1. Find the limit as $(x, y) \to (0,0)$ of $\cos\left(\frac{x^3+y^3}{x^2+y^2}\right)$.

Solution. Switching to polar coordinates, we find that the limit of the expression in parentheses is the same as the limit of $r^3(\cos^3\theta + \sin^3\theta)/r^2$ as $r \to 0$. Since the trigonometric expression is bounded and $r^3/r^2 = r$ goes to 0 as r goes to 0, we obtain a limit of 0 for the expression in parentheses. Since cosine is continuous, the limit of the given function is $\cos 0 = \boxed{1}$.