-1,8	-4,6	F
$N_{1,3}$	-0,5	-1,7	F

Layer2	x_A	x_B	SC
$N_{2,1}$	-0,79	-3,39	F
$N_{2,2}$	-1,37	-4,29	F

Verification of Adversarial Cases

- Conditions for adversarial cases:

$$I^d \in \mathcal{D}^{m \times n}, I \in \mathbb{M}^{m \times n}, \mathcal{D}^{m \times n} \subseteq \mathbb{M}^{m \times n}$$

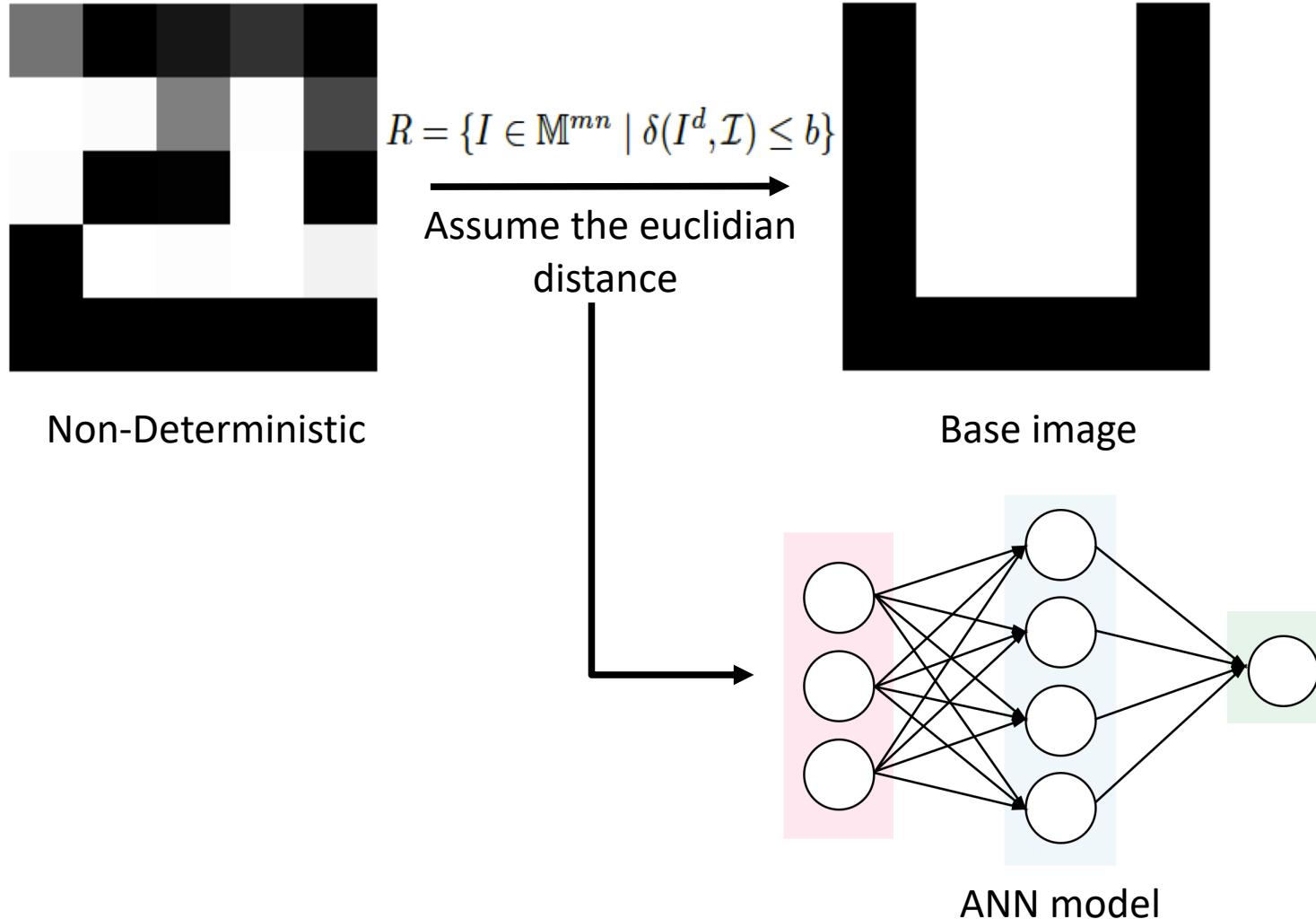
- Restriction:

$$R = \{I \in \mathbb{M}^{mn} \mid \delta(I^d, \mathcal{I}) \leq b\}$$

- Adversarial case definition:

$$\mathcal{Y}^d = \{N(\mathcal{I}) \text{ , } \forall \mathcal{I} \in \mathcal{R}\}$$

Verification of Adversarial Cases



Experimental Evaluation

Benchmark Description:

- ANN that solves the problem of vocalic recognition;
- Dataset composed by: 100 vocalics with noises; 100 characters;
- Backpropagation
- Cross-validation

Experimental Evaluation

- Experiments were conducted on a 8-core 3.40GHz Intel Core i7 with 24 GB of RAM and Linux OS.
- Frameworks versions:
 - CUDA v9.0, cuDNN v5.0, cuBLAS v10.1, and ESBMC-GPU v2.0.

Experimental Objectives

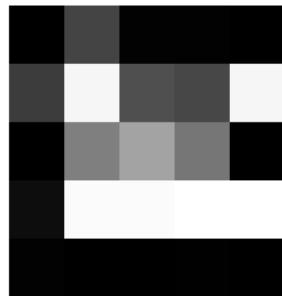
1. Evaluate the performance and correctness of our symbolic verification algorithms to check all four covering methods;
2. Evaluate the performance and correctness of our verification algorithm checkNN to verify adversarial cases obtained from changing input images and parameter proximity.

Results

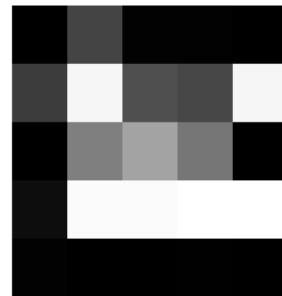
- The four properties specify that 80% of all neurons must be covered by a set of inputs.
- Verification time of the dataset w.r.t all 4 covering methods is around 20 minutes. It correctly verified the 4 properties.
- The verification of two inputs did not take more than a few seconds.

Results

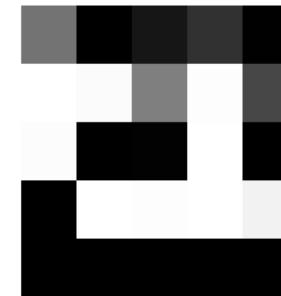
- Adversarial cases for label “E” missclassified as label “O”:



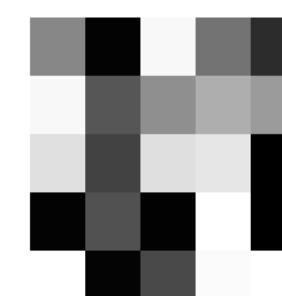
$b = 0.5$



$b = 1.5$

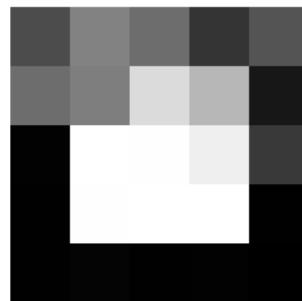


$b = 2.5$

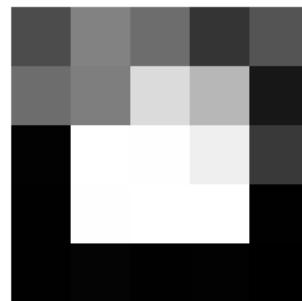


$b = 3.5$

- Label “U” missclassified as label “O”:



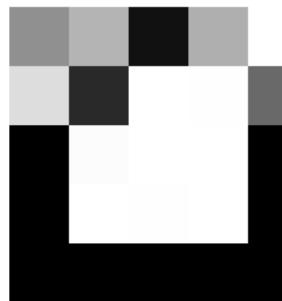
$b = 0.3$



$b = 0.5$



$b = 1.0$



$b = 1.5$

Results

Benchmark	Image	λ	Verification Time (hours)
Ex1	Vocalic O	0.5	1
Ex2	Vocalic O	1.5	4
Ex3	Vocalic O	2.5	8
Ex4	Vocalic O	3.5	6
Ex5	Vocalic E	0.5	25
Ex6	Vocalic E	0.7	25
Ex7	Vocalic E	1.5	14

Ex8	Vocalic E	3.0	12
Ex9	Vocalic U	0.3	6
Ex10	Vocalic U	0.5	5
Ex11	Vocalic U	1.0	20
Ex12	Vocalic U	1.5	19
Ex13	Vocalic A	1.0	63
Ex14	Vocalic A	1.5	63

Conclusion

- Our verification method is able to find adversarial cases for different input images and proximity parameter values;
- Our approach exhaustively verifies all possible adversarial cases inside the proximity parameter b ;
- The verification of covering methods is able to verify our dataset correctly. The dataset is not able to cover 80% of neuron w.r.t. a covering method;
- There is a high verification time in some benchmarks due to bitaccurate verification.

Future Work

- Support convolutional layers;
- Implement some techniques as invariant inference to prune the state space exploration;
- Investigate fault localization and repair techniques;

Questions?

Luiz Henrique Coelho Sena
lhcs@icomp.ufam.edu.br

Universidade Federal do Amazonas