CS 512 – ASSIGNMENT 1
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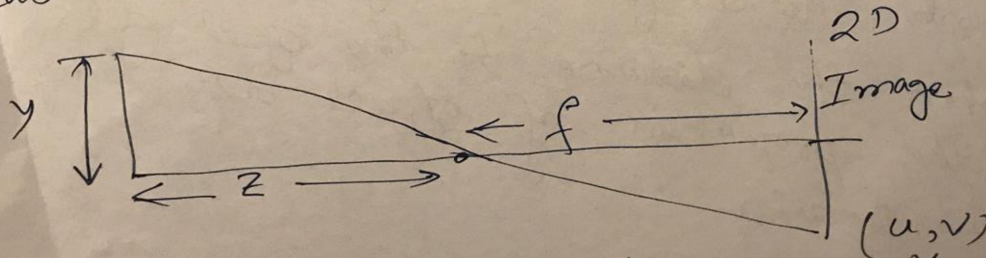
1) (a) $f = 10$ and $p = (3, 2, 1)$ 1

So, the coordinate of the point p when projecting it, $u = -\frac{x}{z} f \Rightarrow u = -\frac{3}{1} \times 10 = -30$

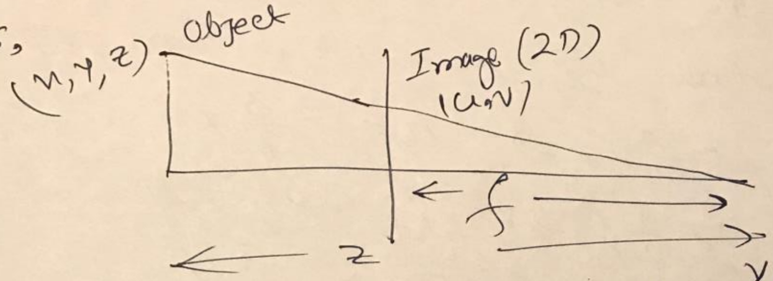
$v = -\frac{y}{z} f \Rightarrow v = -2/1 \times 10 = -20$

b) This is a real camera model when a pinhole camera and its image plane is behind the center of the projection. This camera model is actually used in real life whereas the pinhole camera model where image plane is in front of center of projection is not used that much. real world camera model can be drawn below,



$u = \frac{-x}{z} f$ and $v = \frac{-y}{z} f$

"Not real world" camera model can be drawn as,



$u = \frac{x}{z} \times f$ and $v = \frac{y}{z} f$

In real world it's really difficult to build a model like above.

(c) We know that focul length is directly proportional to image size. So, as focul length gets bigger the projection of the object onto the image will get bigger too. Whereas when the distance to the oject gets bigger, the projection of object onto image will get smaller.

(d) 2D point $\rightarrow (1,1)$

So, in Homogeneous coordinate this can be represented as,

$$(x,y) \rightarrow (x,y,1)$$

So, one 2DH will be, $(1,1,1)$

we can also have another 2DH like

$$(1,1) \rightarrow (3,3,3)$$

$$(2D) \rightarrow (2DH)$$

(e) 2DH $(1,1,2)$

2D point $\Rightarrow \left(\frac{1}{2}, \frac{1}{2}, 1\right)$

(f) 2DH $(1,1,0)$

This points signifies points of infinity direction.

(g) We know non-homogenous co-ordinates (u,v) can be written (u, v, w) in homogenous coordinate,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$2DH \longrightarrow 3D$$

So, here, $x = [u/w)$ and $y = [v/w)$

So, it's possible to write non-linear projection equation as a linear one in homogeous cordinates.

②

(h) $M = K[I|0]$

Dimension of M would be, $M = 3\times4$
$K = 3\times3$
$I = 3\times3$ and
$O = 3\times1$

i) $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ 3D point P
$P = [1, 2, 3]$

So, by projecting P using M we get,

$$p = MP = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}_{(2DH)}$$

So, p will be $\left(18/10, 46/10\right)$ in 2D point.

2)
(a) $P = (1,1)$

$x' = x + t$ and $x' = \begin{bmatrix} I & t \\ 0 & 1 \end{bmatrix} \hat{x}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

So after translating, $(x', y') = (3, 4)$

(b) $P = (1,1)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

So. after scaling, $(x', y') = (2, 2)$

(c) $p = (1,1)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

$(x', y') \Rightarrow (0, \sqrt{2})$

(d) $p = (2,2)$ $\quad \hat{x} = (1,1)$ and $\theta = 45^\circ$

$R_p(\theta) \Rightarrow R_{(2,2)}(45^\circ) = T(2,2) \cdot R(45^\circ) \cdot T(-2,-2)$

(i) $T(-2,-2) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

(ii) $R(45^\circ) = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

(iii) $T(2,2) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2-\sqrt{2} \\ 1 \end{bmatrix}$

So, coordinates after 45° rotation $(x', y') \Rightarrow (2, 2-\sqrt{2})$

(e) The combined matrix can be expressed

$$P' = TRP$$

(f) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

we can express M as, $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \left[\begin{array}{c|c} S & 0 \\ \hline 0 & 1 \end{array}\right]$

So, by applying this to point $p(x, y)$, p will be scaled by point $(3, 2)$ $p(x', y') = (3x, 2y$

(g) $M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

we can express M as, $\begin{bmatrix} I & t \\ 0 & 1 \end{bmatrix}$ which is a ~~tran~~ formation matrix.

So, translation cordinates are, $(t_x, t_y) \rightarrow (1,$

Applying matrix M to point $p(x, y)$ will translate the point by, $(1, 2)$

$\therefore p(x', y') = (x+1, y+2)$

(h) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

we can clearly see that M is a scale matrix. which can be written as,

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To reverse the effect of M,

$$M^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) $M = R(45)\,T(1,2)$

Inverse of this transformation in terms of rotation and translation will be,

$$M^{-1} = (RT)^{-1} = T^{-1}_{(1,2)}\, R^{-1}_{(4,5)}$$

$$= T^{-1}_{(1,2)}\, R^{T}_{(45)}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \left[R^{-1} = \right.$$

③

So, $M^{-1} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & -1 \\ -\sin 45^\circ & \cos 45^\circ & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$M^{-1} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -1 \\ -1/\sqrt{2} & 1/\sqrt{2} & -2 \\ 0 & 0 & 1 \end{bmatrix}$

d) vector $= (1, 3)$

To find vector perpendicular to it rotated by 90° will be,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$(x', y') \Rightarrow (-3, 1)$ which is perpendicu[lar] to $(3, 1)$

k) $\hat{v} = (1,3)$ and $\hat{x} = (2,5)$

$$\hat{v} \cdot \hat{x} = v_x \hat{x}_x + v_y \hat{y}_y$$

$$= (2+15) = 17$$

So, the projection can be given as,

$$= \left(\frac{\vec{x} \cdot \vec{v}}{\vec{x}} \right) = \frac{17 \times (2,5)}{(\sqrt{29})^2}$$

$$= \left[\frac{34}{(\sqrt{29})^2}, \frac{85}{(\sqrt{29})^2} \right]$$

$$= (34/29, 85/29)$$

3)

a) Projection Matrix:

The projection from 3D(X,Y,Z) camera coordinates into 2D image coordinates (x,y) can be given as,

$$(x, y) = (f\, X/Z ,\ f\, Y/ Z)$$

Now we can rewrite this equation in homogeneous coordinates as

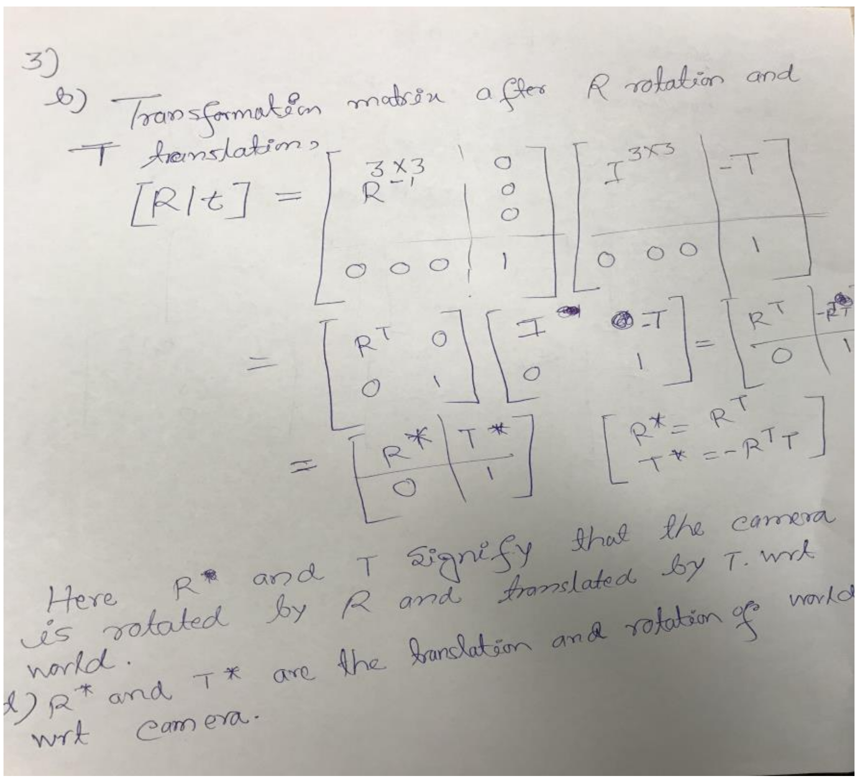
(x, y, 1) = (f X/ Z, f Y/ Z , 1)   and (f X/ Z , f Y/ Z , 1) ≡ (f X, f Y, Z). when Z!=0

Now we can write the project from 3D camera coordinates to 2D image coordinates using 3*4 matrix which is given below,

[fX fY Z]=[[f 0 0 0][0 f 0 0] [0 0 1 0]][X Y Z 1]

So this project matrix maps 4D space into a 3D space using a non-zero vector which is mapped to (0,0,0) and which defines a null space project.

b) Transformation matrix after R rotation and T translation for Camera coordinates:



3)

b) Transformation matrix after R rotation and T translation,

$$[R|t] = \left[\begin{array}{c|c} R^{-1}_{3\times 3} & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I^{3\times 3} & -T \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} R^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R^T & -R^T T \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} R^* = R^T \\ T^* = -R^T T \end{bmatrix}$$

Here R* and T signify that the camera is rotated by R and translated by T. wrt world.

d) R* and T* are the translation and rotation of world wrt camera.

d)   M= [[R* T*],[0,1]]

R* and T* are the rotation and translation of the world with respect to camera.

e)

e) $(u_0, v_0) = (512, 512)$

So, the transformation matrix is given as,

$$M_c = [S(1/k_u, 1/k_v)\ T(-u_0, -v_0)]^{-1}$$

$$= T(u_0, v_0)\ S(k_u, k_v)$$

$$= \begin{bmatrix} 1 & & u_0 \\ & 1 & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_u & & \\ & k_v & \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} k_u & & u_0 \\ & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} k_u & & 512 \\ & k_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$$

f)

Projection Matrix= K*[R*|T*]

Here, K* is the Intrinsic Matrix and [R*|T*] is Extrinsic Matrix

g) 2D Skew Parameter in Camera Model:

We can 2D skew parameter to make camera model more accurate. We normally add skew parameters with Intrinsic matrix when the image coordinates axes are not orthogonal to each other.

h) Radial Distortion:

Radial distortion is caused by the spherical shape of the lens.

In Barrel type radial distortion, image magnification decreases with distance from the optical axis, The apparent effect is that of an image which has been mapped around a sphere.

Whereas in Pincushion type radial distortion, image magnification increases with the distance from the optical axis. The visible effect is that lines that do not go through the center of the image are bowed inwards, towards the center of the image, like a pincushion.

The radial lens distortion introduces non-linearity in a linear image formation equation which is a complication.

i) Weak Perspective Camera:

The affine camera becomes a weak-perspective camera when the rows of $\mathbf{M}$ form a uniformly scaled rotation matrix.

$$\mathbf{T}_{wp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Z_{ave}/f \end{bmatrix}$$

will be,

$$\mathbf{M}_{wp} = \frac{f}{Z_{ave}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{and} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z_{ave}} \begin{bmatrix} X \\ Y \end{bmatrix}$$

The weak-perspective model is valid when the average variation of the depth of the object (delta Z) along the line of sight is small compared to the Zavg and the field of view is small.

4)

b) Radiocity Equation :

Radiosity at surface = Exitance + Radiosity due to other surface,

B(x) = E(x)+ rd (x) B(u) cosqi cosqs pr(x,u) 2 Vis(x,u)dA

c) Albedo of a surface :

The albedo of a surface captures the extent to which it diffusely reflects light.

d) RGB Color Model:

The RGB color model is an additive color model. In this case red, green and blue light are added together in various combinations to reproduce a wide spectrum of colors. The primary purpose of the RGB color model is for the display of images in electronic systems, such as on television screens and computer monitors and it's also used in digital photography. Cathode ray tube, LCD, plasma and LED displays all utilize the RGB model.

e) Colors along the line that connects (0,0,0) with (1,1,1) in RGB cube are given below:

(0,0,0) = black, (1,0,0)= red, (0,1,0)=green, (0,0,1)=blue, (1,1,0)=yellow, (1,0,1)=magenta, (0,1,1)=cyan and (1,1,1)=white

f) RGB Colors in real-world colors:

g) Luminance Component Y in CIE RGB XYZ model:

Component Y indicates the lightness or luminance of the color. The scale for Y extends from the white spot in a line perpendicular to the plane formed by x and y using a scale that runs from 0 to 100. As the Y value increases and the color becomes lighter, the range of color, or gamut, decreases so that the color space at 100 is just a sliver of the original area.

h) Advantages of LAB Color space:

i) Unlike the RGB, Lab color is designed to approximate human vision.
ii) Because the Lab color space is larger than the gamut of computer displays so a bitmap image represented as Lab requires more data per pixel to obtain the same precision as an RGB or CMYK bitmap.
iii) Lab can be used to do a perfect color correction without much change the luminosity.