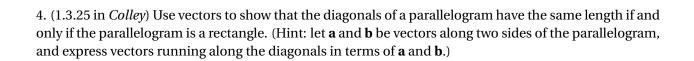
## 18.022 Recitation Handout 10 September 2014

1. For each of the following pairs of vectors  $\mathbf{a}$  and  $\mathbf{b}$ , calculate  $\mathbf{a} \cdot \mathbf{b}$  and  $\|\mathbf{a}\| \|\mathbf{b}\|$ .

(a) $\mathbf{a} = (1,5)$ and $\mathbf{b} = (-2,3)$
(b) $\mathbf{a} = (3, -5)$ and $\mathbf{b} = (2, 0)$
(c) $\mathbf{a} = (-2, 4, 1)$ and $\mathbf{b} = (4, 1, 2)$
(d) Conjecture an inequality relating $ \mathbf{a} \cdot \mathbf{b} $ and $  \mathbf{a}     \mathbf{b}  $ for $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$ .
(e) (Fun/Challenge problem) To prove the inequality conjectured in (d) (called the <i>Cauchy-Schwarz inequality</i> ), expand the left-hand side of the inequality $\ \mathbf{a} + \lambda \mathbf{b}\ ^2 \ge 0$ , where $\lambda$ is any real number.
2. (1.3.20 in <i>Colley</i> ) Suppose that a force $\mathbf{F} = (1, -2)$ is acting on an object moving parallel to the vector $(4, 1)$ . Decompose $\mathbf{F}$ into a sum of vectors $\mathbf{F}_1$ and $\mathbf{F}_2$ , where $\mathbf{F}_1$ points along the direction of motion and $\mathbf{F}_2$ is perpendicular to the direction of motion.
3. (1.3.17 in <i>Colley</i> ) Is it ever the case that the projection of <b>a</b> onto <b>b</b> and the projection of <b>b</b> onto <b>a</b> are the same vector? If so, under what conditions?



5. (1.3.23 in *Colley*) Let A, B, and C denote the vertices of a triangle. Let 0 < r < 1. If  $P_1$  is the point on  $\overline{AB}$  located r times the distance from A to B and  $P_2$  is the point on  $\overline{AC}$  located r times the distance from A to C, use vectors to show that  $\overline{P_1P_2}$  is parallel to  $\overline{BC}$  and has r times the length of  $\overline{BC}$ .

6. (1975 USAMO) Let A, B, C, and D be four points in  $\mathbb{R}^3$ . Use vectors to show that

$$AB^2 + BC^2 + CD^2 + DA^2 \ge AC^2 + BD^2$$
.

(This generalizes the fact that the sum of the squares of the sides of a quadrilateral is at least the sum of the squares of its diagonals.) Make a statement about when equality holds.