

# Reflections in an octogonal mirror maze

By **David Eppstein**

Presented by  
**Chassagne Aurélien** and **Swirydowicz Szymon**

# Hypotheses and properties

## Environment

- Integer grid for mirrors and light ray.
- Horizontal, vertical, diagonal mirrors.
- Start and end points of mirrors do not reflect.
- Fixed initial direction for the light ray.



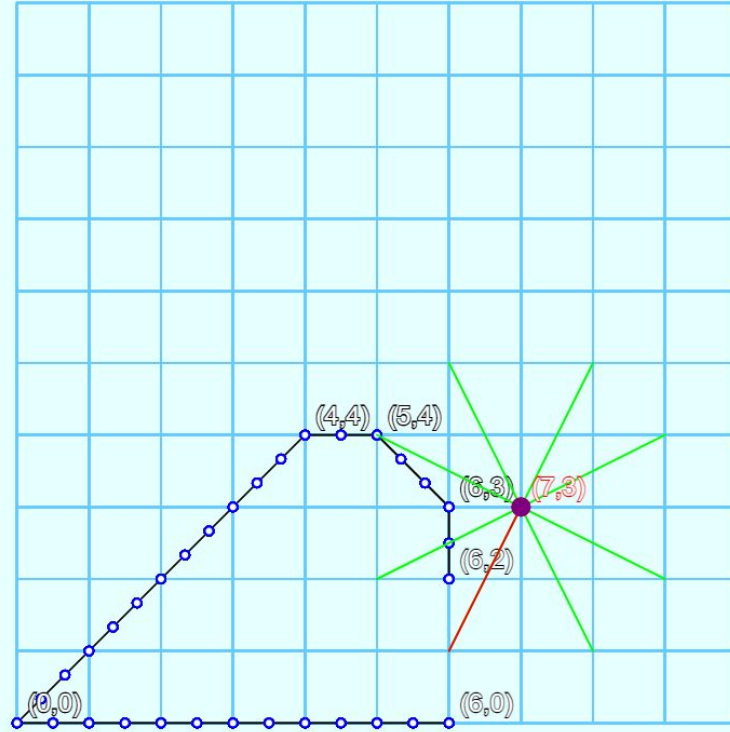
## Mirrors

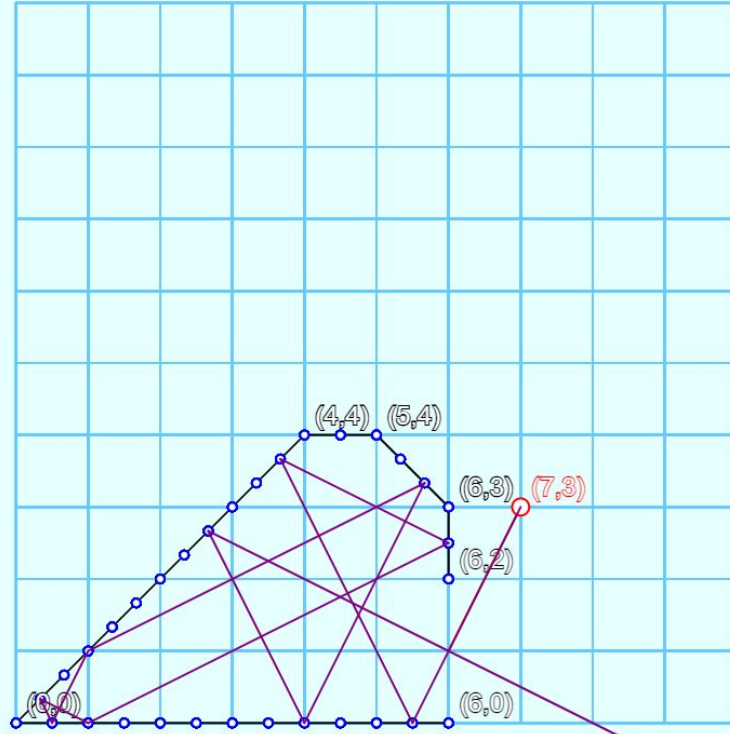
Finite number of hit points.

## Light ray

Only 4 or 8 possible directions.







# Integer interval exchange transformation



Big interval



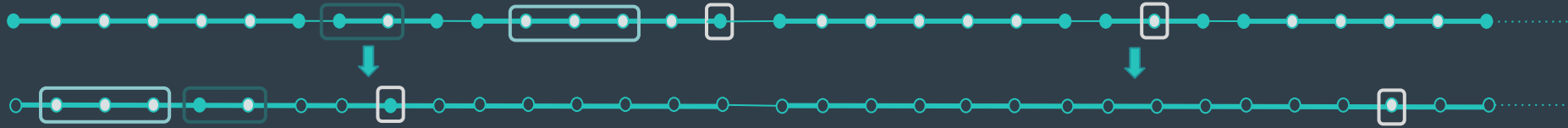
Mapping



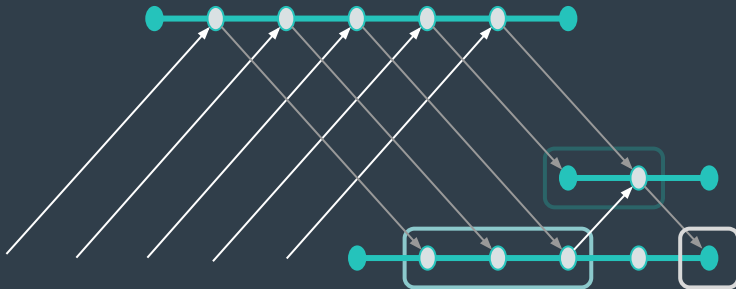
# Integer interval exchange transformation



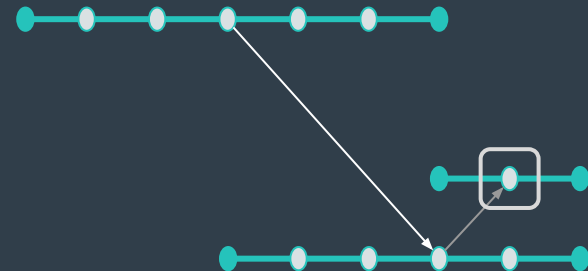
Big interval : Concatenation of the **8 ray's directions**



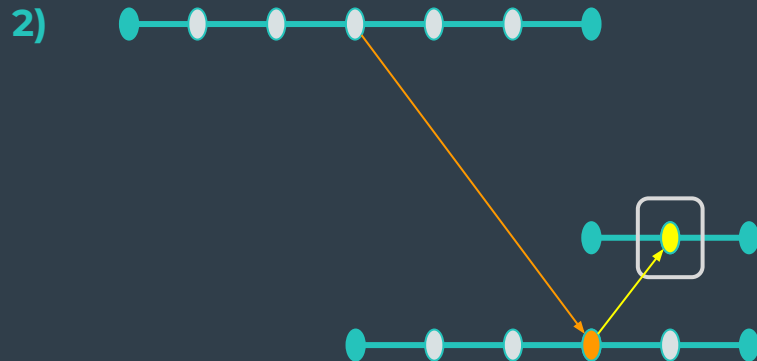
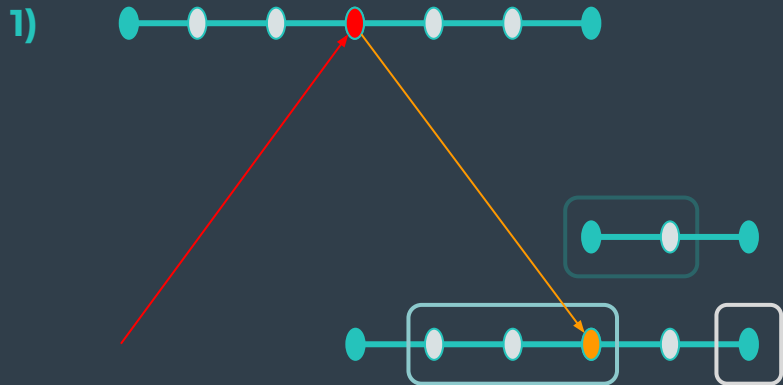
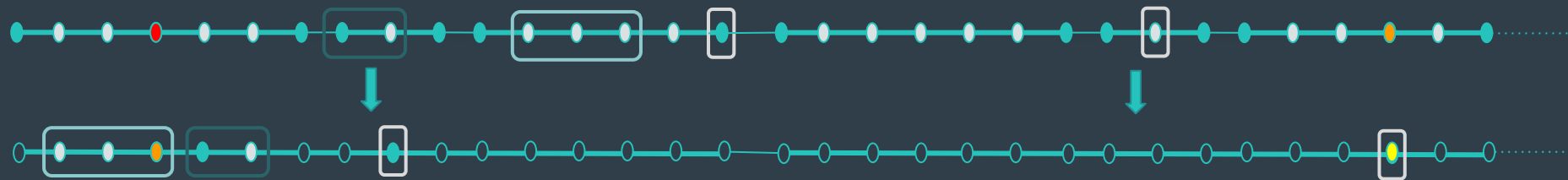
Ray's direction :



Ray's direction :



# Integer interval exchange transformation



# Mapping must be bijective



Problem:

Finite problem,  
Max  $n$  bounces



Apply max  $n$  times  
the transformation

Optimization:

Possible to compute the  $n^{th}$   
**transformation directly**



Mapping is **bijective**



The mapping is **partial**, not bijective yet





# Partial to complete exchange

The mapping is not bijective because of **outgoing rays**, **non-reflective surfaces** and **mirror's endpoints** that are not mapped.

**Goal:** Make the mapping bijective

Outgoing rays



Surrounding environment in a box

Non-reflective surfaces



Considered as virtually reflective

Mirror's endpoints



Considered as flipping back the ray



The mapping is **complete** (bijective) but **false**

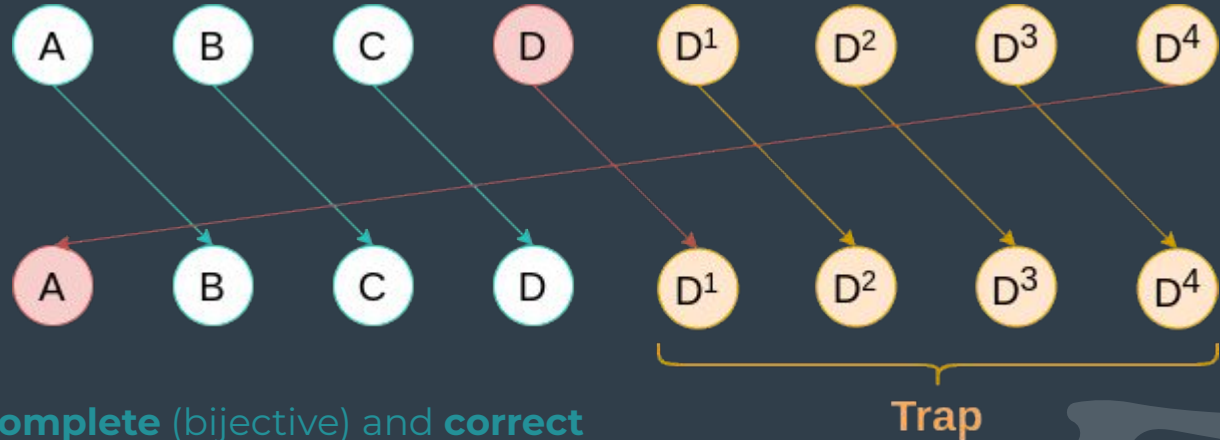
# Repair the mapping

The mapping is bijective but **false** since we mapped non-reflective points

**Goal:** Use “*traps*” when reaching non-reflective points

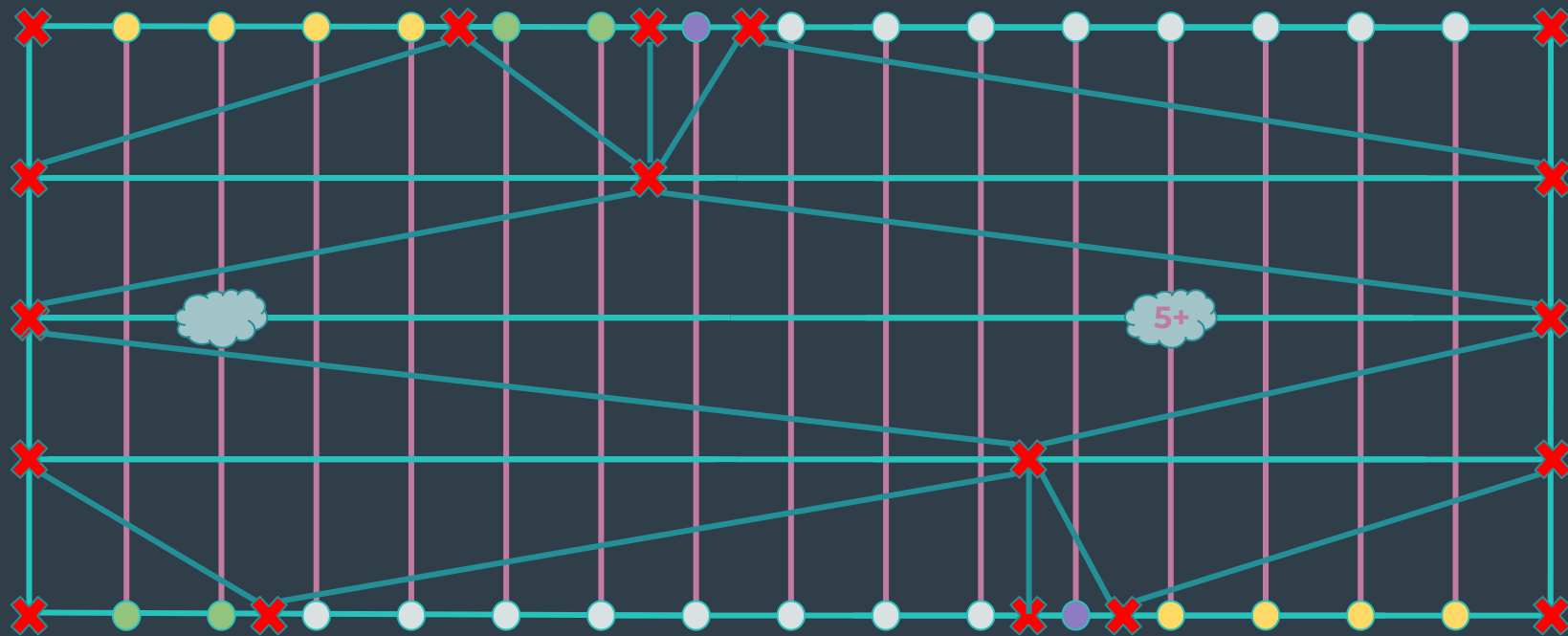
Inserting  $n$  virtual intermediate points between the non-reflective point and its target

E.g. The non-reflective point “D” reflects to point “A”



➡ The mapping is **complete** (bijective) and **correct**

# Triangulation





# Live Nemo



Q&A



Thank you!



# References

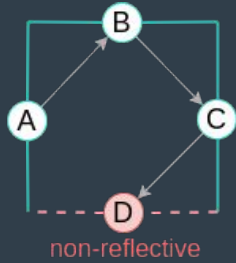
*"Reflections in an octagonal mirror maze"*  
by **David Eppstein**

*"The Complexity of Iterated Reversible Computation"*  
by **David Eppstein**

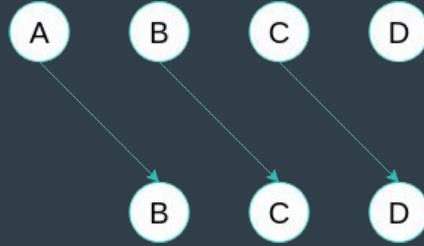
*"Tracing Compressed Curves in Triangulated Surfaces"*  
by **Erickson Jeff** and **Amir Nayyeri**



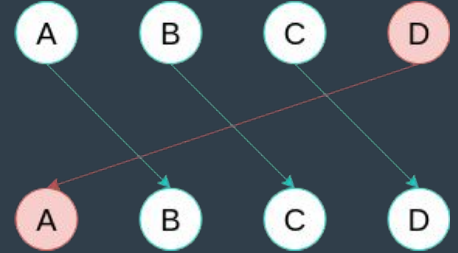
## 1. Environment



## 2. Partial Exchange

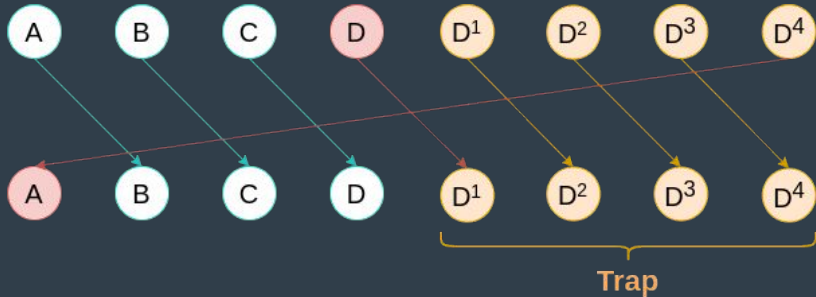


## 3. Non-reflective as reflective



# Algorithm Overview

## 4. Complete Interval Exchange



## 5. Fast iterated exchange

