Problem 1

Describe the qualitative effect on the heatmap shown of the $x(20 - x)$ factor in the density

$$f(x,y) = \frac{3}{4000(3/2)\sqrt{2\pi}}x(20-x)e^{-\frac{1}{2(3/2)^2}\left(y-2-\frac{1}{50}x(30-x)\right)^2}.$$
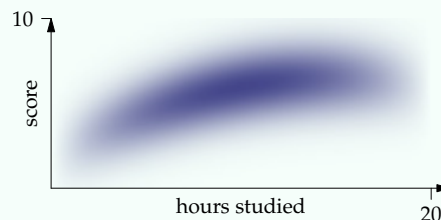
Solution

That factor is responsible for the fadeout near the left and right ends of the figure, since $x(20 - x)$ is small when x is near 0 or near 20.

Problem 2

Suppose that $D_\lambda : \mathbb{R} \to \mathbb{R}$ is a function whose support is equal to $[-\lambda, \lambda]$. What is the support of $K_\lambda(x,y) = D_\lambda(x)D_\lambda(y)$?

(Note: the support of a function is the set of all points where the function is nonzero, together with the boundary of that set.)

Solution

The support of K_λ is the square $[-\lambda, \lambda]^2$, since the value returned by the function is nonzero if and only if both factors are nonzero.