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MATH 19 PROBLEM SET 2 SOLUTIONS

1. a) $\int_0^{2\pi} \sin 2x \cos 3x \, dx$
$= 0$

b) $\int_0^{6\pi} \sin^2(16x)\, dx$
$= 3\pi$

c) $\int_0^{2\pi} \sin x(\cos x - 2\sin x + 3\sin 2x + 4)\, dx$
$= \int_0^{2\pi} \cancel{\sin x\cos x\, dx} - \int_0^{2\pi} 2\sin^2 x\, dx + \int_0^{2\pi} \cancel{3\sin x \sin 2x\, dx} + \int_0^{2\pi} \cancel{4\sin x\, dx}$
$= -2\pi$

d) $\int_0^{2\pi} (\sin x + \cos x + \cdots \sin 10x + \cos 10x)^2\, dx$
$= 20\pi$

2. a) equal because both are periodic from 0 ~~and~~ to 2π and the square makes all integral values positive.

b) $\int_0^{2\pi} \sin^2 x\, dx = \int_0^{2\pi} (1-\cos^2 x)\, dx$
$\int_0^{2\pi} \sin^2 x\, dx = 2\pi - \int_0^{2\pi} \cos^2 x\, dx$
$\Rightarrow \int_0^{2\pi} \sin^2 x\, dx + \int_0^{2\pi} \cos^2 x\, dx = 2\pi$ ✓

↳ *substitute a)
$\int_0^{2\pi} \cos^2 x\, dx + \int_0^{2\pi} \cos^2 x\, dx = 2\pi$

$= 2\int_0^{2\pi} \cos^2 x\, dx = 2\pi$
$\int_0^{2\pi} \cos^2 x\, dx = \int_0^{2\pi} \sin^2 x\, dx = \pi$ ✓

3. $f(x) = x \quad x \in [0, 2\pi]$

$A \Rightarrow$
$\int_0^{2\pi} x \sin x = \int_0^{2\pi} A \sin x (\sin x) + \int_0^{2\pi} \cancel{B\cos x(\sin x)\, dx} + \cdots$
$-2\pi = A\int_0^{2\pi} \sin^2 x\, dx$
$\rightarrow A = -2\pi/\pi = -2$

$B \Rightarrow$
$\int_0^{2\pi} \cancel{x\cos x} = \cdots$
0

$\rightarrow B = 0$

$C \Rightarrow$
$\int_0^{2\pi} x \sin 2x = \int_0^{2\pi} C \sin^2 2x\, dx$
$-\pi = C\int_0^{2\pi} \sin^2 2x\, dx$
$\rightarrow C = \frac{-\pi}{\pi} = -1$

3. cont.

$D \Rightarrow \int_0^{2\pi} \cancel{x\cos(2x)} = \int_0^{2\pi} D\cos^2 2x\,dx$

0

$\rightarrow D = 0$

4. a) $-\frac{1}{24}(3\sin^4 x + 2\sin^2 x + 1)\cos^4 x + c$

OR

$\frac{-72\cos(2x) + 12\cos(4x) + 8\cos(6x) - 3\cos(8x)}{3072} + c$

b) $\frac{1}{192}(12x - 3\sin(2x) - 3\sin(4x) + \sin(6x)) + c$

OR

$\frac{x}{16} - \frac{1}{32}\sin(2x) + \frac{1}{6}\sin^5(x)\cos(x) - \frac{1}{24}\sin^3(x)\cos(x) +$

c) $\frac{1}{3}\sec(x)(\sec^2 x - 3) + c$

OR

$\frac{\sec^3 x}{3} - \sec x + c$

d) $-\frac{1}{5}\cot^5(x) - \frac{\cot^3(x)}{3}$ OR $\frac{1}{15}(\cos(2x) - 4)\cot^3(x)\csc$

5. $*\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$C\cos(x+\alpha) = C[\cos x\cos\alpha - \sin x\sin\alpha]$

$= C\cos x\cos\alpha - C\sin x\sin\alpha$

$= (C\cos\alpha)\cos x + (-C\sin\alpha)\sin x$

so, $C\cos\alpha = 1$ and

$-C\sin\alpha = 1$

To find C, $(C\cos\alpha)^2 + (-C\sin\alpha)^2 = 2$

$C^2\cos^2\alpha + C^2\sin^2\alpha = 2$

$C^2 = 2$

$C = \sqrt{2}$

To find α, $\frac{-C\sin\alpha}{C\cos\alpha} = 1$

$=$ tan a = -1

a= 7π/4

5. cont.

$\Rightarrow \sqrt{2}\,(\cos(x + 3\pi/4))$

$= \sin x + \cos x$

maximum

occurs when $\cos x$ is max (1).

so when max $= \sqrt{2}$

$= \cancel{\frac{7\sqrt{2}\pi}{4}}$

6. $\int \frac{1}{\cos x - \sin x}\, dx$

$= \int \frac{1}{(-1)\sin x + (1)\cos x}\, dx$

$= \int \frac{1}{\sqrt{2}\cos(x+\pi/4)}\, dx$

$= \frac{1}{\sqrt{2}} \int \sec(x+\pi/4)\, dx$

$^*u = x+\pi/4, \quad du = dx$

$= \frac{1}{\sqrt{2}} \int \sec u\, du$

$= \log \frac{(\tan(x+\pi/4) + \sec(x+\pi/4))}{\sqrt{2}}$

or $\sqrt{2}\tanh^{-1}\left(\frac{\tan(x/2)+1}{\sqrt{2}}\right)$

7. a) $\sqrt{x^2-25} - 5\sec^{-1}(x/5) + c$

or $\sqrt{x^2-25} + 5\tan^{-1}(5/\sqrt{x^2-25})$

b) $\int \frac{1}{x^2\sqrt{1-x^2}}\, dx$

$= -\frac{\sqrt{1-x^2}}{x}$

c) $3\pi/8$

d) $\cosh^{-1}(4)$

or $\log(4+\sqrt{15})$

8. The discrepancy between arcsin and arccos is from the lack of definitive bounds on the integral. Technically, both would have $+C$ term, which could potentially differ based on the lower and the upper bound.

9. $\int \frac{1}{(x^2+1)^2-1}\,dx$

$*\ u = x+1,\quad du = dx$

$= \int \frac{1}{u^2-1}\,du$

$= -\int \frac{1}{1-u^2}\,du$

$= -\tanh^{-1}(u) + C$

$= -\tanh^{-1}(x+1) + C$

or $\frac{1}{2}(\log(x) - \log(x+2))$