Continuum Mechanics Equation Sheet

Name	Purpose	Symbols	Tensor Notation	Indicial Notation	Units	Notes
Initial Particle Position	-	$\underset{\sim}{X}$	$\underset{\sim}{X}$	X_i	$length$	Given
Current Particle Position	-	$\underset{\sim}{x}$	$\underset{\sim}{x} = \underset{\approx}{F} \cdot \underset{\sim}{X} + \underset{\sim}{c}$	$x_i = F_{ij}X_j + c_i$	$length$	The vector $\underset{\sim}{c}$ is the displacement vector
Deformation Gradient	Describes local volume, orientation, and shape changes	$\underset{\approx}{F}$	$\underset{\approx}{F} = \frac{\partial \underset{\sim}{x}}{\partial \underset{\sim}{X}} = \underset{\sim}{x}\overleftarrow{\nabla}_0 = \underset{\approx}{R} \cdot \underset{\approx}{U} = \underset{\approx}{V} \cdot \underset{\approx}{R}$	$F_{ij} = \frac{\partial x_i}{\partial X_j} = R_{ik}U_{kj} = V_{ik}R_{kj}$	1	Invertible; Work conjugate with $\underset{\approx}{P} \rightarrow \frac{1}{\rho_0}\underset{\approx}{P} \colon \dot{\underset{\approx}{F}}$
Jacobian	Describes local volume changes	J	$J = \det(\underset{\approx}{F}) = \det(\underset{\approx}{U}) = \det(\underset{\approx}{V}) = \frac{V}{V_0} = \frac{\rho_0}{\rho}$	-	1	Always positive nonzero; 'Jacobian of the motion'
Rotation Tensor	Describes local rotation	$\underset{\approx}{R}$	$\underset{\approx}{R} = \underset{\approx}{F} \cdot \underset{\approx}{U}^{-1}$	$R_{ij} = F_{ik}U_{kj}^{-1}$	1	Orthogonal; Unique; Pure rotation ($\det(\underset{\approx}{R}) = 1$); 'Polar rotation tensor'
Left stretch	Spatial stretch measure	$\underset{\approx}{V}$	$\underset{\approx}{V} = \underset{\approx}{F} \cdot \underset{\approx}{R}^T$	$V_{ij} = F_{ik}R_{jk}$	1	Symmetric; Positive Definite; Unique
Right stretch	Reference stretch measure	$\underset{\approx}{U}$	$\underset{\approx}{U} = \underset{\approx}{R}^T \cdot \underset{\approx}{F}$	$U_{ij} = R_{ki}F_{kj}$	1	Symmetric; Positive Definite; Unique; Unaffected by superimposed rotation; 'Reference stretch'
Left Cauchy-Green Tensor	Reference stretch measure	$\underset{\approx}{B}$	$\underset{\approx}{B} = \underset{\approx}{F} \cdot \underset{\approx}{F}^T = \underset{\approx}{V}^2$	$B_{ij} = F_{ik}F_{jk} = V_{ik}V_{kj}$	1	Symmetric; Positive Definite; 'Finger Tensor'; Inverse called 'Cauchy deformation tensor' $\tilde{\underset{\approx}{B}}$; 'Spatial stretch'
Right Cauchy-Green Tensor	Spatial stretch measure	$\underset{\approx}{C}$	$\underset{\approx}{C} = \underset{\approx}{F}^T \cdot \underset{\approx}{F} = \underset{\approx}{U}^2$	$C_{ij} = F_{ki}F_{kj} = U_{ik}U_{kj}$	1	Symmetric; Positive Definite; Unaffected by superimposed rotation
Euler Strain	Measure of spatial strain	$\underset{\approx}{e}$	$\underset{\approx}{e} = \frac{1}{2}(\underset{\approx}{I} - \underset{\approx}{B}^{-1}) = \frac{1}{2}(\underset{\approx}{I} - \underset{\approx}{F}^{-T} \cdot \underset{\approx}{F}^{-1})$	$e_{ij} = \frac{1}{2}(\delta_{ij} - B_{ij}^{-1}) = \frac{1}{2}(\delta_{ij} - F_{ki}^{-1}F_{kj}^{-1})$	1	Symmetric; Seth-Hill parameter $\kappa = -2$; 'Alamansi-Hamel strain tensor'; 'Eulerian strain tensor'
Logarithmic Strain	Measure of reference strain	$\underset{\approx}{\varepsilon}$	$\underset{\approx}{\varepsilon} = \ln(\underset{\approx}{U})$	-	1	Symmetric; Seth-Hill parameter $\kappa \rightarrow 0$; 'Hencky strain tensor'
Lagrange Strain	Measure of reference strain	$\underset{\approx}{E}, \underset{\approx}{\varepsilon}$	$\underset{\approx}{E} = \frac{1}{2}(\underset{\approx}{C} - \underset{\approx}{I}) = \frac{1}{2}(\underset{\approx}{F}^T \cdot \underset{\approx}{F} - \underset{\approx}{I})$	$E_{ij} = \frac{1}{2}(C_{ij} - I_{ij}) = \frac{1}{2}(F_{ki}F_{kj} - I_{ij})$	1	Symmetric; Seth-Hill parameter $\kappa = 2$; Unaffected by superimposed rotation; Work conjugate with $\underset{\approx}{S} \rightarrow \frac{1}{\rho_0}\underset{\approx}{S} \colon \dot{\underset{\approx}{E}}$; 'Green strain tensor'; 'Green-St. Venant strain tensor'
Velocity Gradient	-	$\underset{\approx}{L}$	$\underset{\approx}{L} = \underset{\sim}{v}\overleftarrow{\nabla}_x = \left(\overrightarrow{\nabla}_x \underset{\sim}{v}\right)^T = \frac{\partial \underset{\sim}{v}}{\partial \underset{\sim}{x}} = \dot{\underset{\approx}{F}} \cdot \underset{\approx}{F}^{-1}$	$L_{ij} = \frac{\partial v_i}{\partial x_j} = \dot{F}_{ik}F_{kj}^{-1}$	$\frac{1}{Second}$	'Spatial velocity gradient'
Symmetric Part of the Velocity Gradient	Strain rate approximation	$\underset{\approx}{D}$	$\underset{\approx}{D} = \frac{1}{2}(\underset{\approx}{L} + \underset{\approx}{L}^T) = \mathrm{sym}(\underset{\approx}{L})$	$D_{ij} = \frac{1}{2}(L_{ij} + L_{ji}) = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})L_{kl}$	1	Generally not a true rate; If $\underset{\approx}{U}(t)$ is diagonal, $\underset{\approx}{D} = \dot{\underset{\approx}{\varepsilon}}^{log}$; Work conjugate with $\underset{\approx}{\sigma} \rightarrow \frac{1}{\rho}\underset{\approx}{\sigma} \colon \underset{\approx}{D}$; 'Deformation rate'
Vorticity Tensor	Measure of 'tumble'	$\underset{\approx}{W}$	$\underset{\approx}{W} = \frac{1}{2}(\underset{\approx}{L} - \underset{\approx}{L}^T) = \mathrm{skw}(\underset{\approx}{L})$	$W_{ij} = \frac{1}{2}(L_{ij} - L_{ji}) = \frac{1}{2}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})\underset{\approx}{L}_{kl}$	$\frac{1}{time}$	Skew; 'Spin Tensor'
Vorticity Vector	Measure of 'tumble'	$\underset{\sim}{w}$	$\underset{\sim}{w} = -\frac{1}{2}\underset{\approx}{\underset{\sim}{\varepsilon}} \colon \underset{\approx}{W} = \frac{1}{2}\overrightarrow{\nabla} \times \underset{\sim}{v}$	$w_i = -\frac{1}{2}\varepsilon_{ijk}W_{jk} = \frac{1}{2}\varepsilon_{ijk}\frac{\partial v_k}{\partial x_j}$	1	Axial vector of $\underset{\approx}{W} \rightarrow W_{ij} = -\varepsilon_{ijk}w_k$
Cauchy Stress	Current force per unit deformed area	$\underset{\approx}{\sigma}$	-	-	$\frac{Force}{Area}$	Symmetric; Work conjugate with $\underset{\approx}{D} \rightarrow \frac{1}{\rho}\underset{\approx}{\sigma} \colon \underset{\approx}{D}$; Defined by $\underset{\sim}{t} = \underset{\approx}{\sigma} \cdot \underset{\sim}{n}$
First Piola-Kirchhoff Stress	Current force per unit undeformed area	$\underset{\approx}{P}, \underset{\approx}{t}$	$\underset{\approx}{P} = \underset{\approx}{\sigma} \cdot \underset{\approx}{F}^c = J\underset{\approx}{\sigma} \cdot \underset{\approx}{F}^{-T}$	$P_{ij} = \sigma_{ik}F_{kj}^c = J\sigma_{ik}F_{jk}^{-1}$	$\frac{Force}{Area}$	'Nominal stress tensor'; 'Lagrangian stress tensor'; Work conjugate with $\underset{\approx}{F} \rightarrow \frac{1}{\rho_0}\underset{\approx}{P} \colon \dot{\underset{\approx}{F}}$; Solve using Nanson's Relation ($d\underset{\sim}{A} = \underset{\approx}{F}^c \cdot d\underset{\sim}{A}_0$))
Second Piola-Kirchhoff Stress	Transformed (unrotated) current force per unit undeformed area	$\underset{\approx}{S}$	$\underset{\approx}{S} = \underset{\approx}{F}^{-1} \cdot \underset{\approx}{P} = J\underset{\approx}{F}^{-1} \cdot \underset{\approx}{\sigma} \cdot \underset{\approx}{F}^{-T}$	$S_{ij} = F_{ik}^{-1}P_{kj} = JF_{ik}^{-1}\sigma_{kl}F_{lj}^{-T}$	$\frac{Force}{Area}$	Symmetric; Work conjugate with $\underset{\approx}{E} \rightarrow \frac{1}{\rho_0}\underset{\approx}{S} \colon \dot{\underset{\approx}{E}}$; Unaffected by superimposed rotation
Stiffness	Constitutive relation	$\underset{\approx}{\underset{\approx}{C}}, \underset{\approx}{\underset{\approx}{\mathbb{E}}}$	$\underset{\approx}{\underset{\approx}{C}} = \frac{\partial \underset{\approx}{\sigma}}{\partial \underset{\approx}{\varepsilon}}$	$C_{ijkl} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}}$	$\frac{Force}{Area}$	For symmetric $\underset{\approx}{\sigma}$ and $\underset{\approx}{\varepsilon}$, $\underset{\approx}{\underset{\approx}{C}}$ is minor symmetric; 'Elastic tangent stiffness'
Compliance	Constitutive relation	$\underset{\approx}{\underset{\approx}{S}}, \underset{\approx}{\underset{\approx}{H}}$	$\underset{\approx}{\underset{\approx}{S}} = \frac{\partial \underset{\approx}{\varepsilon}}{\partial \underset{\approx}{\sigma}}$	$S_{ijkl} = \frac{\partial \varepsilon_{ij}}{\partial \sigma_{kl}}$	$\frac{Area}{Force}$	For symmetric $\underset{\approx}{\sigma}$ and $\underset{\approx}{\varepsilon}$, $\underset{\approx}{\underset{\approx}{S}}$ is minor symmetric
Specific Kinetic Energy	-	k	$k = \frac{1}{2}\underset{\sim}{v} \cdot \underset{\sim}{v}$	$k = \frac{1}{2}\underset{\sim}{v} \cdot \underset{\sim}{v}$	$\frac{Length^2}{Second^2}$	$\underset{\sim}{v} \cdot \underset{\sim}{a} = \underset{\sim}{v} \cdot \dot{\underset{\sim}{v}} = \dot{k}$
Isotropic Stress	Measure of 'average' stress	$iso\underset{\approx}{A}$	$iso\underset{\approx}{A} = \frac{1}{3}tr(\underset{\approx}{A})$	$iso\underset{\approx}{A} = \frac{1}{3}tr(\underset{\approx}{A})$	–	A.K.A. 'Spherical Stress', 'Hydrostatic Stress'
Deviatoric Stress	Measure of 'shear' stress	$dev\underset{\approx}{A}$	$dev\underset{\approx}{A} = \underset{\approx}{A} - iso\underset{\approx}{A} = \underset{\approx}{A} - \frac{1}{3}tr\underset{\approx}{A}$	$dev\underset{\approx}{A} = \underset{\approx}{A} - iso\underset{\approx}{A} = \underset{\approx}{A} - \frac{1}{3}tr\underset{\approx}{A}$	–	-
Spherical Deformation	-	-	$\underset{\approx}{F} = \alpha\underset{\approx}{I}$	$\underset{\approx}{F} = \alpha\underset{\approx}{I}$	-	Volume change without shape change
Isochoric Deformation	-	-	$J = det\underset{\approx}{F} = 1$	$J = det\underset{\approx}{F} = 1$	-	Shape change without volume change
Hooke's Law	Relate stress to strain	-	$\underset{\approx}{\sigma} = \lambda(tr\underset{\approx}{\varepsilon})\underset{\approx}{I} + 2G\underset{\approx}{\varepsilon}$	$\underset{\approx}{\sigma} = \lambda(tr\underset{\approx}{\varepsilon})\underset{\approx}{I} + 2G\underset{\approx}{\varepsilon}$	-	λ = Lamé constant, G = Shear modulus, $K = \lambda + \frac{2}{3}G$ = Bulk modulus
Reynolds Transport	-	-	$\frac{D}{Dt}\int_\Omega f\rho dV = \int_\Omega \dot{f}\rho dV$	$\frac{D}{Dt}\int_\Omega f\rho dV = \int_\Omega \dot{f}\rho dV$	-	$\dot{\phi} = \frac{D\phi}{Dt} = (\frac{\partial \phi}{\partial t})_{\underset{\sim}{X}}$ Lagrange rate
Material Velocity (Lagrangian)	-	$\underset{\sim}{v}(\underset{\sim}{X}, t)$	$\underset{\sim}{v} = (\frac{\partial \underset{\sim}{x}}{\partial t})_{\underset{\sim}{X}}$	$\underset{\sim}{v} = (\frac{\partial \underset{\sim}{x}}{\partial t})_{\underset{\sim}{X}}$	$\frac{Length}{Second}$	This is the Lagrangian (particle tracking) velocity $\underset{\sim}{v}(\underset{\sim}{X}, t)$
Material Velocity (Eulerian)	-	$\underset{\sim}{v}(\underset{\sim}{x}, t)$	$\underset{\sim}{v} = (\frac{\partial \underset{\sim}{x}}{\partial t})_{\underset{\sim}{X}}$, Substitute $\underset{\sim}{X}(\underset{\sim}{x}, t)$ for $\underset{\sim}{X}$	$\underset{\sim}{v} = (\frac{\partial \underset{\sim}{x}}{\partial t})_{\underset{\sim}{X}}$, Substitute $\underset{\sim}{X}(\underset{\sim}{x}, t)$ for $\underset{\sim}{X}$	$\frac{Length}{Second}$	This is Eulerian (stationary observer) velocity $\underset{\sim}{v}(\underset{\sim}{x}, t)$
Reference Backward Gradient	Alternate method to find $\dot{\underset{\approx}{F}}$	$\underset{\sim}{v}\overleftarrow{\nabla}_0$	$\underset{\sim}{v}\overleftarrow{\nabla}_0 = (\frac{\partial \underset{\sim}{v}}{\partial \underset{\sim}{X}})_t = \dot{\underset{\approx}{F}}$	$\underset{\sim}{v}\overleftarrow{\nabla}_0 = (\frac{\partial \underset{\sim}{v}}{\partial \underset{\sim}{X}})_t = \dot{\underset{\approx}{F}}$	$\frac{1}{Second}$	Use the Lagrange material velocity $\underset{\sim}{v}(\underset{\sim}{X}, t)$
Spatial Backward Gradient	Alternate method to find the velocity gradient ($\underset{\approx}{L}$)	$\underset{\sim}{v}\overleftarrow{\nabla}$	$\underset{\sim}{v}\overleftarrow{\nabla} = (\frac{\partial \underset{\sim}{v}}{\partial \underset{\sim}{x}})_t = \underset{\approx}{L}$	$\underset{\sim}{v}\overleftarrow{\nabla} = (\frac{\partial \underset{\sim}{v}}{\partial \underset{\sim}{x}})_t = \underset{\approx}{L}$	$\frac{1}{Second}$	Use the Lagrange material velocity $\underset{\sim}{v}(\underset{\sim}{X}, t)$
Polar Decomposition	Decomp $\underset{\approx}{F}$ into a 'stretch' and 'rotate'	-	$\underset{\approx}{F} = \underset{\approx}{R} \cdot \underset{\approx}{U} = \underset{\approx}{V} \cdot \underset{\approx}{R}$	$\underset{\approx}{F} = \underset{\approx}{R} \cdot \underset{\approx}{U} = \underset{\approx}{V} \cdot \underset{\approx}{R}$	-	
Right Stretch Tensor	Used in polar decomposition	$\underset{\approx}{U}$	$\underset{\approx}{U} = \underset{\approx}{C}^{\frac{1}{2}}$	$\underset{\approx}{U} = \underset{\approx}{C}^{\frac{1}{2}}$	1	$\underset{\approx}{U} = \underset{\approx}{R}^T \cdot \underset{\approx}{F}$
Left stretch tensor	Used in polar decomposition	$\underset{\approx}{V}$	$\underset{\approx}{V} = \underset{\approx}{B}^{\frac{1}{2}}$	$\underset{\approx}{V} = \underset{\approx}{B}^{\frac{1}{2}}$	1	$\underset{\approx}{V} = \underset{\approx}{F} \cdot \underset{\approx}{R}^T$
		$\underset{\approx}{H}$	$\underset{\approx}{H} = \underset{\sim}{u}\overleftarrow{\nabla}_0 = (\frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{X}})_t$	$\underset{\approx}{H} = \underset{\sim}{u}\overleftarrow{\nabla}_0 = (\frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{X}})_t$	1	$\underset{\sim}{u} = \underset{\sim}{x} - \underset{\sim}{X}$, $\underset{\approx}{E} = \frac{1}{2}(\underset{\approx}{H} + \underset{\approx}{H}^T + \underset{\approx}{H}^T \cdot \underset{\approx}{H})$
		$\underset{\approx}{h}$	$\underset{\approx}{h} = \underset{\sim}{u}\overleftarrow{\nabla} = (\frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{x}})_t$	$\underset{\approx}{h} = \underset{\sim}{u}\overleftarrow{\nabla} = (\frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{x}})_t$	1	$\underset{\sim}{u} = \underset{\sim}{x} - \underset{\sim}{X}$, $\underset{\approx}{e} = \frac{1}{2}(\underset{\approx}{h} + \underset{\approx}{h}^T - \underset{\approx}{h}^T \cdot \underset{\approx}{h})$
Spatial Gradient	S.G. of scalar field	$\overrightarrow{\nabla}\phi$	$\overrightarrow{\nabla}\phi = \frac{\partial \phi}{\partial \underset{\sim}{x}}$	$\overrightarrow{\nabla}\phi = \frac{\partial \phi}{\partial \underset{\sim}{x}}$	-	
Spatial Gradient	S.G. of vector field	$\overrightarrow{\nabla}\underset{\sim}{\phi}, \underset{\sim}{\phi}\overleftarrow{\nabla}$	$(\overrightarrow{\nabla}\underset{\sim}{\phi})_{ij} = \frac{\partial \phi_j}{\partial x_i}$, $(\underset{\sim}{\phi}\overleftarrow{\nabla})_{ij} = \frac{\partial \phi_i}{\partial x_j}$	$(\overrightarrow{\nabla}\underset{\sim}{\phi})_{ij} = \frac{\partial \phi_j}{\partial x_i}$, $(\underset{\sim}{\phi}\overleftarrow{\nabla})_{ij} = \frac{\partial \phi_i}{\partial x_j}$	-	$\overrightarrow{\nabla}r = \frac{\underset{\sim}{x}}{r}$, $\overrightarrow{\nabla}\underset{\sim}{\phi} = (\underset{\sim}{\phi}\overleftarrow{\nabla})^T$
Divergence	Measures magnitude of outward flux of a vector field	$\overrightarrow{\nabla} \cdot \underset{\sim}{\phi} = \underset{\sim}{\phi}\overleftarrow{\nabla}$	$(\overrightarrow{\nabla} \cdot \underset{\sim}{\phi}) = (\underset{\sim}{\phi}\overleftarrow{\nabla}) = \frac{\partial \phi_i}{\partial x_i}$	$(\overrightarrow{\nabla} \cdot \underset{\sim}{\phi}) = (\underset{\sim}{\phi}\overleftarrow{\nabla}) = \frac{\partial \phi_i}{\partial x_i}$	-	
Curl	Describes the 'rotation' of a vector field	$\overrightarrow{\nabla} \times \underset{\sim}{\phi}$	$(\overrightarrow{\nabla} \times \underset{\sim}{\phi})_i = \varepsilon_{ijk}\frac{\partial \phi_k}{\partial x_j}$, $(\underset{\sim}{\phi} \times \overleftarrow{\nabla})_i = \varepsilon_{ijk}\frac{\partial \phi_j}{\partial x_k}$	$(\overrightarrow{\nabla} \times \underset{\sim}{\phi})_i = \varepsilon_{ijk}\frac{\partial \phi_k}{\partial x_j}$, $(\underset{\sim}{\phi} \times \overleftarrow{\nabla})_i = \varepsilon_{ijk}\frac{\partial \phi_j}{\partial x_k}$	-	$\overrightarrow{\nabla} \times \underset{\sim}{\phi} = -(\underset{\sim}{\phi} \times \overleftarrow{\nabla})$
Direction Cosine Matrix	Basis transformation	$\underset{\approx}{L}$	$L_{ij} = e_i^A e_j^B = \cos\theta_{ij}^{AB}$	$L_{ij} = e_i^A e_j^B = \cos\theta_{ij}^{AB}$	1	$\underset{\approx}{L} = \underset{\approx}{L}^T$
Cayley-Hamilton Theorem	-	-	$\underset{\approx}{A}^3 - I_1\underset{\approx}{A}^2 + I_2\underset{\approx}{A} - I_3\underset{\approx}{I} = \underset{\approx}{0}$	$A_{ik}A_{kl}A_{lj} - I_1A_{ik}A_{kj} + I_2A_{ij} - I_3\delta_{ij} = 0_{ij}$	-	$\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0$
Internal Dissipation	-	$\mathcal{D}$	$\mathcal{D} = \mathcal{P}_s + \theta\dot{\eta} - \dot{e}$	$\mathcal{D} = \mathcal{P}_s + \theta\dot{\eta} - \dot{e}$	1	$\mathcal{D} \geq 0$ and η is entropy per mass
Isotropic Tensors	Second and fourth order	-	$A_{ij} = \alpha I_{ij}$, $A_{ijkl} = \alpha\delta_{ij}\delta_{kl} + \beta\delta_{ik}\delta_{jl} + \gamma\delta_{il}\delta jk$	$A_{ij} = \alpha I_{ij}$, $A_{ijkl} = \alpha\delta_{ij}\delta_{kl} + \beta\delta_{ik}\delta_{jl} + \gamma\delta_{il}\delta jk$	-	if A_{ijkl} is minor symmetric $\beta = \gamma$
Isotropic Function	-	-	$\phi(\underset{\approx}{Q} \cdot \underset{\sim}{v}) = \phi(\underset{\sim}{v}) \quad forall\underset{\approx}{Q} = \underset{\approx}{Q}^{-T}$	$\phi(\underset{\approx}{Q} \cdot \underset{\sim}{v}) = \phi(\underset{\sim}{v}) \quad forall\underset{\approx}{Q} = \underset{\approx}{Q}^{-T}$	-	For tensors $\phi(\underset{\approx}{Q} \cdot \underset{\approx}{T} \cdot \underset{\approx}{Q}^T)$
Stress Power	-	$\mathcal{P}_s$	$\mathcal{P}_s = \frac{1}{\rho}\underset{\approx}{\sigma} : \underset{\approx}{L} = \frac{1}{\rho_0}\underset{\approx}{S} : \dot{\underset{\approx}{E}}$	$\mathcal{P}_s = \frac{1}{\rho}\underset{\approx}{\sigma} : \underset{\approx}{L} = \frac{1}{\rho_0}\underset{\approx}{S} : \dot{\underset{\approx}{E}}$	$\frac{Watt}{kg}$	1)Product Rule 2)use equation of motion $\underset{\approx}{\sigma} \cdot \overleftarrow{\nabla} = \rho\underset{\sim}{a} - \rho\underset{\sim}{b}$
Mechanical Power		$\mathcal{P}_m$	$\mathcal{P}_m = \int_{\partial\Omega} \underset{\sim}{t} \cdot \underset{\sim}{v}dS + \int_\Omega \underset{\sim}{b} \cdot \underset{\sim}{v}\rho dV$	$\mathcal{P}_m = \int_{\partial\Omega} \underset{\sim}{t} \cdot \underset{\sim}{v}dS + \int_\Omega \underset{\sim}{b} \cdot \underset{\sim}{v}\rho dV$	$\frac{Watt}{kg}$	
Nanson's Relation	Tracks area	-	$d\underset{\sim}{A} = J\underset{\approx}{F}^{-T} \cdot d\underset{\sim}{A}_0 \quad dA\underset{\sim}{n} = J\underset{\approx}{F}^{-T} \cdot \underset{\sim}{N}dA_0$	$d\underset{\sim}{A} = J\underset{\approx}{F}^{-T} \cdot d\underset{\sim}{A}_0 \quad dA\underset{\sim}{n} = J\underset{\approx}{F}^{-T} \cdot \underset{\sim}{N}dA_0$	-	let $\underset{\sim}{u} = d\underset{\sim}{X}^1$ and $\underset{\sim}{v} = d\underset{\sim}{X}^2$, $(\underset{\sim}{u} \times \underset{\sim}{v}) = d\underset{\sim}{A}_0$
Local Equation of Motion	Continuum analog of $F = ma$	-	$\overrightarrow{\nabla} \cdot \underset{\approx}{\sigma} + \rho\underset{\sim}{b} = \rho\underset{\sim}{a}$	$\overrightarrow{\nabla} \cdot \underset{\approx}{\sigma} + \rho\underset{\sim}{b} = \rho\underset{\sim}{a}$	-	(surface forces)+(body forces) = (rate of momentum)
Continuity	Local form of conservation of mass	-	$\dot{\rho} + \rho\overrightarrow{\nabla} \cdot \underset{\sim}{v} = 0$	$\dot{\rho} + \rho\overrightarrow{\nabla} \cdot \underset{\sim}{v} = 0$	-	alternate form: $\rho_{,t} + \overrightarrow{\nabla} \cdot (\rho\underset{\sim}{v}) = 0$
First Law of Thermodynamics	Conservation of energy	-	$\dot{e} = \frac{1}{\rho}\underset{\approx}{\sigma} : \underset{\approx}{D} + \xi - \frac{1}{\rho}\overrightarrow{\nabla} \cdot \underset{\sim}{q}$	$\dot{e} = \frac{1}{\rho}\underset{\approx}{\sigma} : \underset{\approx}{D} + \xi - \frac{1}{\rho}\overrightarrow{\nabla} \cdot \underset{\sim}{q}$	-	ξ ='microwave heat', e =internal energy per mass, $\underset{\sim}{q}$ =heat flux
Second Law of Thermodynamics - Local Form	Disorder tends to increase.	-	$\dot{\eta} \geq \frac{\xi}{\theta} - \frac{1}{\rho}\overrightarrow{\nabla} \cdot (\frac{\underset{\sim}{q}}{\theta})$	$\dot{\eta} \geq \frac{\xi}{\theta} - \frac{1}{\rho}\overrightarrow{\nabla} \cdot (\frac{\underset{\sim}{q}}{\theta})$	-	A.K.A. Clausius-Duhem Inequality
Traction	-	$\underset{\sim}{t}$	$\underset{\sim}{t} = \underset{\approx}{\sigma} \cdot \underset{\sim}{n}$	$\underset{\sim}{t} = \underset{\approx}{\sigma} \cdot \underset{\sim}{n}$	$\frac{Force}{Area}$	Cauchy Tetrahedron Argument proves $\underset{\sim}{t}$ is linear
Eulerian Rate	Rate seen by fixed observer (optical sensor)	$\phi_{,t}$	$\phi_{,t} = (\frac{\partial \phi}{\partial t})_{\underset{\sim}{x}}$	$\phi_{,t} = (\frac{\partial \phi}{\partial t})_{\underset{\sim}{x}}$	$\frac{1}{Second}$	$(\frac{\partial \phi}{\partial t})_{\underset{\sim}{X}} = (\frac{\partial \phi}{\partial t})_{\underset{\sim}{x}} + (\frac{\partial \phi}{\partial \underset{\sim}{x}})_t \cdot (\frac{\partial \underset{\sim}{x}}{\partial t})_{\underset{\sim}{X}}$
Lagrange Rate/Material Rate	Rate as experienced by discrete particles	$\dot{\phi}$	$\dot{\phi} = \frac{D\phi}{Dt} = (\frac{\partial \phi}{\partial t})_{\underset{\sim}{X}}$	$\dot{\phi} = \frac{D\phi}{Dt} = (\frac{\partial \phi}{\partial t})_{\underset{\sim}{X}}$	$\frac{1}{Second}$	Lagrange rates are the 'usual' rates
Leibniz Theorem	Lemma for Reynold's Transport	-	$\frac{d}{dt}\int_{\Omega(t)} f(\underset{\sim}{x}, t)dV = \int_{\Omega(t)} \frac{\partial f(\underset{\sim}{x}, t)}{\partial t}dV + \int_{\Omega(t)} f(\underset{\sim}{x}, t)\underset{\sim}{v}_x \cdot \underset{\sim}{n}dS$	$\frac{d}{dt}\int_{\Omega(t)} f(\underset{\sim}{x}, t)dV = \int_{\Omega(t)} \frac{\partial f(\underset{\sim}{x}, t)}{\partial t}dV + \int_{\Omega(t)} f(\underset{\sim}{x}, t)\underset{\sim}{v}_x \cdot \underset{\sim}{n}dS$	-	Rate of an integral over a time varying domain. dS = surface velocity.
Material Rate	-	$\dot{\underset{\approx}{F}}$	$\dot{\underset{\approx}{F}} = (\frac{\partial \underset{\approx}{F}}{\partial t})_{\underset{\sim}{X}}$	$\dot{\underset{\approx}{F}} = (\frac{\partial \underset{\approx}{F}}{\partial t})_{\underset{\sim}{X}}$	$\frac{1}{Second}$	This is the Lagrangian rate (as $\underset{\sim}{X}$ is constant)

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