18.022 Recitation Quiz (with solutions)
03 December 2014

1. Calculate the flux of the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ across the surface S given by

$$z = \sqrt{1 - x^2 - y^2},$$

where (x, y) ranges over the unit disk.

Solution. Note that the surface is the unit upper hemisphere, which means that the unit normal vector is (x, y, z). Therefore, the flux is

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (x, y, z) \cdot (x, y, z)\, dS = \iint_S 1\, dS,$$

which is the surface area of the unit upper hemisphere. Since the surface area of a sphere of radius r is $4\pi r^2$, this area is $\boxed{2\pi}$. □