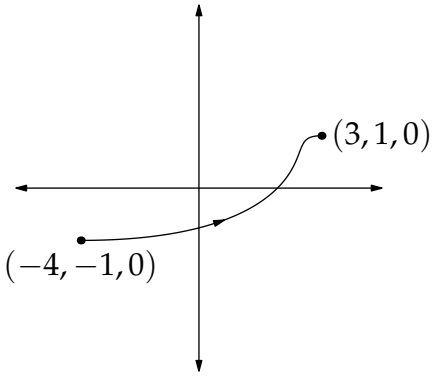
18.022 Recitation Quiz (with solutions)
24 November 2014

1. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x, y, z) = x^3y + \dfrac{xy^2}{1 + z^2}$.

(a) Find $\nabla \times (\nabla f)$.

(b) Calculate $\int_C \nabla f \cdot d\mathbf{s}$, where C is the path contained in the x-y plane shown below.



Solution. (a) The answer is $\boxed{0}$, because conservative vector fields are curl-free.

(b) The answer is $f((3, 1, 0)) - f((-4, -1, 0)) = 30 - 60 = \boxed{-30}$, because $\int_{\mathbf{a}}^{\mathbf{b}} \nabla f \cdot d\mathbf{s} = f(\mathbf{b}) - f(\mathbf{a})$. □