Preliminaries

February

1. If G is a simple graph with n vertices, what is the maximum number of edges that G can have?

Solution: The maximum number of edges that a simple graph with n vertices can have is $\frac{n(n-1)}{2}$.

2. If G is a simple graph with n vertices, what is the minimum number of edges that G can have?

Solution: The minimum number of edges that a simple graph with n vertices can have is 0.

3. Prove that for any graph G of order at least 2, the degree sequence has at least two equal entries.

Solution: The degree sequence of a graph is the list of the degrees of the vertices in the graph. The degree of a vertex is the number of edges incident to it.

(a) The degree of a vertex is a non-negative integer.

(b) The degree sequence of a graph is a list of non-negative integers.

(c) If a graph has at least two vertices, then the degree sequence has at least two equal entries: pigeonhole principle.

4. Prove that every connected graph contains at least one spanning tree.

Solution: A connected graph is a graph in which there is a path between every pair of vertices. A spanning tree of a graph is a subgraph that is a tree and connects all the vertices together.

(a) A graph with n vertices and $n-1$ edges is a tree.

(b) A connected graph with n vertices and $n-1$ edges is a tree.

(c) A connected graph with n vertices and $n-1$ edges contains a spanning tree.

5. Give the adjacency matrix for K_n.

Solution: The adjacency matrix for K_n is the $n \times n$ matrix with all entries equal to 1, except for the diagonal entries which are all 0.

6. Give the adjacency matrix for P_n (reminder: P_n is the simple chain, path, of n vertices.).

Solution: The adjacency matrix for P_n is the $n \times n$ matrix with 1s on the diagonal and 1s on the off-diagonal entries that are adjacent.

7. Give the adjacency matrix for C_n (reminder: C_n is the simple cycle, or ring, of n vertices).

Solution: The adjacency matrix for C_n is the $n \times n$ matrix with 1s on the diagonal and 1s on the off-diagonal entries that are adjacent. The entry in the top right corner is a 1.

8. Give the adjacency matrix for $K_{m,n}$.

Solution: The adjacency matrix for $K_{m,n}$ is the $(m+n) \times (m+n)$ matrix with the first m rows and columns corresponding to the vertices in the first part of the bipartite graph and the last n rows and columns corresponding to the vertices in the second part of the bipartite graph. The entries are 1 if the vertices are adjacent and 0 otherwise.

Further reading