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MATH 19 PRACTICE FINAL FALL 2016 BROWN UNIVERSITY SAMUEL S. WATSON

1 (10 points) Find $\int \cos^2 x + \cos^{10} x \sin x + xe^x dx$.

2 (10 points) Consider the function

$$f(x) = \int_0^x \sqrt{4\cos^2 t - 1} \, dt.$$

Find the arclength of the graph of f over the interval $[0, \pi/2]$.

 $\fbox{ \ \ \, }$ (10 points) Find the function f which satisfies f(0)=3, f'(0)=6, and

$$f''(x) + 2f(x) = 2f'(x) + e^x.$$

4 (10 points) Determine the convergence or divergence of each of the following series.

(a)
$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

(b)
$$1 + \frac{1}{1+2} + \frac{1}{1+2+4} + \frac{1}{1+2+4+8} + \cdots$$

[5] (10 points) Determine the convergence or divergence of each of the following series.

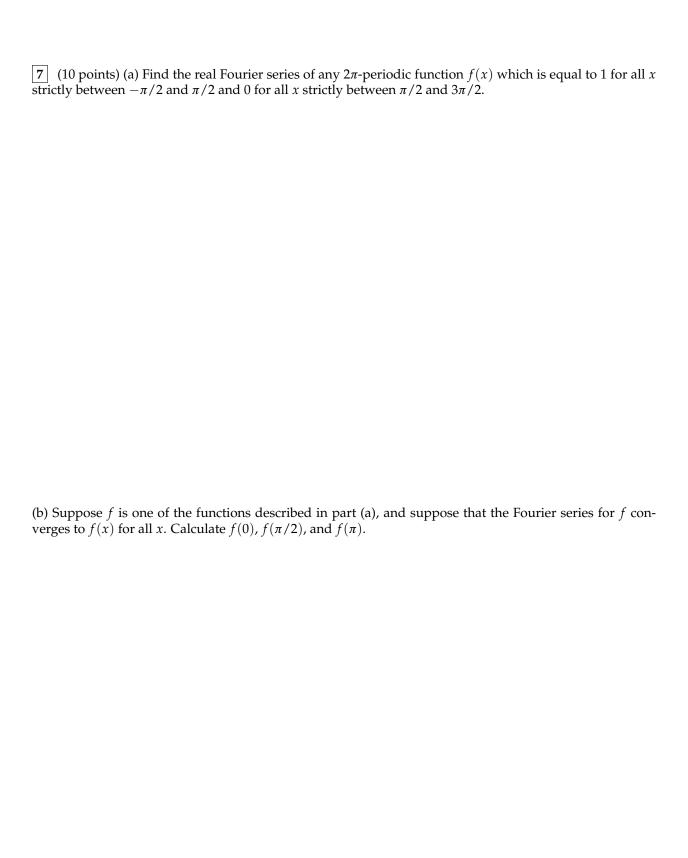
(a)
$$\sum_{n=1}^{\infty} \frac{n^n}{n^{n^2}}$$

(b)
$$\sum_{n=0}^{\infty} (-1)^n \frac{n^2 + 1 + 2^{-n}}{n^2 - 10}$$

6 (10 points) (a) Suppose f is a continuous function from $[0,\infty)$ to $\mathbb R$ and that $\int_0^\infty f(x)dx=21$ and $\int_0^1 f(x)dx=16$. Find

$$\lim_{b\to\infty}\int_1^b f(x)\,dx.$$

(b) Suppose that the integrals $\int_0^1 x^p dx$ and $\int_1^\infty x^p dx$ are both improper and divergent. Find p.



 $\fbox{8}$ (10 points) Consider a physical system which responds to a periodic stimulus V(t) by behaving according to the periodic solution Q of the differential equation

$$Q''(t) + 6Q'(t) + 13Q(t) = V(t).$$

Express V(t) as a real Fourier series given that

$$Q(t) = \sum_{n=-\infty}^{\infty} \frac{1}{\pi(n^2+1)} e^{int}.$$

Note: You may assume that *Q* is twice differentiable (it is).

- 9 (10 points) Consider the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n} (x-3)^n$.
- (a) Find the radius of convergence of this power series.

(b) Let us define $f(x) = \sum_{n=0}^{\infty} \frac{n!}{n^n} (x-3)^n$ for all x such that the infinite series on the right-hand side converges. Calculate $f^{(4)}(3)$.

10 (10 points) (a) Writing an arbitrary complex number z as x + iy where x and y are real, show that $e^z \neq 0$ for all complex numbers z.

(b) Show that for all real numbers x, we have

$$1 - \frac{x^2}{2} + \frac{x^4}{2^2 \cdot 2!} - \frac{x^6}{2^3 \cdot 3!} + \frac{x^8}{2^4 \cdot 4!} - \dots \neq 0.$$