1. Describe the upper half of the unit sphere centered at the origin using cylindrical coordinates.

Solution. We begin with the coordinate r, which describes the distance from the z-axis. The smallest and largest such distances for points on the unit sphere are 0 and 1, so we have $0 \le r \le 1$. For each value of r, the sphere intersects the cylinder of radius r, so θ ranges from 0 to 2π. For fixed r and θ, the vertical line passing through $(r, \theta, 0)$ intersects the half-sphere in the interval $\left[0, \sqrt{1-r^2}\right]$, by the Pythagorean theorem. Therefore, the region is described by

$$0 \le r \le 1$$
$$0 \le \theta \le 2\pi$$
$$0 \le z \le \sqrt{1-r^2}.$$