Brown University
Problem Set 9
Instructor: Samuel S. Watson
Due: 17 November 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

Find $\iint_R \frac{y}{x}\,dA$ where R is the region bounded by the curves $y = 0$, $y = x/2$, $x^2 - y^2 = 1$, and $x^2 - y^2 = 4$, by changing coordinates. (Hint: try letting y/x be one of your new coordinates.)

Solution

Problem 2

Find

$$\iint_R \frac{x-2y}{3x-y}\,dA,$$

where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, and $3x-y=8$.

Solution

Problem 3

Consider the functions f, w_1, and w_2 from $[1,2]^2 \to \mathbb{R}$ defined by

$$\begin{aligned} f(x,y) &= xy, \\ w_1(x,y) &= x+y, \\ w_2(x,y) &= 4-x-y. \end{aligned}$$

(a) Describe qualitatively how you can guess which of the following is greater, before doing any calculation (i) the w_1-weighed average of f, or (ii) the w_2-weighted average of f.

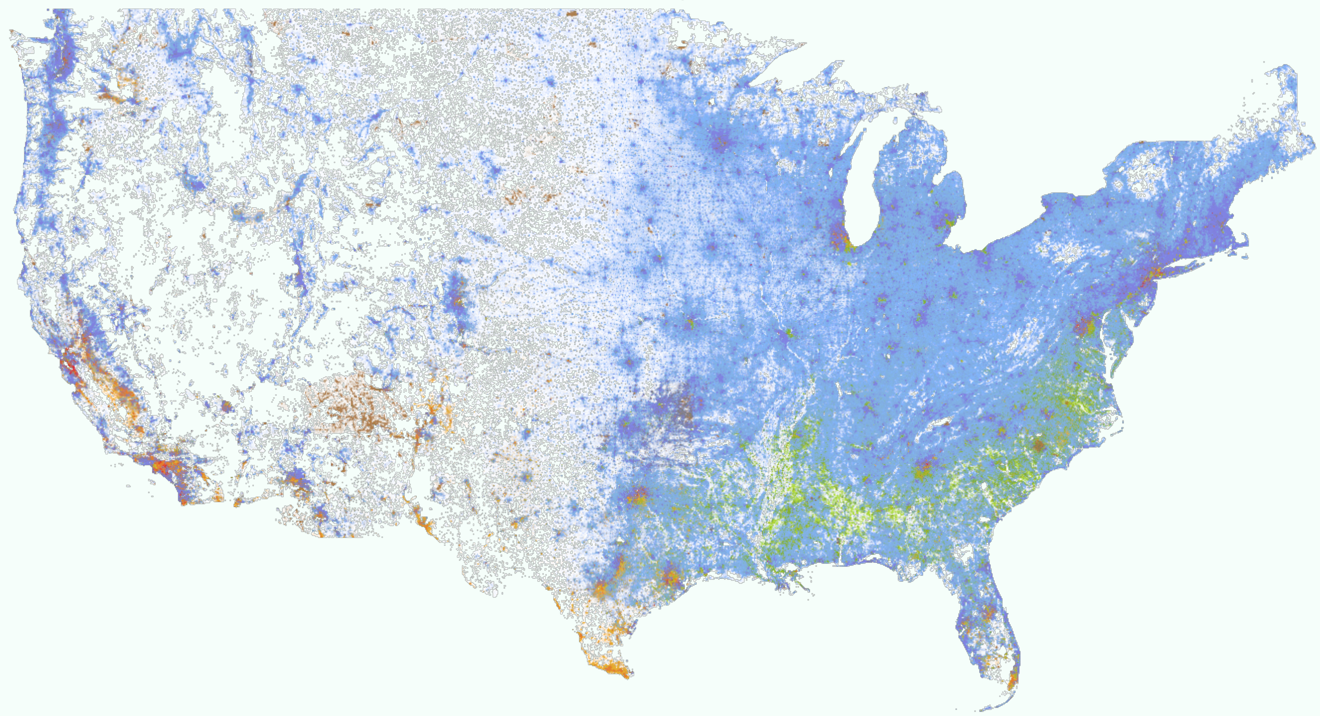
(b) Calculate the w_1-weighted average of f and the w_2-weighted average of f.

(c) Confirm that both weighted averages of f computed above lie between the maximum and minimum values of f.

Solution

Problem 4

The map below was obtained by placing one dot at the residence of each person in the contiguous United States in the 2010 US Census (with color indicating race identification) and then representing each of many small squares in a grid with the average of the colors of the dots in the square.



You can interact with the underlying data *at the level of individual dots* at `https://demographics.virginia.edu/DotMap/`

Suppose you're given the 764×1366 grid of numbers which represent how darkly colored in each little square is (see the figure to the right for a small piece of this grid). These numbers are roughly proportional to the number of people living in each square.

0.18	0.3	0.29	0.3	0.28	0.27	0.27	0.31	0.32	0.3	0.36	0.34	0.3	0.29	0.38	0.37	0.36	0.34	0.38	0.36	0.27
0.27	0.23	0.28	0.3	0.22	0.23	0.34	0.36	0.3	0.3	0.36	0.37	0.4	0.36	0.3	0.35	0.32	0.32	0.28	0.34	0.31
0.29	0.27	0.3	0.21	0.3	0.3	0.37	0.35	0.3	0.31	0.3	0.35	0.38	0.34	0.3	0.33	0.31	0.4	0.15	0.26	0.14
0.04	0.33	0.3	0.3	0.29	0.26	0.31	0.34	0.33	0.25	0.24	0.3	0.3	0.02	0.26	0.33	0.36	0.25	0.28	0.22	0.0
0.09	0.25	0.3	0.3	0.38	0.3	0.31	0.35	0.31	0.3	0.3	0.3	0.22	0.12	0.07	0.27	0.19	0.18	0.32	0.19	0.01
0.31	0.31	0.3	0.3	0.31	0.3	0.3	0.32	0.31	0.3	0.3	0.31	0.3	0.21	0.23	0.29	0.27	0.11	0.32	0.31	0.31
0.3	0.41	0.31	0.05	0.27	0.28	0.11	0.28	0.3	0.36	0.3	0.35	0.28	0.27	0.3	0.19	0.08	0.21	0.07	0.22	0.3
0.0	0.31	0.31	0.33	0.18	0.15	0.29	0.3	0.28	0.31	0.47	0.35	0.3	0.29	0.28	0.29	0.27	0.3	0.3	0.3	0.33
0.48	0.3	0.41	0.3	0.3	0.12	0.06	0.22	0.29	0.21	0.25	0.36	0.39	0.3	0.3	0.3	0.25	0.26	0.31	0.3	0.39
0.43	0.41	0.36	0.37	0.3	0.21	0.03	0.04	0.13	0.22	0.3	0.34	0.32	0.31	0.27	0.3	0.25	0.17	0.33	0.33	0.36
0.35	0.33	0.31	0.37	0.33	0.27	0.24	0.05	0.06	0.27	0.13	0.36	0.3	0.3	0.28	0.26	0.16	0.21	0.3	0.31	0.3
0.0	0.3	0.3	0.33	0.42	0.3	0.3	0.25	0.1	0.11	0.25	0.31	0.26	0.29	0.3	0.2	0.2	0.31	0.29	0.3	0.3
0.02	0.0	0.33	0.31	0.35	0.31	0.35	0.3	0.28	0.2	0.25	0.26	0.11	0.22	0.4	0.34	0.33	0.3	0.3	0.3	0.18
0.28	0.3	0.3	0.3	0.33	0.35	0.34	0.3	0.3	0.3	0.19	0.3	0.28	0.33	0.26	0.25	0.21	0.36	0.37	0.37	0.31
0.3	0.31	0.49	0.33	0.3	0.35	0.34	0.36	0.4	0.27	0.13	0.2	0.2	0.08	0.04	0.11	0.24	0.42	0.52	0.39	0.3
0.3	0.32	0.44	0.37	0.36	0.45	0.34	0.35	0.3	0.48	0.31	0.18	0.0	0.09	0.03	0.12	0.3	0.38	0.4	0.13	0.3
0.31	0.3	0.5	0.33	0.42	0.4	0.48	0.42	0.3	0.53	0.4	0.34	0.2	0.24	0.13	0.06	0.33	0.35	0.32	0.22	0.42
0.33	0.31	0.3	0.31	0.35	0.47	0.5	0.36	0.52	0.47	0.39	0.34	0.35	0.3	0.3	0.3	0.3	0.3	0.16	0.29	0.28
0.3	0.3	0.3	0.3	0.33	0.31	0.43	0.33	0.43	0.51	0.36	0.35	0.37	0.3	0.42	0.39	0.36	0.3	0.3	0.05	0.29
0.3	0.31	0.31	0.32	0.3	0.31	0.3	0.35	0.53	0.45	0.47	0.45	0.37	0.35	0.36	0.44	0.31	0.31	0.28	0.26	0.3
0.3	0.3	0.38	0.31	0.3	0.3	0.32	0.35	0.53	0.49	0.08	0.38	0.31	0.33	0.46	0.3	0.29	0.24	0.29	0.29	0.31

(a) Using the center-of-mass formula and Riemann approximations thereof, explain what calculation you could perform with these numbers to approximate the population-weighted center of mass of the United States. Apply your procedure to the small 3×3 portion of the grid below to show how it works.

0.35	0.33	0.3
0.0	0.3	0.3
0.02	0.0	0.18

(b) In what state would you guess the population-weighted center of mass is?

Note: click here to access the full matrix if you want to do some of these computations yourself.

Solution

Problem 5

The dart thrower in Example 5.5.3 in the book is terribly unlikely to hit the triple-20. What a shame. Let's see how they can increase their chances by becoming less accurate.

(a) Suppose that the probability density function of the dart's location is given by

$$f_\alpha(x,y) = \frac{1}{\pi\alpha}\mathrm{e}^{-\frac{x^2+y^2}{\alpha}},$$

where $\alpha > 0$ is an accuracy parameter. If a player becomes more accurate, does their α value increase or decrease? (You'll need to explain how a change in α affects the shape of the above density).

(b) Explain in intuitive terms why a thrower with accuracy α is extremely unlikely to hit the triple-20 either when α is very small or when α is very large.

(c) Find the value of α that maximizes the probability of hitting the triple-20.

Solution

Final answer: