Απαντήσεις στις Ασκήσεις του Κεφαλαίου 1

1. (i) Δεν ορίζεται, (ii) 4×2, (iii) Δεν ορίζεται, (iv) Δεν ορίζεται, (v) 5×5, (vi) 5×2, (vii) 5×5, $(viii)$ Δεν ορίζεται.

2.

(i) $\begin{pmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{pmatrix}$, (ii) $\begin{pmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$, (iii) $\begin{pmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{pmatrix}$, (iv) $\begin{pmatrix} -7 & -28 & -14 \\ -21 & -7 & -35 \end{pmatrix}$, (v) Δεν ορίζεται,

(vi) $\begin{pmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{pmatrix}$, (vii) $\begin{pmatrix} -39 & -21 & -24 \\ 9 & -6 & -15 \\ -33 & -12 & -30 \end{pmatrix}$, $(viii)$ $\mathbb{O}$, (ix) $\begin{pmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{pmatrix}$, (x) Δεν ορίζεται,

(xi) $\begin{pmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{pmatrix}$, (xii) $\begin{pmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{pmatrix}$.

3. $a = 4$, $b = -6$, , $d = 1$, $c = -1$.

4.

(i) $\begin{pmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix}$, (ii) $\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{pmatrix}$, (iii) $\begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$, (iv) $\begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}$

5. (i) όχι, (ii) όχι, (iii) ναι.

6. $a = -8$.

8.

(i) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$, (ii) $\begin{pmatrix} 1/5 & 3/10 \\ -1/5 & 1/10 \end{pmatrix}$, (iii) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

10. $X = CB - A$.

11. $B = P^{-1}AP$.

12. (i) B^{-1}, (ii) CD^{-1}.

13.

(i) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$, (ii) $\begin{pmatrix} 1 & 0 & 7/8 & 3/4 & 0 \\ 0 & 1 & 5/8 & -22/8 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$.

14.

(i) $\begin{pmatrix} -7 & 2 \\ 4 & -1 \end{pmatrix}$, (ii) $\begin{pmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \end{pmatrix}$, (iii) $\begin{pmatrix} 1/5 & 0 & 0 \\ -3/35 & -1/7 & 0 \\ 41/35 & -5/7 & -1 \end{pmatrix}$, (iv) Μη αντιστρέψιμος,

(v) Μη αντιστρέψιμος, (vi) $\begin{pmatrix} 1/4 & 1/2 & -3 & 0 \\ -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 1/40 & -1/20 & -1/10 & -1/5 \end{pmatrix}$.

15. $c \in \mathbb{R} \setminus \{0, 1\}$.