CS 512 HW 6
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1)
a)
3D Motion Vectors: These are 3D vectors which tell us about moving a rigid object in a particular direction. Basically, these vectors tell us about how to move any rigid object.

2D Projected Motion Vectors: This is the 2D project of a 3D motion in the form of a video or picture.

2D Motion Vectors: This is the optical flow which is basically noisy estimation of the true 2D projection. It can either zero or any values based on the 2D project motion**.**

Yes, it's possible that motion in 3D will not produce any optical flow vectors at all. We might have this scenario when the object is a point or it's quite far.

b) In this case, motion field will have parallel motion vectors. So the points which are close will have large project motion vectors.

c)In this case, the motion field will not be uniform. So the points close to runaway will have larger projected motion vectors.

d)
Projected equation can be given as,

$$P = f/z\, P'$$

where, $P \rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$ [2D Coordinate]

$P' \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ [3D "]

So, motion projection equation given,

$V \rightarrow$ 3D motion vector
$v \rightarrow$ 2D projected motion vector
$f \rightarrow$ focal length
$P \rightarrow$ position of object point
$z \rightarrow$ z co-ordinate of object pt
$V_z \rightarrow$ z component of 3D motion vector.

$$\boxed{v = f/z^2 \left(Vz - \frac{1}{z^2} V_z P'\right)}$$

Here,

$$\boxed{V_z = \frac{f}{z^2}\left(V_z z - V_z z\right) = 0}$$

e)

$$\boxed{V = \omega \times v + Y}$$

g) Instantaneous epipole equation ⇒

$$\boxed{\begin{aligned} x_0 &= \frac{Z_x}{Z_z} f \\ y_0 &= \frac{Z_y}{Z_z} f \end{aligned}}$$

h) It's apparent motion of two instantaneously coincident points.

Relative motion field equations ⇒

Suppose we have two points P and P',

motion equation for point P →

$$V_x = V_x^{(z)} + V_x^{(\omega)}$$
$$V_y = V_y^{(z)} + V_y^{(\omega)}$$

Similarly for P' ⇒

$$\bar{V}_x = \bar{V}_x^{(z)} + \bar{V}_x^{(\omega)}$$
$$\bar{V}_y = \bar{V}_y^{(z)} + \bar{V}_y^{(\omega)}$$

f) In pure translation motion, there is no rotation. So, ω becomes 0 in motion component. there is only translation motion which can be given as,

$$V_x - V_x(t) = \frac{T_z x - T_x f}{Z}$$

$$V_y - V_y(t) = \frac{T_z y - T_y f}{Z}$$

Case I $\Rightarrow$ $T_z \neq 0$

Example $\Rightarrow$ When plane is landing

Case II $\Rightarrow$ $Z_t = 0$

Example $\Rightarrow$ Driving a car.

2)

b)

Aperture Problem: The aperture problem refers to the fact that the motion of a one-dimensional spatial structure, such as a bar or edge, cannot be determined unambiguously if it is viewed through a small aperture such that the ends of the stimulus are not visible.

We hope to recover the image difference with respect to time by taking frame difference between two frames as time derivatives.

c) **Block-based Optical Flow:**
Block based methods don't use the second order derivatives to find optical flow but they use a neighborhood to estimate the optical flow at a specific location.

a) $$\boxed{\frac{d}{dt} I(x(t), y(t), t) = 0}$$

OFCE Equation

The basic assumption to derive this equation is that the image brightness of object is constant.

So, $I(x(t), y(t), t) = C$

d) Objective function, (finding v in local neighborhood)

$$E(v) = \sum_{(x,y)\in \text{Path}} \left(\nabla I(x,y) \cdot v + I_t\right)^2$$

$$v^* = \arg\min_v E(v)$$

The system of equation that has to be solved,

$$\nabla E(v) = 0$$

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}^{-1} \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix}$$

The purpose of weighted block method is to give the highest weight to the center of the window.

It will decrease the weight as you go further away.

$$E(V) = \sum \omega(x,y)\left(I_x \cdot I_t + I_y \cdot I_t + I_t\right)^2$$

$$\begin{bmatrix} \sum \omega I_x^2 & \sum \omega I_x I_y \\ \sum \omega I_x I_y & \sum \omega I_y^2 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} -\sum \omega I_x I_t \\ -\sum \omega I_y I_t \end{bmatrix}$$

f) Global motion estimation (Horn-Schunck)

It solves all the optical flow vectors at same time and it eliminates all the motion vectors at the same time also. To find this we used regularizat

$$E(V(x,y,t)) = \iint_{img} E_{op}^2 (V(x,y,t)) - \alpha^2 E_s^2 (V(x,y,t))$$

Here a is selected by user

$$V^* = \underset{V}{\operatorname{argmin}} E(V(x,y,t))$$

$$E_{OF} = (I_x \cdot u + I_y v + I_t)^2$$

$$E_s = ||\nabla u||^2 + ||\nabla v||^2$$

$$\left[\because u \to x_t \text{ and } v \to y_t \right]$$

a) In order to find $(\bar{u}, \bar{v})$ motion vectors, we need an iterative solution. So, problem here is to find u, v, we need $\bar{u}$ and $\bar{v}$.

Here we start with a guess for u, v and iterate to return values. To make an informed decision for initial values we either use Lucas-Kanade or affine flow method.

$$u^{n+1} = \bar{u}^{(n)} - \frac{(I_x \bar{u}^{(n)} + I_y \bar{v}^{(n)} + I_t) I_x}{I_x^2 + I_y^2 + a^2}$$

$$v^{n+1} = \bar{v}^{(n)} - \frac{(I_x \bar{u}^{(n)} + I_y \bar{v}^{(n)} + I_t) I_y}{I_x^2 + I_y^2 + a^2}$$

We will use this equation in iterative to find u and v.