1. $f''(x) + 5f(x) = 4 f'(x)$.

The characteristic equation is $\lambda^2 - 4\lambda + 5 = 0$.

$\lambda = 2 \pm i$.

$f(x) = A e^{2x} \cos x + B e^{2x} \sin x$.

$\because f(0) = 1$.

$\therefore f(0) = A \cdot \cos 0 + B \cdot 0 = A = 1$.

So $\boxed{f(x) = e^{2x} \cos x + B e^{2x} \sin x.}$

B can be any real number.