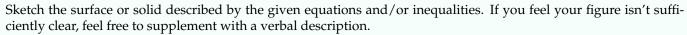
BROWN UNIVERSITY PROBLEM SET 3

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Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1
An astronaut is using a rope to move in space in such a way that her position at time t is given by $\mathbf{r}(t) = (2+t)\mathbf{i} + (2+\ln t)\mathbf{j} + \left(7-\frac{4}{t^2+1}\right)\mathbf{k}$. The coordinates of the space station doorway are $(5,4,9)$. When should the astronaut let go of the rope so as to drift into the doorway?
Solution
Solution
Problem 2
Find the curvature of the helix $\mathbf{r}(t) = \langle a \cos t, a \sin t, bt \rangle$.
Solution

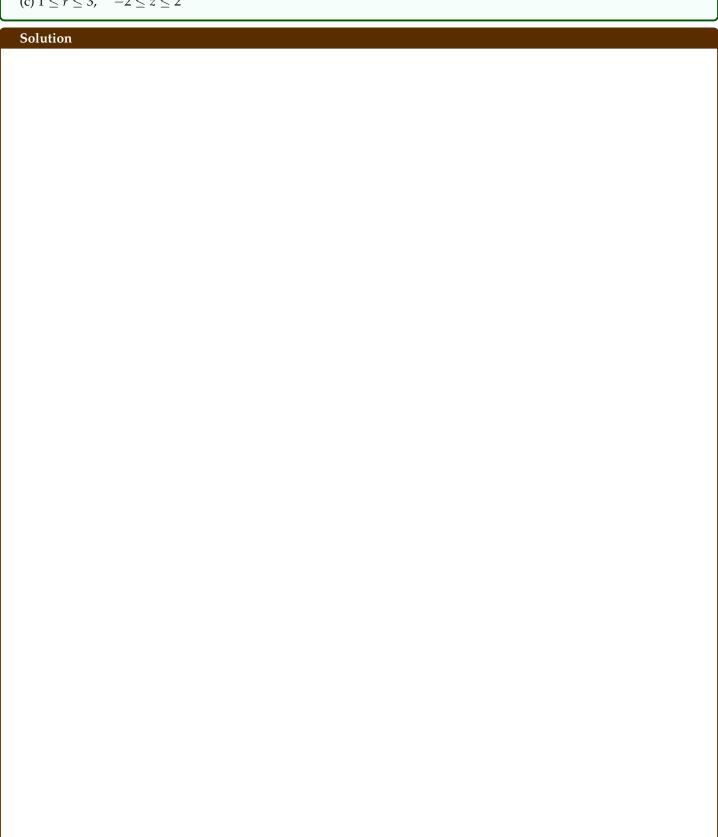
Problem 3



(a)
$$r = 3$$
, $-1 \le z \le 1$

(b)
$$\rho = 2$$
, $\pi/3 \le \phi \le 2\pi/3$

(c)
$$1 \le r \le 3$$
, $-2 \le z \le 2$



Problem 4

Suppose that $\mathbf{r}:[a,b]\to\mathbb{R}^3$ is a curvy path with no straight portions. Given any positive integer n and real numbers t_0,\ldots,t_n satisfying

$$a = t_0 < t_1 < \cdots < t_n = b$$
,

explain briefly and in simple terms why

$$|\mathbf{r}(t_1) - \mathbf{r}(t_0)| + |\mathbf{r}(t_2) - \mathbf{r}(t_1)| + \cdots + |\mathbf{r}(t_n) - \mathbf{r}(t_{n-1})|$$

is less than the length of r. Hint: draw a figure wherein the above expression has a natural geometric interpretation.

Solution

Problem 5

- (a) The set of points satisfying $z = x^2$ and y = 0 is revolved around the *z*-axis. Write an equation for the surface generated in rectangular coordinates, and in cylindrical coordinates.
- (b) The set of points satisfying $4x^2 + y^2 = 1$ and z = 0 is revolved around the *y*-axis. Find an equation in rectangular coordinates for the resulting ellipsoid.

Solution

Additional space	