Graph Skills

February

Note, this is graph only material - not networks, so this includes Eulerian, Hamiltonian and planar issues, but not Dijkstra, TSP.

1. How many arcs in:
   (a) K_6
   (b) C_7
   (c) $K_{3,4}$
   (d) K_n
   (e) C_n
   (f) $K_{m,n}$
   (g) a MST on a graph with n vertices.
2. Draw example graphs that are:
   (a) simple and connected
   (b) simple but not connected
   (c) connected but not simple
   (d) neither simple nor connected
3. True or False: A minimal spanning tree is a trail.
4. State Euler's formula in terms of a planar graph.
5. Prove this theorem[1]: The sum of all the degrees of the faces/regions of a connected planar graph is equal to twice the number of arcs ($\Sigma degree(f) = 2e$).
6. If G is a planar simple graph, prove that $e \leq 3v - 6$
7. Show that $K_{2,n}$ is planar, for any value of n.
8. State Kuratowksi's Theorem.
9. Use Kuratowksi's theorem to prove that all complete graphs for K_n where $n \geq 5$ are non-planar.
10. Prove that a graph is Eulerian if and only if it is connected and every vertex has even degree.
11. State Ore's Theorem.
12. Prove Ore's Theorem.

[1] known as The Handshaking Theorem for Planar Graphs