# PSet4 Sol

1. $$\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} \sim \begin{pmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{pmatrix}$$

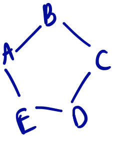
$$\sim \begin{pmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & 0 & c^2-a^2-\frac{c-a}{b-a}(b^2-a^2) \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & 0 & (c-a)(c-b) \end{pmatrix}.$$

Since $b-a \neq 0$, $c-a \neq 0$, $c-b \neq 0$, this matrix has 3 pivots.

2. (a) 500 cars go in & 400 come out, so something is wrong.

(b) Change 200 out to 300, at the top. Then

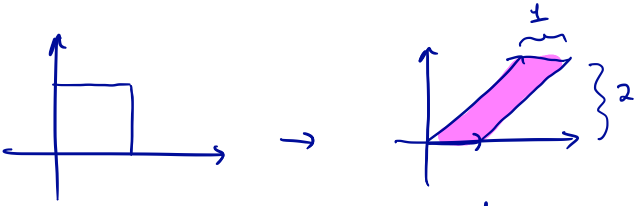
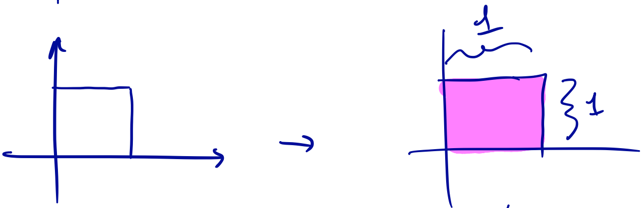
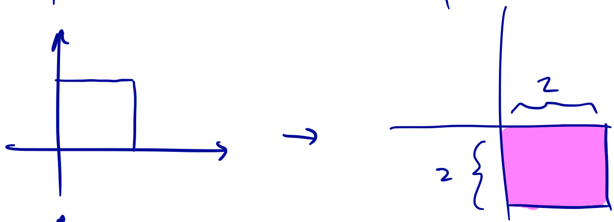
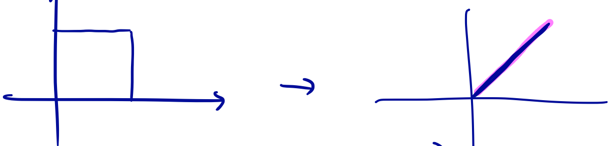
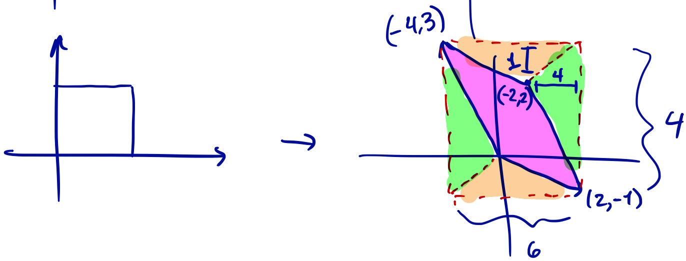
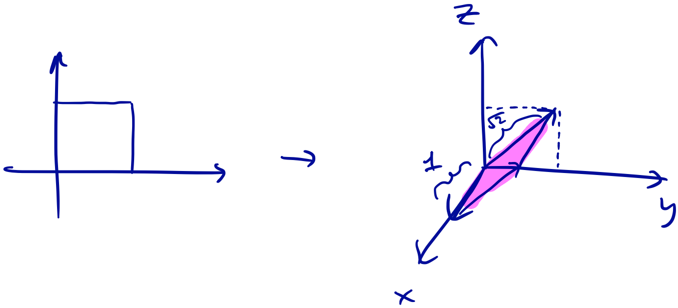



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 300 \\ 0 & 0 & 0 & -1 & 1 & 300 \\ 0 & 0 & 1 & 1 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 100 \\ -1 & 1 & 0 & 0 & 0 & 200 \end{array}\right) \end{array} \overset{\text{Julia}}{\sim} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 300 \\ 0 & 1 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & 0 & 1 & 400 \\ 0 & 0 & 0 & 1 & -1 & -300 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{c} \\ \\ \\ \\ \leftarrow \text{no bad rows} \end{array}$$

free variable

so $\infty$ many sols.

3.

| | | area |
|---|---|---|
|  |  | 2 |
|  |  | 1 |
|  |  | 4 |
|  | | 0 |
|  |  | $24 - \quad - \quad$<br>$= 24 - 2\cdot\frac{1}{2}\cdot 6\cdot 1 - 2\cdot\frac{1}{2}\cdot 4\cdot 4$<br>$= \boxed{2}$ |
|  |  | $(\sqrt{2})(1)$<br>$= \sqrt{2}$ |

4. We have

$$c_1 T(\vec{v}_1) + \cdots + c_n T(\vec{v}_n) = \vec{0}$$

for some $c_1, \ldots, c_n$ not all zero, since $\{T(\vec{v}_1), \ldots, T(\vec{v}_n)\}$ is linearly dependent. Thus

$$T(c_1\vec{v}_1 + \cdots + c_n\vec{v}_n) = \vec{0},$$

by linearity of $T$. Also, $c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$ is not $\vec{0}$ because $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is lin. ind. and not all the $c$'s are zero. So $c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$ is a nonzero vector satisfying $T(\vec{x}) = \vec{0}$.

5. $T$ cannot be surjective (because it can't have 5 pivots), but it can be injective if it has 3 pivots

$S$ cannot be injective (again, can't have five pivots) but it can be surjective if it has 3 pivots.