1)
$$\begin{bmatrix} 3 & 0 & -2 & 1 & 3 \\ 0 & 0 & 4 & 2 & 2 \\ 1 & 0 & 2 & 0 & 0 \end{bmatrix} \xrightarrow[\text{rows 1 and 3}]{\text{Interchange}} \begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & 2 & 2 \\ 3 & 0 & -2 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow[\text{to row 3}]{\text{Add } -3 \text{ times row 1}} \begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & 2 & 2 \\ 0 & 0 & -8 & -2 & 3 \end{bmatrix} \xrightarrow[\text{row 2 to row 3}]{\text{Add 2 times}}$$

$$\begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & 2 & 2 \\ 0 & 0 & 0 & 2 & 7 \end{bmatrix} \xrightarrow[\text{row 3 to row 2}]{\text{Add } -1 \text{ times}} \begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & 0 & -5 \\ 0 & 0 & 0 & 2 & 7 \end{bmatrix}$$

$$\xrightarrow[\text{by } \frac{1}{2} \text{ and row 2 by } \frac{1}{4}]{\text{Multiply row 3}} \begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -5/4 \\ 0 & 0 & 0 & 1 & 7/2 \end{bmatrix} \xrightarrow[\text{row 3 to row 1}]{\text{Add } -1 \text{ times}}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -7/2 \\ 0 & 0 & 1 & 0 & -5/4 \\ 0 & 0 & 0 & 1 & 7/2 \end{bmatrix} \xrightarrow[\text{to row 1}]{\text{Add } -2 \text{ times row 2}} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -5/4 \\ 0 & 0 & 0 & 1 & 7/2 \end{bmatrix}$$

2)

shaded region with weights in $[0,1]$ is
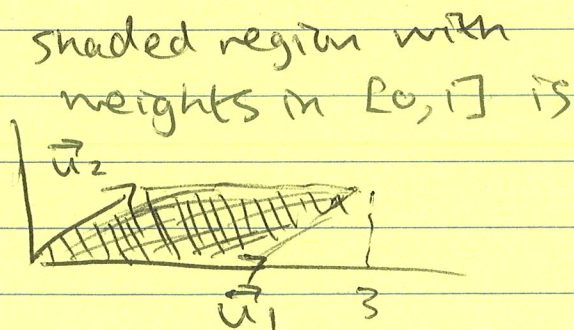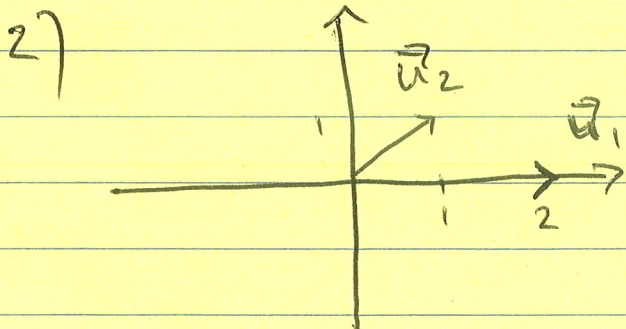
3) By Theorem 6 on Page 47 we know $A\vec{x} = \vec{c}$ has at most one solution (the only solution to $A\vec{x} = \vec{0}$ is $\vec{x} = \vec{0}$). Also $A\vec{x} = \vec{c}$ has exactly one solution as the reduced echelon form looks like
$$\begin{bmatrix} 1 & 0 & 0 & \cdot \\ 0 & 1 & 0 & \cdot \\ 0 & 0 & 1 & \cdot \end{bmatrix}$$
where the dots depend on what $\vec{c}$ is.

4a) $3\vec{u}$ represents how much mine A produces in 3 days

4b)

$$A\vec{u} + B\vec{v} = \begin{bmatrix} 150 \\ 2825 \end{bmatrix}$$

where A, resp. B, is the number of days mine A, resp. mine B should operate.

4c) Another way to ~~wtt~~ write the above is

$$\begin{bmatrix} \vec{u} & \vec{v} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 150 \\ 2825 \end{bmatrix}$$

Solve this by performing row operations on $\begin{bmatrix} \vec{u} & \vec{v} & 150 \\ & & 2825 \end{bmatrix}$ to obtain the REF:

$$\begin{bmatrix} 20 & 30 & 150 \\ 550 & 500 & 2825 \end{bmatrix} \xrightarrow[\text{by } 1/20]{\substack{\text{Multiply} \\ \text{row 1}}} \begin{bmatrix} 1 & 3/2 & 15/2 \\ 550 & 500 & 2825 \end{bmatrix}$$

$$\xrightarrow[\text{1 to row 2}]{\text{Add } -550 \text{ times row}} \begin{bmatrix} 1 & 3/2 & 15/2 \\ 0 & -325 & -1300 \end{bmatrix}$$

$$\xrightarrow[\text{2 by } -\frac{1}{325}]{\text{Multiply row}} \begin{bmatrix} 1 & 3/2 & 15/2 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow[\text{row 2 to row 1}]{\text{Add } -\frac{3}{2} \text{ times}}$$

$$\begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & 4 \end{bmatrix} \Rightarrow$$ Mine A needs a day and a half, while Mine B needs 4 days.