

CS 512 HW5
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1)

a)

Sparse and dense matching algorithms give different results on narrow and wide baseline stereo pairs. Sparse matching algorithms are used to establish a set of robust matches between an image pair. These sparse matches may then be used to compute the epipolar geometry, using techniques such as the RANSAC (random sampling) method. Dense matching algorithms are used to find matches for all points in the images. The search for a match is constrained by the epipolar geometry derived from the set of sparse matches

c)

$$Z = f \frac{I}{d}$$

So, depth =

$$f = 10, T = 110$$

$$P_L = (100, 200)$$

$$P_R = (103, 200)$$

$$\rightarrow = \frac{100}{(103-100) \cdot (200-200)}$$

$$= \frac{1000}{3} = 333.33$$

c) $R_L, T_L \rightarrow$ Rotation and translation of left camera w.r.t to world

$R_R, T_R \rightarrow$ Rotation and translation of right camera w.r.t to world

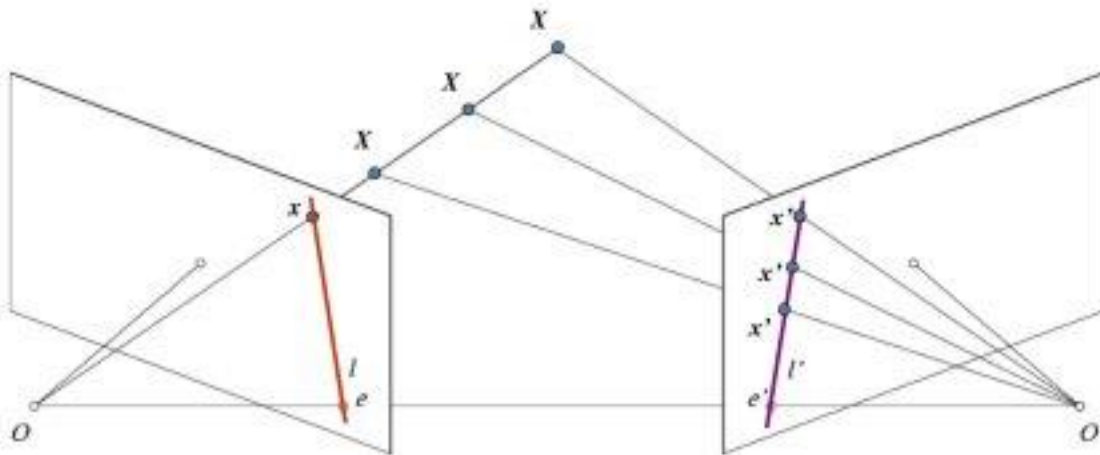
expression for the rotation and translation of right camera w.r.t to left,

$$[R = R_L^T R_R] \quad [T = R_L^T (T_R - T_L)]$$

d) Ambiguity Problem:

The point ambiguity owing to the ambiguous local appearances of image points is the one of the main causes making the stereo problem difficult. Under the point ambiguity, local similarity measures are easy to be ambiguous and this results in false matches in ambiguous regions.

2)
a)



From the setup given above, you can see that projection of right camera O' is seen on the left image at the point, e . It is called the **epipole**. Epipole is the point of intersection of line through camera centers and the image planes. Similarly, epipole of the left camera. In some cases, you won't be able to locate the epipole in the image, they may be outside the image (which means, one camera doesn't see the other).

b) Essential matrix (E) \Rightarrow

$$E = R^T [T]_x$$

(3×3 matrix)

Epipolar constraint equation using essential matrix E , given P_L, P_R

$$P_R^T E P_L = 0$$

$$P_R^T E P_L = 0$$

c) Fundamental matrix (F) \Rightarrow

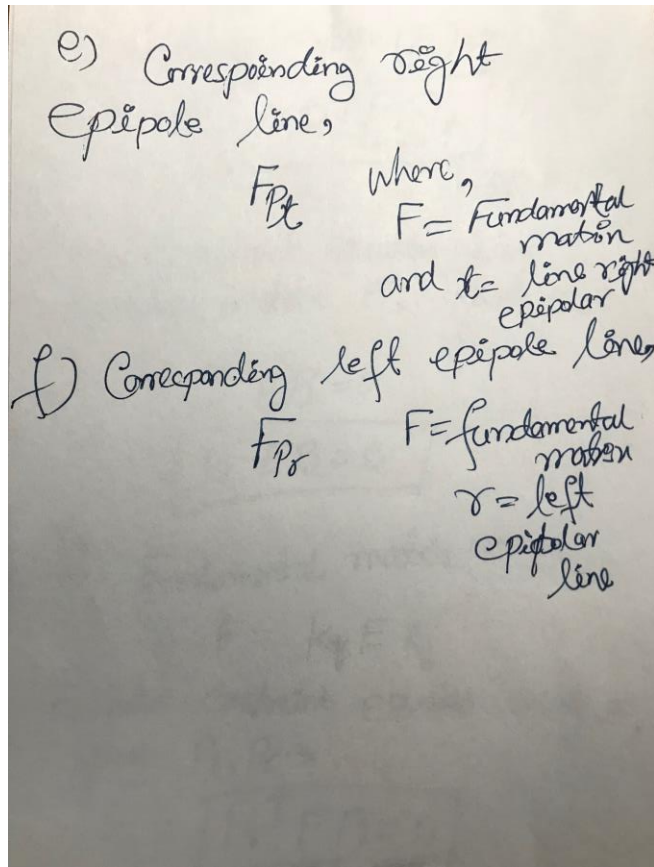
$$F = K_L^T E K_L$$

Epipolar constraint equation using F , given $P_L, P_R \Rightarrow$

$$P_R^T F P_L = 0$$

d)

Both Essential and Fundamental matrix are rank 2 matrix as both these matrices have T in them which is a rank 2 matrix.



g)

Weak calibration method requires there to be a set of known correspondences beforehand in order to calculate the fundamental matrix. This is widely used to determine a fundamental matrix which can then be used to find epipolar lines. These lines can then be used in pre-existing algorithms by rectifying the stereo images so that the scan lines are the epipolar lines as well.

$$h) P_L = (100, 200) P'$$

$$P_R = (50, 100) (P)$$

First line in matrix will be

$$\begin{bmatrix} x_1 x' & x_1 y_1' & x_1 & y_1 x_1' & y_1 x_1' & y_1 y_1' & y_1 \\ & & & & & x_1' y_1' & 1 \\ 1 & \dots & 1 & 1 & 1 & 1 & 1 \\ 1 & & 1 & 1 & 1 & & \end{bmatrix}$$

First row \Rightarrow

$$= \begin{bmatrix} 5000 & 10000 & 50 & 10000 & 20000 & 100 & 100 & 200 \end{bmatrix}$$

i) To recover epipole from fundamental matrix \rightarrow

Right epipole (e_r) is left null space of fundamental matrix F . where left null

Space of F is the last column of $U (F = U \Delta V^T)$ for the left epipole

$$P_r^T F e_r = 0 \Rightarrow \boxed{F e_r = 0}$$

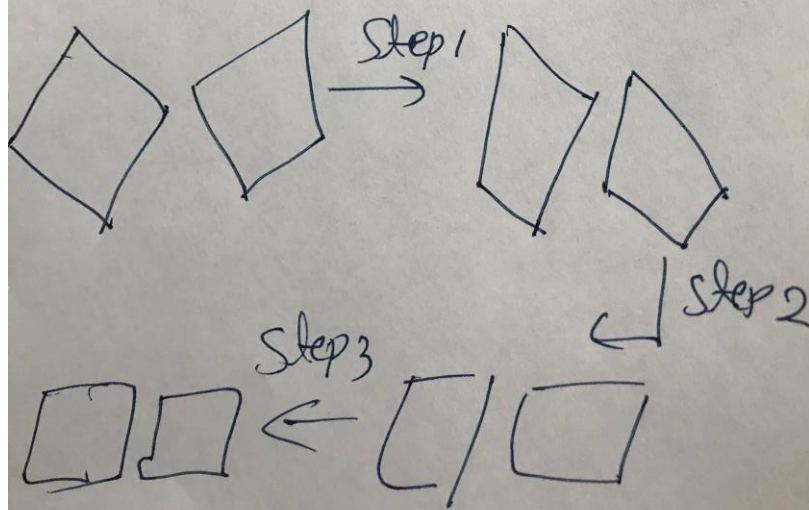
left epipole e_l is the right null space of F which is the right column of $V (F = U \Delta V^T)$

5) (a) Rectifying Sterea pair \Rightarrow

Step 1: align right with left image
that means they should have same
Orientation.

Step 2: Align both images with
baseline.

Step 3: Make w ploner



before starting the process we need to move points to camera coordinates.

$$R_{\text{left-rot}} = K_l^+ R_{\text{ref}} (K_l^+)^{-1}$$

$$R_{\text{right-rot}} = K_r^+ R_{\text{ref}} R^T (K_r^+)^{-1}$$

b) Reconstruction Approaches \Rightarrow

(a) Euclidean Reconstruction

(b) Reconstruction up to unknown 3D projective map

c) given $R \rightarrow$ rotation

$T \rightarrow$ translation

P_l and $P_r \rightarrow$ left and right
coordinates
to solve (a, b, c) we need
to solve,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1} T$$

$$\text{Where, } A = \begin{bmatrix} P_l - R P_r & R \times R P_r \end{bmatrix}$$

d) using coefficient (a, b, c)
of the triangulated point

$$P = aP + bT + \frac{1}{2}CW$$

$$P = \frac{1}{2}[aP + bRPr + T]$$

e) f) Matrix can be
normalized to have a baseline
of 1 by \rightarrow

$$E = \frac{E}{\text{tr}(E^T E)}$$

Since we do not know R and T ,
the normalized matrix is to
be expressed in linear of
coefficient of E only

g) To determine the unknown
sign of rotation and translation
for euclidean reconstruction, we
use all four combinations,

$(++)$, $(--)$, $(+-)$ and $(-+)$
to reconstruct the point and
only use the combination
for which all three co-ord
are positive for point P.