MATH 19 RECITATION
3 NOVEMBER 2016
BROWN UNIVERSITY
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1. Determine whether the following sum converges.

$$\overset{n=1}{\frac{5}{2}} + \overset{n=2}{\frac{5\cdot 7}{2\cdot 5}} + \overset{n=3}{\frac{5\cdot 7\cdot 9}{2\cdot 5\cdot 8}} + \overset{n=4}{\frac{5\cdot 7\cdot 9\cdot 11}{2\cdot 5\cdot 8\cdot 11}} + \cdots$$

Each time we multiply by $\frac{2n+3}{3n-1}$ to get from the $(n-1)^{\text{st}}$ term to the n^{th}. Since

$$\lim_{n\to\infty} \frac{2n+3}{3n-1} = \frac{2}{3} < 1,$$

the ratio test implies that the series converges.

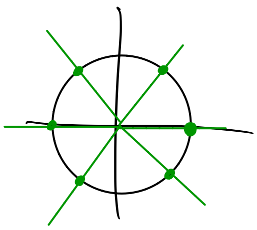
2. The *root test* says that if $\sqrt[n]{|a_n|} = |a_n|^{1/n}$ converges to a number less than 1, then $\sum a_n$ converges. Use the root test to show that $\sum \frac{n^2}{1.01^n}$ converges. (Note: it's handy to know that $n^{1/n} \to 1$ as $n \to \infty$.) Which is easier for this problem, the root test or the ratio test?

$$\lim_{n\to\infty}\left(\frac{n^2}{1.01^n}\right)^{1/n} = \frac{\lim_{n\to\infty}(n^{1/n})^2}{\lim_{n\to\infty} 1.01} = \frac{1}{1.01} < 1,$$

So converges. The root test is easier.

3. Show that $\sum_{n=1}^{\infty} \frac{\sin(n\pi/3)}{n}$ converges.

$$\sum_{n=1}^{\infty} \frac{\sin(n\pi/3)}{n} = \frac{\sqrt{3}/2}{1} + \frac{\sqrt{3}/2}{2} - \frac{\sqrt{3}/2}{4} - \frac{\sqrt{3}/2}{5} + \frac{\sqrt{3}/2}{7} + \frac{\sqrt{3}/2}{8} - \cdots$$



$$= \frac{\sqrt{3}}{2} \sum_{k=0} (-1)^k \left[\frac{1}{(3k+1)} + \frac{1}{(3k+2)}\right].$$

this sum converges because $\frac{1}{3k+1} + \frac{1}{3k+2}$ decreases to 0 as $k \to \infty$.
(by the alternating series test)

4. Show that $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ *without using the alternating series test* by grouping terms into consecutive pairs and showing that the infinite sum of these "pair-sums" converges.

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots$$

$$= \sum_{k=1}^{\infty} \left[\frac{-1}{2k-1} + \frac{1}{2k}\right]$$

$$= \sum_{k=1}^{\infty} \frac{-2k + 2k - 1}{(2k-1)(2k)}$$

$$= \sum_{k=1}^{\infty} \frac{-1}{(2k-1)(2k)}.$$

this sum converges by comparison to $\sum_{k=1}^{\infty} \frac{1}{k^2}$ which converges by the integral test