CS-E4895 Gaussian Processes
Lecture 0: Example

A.N. Onymous

Aalto University

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Agenda for today

1. Motivation for Gaussian processes
2. Course content, format, and evaluation
3. Warm up for Gaussian processes: Review of the multivariate Gaussian distribution
4. First assignment

The multivariate Gaussian distribution

- **Definition** A random vector $\boldsymbol{x} = [x_1, x_2, \cdots, x_D]^\top$ is said to have the multivariate Gaussian distribution if all linear combinations of $\boldsymbol{x}$ are Gaussian distributed:

$$y = \boldsymbol{a}^\top \boldsymbol{x} = a_1 x_1 + a_2 x_2 + \cdots + a_D x_D \sim \mathcal{N}(m, v) \tag{1}$$

for all $\boldsymbol{a} \in \mathbb{R}^D$, where $\boldsymbol{a} \neq \mathbf{0}$

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- The multivariate Gaussian density for a variable $\boldsymbol{x} \in \mathbb{R}^D$:

$$N(x|\mu, S) = (2\pi)^{-\frac{D}{2}} |S|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(x-\mu)^\top S^{-1}(x-\mu)\right] \quad \in \mathbb{R}_{\geq 0} \tag{2}$$

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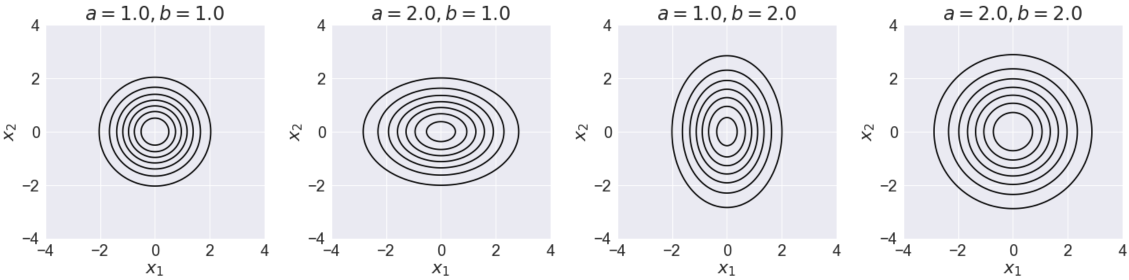
- $(\boldsymbol{\Sigma})_{ij}$ is the covariance between the i'th and j'th elements x_i and x_j of $\boldsymbol{x}$

Interpretation of the covariance matrix - 2D examples

The diagonal of the covariance controls the scaling/marginal variances

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{\Sigma} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{4}$$



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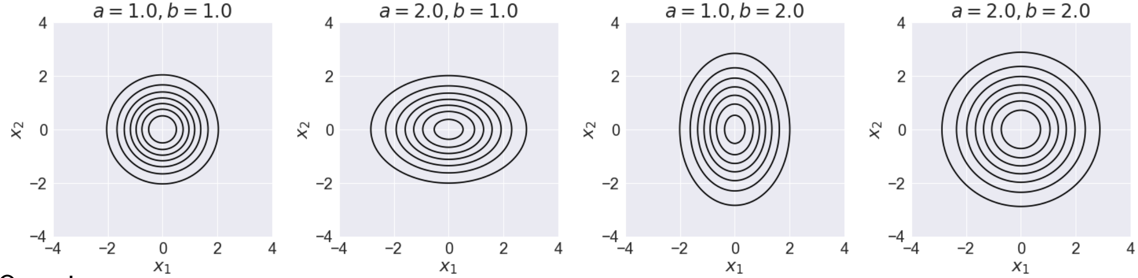

Questions:

1. If $\boldsymbol{\Sigma}$ is diagonal, then x_1 and x_2 are uncorrelated? True or false?
2. If $\boldsymbol{\Sigma}$ is diagonal, then x_1 and x_2 are independent? True or false?
3. What is the volume (integral) of density?
4. Which of the four densities has the highest peak and why?

More slides...

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  - We will introduce Gaussian processes more formally

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