Homework #1
Solutions

#1 We get four equations for the four nodes:

$$\begin{cases} A = \frac{1}{4}(20+10+B+C) \\ B = \frac{1}{4}(20+40+D+A) \\ C = \frac{1}{4}(10+30+D+A) \\ D = \frac{1}{4}(40+30+B+C) \end{cases}$$

Multiplying by 4 and clearing to the left side,

$$\begin{cases} 4A - B - C = 30 \\ -A + 4B - D = 60 \\ -A + 4C - D = 40 \\ -B - C + 4D = 70 \end{cases}$$

which in augmented form is $\begin{pmatrix} 4 & -1 & -1 & 0 & 30 \\ -1 & 4 & 0 & -1 & 60 \\ -1 & 0 & 4 & -1 & 40 \\ 0 & -1 & -1 & 4 & 70 \end{pmatrix}$.

Row reducing:

$$\begin{pmatrix} 4 & -1 & -1 & 0 & 30 \\ -1 & 4 & 0 & -1 & 60 \\ -1 & 0 & 4 & -1 & 40 \\ 0 & -1 & -1 & 4 & 70 \end{pmatrix} \to \begin{pmatrix} 1 & -4 & 0 & 1 & -60 \\ 4 & -1 & -1 & 0 & 30 \\ -1 & 0 & 4 & -1 & 40 \\ 0 & -1 & -1 & 4 & 70 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -4 & 0 & 1 & -60 \\ 0 & 15 & -1 & -4 & 270 \\ 0 & -4 & 4 & 0 & -20 \\ 0 & -1 & -1 & 4 & 70 \end{pmatrix} \to \begin{pmatrix} 1 & -4 & 0 & 1 & -60 \\ 0 & -1 & -1 & 4 & 70 \\ 0 & -4 & 4 & 0 & -20 \\ 0 & 15 & -1 & -4 & 270 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -4 & 0 & 1 & -60 \\ 0 & -1 & -1 & 4 & 70 \\ 0 & 0 & 8 & -16 & -300 \\ 0 & 0 & -16 & 56 & 1320 \end{pmatrix} \to \begin{pmatrix} 1 & -4 & 0 & 1 & -60 \\ 0 & -1 & -1 & 4 & 70 \\ 0 & 0 & 8 & -16 & -300 \\ 0 & 0 & 0 & 24 & 720 \end{pmatrix}$$

So $D = \frac{720}{24} = 30$, $C = \frac{1}{8}(-300 + 16D) = 22.5$

$B = -70 - C - 4D = 27.5$, and $A = -60 + 4B - C = 20$. So:

$$(A, B, C, D) = (20, 27.5, 22.5, 30)$$

[2] The info $p(1) = 12$ tells us that

$$12 = a_0 + a_1 + a_2$$

Similarly, $15 = a_0 + 2a_1 + 4a_2$, &

$$16 = a_0 + 3a_1 + 9a_2.$$

So we solve

$$\begin{pmatrix} 1 & 1 & 1 & 12 \\ 1 & 2 & 4 & 15 \\ 1 & 3 & 9 & 16 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & 8 & 4 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 2 & -2 \end{pmatrix}.$$

So $a_2 = \frac{-2}{2} = -1$, $a_1 = 3 - 3a_2 = 6$,

and $a_0 = 12 - a_1 - a_2 = 7$. So

$$(a_0, a_1, a_2) = (7, 6, -1).$$