There Is No Largest Prime Number

With an introduction to a new proof technique

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Simple Text Orgnazition

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Outline

1. Motivation
   - What Are Prime Numbers?

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   - What Are Prime Numbers?

2. Results
   - Proof of the Main Theorem

3. What's More
   - An Open Question
   - Another Open Question
   - An Algorithm For Finding Primes Numbers

What Are Prime Numbers?

Definition

A prime number is a number that has exactly two divisors.

Example

- 2 is prime (two divisors: 1 and 2).

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- 3 is prime (two divisors: 1 and 3).

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Example

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (three divisors: 1, 2, and 4).

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Proof of the Main Theorem

The proof uses *reductio ad absurdum*

Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.

4. But $q+1$ is divisible by some prime number not in the first p numbers. $\square$

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An Open Question

Answered Questions

How many primes are there?

Open Questions

Is every even number the sum of two primes? [1]

[Goldbach, 1742] Christian Goldbach.
A problem we should try to solve before the ISPN '43 deadline,
Letter to Leonhard Euler, 1742.

Another Open Question

In complex plane, define $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$. Then, are all zeros of ζ in the strip $0 \leq \mathrm{Re}\,(s) \leq 1$ lie on the line $\mathrm{Re}\,(s) = \frac{1}{2}$?

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Some Observation

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If $\mathrm{Re}\,(s) > 1$, then

$$\begin{aligned}\zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s}\\ &= \prod_{p} \frac{1}{1-p^{-s}}.\end{aligned}$$

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$$\begin{aligned}\zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s} \\ &= \prod_{p} \frac{1}{1 - p^{-s}}.\end{aligned}$$

Thus ζ does not vanish when $\mathrm{Re}\,(s) > 1$.

An Algorithm For Finding Primes Numbers

```
int main (void)
{
 std::vector<bool> is_prime (100, true);
 for (int i = 2; i < 100; i++)






 return 0;
}
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int main (void)
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int main (void)
{
 std::vector<bool> is_prime (100, true);
 for (int i = 2; i < 100; i++)
 if (is_prime[i])
 {
 std::cout << i << " ";
 for (int j = i; j < 100;
 is_prime [j] = false, j+=i);
 }
 return 0;
}
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Note the use of `std::`.