1. Show that if S is a square in the plane with sides parallel to the axes and $A = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$, then AS has the same area as S.

Solution. If S has opposite corners at (x, y) and $(x + h, y + h)$, then A maps S to the parallelogram with opposite corners at $(x + \lambda y, y)$ and $(x + h + \lambda(y + h), y + h)$. This parallelogram has a base of $(x + h + \lambda y - (x + \lambda y) = h$ and a height of $y + h - y = h$, so its area is h^2. Since the area of S is also h^2, we conclude that AS and S have the same area. Note: A is an example of a *shear* transformation.