1. Find the second order Taylor polynomial for $f(x, y) = \cos(x + 2y)$ at the origin. What is the second order Taylor polynomial for $g(\theta) = \cos\theta$ at $\theta = 0$?

2. (a) Find the critical points of $f(x, y) = x^2 + 4xy + y^2$. Use the second derivative test for local extrema to determine whether the point is a local maximum, a local minimum, or a saddle point.

(b) Find the critical points of $g(x, y) = x^2 + xy + y^2$. Use the second derivative test for local extrema to determine whether the point is a local maximum, a local minimum, or a saddle point.

3. (a) What theorem ensures that the function $f(x, y) = x\sin(x + y)$ defined on the rectangle $\{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq 7\}$ has an absolute maximum and an absolute minimum? Verify the hypotheses of that theorem.

(b) Find the absolute extrema of f. You are given that there are no absolute extrema on the top or bottom of the rectangle; see the surface plot below to guide your intuition.