1.

Using the definition of Q function and Bellman equation we obtain the following:

$$Q^\pi(s,a) = E_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s, A_t = a] =$$

$$= E_\pi[R_{t+1} + \sum_{k=1}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s, A_t = a] * ;$$

$$* \; E_\pi[R_{t+1} \mid S_t = s, A_t = a] = E_\pi r(s,a);$$

$$E_\pi[\sum_{k=1}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s, A_t = a] =$$

$$= E_\pi[\gamma \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_{t+1} = s', A_{t+1} = a'] = E_\pi[\gamma Q_\pi(s',a')];$$

$$\Rightarrow Q^\pi(s,a) = E_{(s',a') \sim P(\cdot|s,a)}[r(s,a) + \gamma Q^\pi(s',a')]$$

2.

$$Q^*(s,a) = \max_\pi q_\pi(s,a) = \max_\pi [E_{(s',a') \sim P(\cdot|s,a)}[r(s,a) + \gamma Q^\pi(s',a')]]$$

$$= E_{(s',a') \sim \pi^*}[r(s,a) + \gamma \max_{a'} Q^{\pi^*}(s',a')] =$$

$$= E_{(s',a') \sim \pi^*(\cdot|s,a)}[r(s,a) + \gamma \max_{a'} Q^\pi(s',a')]$$

3. A plausable objective minimizes the difference between current Q-function and the optimal Q-function

$$\ell(\theta) = \mathbb{E} \, || Q^*(s, a, \theta) - Q(s, a, \theta) ||^2$$

$$Q^*(s, a, \theta) = E_{s' \sim \pi^*(\cdot | s, a)} [r(s, a) + \gamma \max_{a'} Q^*(s', a')) \} =$$

$$= E_{s' \sim \pi^*(\cdot | s, a)} \{ r(s, a) + \gamma \max \max_{a'} Q(s', a')) \}$$

$$\Rightarrow \ell(\theta) = E_{s' \sim \pi^*(\cdot | s, a)} || r + \gamma \max \max_{a'} Q(s', a', \theta) - Q(s, a, \theta) ||^2$$