PCA – przykład

Dane: X_c

x_1	x_2
-2	-2
-1	0
0	1
1	-1
2	2
avg 0	0

Macierz kowariancji: S_x

	x_1	x_2
x_1	2.5	1.75
x_2	1.75	2.5

Wykres:
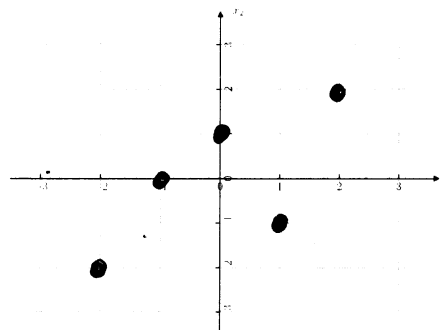
Miejsce na obliczenia:

$$S_x = \frac{1}{4}\left(X_c^T \times X_c\right)$$

$$\begin{vmatrix} 2.5-\lambda & 1.75 \\ 1.75 & 2.5-\lambda \end{vmatrix} \rightarrow (2.5-\lambda)^2 - 1.75^2 \rightarrow \lambda^2 - 5\lambda - 3.20 \rightarrow \sqrt{\Delta} \approx 3.5 \rightarrow \lambda_1 = 4.25 \quad \lambda_2 = 0.75$$

$$\lambda_1 \rightarrow x=y \rightarrow V = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow V = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

$$\lambda_2 \rightarrow x=-y \rightarrow V = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \rightarrow V = \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix}$$

$$S_y = \frac{1}{4}\left(Y_c^T \times Y_c\right)$$

Posortowane malejąco wartości i wektory własne (znormalizowane):

$\lambda_1 = $ __4.25__ $\qquad \lambda_2 = $ __0.75__

$$k_1 = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix} \qquad k_2 = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix} \qquad K = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$
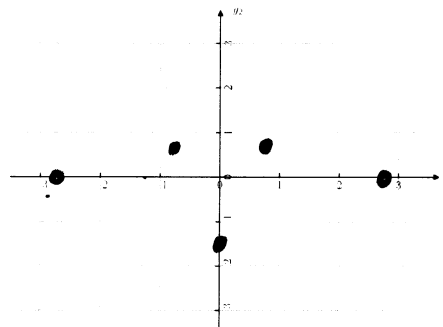
Dane po PCA: Y_c

$$Y = XK = \begin{bmatrix} -2.83 & 0 \\ -0.707 & 0.707 \\ 0.707 & 0.707 \\ 0 & -1.414 \\ 2.83 & 0 \end{bmatrix}$$

Macierz kowariancji: S_y

	y_1	y_2
y_1	4.25	0
y_2	0	0.75

Wykres:

$$y_1(x_1, x_2) = \underline{0.707}\, x_1 + \underline{0.707}\, x_2$$
$$y_2(x_1, x_2) = \underline{-0.707}\, x_1 + \underline{0.707}\, x_2$$